**ORIGINAL RESEARCH PAPER**

# Investigating Carbon Emissions in a Single-Manufacturer Multi-Retailer System with Stochastic Demand and Hybrid Production Facilities

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## Abstract

This paper presents a mathematical model to investigate carbon emissions reduction in a supply chain system comprising a manufacturer and multiple retailers. The demand at the retailer side follows a normal distribution, and the lead time is variable. The proposed model considered emissions arising from production, storage, and transportation. To comply with the carbon tax regulation imposed by the government, the manufacturer operates a hybrid production system composed of two facilities, where one of them adopts a green technology. The objective of the model is to determine the allocation factor, the number of shipments, safety factor, shipment lot, and production rate that minimize the joint total cost. An efficient algorithm is also proposed to obtain the solutions. Some numerical examples are provided to illustrate the application of the model and to compare the model with the one with an identical lead time. Sensitivity analysis is finally carried out to study how the model behaves against the changes in some key parameters. The results obtained indicate that green production facilities in the manufacturing system have proven to offer significant benefits, especially in reducing emissions. In addition, the emissions from the manufacturer can be managed by adjusting the production rate and allocation factor. The emissions from the retailers can be managed by controlling the shipment lot, the number of deliveries, and the safety factor. Finally, the proposed model performs better in increasing economic and environmental performances of the supply chain system compared to the model with an identical lead time.

**Keywords** Multi-retailer · Lot sizing · Supply chain · Green technology · Carbon tax · Emissions

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## Introduction

The increase in global warming and the depletion of natural resources have threatened life on earth. Over the past few years, environmental issues have become a significant agenda of countries and companies worldwide. Yet, environmental protection still requires continuous and global efforts to reduce the negative impact of human activities on ecosystems. Industries have to keep developing renewed environmental awareness to drive radical changes in everyday life, government policies, and industrial systems (Hua et al. 2011). Industries, known as the largest carbon emitters, must firmly commit to reducing these emissions from their operations. In addition, the government needs the commitment to encourage the creation of a green industry as evidenced by the issuance of emission restriction policies to ensure the achievement of the global emission reduction target.

Since several years ago, companies from various sectors have implemented new practices for more environmentally

conscious operations. Companies are looking for a way to reduce emissions resulting from their activities by using green technologies (Jiang et al. 2021). For instance, H&M is one of the biggest global fashion industries that has adopted various green technologies to lessen carbon emissions. The Home Depot (2018) and Panasonic Group (2020) have also adopted green technology and have proven that they can realize significant emission reductions by utilizing this type of technology. Although green technology in production systems can reduce emission levels, replacing existing production facilities with greener alternatives increases the total operational cost (Entezaminia et al. 2020). Therefore, it will be more sensible for the companies to operate both facilities together in a hybrid system to minimize costs. In addition to green technology, companies have also tried to optimize strategic and operational decisions in the supply chain system by accounting for GHG emissions. The integration of environmental influences, such as GHG emissions, on operational decisions at the supply chain level is called green supply chain management (GSCM) (Sarkis 2012). Storage, transportation, production, and warehousing activities are the main emitting activities in the supply chain that should be planned and controlled optimally (Chelly et al. 2019; Ahmed and Sarkar 2018). In this regard, inventory control is critical in optimizing the operational management of these four activities. Therefore, inventory management policies become vital determinants that reduce GHG emissions along the supply chain (Hovelaque and Bironneau 2015).

To determine the correct inventory decisions, managers need to choose what suitable models have been proven to minimize costs and increase collaboration between parties. A joint economic lot size (JELS) problem is a model that has been proven to solve various inventory problems. The essence of the model is to integrate inventory decisions and minimize the joint total cost in the supply chain. The model was pioneered by Goyal (1976), who proposed an integrated inventory model involving a vendor and a buyer under deterministic demand. Then, this model was extended by many researchers to accommodate various conditions, including carbon emissions reduction (Dwicahyani et al. 2017; Hariga et al. 2019; Ganesh Kumar and Uthayakumar 2019; Rout et al. 2021; Konstantaras et al. 2021) and stochastic environments (Hallak et al. 2021; Sarkar and Giri 2021; Tiwari et al. 2020; Taleizadeh et al. 2020a).

Our review on the latest research addressing the JELS model shows that no publication deals with incorporating green production to lessen emissions. In addition, the investigation of carbon emissions and the implementation of carbon tax regulation are rarely discussed in a situation in which a supply chain consists of a manufacturer and multiple retailers. The implementation of hybrid production in the manufacturer and the consideration of multi-retailer case makes the issues more complex. In addition to the need for a

strategy to coordinate production decisions on the manufacturer's side and purchasing decisions on the retailers' side, a plan to synchronize retailers' order cycles needs to be optimized. As a summary, the research questions to be answered here are as follows:

1) How to determine the optimal production allocation in the hybrid production system and to synchronize it with retailers' replenishment decisions?
2) How to determine the optimal common replenishment cycle and the shipment decisions for retailers that can minimize the total cost?
3) How does carbon tax affect the inventory decisions in the supply chain involving a manufacturer and multiple retailers?

The remainder of the paper is organized as follows. "Literature Review" reviews the JELS literature dealing with stochastic demand, multi-retailer cases, and carbon emissions. "Problem Descriptions, Notations, and Assumptions" presents the description of the investigated system and the notations and assumptions used to formulate the model. "Model Formulation" explains the proposed stochastic-JELS model. "Solution Method" describes the methodology to obtain the solutions. "Numerical Examples" illustrates the results, and "Sensitivity Analysis" investigates the model's behavior through a sensitivity analysis. "Managerial Insights" presents the managerial insights. Finally, "Conclusions" concludes the paper and suggests a direction for future research.

## Literature Review

In the existing JELS literature, three different streams of research are found. The first is the JELS model that works under stochastic environments; the second is the JELS model with the multi-buyer case; and the third is the JELS model that considers carbon emissions reduction. The discussions of each stream of research are provided in the following sub-sections.

### JELS with Stochastic Demand

The first stream of research tackles a situation in which the market demand is stochastic in nature, and the lead time is variable and depends on the shipment lot size. Some researchers have made valuable works on models to determine the optimal decisions under a situation where demand fluctuates over time. Ben-Daya and Hariga (2004) were noted to be one of the first to introduce stochastic JELS under variable lead time. They assumed that the lead time consists of productive and non-productive time, and its

value depends on shipment lot size. Hsiao (2008) suggested a new formulation of lead time, which is a lead time that depends on the production and transportation times for the first shipments and depends only on the transportation time for the 2-nd, …, j-th shipments. Glock (2012) used Hsiao's model to formulate strategies to shorten the lead time in a condition in which the production rate is adjustable and influences the lead time. Jauhari (2012) developed raw material procurement strategies for a three-echelon supply chain system. Heydari et al. (2018) determined the optimal inventory decisions for the supply chain and examined how different shipping modes can affect the system's decisions and performances. Christy et al. (2017) formulated a closed-loop supply chain and modeled the demand as a function of selling price and quality level. Jauhari and Saga (2017) investigated an imperfect production process and inspection errors in a two-level supply chain system under a periodic review policy. Recently, Saga et al. (2019) investigated the influence of imperfect production and inspection errors on production and inventory decisions. A periodic review policy with service-level constraint is adopted to control the inventory level of the buyer. Sarkar et al. (2020) proposed a JELS model composed of a single vendor and single buyer with a time value of money and partial backorders. Additional investment is included in the mathematical model to control the replenishment lead time. More recently, Roy and Sana (2021) proposed a JELS model for a two-stage supply chain with an investment to reduce setup costs. They also consider a situation where the vendor offers a fixed credit period to the buyer to complete the account. Barman and Mahata (2022) used a continuous review policy to manage inventory in the vendor–buyer system. The buyer's lead time is dependent on ordering lot and can be reduced by crashing cost.

## JELS with Multi-Buyer Case

The second stream of research focused on the incorporation of the multi-buyer cases in the JELS model. Islam et al. (2017) studied the impact of adopting consignment policy and inventory decisions in a supply chain system comprising a supplier, a manufacturer, and a retailer. Later, Ben-Daya et al. (2019) formulated a mathematical inventory model for single-vendor and multi-buyer systems under remanufacturing and consignment stock policy. They proposed a shipping strategy, namely joint replenishment, that synchronizes the shipments of products to the buyers. Chan et al. (2018) proposed a coordination policy to synchronize the production and buyer replenishment cycles in a single-vendor multi-buyer supply chain system. They proved that the proposed approach performs better than the decentralized model and the common cycle model. Chan et al. (2020) extended the model of Chan et al. (2018) by incorporating

the costs associated with remanufacturing and shipments to pick up the returned products from buyers. Hoque (2020) investigated the influence of the various distribution of lead times to ship equal lots on the inventory decisions in a supply chain system consisting of a manufacturer and multi-buyer. Subsequently, Hoque (2021) examined the impact of the normal distribution of lead times on the inventory decisions under a situation in which the lots are delivered to the buyers with equal and unequal shipment policies. Darwish et al. (2018) addressed a two-stage newsvendor inventory model for general demand distribution. They determined the optimal ordering quantities at two different times in the selling period, minimizing the total cost. Juman et al. (2023) proposed an unequal-sized shipment policy to synchronize production flow and deliveries in the multi-vendor multi-buyer supply chain system. They showed that the integrated model performs better in reducing joint total cost than the independent model. Biswas and Sarker (2021) formulated a multi-vendor and multi-buyer inventory model as an integer non-linear programming problem and proposed a heuristic approach to obtain the solution. A just-in-time delivery is adopted to manage the raw material deliveries and product deliveries in the investigated system.

## JELS with Carbon Emissions

The third stream of research deals with the development of the JELS model by taking GHG emissions into account. Wahab et al. (2011) were the first to study the impact of GHG emissions on inventory decisions. They addressed the problem of determining the optimal shipment and production policies for the two-echelon system in domestic and international supply chains. Halat and Hafezalkotob (2019) proposed a coordination model of the supply chain system and investigated the impact of different carbon policies on inventory decisions under coordinated and independent scenarios. Huang et al. (2020) considered some carbon policies, such as carbon tax, carbon cap and trade, and carbon constraint in the inventory modeling. Marchi et al. (2019) developed a JELS model involving a manufacturer and a buyer with an imperfect production process. They determined the shipment and production decisions by incorporating the emissions and energy consumption from production and transportation activities. Kumar and Uthayakumar (2019) developed a vendor–buyer system with vendor-managed inventory (VMI) and unequal shipment policies. The emissions from production are considered in the model and are minimized using an emissions-trading scheme. Some researchers also focused on studying the emissions impact in the JELS under stochastic demand. Wangsa (2017) and Saga et al. (2019) used penalties and incentive policies to lessen the emissions from the supply chain involved of a manufacturer and a retailer. Ghosh et al. (2017) implemented a strict

carbon policy to manage the emissions resulting from transportation, storage, and production. Taleizadeh et al. (2019) investigated the influences of carbon emission reduction, promotional efforts, and return policy on inventory decisions in the closed-loop supply chain system. A trade and cap policy and technology investment are applied to minimize the emissions produced. Later, Taleizadeh et al. (2021) proposed a cost-sharing contract to coordinate manufacturer and distributor in the supply chain system under a stochastic environment and green investment. Manupati et al. (2019) evaluated the emission generation using three policies: the carbon cap and trade, carbon cap, and carbon tax in a multi-echelon supply chain system. Taleizadeh et al. (2020b) used a carbon tax mechanism to control the emissions produce from replenishment, storage, and units purchased or produced. Alizadeh-Basban and Taleizadeh (2020) proposed three policies to ease the emissions, which are green production investment, hybrid manufacturing, and carbon cap-and-trade. A dual-channel system comprising of online channel and distributor channel is employed to attract the customers to buy products. Yadav et al. (2021) considered emissions and waste reduction in a two-level supply chain with cross-price elasticity of demand and preservation technology investment. Ramandi and Bafruei (2020) proposed a supplier and retailer's two-stage supply chain system under a periodic review policy. The government controls the emissions resulting from the system by applying carbon subsidies and penalties. Taleizadeh et al. (2022) formulated an inventory model by taking into account partial trade credit, price- and emission-dependent demand, and environmental impacts. The carbon tax and carbon cap-and-trade policies are used to cut down the emissions released from some operations in the investigated system.

## Research Contributions

As this paper considers a single-manufacturer-multi-retailer system, it is worth pointing out that the JELS model was discussed extensively in the literature and cases involving the efforts to reduce emissions. However, to the best of the author's knowledge, no research has been conducted to investigate the impact of implementing green technology in the production process, and its impact on production and ordering decisions in the supply chain consists of a manufacturer and multiple retailers. Table 1 shows how the proposed model differs from other models. Compared with the previous research, the contributions of our study are described as follows:

1)  Previous research, including Wangsa (2017), Saga et al. (2019), and Ramandi and Bafruei (2020), generally disregarded the use of green technology in production and only used a single type of production facility. The use

of this type of production facility becomes less efficient when the government implements carbon limitation policies. The use of traditional technology in production facilities will cause high emission levels which result in a large emission tax burden. In a situation where manufacturers are faced with strict carbon policies, a hybrid production system is more suitable to operate in order to minimize the total emissions produced. Here, we proposed a hybrid production system, comprising green production and regular production. By adopting green production and aligning it with regular production, the manufacturer will have more opportunities to control the amounts of emissions produced. To synchronize the operation of regular and green productions in the hybrid system, we propose a new decision variable, a production allocation factor, to set the production rate on both systems.

2)  We propose a supply chain system incorporating multiple retailers and environmental investigations into the model. This environmental investigation will add to the complexity of the problems, dealing with determining the optimal ordering quantity and the number of deliveries to different retailers. Thus, to maintain the total cost and the total emissions, the manufacturer's production allocations are aligned with ordering decisions on the retailers' side. Unlike Islam et al. (2017), Chan et al. (2018), and Chan et al. (2020), the emissions resulting from production, transportation, and storage activities are incorporated into the model.

3)  Unlike Ben-Daya and Hariga (2004) and Jauhari and Saga (2017) that used identical lead times for all shipments, we consider two different reorder points on the retailers to deal with different lead times. The use of identical lead times in the inventory modeling causes the inventory calculation to be inaccurate because it does not match the real conditions. In fact, the length of lead time for the first shipment is different from the lead time for the next shipments. To make the first shipment, the shipper must wait for the manufacturer to finish producing the first lot. Meanwhile, for the subsequent shipments, the shipper does not need to wait for lot production because at that time the lot has been produced by the manufacturer. This is because the manufacturer's production rate is greater than the demand rate. Unlike the first shipment which considers the production time and transportation time, the subsequent shipments only consider transportation time. By taking into account the different lead times for the two shipments, the expected shortages at retailers will be lower, thereby reducing the total cost. To the best of our knowledge, this is the first research in which different lead times are incorporated in a multi-retailer case. To operate the inventory system at the retailers,

**Table 1** The comparison between the proposed work and the previous works

| No | Authors (year) | Demand | Multi-retailer case | Retailers' policy | Green production | Adjustable production | Emissions sources | Carbon regulations |
|---|---|---|---|---|---|---|---|---|
| 1 | Ben-Daya and Hariga (2004) | Stochastic | No | Continuous review | No | No | No | No |
| 2 | Glock (2012) | Stochastic | No | Continuous review | No | No | No | No |
| 3 | Jauhari and Saga (2017) | Fuzzy | No | Periodic review | No | Yes | No | No |
| 4 | Roy and Sana (2021) | Stochastic | No | Continuous review | No | Yes | No | No |
| 5 | Sarkar et al. (2020) | Stochastic | No | Continuous review | No | Yes | No | No |
| 6 | Ben-Daya et al. (2019) | Deterministic | Yes | EOQ | No | Yes | No | No |
| 7 | Chan et al. (2018) | Stochastic | Yes | Continuous review | No | Yes | No | No |
| 8 | Islam et al. (2017) | Random | Yes | EOQ | No | Yes | No | No |
| 9 | Chan et al. (2020) | Stochastic | Yes | Continuous review | No | Yes | No | No |
| 10 | Hoque (2021) | Deterministic | Yes | EOQ | No | Yes | No | No |
| 11 | Wahab et al. (2011) | Deterministic | No | EOQ | No | No | Transportation | Carbon tax |
| 12 | Marchi et al. (2019) | Deterministic | No | EOQ | No | Yes | Production, transportation | Carbon tax |
| 13 | Wangsa (2017) | Stochastic | No | Continuous review | No | No | Transportation | Carbon penalty and incentive |
| 14 | Saga et al. (2019) | Fuzzy | No | Periodic review | No | Yes | Transportation | Carbon penalty and incentive |
| 15 | Ghosh et al. (2017) | Stochastic | No | Continuous review | No | No | Production, inventory, transportation | Strict carbon cap |
| 16 | Ramandi and Bafruei (2020) | Stochastic | No | Periodic review | No | No | Transportation | Subsidy and penalty |
| 17 | Proposed model | Stochastic | Yes | Continuous review | Yes | Yes | Production, transportation, holding items | Carbon tax |

**Fig. 1** The proposed single-manufacturer multi-retailer supply chain inventory system
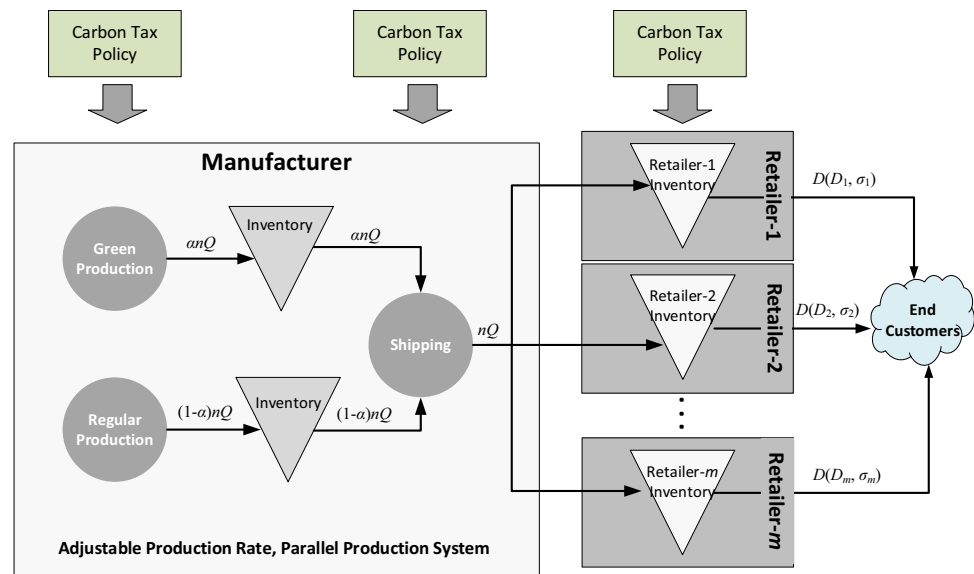


we use a common cycle and the same level of safety factor for the first shipment for all retailers.

4) We allow the manufacturer's production rate to be adjusted flexibly to control production costs and production emissions. In addition, the hybrid production system faces a situation where the production costs and the emissions resulting from regular and green production facilities are different. The regular production facility has a lower production cost than the green production but produces more emissions. Obtaining the optimal production rate in this hybrid system is challenging since it must accommodate complex trade-offs. Previous research dealing with JELS mostly treated the production rate as input parameters. The developed model does not allow decision-makers to control the emission levels and the production costs.

## Problem Descriptions, Notations, and Assumptions

### Problem Descriptions

The investigated system considers a supply chain composing a single manufacturer and multiple retailers as depicted in Fig. 1. Retailers face a stochastic demand that is normally distributed with a mean of $D_m$ and a standard deviation of $\sigma_m$. To fulfill the demand, the retailers order $nQ_m$ units of product to the manufacturer and sell them to the end customers. A common cycle is used to coordinate

the replenishment cycles of all retailers. All retailers use a continuous review policy to control the inventories stored in warehouses. Two types of safety factors are used to determine safety stock and shortage at 1st shipment and 2nd, …, to $n$th shipments. The shortages are allowed in the model and assumed to be fully backordered. The manufacturer operates a hybrid system containing green production and regular production facilities to produce final products and ship them to $M$ retailers. Green production produces lower emissions than regular production but requires higher production costs. Conversely, regular production produces higher emissions than green production but requires lower production costs. The amount of $nQ$ units are produced by the manufacturer at each production cycle. Given the production allocation factor is $\alpha$, the amount of $\alpha nQ$ units and $(1 - \alpha)nQ$ units result from regular production and green production facilities, respectively, for each production cycle.

Emissions resulting from manufacturer and retailers' operations are incorporated into the model. The manufacturer incurs some costs for each emission released from production and storage. Retailers bear their emissions costs for each emission generated from holding the product in a warehouse and transportation. The number of emissions resulting from production is formulated as a quadratic function that depends on the production rate. Emissions from storage are calculated based on the inventories kept in the warehouse. The regulator implements a carbon tax policy to cut down the overall emissions from the supply chain. Furthermore, the production rate is flexibly adjustable so that it gives opportunities for decision-makers to control both emissions and production costs at the manufacturer.

## Notations

To develop the proposed mathematical model, we use the following notations:

Parameters:

| | |
|---|---|
| $A_m$ | ordering cost for retailer-$m$ (\$/order) |
| $a_g(a_r)$ | emission parameter for green production (regular production) process (kg year$^2$/unit$^3$) |
| $b_g(b_r)$ | emission parameter for green production (regular production) process (kg year/unit$^2$) |
| $c_g(c_r)$ | emission parameter for green production (regular production) process (kg /unit) |
| $CMG_{storage}$ | amount of carbon emissions generated from green production's storage (kg $CO_2$) |
| $CMG_{prod}$ | amount of carbon emissions generated from green production's facility (kg $CO_2$) |
| $CMR_{storage}$ | amount of carbon emissions generated from regular's production's storage (kg $CO_2$) |
| $CMR_{prod}$ | amount of carbon emissions generated from regular production's facility (kg $CO_2$) |
| $CR_{storage}$ | amount of carbon emissions generated from retailer-$m$'s storage (kg $CO_2$) |
| $CR_{transport}$ | amount of carbon emissions generated from transportation (kg $CO_2$) |
| $D_m$ | demand for retailer-$m$ (units/year) (with $m = \{1, 2, …, M\}$) |
| $D$ | total demand for $M$ retailers (units/year) |
| $E$ | carbon tax (\$/kg $CO_2$) |
| $ES_m^1$ | expected shortage for first shipment (units) |
| $ES_m^2$ | expected shortage for $n$th shipments (units) |
| $F_m$ | transportation cost for retailer-$m$ (\$/shipment) |
| $h_m^b$ | holding cost for retailer-$m$ (\$/unit/year) |
| $h_g(h_r)$ | holding cost for green production (regular production) (\$/unit/year) |
| $IVR_m$ | average inventory per unit time at the retailer-$m$ (units) |
| $IVM_g$ | average inventory per unit time at the green production (units) |
| $IVM_r$ | average inventory per unit time at the regular production (units) |
| $JTC$ | joint total cost (\$/year) |
| $J_m$ | distance from manufacturer to retailer-$m$ (km) |
| $K_g(K_r)$ | setup cost for green production (regular production) (\$/setup) |
| $M$ | number of retailers |
| $P_{min}$ | minimum production rate (units/year) |
| $P_{max}$ | maximum production rate (units/year) |
| $Prod_g$ | production cost for green production (\$/year) |
| $Prod_r$ | production cost for regular production (\$/year) |
| $T^p$ | production time and non-productive time (year) |
| $T_m^s$ | transportation time for retailer-$m$ (year) |

| | |
|---|---|
| $TCR$ | total cost for retailers (\$/year) |
| $TCM$ | total cost for manufacturer (\$/year) |
| $W^b$ | amount of carbon emissions from retailers' storage (kg $CO_2$/unit/year) |
| $W^f$ | amount of carbon emissions from manufacturer's storage (kg $CO_2$/unit/year) |
| $x$ | weight of the product (kg/unit) |
| $X_{g1}(X_{r1})$ | green production's (regular production's) per unit time cost for running the machine independent of production rate (\$/year) |
| $X_{g2}(X_{r2})$ | the increase in green production's (regular production's) unit machining cost due to one unit increase in production rate (\$ year/unit$^2$) |
| $\sigma_m$ | standard deviation of demand for retailer-$m$ (units/year) |
| $\pi_m$ | back-ordering cost for retailer-$m$ (\$/unit) |
| $\vartheta_{T1}$ | indirect emission factor for transportation (kg $CO_2$/liter) |
| $\vartheta_{T2}$ | direct emission factor for transportation (kg $CO_2$/kg) |
| $\varepsilon$ | fuel consumption (liters/km) |

Decision variables:

| | |
|---|---|
| $k^1$ | safety factor of the 1st shipment |
| $k_m^p$ | safety factor of the 2nd, …, to $n$th shipments for retailer-$m$ (with $m = \{1, 2, …, M\}$) |
| $n$ | number of shipments per production batch |
| $P$ | production rate (unit/year) |
| $Q$ | shipment quantity (unit) |
| $\alpha$ | hybrid production system allocation factor |

## Assumptions

1) The investigated system consists of a manufacturer and $M$ retailers.
2) The retailers employ a continuous review policy to control their inventory levels.
3) The demand at the retailers' side follows a normal distribution with a mean of $D_m$ and a standard deviation of $\sigma_m$.
4) All retailers have the same ordering cycle, $Q/D = Q_m/D_m$, where $D = \sum_{m=1}^{M} D_m$ and $Q = \sum_{m=1}^{M} Q_m$.
5) The $M$-retailers order products with a quantity of $nQ_m$, and the manufacturer produces a batch of $nQ_m$ units for each production run. The manufacturer will deliver $Q$ units over $n$ time to fulfill the demand from all retailers.
6) The shipment lot and production rate for the green facility are given by $Q_g = Q(1 - \alpha)$ and $P_g = P(1 - \alpha)$, whereas the shipment lot and production rate for the regular production facility are given by $Q_r = Q\alpha$ and $P_g = P\alpha$. The green production lot depends on the ratio

of the green production rate to the total system production rate, $nQ_g = nQ\frac{P_g}{P}$, and the regular production lot depends on the ratio of the regular production rate to the total system production rate, $nQ_r = nQ\frac{P_r}{P}$.

7) The green production facility generates a lower emission than the regular production facility and requires a higher production cost.

8) The green production and the regular production facilities produce the same quality products.

## Model Formulation

In this section, an integrated inventory model for the supply chain composed of a manufacturer and retailers is formulated. The inventory profile is depicted in Fig. 2. The regulator imposes a carbon tax to reduce the emissions released from production, transportation, and storage. We initially derive the mathematical model for the retailers and then develop the model for the manufacturer.

### Total Cost for the Retailers

To formulate the retailer inventory model, we refer to the basic model proposed by Hsiao (2008). The lead time of the first shipment for retailer $m$, $L_m^1$, is expressed by $L_m^1 = T^p + T_m^s$. The lead time for the 2nd,.., to $n$th shipments, $L_m^2$, is expressed by $L_m^2 = T_m^s$. In the stochastic inventory model, the determination of safety stock must consider the lead time. The lead time for the first shipment is calculated by considering production time and transportation time while the lead time for the 2nd, …, to $n$th shipments is calculated by considering transportation time. Due to the difference in lead times, the safety stock for the first shipment will be different from the safety stock for the 2nd, …, to $n$th shipments. Here, we assume that all retailers use the same safety factor for the first shipment. Thus, the demand during lead time, the standard deviation, and the safety stock of the first shipment for retailer-$m$ are formulated by $D_m(T^p + T_m^s)$, $\sigma_m\sqrt{T^p + T_m^s}$, and $k^1\sigma_m\sqrt{T^p + T_m^s}$, respectively. The demand during lead time, the standard deviation and the safety stock of the $n$th shipment are given by $D_m T_m^s$, $\sigma_m\sqrt{T_m^s}$, and $k_m^2\sigma_m\sqrt{T_m^s}$, respectively. The expected shortages for the first and $n$th shipments are given by Eqs. (1) and (2), respectively.

$$ES_m^1 = \sigma_m\sqrt{T^p + T_m^s}\,\psi\left(k^1\right) \tag{1}$$

$$ES_m^2 = \sigma_m\sqrt{T_m^s}\,\psi\left(k_m^p\right) \tag{2}$$

with, $\psi(k^1) = f_s(k^1) - k^1[1 - F_s(k^1)]$ and $\psi(k_m^p) = f_s(k_m^p) - k_m^p[1 - F_s(k_m^p)]$.

By considering the above safety stock formulations, $k_m^p$ can be formulated as follows:

$$k_m^p = k^1\sqrt{\frac{T^p + T_m^s}{T_m^s}} \tag{3}$$

The retailers use a continuous review policy to control the inventory level. Thus, whenever the inventory level reaches the reorder point, the retailers place an order of $nQ_m$ items to the manufacturer (See Hadley and Within 1963). Thus, the average inventory level per unit of time is determined by summing up the average of the number of products stored in the warehouse and the safety stock. The average inventory per unit time at the retailer-$m$ is given by the following:

$$IVR_m = \frac{Q}{2}\frac{D_m}{D} + k^1\sigma_m\sqrt{T^p + T_m^s} \tag{4}$$

The carbon emissions are generated from storage and transportation. The number of emissions resulted from storage depends on the number of products stored in the warehouse. In the warehouse, the emissions are produced from some activities e.g., loading, unloading, and energy consumption e.g., for lightening, heating, or cooling. The emissions generated from transportation are classified into two types, namely direct emissions and indirect emissions. The direct emissions are influenced by the weight of the product while the indirect emissions are influenced by the fuel consumption, number of shipments, and distance. Equations (5) and (6) present the carbon emissions generated from storage and transportation.

$$CR_{storage} = W^b\left(\frac{Q}{2}\frac{D_m}{D} + k^1\sigma_m\sqrt{T^p + T_m^s}\right) \tag{5}$$

$$CR_{transport} = \frac{D}{Q}\vartheta_{T1}\varepsilon J_m + \vartheta_{T2}xD_m \tag{6}$$

Hence, the total cost incurred by all retailers, including ordering cost and transportation cost, is derived as follows:

$$\begin{aligned} TCR = &\frac{D}{nQ}\sum_{m=1}^{M}\left(A_m + nF_m\right) + \sum_{m=1}^{M}\left(h_m^b + EW^b\right)\left(\frac{Q}{2}\frac{D_m}{D} + k^1\sigma_m\sqrt{T^p + T_m^s}\right) \\ &+ \frac{D}{nQ}\sum_{m=1}^{M}\pi_m\sigma_m\left[\sqrt{T^p + T_m^s}\psi(k^1) + (n-1)\sqrt{T_m^s}\psi(k_m^p)\right] \\ &+ \sum_{m=1}^{M}E\left(\frac{D}{Q}\vartheta_{T1}\varepsilon J_m + \vartheta_{T2}xD_m\right) \end{aligned} \tag{7}$$

### Expected Total Cost for the Manufacturer

To fulfill demand from the retailers, the manufacturer operates a hybrid production system composed of a green production facility and a regular production facility. Let $nQ_g$ and $nQ_r$ be the production batch for green production and

**Fig. 2** Inventory profile of the investigated single-manufacturer multi-retailer system
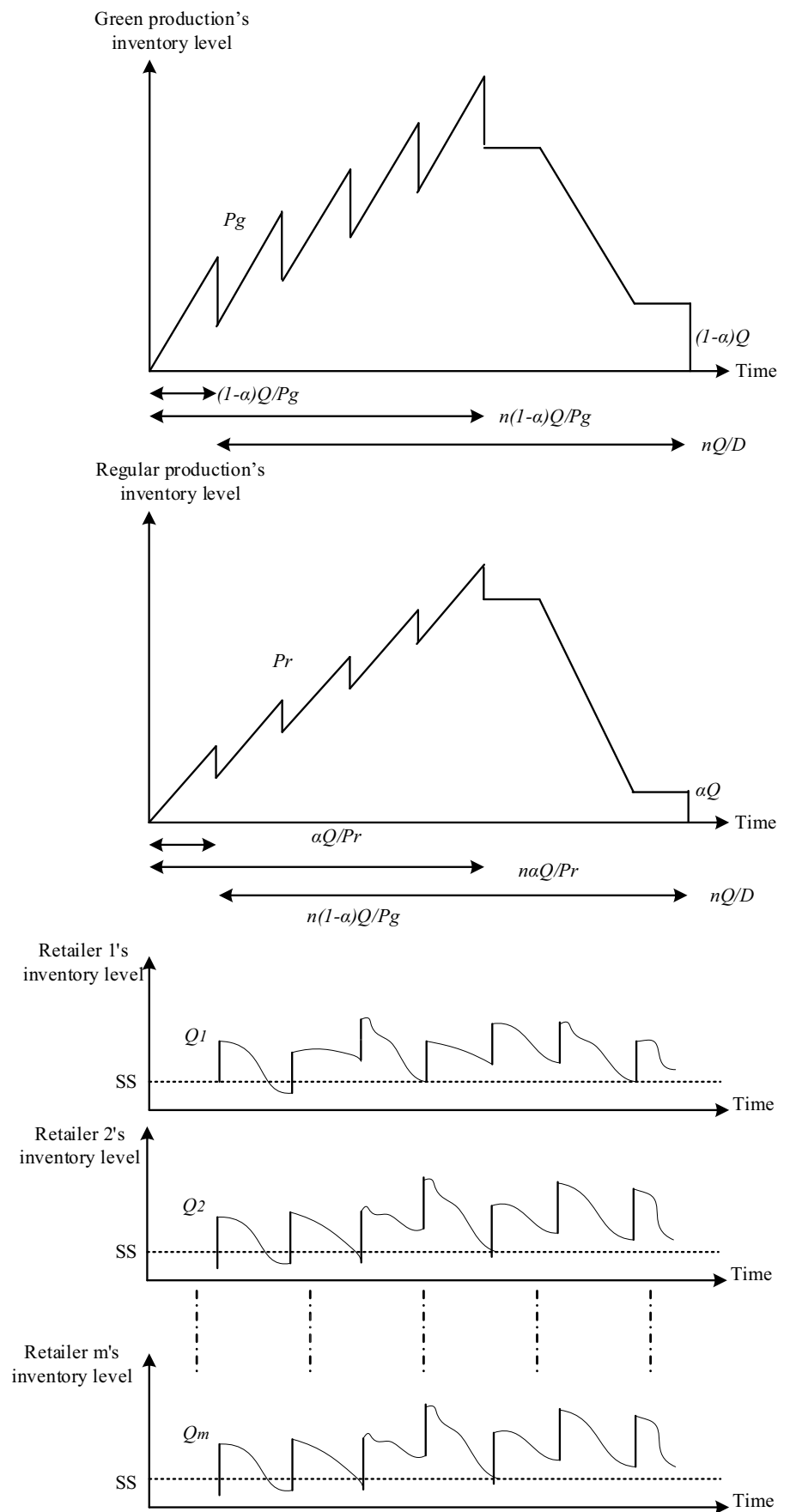
regular production, respectively, and ${^{nQ_g}}/{_{P_g}}$ and ${^{nQ_r}}/{_{P_r}}$ be the length of production time for green production and regular production, respectively. The average inventory level of both production systems can be calculated by subtracting the accumulative shipment from the manufacturer's accumulated production. Equations (8) and (9) express the inventory levels for both systems.

$$IVM_g = \frac{Q_g}{2}\left(n\left[1 - \frac{(1-\alpha)D}{P_g}\right] - 1 + \frac{2(1-\alpha)D}{P_g}\right) \quad (8)$$

$$IVM_r = \frac{Q_r}{2}\left(n\left[1 - \frac{\alpha D}{P_r}\right] - 1 + \frac{2\alpha D}{P_r}\right) \quad (9)$$

The manufacturer spends a certain amount of costs to produce the items for both green and regular productions. To model the production cost, we refer to Khouja and Mehrez's (1994) formulation.

$$Prod_g = \left(\frac{X_{g1}}{P_g} + X_{g2}P_g\right)(1-\alpha)D \quad (10)$$

$$Prod_r = \left(\frac{X_{r1}}{P_r} + X_{r2}P_r\right)\alpha D \quad (11)$$

The production cost function is influenced by the production rate of the manufacturing system. $X_{g1}$ and $X_{r1}$ represent labor costs. If the production rate increases, the labor cost will decrease. When a worker is scheduled to operate a machine, the more products that are produced, the lower the wage per unit of time incurred. $X_{g2}$ and $X_{r2}$ represent tools and rework costs. If the production rate gets higher, the number of defective products will increase due to the increase in tool wear.

The carbon emissions are calculated from storage and production activities. The amount of carbon emission from storage depends on the manufacturer's inventory level. Equations (12) and (13) formulate the emissions released from both systems.

$$CMG_{storage} = W^f \frac{Q_g}{2}\left(n\left[1 - \frac{(1-\alpha)D}{P_g}\right] - 1 + \frac{2(1-\alpha)D}{P_g}\right) \quad (12)$$

$$CMR_{storage} = W^f \frac{Q_r}{2}\left(n\left[1 - \frac{\alpha D}{P_r}\right] - 1 + \frac{2\alpha D}{P_r}\right) \quad (13)$$

To derive the emissions from production, we refer to Bogaschewsky (1995)'s formulation. The number of emissions depends on the production rate and its formulation is expressed in quadratic forms. The emissions from both production activities are given by the following:

$$CMG_{prod} = a_g P_g{}^2 - b_g P_g + c_g \quad (14)$$

$$CMR_{prod} = a_r P_r{}^2 - b_r P_r + c_r \quad (15)$$

Based on the experiments conducted by Narita (2012), Eqs. (14) and (15) can approximate the level of carbon emissions resulting from machine tool operation. Narita (2012) analyzed the environmental burden, in the form of carbon emissions, caused by the operation of machine tools. Experimental results show that the increase in cutting speed will accelerate tool wear and shorten tool life, which will lead to an increase in the environmental burden.

Therefore, the total cost for the manufacturer, including the setup cost, can be calculated by the following:

$$
\begin{aligned}
TCM = {} & \frac{(1-\alpha)DK_g}{n(1-\alpha)Q} + \left(h_g + EW^f\right)\frac{(1-\alpha)Q}{2}\left(n\left[1 - \frac{(1-\alpha)D}{P_g}\right] - 1 + \frac{2(1-\alpha)D}{P_g}\right) \\
& + E(1-\alpha)D\left(a_g P_g^2 - b_g P_g + c_g\right) + \left(\frac{X_{g1}}{P_g} + X_{g2}P_g\right)(1-\alpha)D + \frac{\alpha DK_r}{n\alpha Q} \\
& + \left(h_r + EW^f\right)\frac{\alpha Q}{2}\left(n\left[1 - \frac{\alpha D}{P_r}\right] - 1 + \frac{2\alpha D}{P_r}\right) + E\alpha D\left(a_r P_r^2 - b_r P_r + c_r\right) \\
& + \left(\frac{X_{r1}}{P_r} + X_{r2}P_r\right)\alpha D
\end{aligned}
\quad (16)
$$

## Joint Total Cost

The joint total cost can be calculated by summing up the total cost incurred by the retailers and the total cost incurred by the manufacturer as given below.

$$
\begin{aligned}
JTC = {} & \frac{D}{nQ}\sum_{m=1}^{M}\left(A_m + nF_m\right) + \sum_{m=1}^{M}\left(h_m^b + EW^b\right)\left(\frac{Q}{2}\frac{D_m}{D} + k^1\sigma_m\sqrt{T^p + T_m^s}\right) \\
& + \frac{D}{nQ}\sum_{m=1}^{M}\pi_m\sigma_m\left[\sqrt{T^p + T_m^s}\,\psi\left(k^1\right) + (n-1)\sqrt{T_m^s}\,\psi\left(k_m^p\right)\right] \\
& + \sum_{m=1}^{M}E\left(\frac{D}{Q}\vartheta_{T1}\varepsilon J_m + \vartheta_{T2}xD_m\right)\frac{(1-\alpha)DK_g}{n(1-\alpha)Q} \\
& + \left(h_g + EW^f\right)\frac{(1-\alpha)Q}{2}\left(n\left[1 - \frac{(1-\alpha)D}{P_g}\right] - 1 + \frac{2(1-\alpha)D}{P_g}\right) \\
& + E(1-\alpha)D\left(a_g P_g^2 - b_g P_g + c_g\right) + \left(\frac{X_{g1}}{P_g} + X_{g2}P_g\right)(1-\alpha)D + \frac{\alpha DK_r}{n\alpha Q} \\
& + \left(h_r + EW^f\right)\frac{\alpha Q}{2}\left(n\left[1 - \frac{\alpha D}{P_r}\right] - 1 + \frac{2\alpha D}{P_r}\right) + E\alpha D\left(a_r P_r^2 - b_r P_r + c_r\right) \\
& + \left(\frac{X_{r1}}{P_r} + X_{r2}P_r\right)\alpha D
\end{aligned}
\quad (17)
$$

## Solution Method

For fixed $n$ and $\alpha$, the minimum joint total cost occurs at point $(Q, k^1, P)$ that satisfies $\frac{\partial JTC}{\partial Q} = 0$, $\frac{\partial JTC}{\partial k^1}$, and $\frac{\partial JTC}{\partial P}$, simultaneously. By taking the first partial derivative of the joint total cost with respect to $Q, k^1$, and $P$, we obtain the following equations:

$$\frac{\partial JTC}{\partial Q} = -\frac{D}{Q^2} \sum_{m=1}^{M} \left( \frac{A_m}{n} + F_m \right) + \frac{1}{2} \sum_{m=1}^{M} \left( h_m^b + EW^b \right) \frac{D_m}{D}$$
$$- \frac{D}{nQ^2} \sum_{m=1}^{M} \pi_m \sigma_m \left[ \sqrt{T^p + T_m^s} \psi(k^1) + (n-1)\sqrt{T_m^s} \psi(k_m^p) \right]$$
$$- \frac{D}{Q^2} E\vartheta_{T1}\varepsilon \sum_{m=1}^{M} J_m - \frac{DK_g}{nQ^2} + \left( h_g + EW^f \right) \frac{(1-\alpha)}{2} \left( n\left[ 1 - \frac{D}{P} \right] - 1 + \frac{2D}{P} \right)$$
$$- \frac{DK_r}{nQ^2} + \left( h_r + EW^f \right) \frac{\alpha}{2} \left( n\left[ 1 - \frac{D}{P} \right] - 1 + \frac{2D}{P} \right) \tag{18}$$

$$\frac{\partial JTC}{\partial k^1} = \left( h_m^b + EW^b \right)\sigma_m\sqrt{T^p + T_m^s} - \frac{\pi_m D\sigma_m}{nQ}\sqrt{T^p + T_m^s}\left[ \overline{F}(k^1) + (n-1)\overline{F}(k_m^p) \right] \tag{19}$$

$$\frac{\partial JTC}{\partial P} = \frac{D(n-2)}{P^2}\left( h_g + EW^f \right)\frac{(1-\alpha)Q}{2} + E(1-\alpha)D\left( 2a_g(1-\alpha)^2 P - (1-\alpha)b_g \right)$$
$$+ \frac{D(n-2)}{P^2}\left( h_r + EW^f \right)\frac{\alpha Q}{2} + E\alpha D\left( 2a_r\alpha^2 P - \alpha b_r \right) + \left( X_{g2}(1-\alpha)^2 - \frac{X_{g1}}{P^2} \right)D$$
$$+ \left( X_{r2}\alpha^2 - \frac{X_{r1}}{P^2} \right)D \tag{20}$$

By setting Eqs. (18)–(20) equal to zero, we obtain the following expressions:

$$Q = \sqrt{\frac{2D\left\{ \begin{array}{c} \sum_{m=1}^{M} \left( \frac{A_m}{n} + F_m + \frac{\pi_m\sigma_m\psi(k^1)\sqrt{T^p+T_m^s} + \pi_m\sigma_m(n-1)\sqrt{T_m^s}\psi(k_m^p)}{n} \right) \\ + E\vartheta_{T1}\varepsilon\sum m=1^M J_m + \frac{K_g}{n} + \frac{K_r}{n} \end{array} \right\}}{\sum_{m=1}^{M}\left( \left( h_m^b + EW^b \right)\frac{D_m}{D} \right) + \left( h_g + EW^f \right)\left( n\left[ 1 - \frac{D}{P} \right] - 1 + \frac{2D}{P} \right) + \left( h_r + EW^f \right)\left( n\left[ 1 - \frac{D}{P} \right] - 1 + \frac{2D}{P} \right)}} \tag{21}$$

$$\overline{F}(k^1) + (n-1)\overline{F}\left( k^1\sqrt{\frac{T^p + T_m^s}{T_m^s}} \right) = \frac{\left( h_m^b + EW^b \right)nQ}{\pi_m D} \tag{22}$$

$$P = \sqrt{\frac{\begin{array}{c} X_{g1} + X_{r1} - (n-2)\left( h_g + EW^f \right)\frac{(1-\alpha)Q}{2} \\ -(n-2)\left( h_r + EW^f \right)\frac{\alpha Q}{2} \end{array}}{\begin{array}{c} E(1-\alpha)\left( 2a_g(1-\alpha)^2 P - (1-\alpha)b_g \right) + E\alpha\left( 2a_r\alpha^2 P - \alpha b_r \right) \\ + X_{g2}(1-\alpha)^2 + X_{r2}\alpha^2 \end{array}}} = \sqrt{\gamma} \tag{23}$$

For a condition when $\delta < 0$, we set $P = P_{min}$. Therefore, we obtain the following:

$$P = max\left[ P_{min}, min\left( P_{max}, \sqrt{\gamma} \right) \right] \tag{24}$$

We suggest an efficient algorithm based on the procedure developed by Ben-Daya and Hariga (2004) and Glock (2012) to solve the proposed inventory problem. The algorithm is presented below:

1. Set $\alpha = 0.01$, $u_\alpha = 1$ and $JTC(P, Q, k^1, n, u_\alpha - 1) = \infty$.
2. Set $n = 1$ and $JTC_{n-1}\left( P_{n-1}, Q_{n-1}, k_{n-1}^1, n-1, u_\alpha \right) = \infty$.
3. Calculate $P$ by using the equation below.

$$P = \sqrt{\frac{X_{g1} + X_{r1}}{X_{g2}(1-\alpha)^2 + X_{r2}\alpha^2}} \tag{25}$$

4. Calculate $Q$ by considering the previous value of $P$ into Eq. (21).
5. Calculate $k^1$ by substituting $Q$ into Eq. (22).
6. Update the value of $P$ by substituting the previous values of $Q$ and $k^1$ into Eq. (23).
7. Repeat steps 4−6 until no change occurs in the values of $P$, $Q$ and $k^1$.
8. Set $P_n = P$, $Q_n = Q$ and $k_n^1 = k^1$. Calculate $JTC_n\left( P_n, Q_n, k_n^1, n, u_\alpha \right)$ using Eq. (17).
9. If $JTC_n\left( P_n, Q_n, k_n^1, n, u_\alpha \right) \leq JTC_{n-1}\left( P_{n-1}, Q_{n-1}, k_{n-1}^1, n-1, u_\alpha \right)$ repeat steps 3−8 with $n = n + 1$, otherwise go to step 10.
10. Compute $JTC\left( P, Q, k^1, n, u_\alpha \right) = JTC_{n-1}\left( P_{n-1}, Q_{n-1}, k_{n-1}^1, n-1, u_\alpha \right)$.
11. If $JTC\left( P, Q, k^1, n, u_\alpha \right) \leq JTC(P, Q, k^1, n, u_\alpha - 1)$ repeat steps 2−10 with $\alpha = \alpha + 0.01$ and $u_\alpha = u_\alpha + 1$, otherwise go to step 12.
12. Set $JTC(P, Q, k^1, n, u_\alpha - 1)$ as the minimum value of joint total cost and $P$, $Q$, $k^1$, $n$, $\alpha$ are the solutions.

## Numerical Examples

The proposed model is typically applicable to industries that employ various types of production technology. However, in this numerical example, we are referring to an Indonesian manufacturing industry that produces numerous types of machine spare parts. The company operates two production facilities to produce the spare parts. The first facility consists of some conventional machines, such as milling machines, grinding machines, lathe machines, and drilling machines. The second facility is more advanced since it has a variety of computer numerical control (CNC) machines. CNC machines are machines that use computer numerical control to regulate machining processes on various equipment such as routers, lathes, mills, or grinders. Computer numerical control allows precise control of features such as location, speed, and speed feed. Conventional machines are typically less expensive and are used for small production projects whereas CNC machines are usually more expensive than traditional machines and are best suited for large production projects. Green CNC machines are typically outfitted with decarbonization and energy-saving technologies to control emissions and energy usage. As a result, while these machines are more expensive than the conventional machines, they emit fewer emissions.

In this section, we present two numerical examples to justify the feasibility of the proposed model. The parameter values are mainly adapted from Jauhari and Saga (2017), Ben-Daya and Hariga (2004), and Hoque (2021). The fuel consumption and carbon tax data are taken from the Volvo Truck report (2018) and Chan (2009), respectively.

**Table 2** Input parameters for example 1

| Parameters | Values |
|---|---|
| $K_g$ | \$400/setup |
| $K_r$ | \$400/setup |
| $h_g$ | \$0.5/unit/year |
| $h_r$ | \$0.5/unit/year |
| $W^f$ | 8 kg $CO_2$/unit/year |
| $W^b$ | 10 kg $CO_2$/unit/year |
| $a_g$ | 0.0000007 kg year$^2$/unit$^3$ |
| $b_g$ | 0.0012 kg year/unit$^3$ |
| $c_g$ | 1.4 kg/unit |
| $a_r$ | 0.0000012 kg year$^2$/unit$^3$ |
| $b_r$ | 0.0008 kg year/unit$^3$ |
| $c_r$ | 8.4 kg/unit |
| $\vartheta_{T1}$ | 2.6 kg $CO_2$/l |
| $\vartheta_{T2}$ | 2.5 kg $CO_2$/kg |
| $\varepsilon$ | 0.3 l/km |
| $T^p$ | 0.09 year |
| $E$ | \$0.0618/kg $CO_2$ |
| $X_{g1}$ | \$2500/unit |
| $X_{r1}$ | \$2000/unit |
| $X_{g2}$ | \$0.0008 year/unit$^2$ |
| $X_{r2}$ | \$0.0004 year/unit$^2$ |
| $P_{min}$ | 2500 units/year |
| $P_{max}$ | 3800 units/year |

**Table 4** Input parameters for example 2

| Parameters | Values |
|---|---|
| $K_g$ | \$600/setup |
| $K_r$ | \$600/setup |
| $h_g$ | \$0.75/unit/year |
| $h_r$ | \$0.75/unit/year |
| $W^f$ | 12 kg $CO_2$/unit/year |
| $W^b$ | 15 kg $CO_2$/unit/year |
| $a_g$ | 0.0000007 kg year$^2$/unit$^3$ |
| $b_g$ | 0.0012 kg year/unit$^3$ |
| $c_g$ | 1.4 kg/unit |
| $a_r$ | 0.0000012 kg year$^2$/unit$^3$ |
| $b_r$ | 0.0008 kg year/unit$^3$ |
| $c_r$ | 8.4 kg/unit |
| $\vartheta_{T1}$ | 2.6 kg $CO_2$/litter |
| $\vartheta_{T2}$ | 2.5 kg $CO_2$/kg |
| $\varepsilon$ | 0.3 l/km |
| $T^p$ | 0.3 year |
| $E$ | \$0.0618/kg $CO_2$ |
| $X_{g1}$ | \$2500/unit |
| $X_{r1}$ | \$2000/unit |
| $X_{g2}$ | \$0.0024 year/unit$^2$ |
| $X_{r2}$ | \$0.0004 year/unit$^2$ |
| $P_{min}$ | 2000 units/year |
| $P_{max}$ | 3800 units/year |

## Example 1

To validate the proposed model and results, the following numerical experiment is considered. The supply chain is characterized by the parameters presented in Table 2. The parameters for retailers are given in Table 3.

By applying the proposed procedure, we obtain the following results. The optimal allocation factor, shipment frequency, and shipment lot are 0.55, 3 shipments, and 629.55 units, respectively. The optimal safety factor for the first shipment and the production rate are 1.478 and 3393.75

units/year, respectively. The number of emissions generated from the retailers and the manufacturer is 3117.57 kg $CO_2$ and 9322.19 kg $CO_2$, respectively. The total cost incurred by the retailers, manufacturer, and supply chain are \$1205.63, \$3654.01, and \$4859.64, respectively.

## Example 2

To validate the theoretical model, the following numerical experiment is presented. The input parameters' values for the inventory problem are presented in Tables 4 and 5. The

**Table 3** Input parameters for retailers in example 1

| Parameters | Retailers | | | | |
|---|---|---|---|---|---|
| | Retailer 1 | Retailer 2 | Retailer 3 | Retailer 4 | Retailer 5 |
| $D_m$ | 200 units/year | 250 units/year | 150 units/year | 220 units/year | 180 units/year |
| $\sigma_m$ | 5 units/year | 8 units/year | 6 units/year | 10 units/year | 7 units/year |
| $A_m$ | \$90/order | \$95/order | \$100/order | \$85/order | \$90/order |
| $F_m$ | \$40/shipment | \$42/shipment | \$54/shipment | \$56/shipment | \$30/shipment |
| $h_m^b$ | \$1.2/unit/year | \$1/unit/year | \$1.25/unit/year | \$0.98/unit/year | \$0.96/unit/year |
| $\pi_m$ | \$50/unit | \$55/unit | \$48/unit | \$42/unit | \$40/unit |
| $T_m^s$ | 0.01 year | 0.005 year | 0.006 year | 0.007 year | 0.002 year |
| $J_m$ | 100 km | 50 km | 60 km | 64 km | 25 km |

**Table 5** Input parameters for retailers for example 2

| Parameters | Retailers | | | | |
|---|---|---|---|---|---|
| | Retailer 1 | Retailer 2 | Retailer 3 | Retailer 4 | Retailer 5 |
| $D_m$ | 210 units/year | 220 units/year | 120 units/year | 240 units/year | 190 units/year |
| $\sigma_m$ | 15 units/year | 18 units/year | 16 units/year | 11 units/year | 17 units/year |
| $A_m$ | $90/order | $95/order | $100/order | $85/order | $90/order |
| $F_m$ | $40/shipment | $42/shipment | $54/shipment | $56/shipment | $30/shipment |
| $h_m^b$ | $3.2/unit/year | $2/unit/year | $3.25/unit/year | $2.98/unit/year | $1.96/unit/year |
| $\pi_m$ | $50/unit | $55/unit | $48/unit | $42/unit | $40/unit |
| $T_m^s$ | 0.01 year | 0.005 year | 0.006 year | 0.007 year | 0.002 year |
| $J_m$ | 100 km | 50 km | 60 km | 64 km | 25 km |

optimization results are as follows. The optimal shipment frequency, shipment lot, and safety factor for the first shipment are 5 times, 386.66 units, and 1.029, respectively. The optimal production allocation is 77% for regular production and 23% for green production. With this production allocation, the optimal production rates for green production and regular production are 605.8 units/year and 2028.13 units/year. The total emissions released from the retailers and the manufacturer are 4648.14 kg $CO_2$ and 11,836.18 kg $CO_2$, respectively. The total cost borne by the retailers,

**Table 6** Optimization results for the proposed model and IL model

| Decision variables, emissions, and costs | Proposed model | IL model |
|---|---|---|
| Allocation factor | 0.77 | 0.77 |
| Number of shipments | 5 | 4 |
| Shipment lot | 386.65 | 479.22 |
| Production rate | 2633.93 | 2685.09 |
| Production rate of green prod | 605.80 | 617.57 |
| Production rate of regular prod | 2028.13 | 2067.52 |
| Safety factor of 1st shipment | 1.029 | 1.035 |
| Emissions from retailers' storage | 3267.94 | 3964.09 |
| Emissions from transportation | 1380.21 | 1155.87 |
| Total emissions from retailers | 4648.14 | 5119.96 |
| Emissions from manufacturer's storage | 2787.60 | 2719.62 |
| Emissions from production | 9048.58 | 9169.98 |
| Total emissions from manufacturer | 11,836.18 | 11,889.60 |
| Total emissions from supply chain | 16,484.32 | 17,009.56 |
| Total cost of retailer 1 | 368.51 | 384.96 |
| Total cost of retailer 2 | 324.65 | 330.62 |
| Total cost of retailer 3 | 339.58 | 334.24 |
| Total cost of retailer 4 | 397.84 | 410.16 |
| Total cost of retailer 5 | 282.80 | 287.23 |
| Total cost of green production | 1766.25 | 1752.78 |
| Total cost of regular production | 2926.62 | 2916.94 |
| Joint total cost | 6406.25 | 6416.93 |

manufacturer, and supply chain are $1713.38, $4692.87, and $6406.25, respectively.

## Comparison of the Proposed Model with the Model with Identical Lead Time (IL Model)

In this sub-section, we compare the performance of the proposed model with the model that uses identical lead time (IL model). For the case of identical lead time, the length of the lead time of the first delivery is the same as the length of the lead time of subsequent shipments (see Ben-Daya and Hariga 2004). The total inventory cost incurred by retailers per unit of time for the IL model is expressed by the following:

$$
\begin{aligned}
TCR^{IL} = \frac{D}{nQ} \sum_{m=1}^{M} \left( A_m + nF_m \right) + \sum_{m=1}^{M} \left( h_m^b + EW^b \right) \left( \frac{Q}{2} \frac{D_m}{D} + k^1 \sigma_m \sqrt{T^p + T_m^s} \right) \\
+ \frac{D}{Q} \sum_{m=1}^{M} \pi_m \sigma_m \sqrt{T^p + T_m^s} \psi\left(k^1\right) + \sum_{m=1}^{M} E\left( \frac{D}{Q} \vartheta_{T1} \varepsilon J_m + \vartheta_{T2} x D_m \right)
\end{aligned}
\tag{26}
$$

By using a similar approach as described in "Solution Method," the shipment lot ($Q^{IL}$) is given by the following:

$$
Q^{IL} = \sqrt{ \frac{ 2D \left\{ \begin{array}{l} \sum_{m=1}^{M} \left( \frac{A_m}{n} + F_m + \pi_m \sigma_m \psi\left(k_m^1\right) \sqrt{T^p + T_m^s} \right) \\ + E\vartheta_{T1}\varepsilon \sum_{m=1}^{M} J_m + \frac{K_g}{n} + \frac{K_r}{n} \end{array} \right\} }{ \sum_{m=1}^{M} \left( \left( h_m^b + EW^b \right) \frac{D_m}{D} \right) + \left( h_g + EW^f \right) \left( n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P} \right) + \left( h_r + EW^f \right) \left( n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P} \right) } }
\tag{27}
$$

We note that the formulation for the total cost for the manufacturer and the formulation for the production rate remain unchanged.

The comparison of the proposed model and the IL model is summarized in Table 6. As can be seen in the table that the proposed performs better in reducing total cost compared to the IL model. It is observed that the joint total cost for the proposed model is $6406.25 and the joint total cost for the IL model is $6,416.93. Although the IL model results in a lower total manufacturer cost, the retailers' total cost is higher than the retailers' total cost in the proposed model. This makes sense since the expected shortages in the IL model is always greater than that of in the proposed model. Besides having better economic

**Table 7** The impact of demand on the model's solution

| % Change in demand | − 80% | − 40% | 0% | 40% | 80% | 120% |
|---|---|---|---|---|---|---|
| Allocation factor | 0.55 | 0.55 | 0.55 | 0.55 | 0.55 | 0.55 |
| Number of shipments | 2 | 3 | 3 | 4 | 5 | 10 |
| Shipment lot | 364.97 | 479.93 | 629.55 | 642.79 | 674.83 | 672.17 |
| Production rate | 3500.96 | 3419.79 | 3393.75 | 3276.18 | 3136.33 | 2500.00 |
| Production rate of green prod | 1575.43 | 1538.90 | 1527.19 | 1474.28 | 1411.35 | 1125.00 |
| Production rate of regular prod | 1925.53 | 1880.88 | 1866.56 | 1801.90 | 1724.98 | 1375.00 |
| Safety factor of 1st shipment | 1.1101 | 1.3518 | 1.4783 | 1.4926 | 1.4822 | 1.2183 |
| Total cost of retailer 1 | 120.53 | 186.95 | 240.38 | 271.22 | 298.58 | 306.65 |
| Total cost of retailer 2 | 131.50 | 201.03 | 258.21 | 289.32 | 317.22 | 323.90 |
| Total cost of retailer 3 | 115.44 | 186.94 | 239.26 | 276.67 | 308.33 | 321.47 |
| Total cost of retailer 4 | 128.10 | 203.62 | 260.71 | 299.89 | 334.13 | 351.47 |
| Total cost of retailer 5 | 103.54 | 161.48 | 207.06 | 234.01 | 257.28 | 261.88 |
| Total cost of green prod | 454.44 | 1153.85 | 1771.81 | 2382.56 | 2961.82 | 3573.95 |
| Total cost of regular prod | 485.26 | 1232.52 | 1882.21 | 2521.07 | 3112.70 | 3632.22 |
| Joint total cost | 1538.80 | 3326.39 | 4859.64 | 6274.74 | 7590.05 | 8771.54 |



**Fig. 3** The effects of the change in demand on the inventory level and emissions
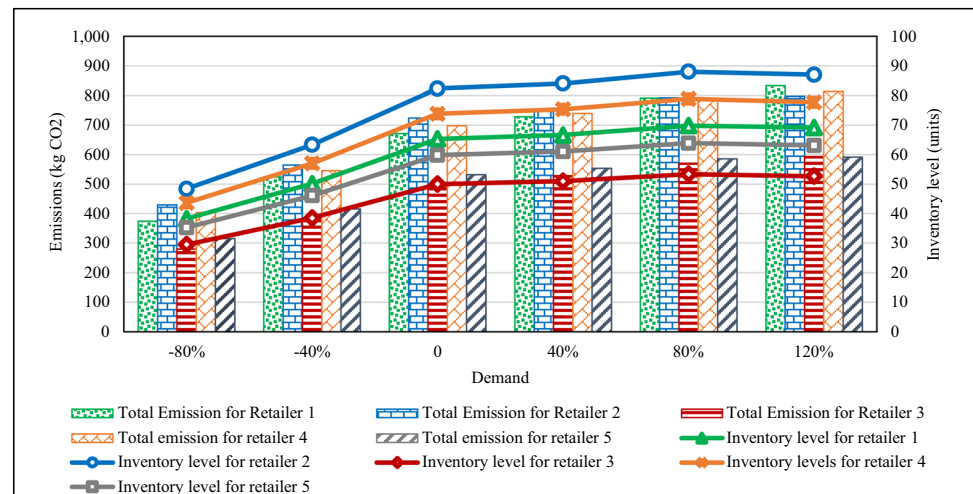
performance, the proposed model also has better environmental performance than the IL model. The results from Table 6 show that the total emissions resulted from the supply chain in the proposed model and IL model are 16,484.32 kg $CO_2$ and 17,009.56 kg $CO_2$, respectively. We also observe that the total emissions released from retailers in the proposed model are 4648.14 kg $CO_2$, which means 9.22% lower than the total emissions generated from retailers in the IL model. The total emissions released from the manufacturer in the proposed model are slightly lower (0.45%) than that in the IL model.

## Sensitivity Analysis

In this section, we perform a sensitivity analysis to investigate the behavior of the model. We focus on investigating the influence of demand, buyer's holding cost, carbon tax, and

production cost on the model's solution and total cost. We use numerical values in Example 1 as a base for performing a sensitivity analysis. First, we investigate the impact of the changes in demand on the proposed model, and the result is given in Table 7. We observe that when the demand increases by 120%, the shipment frequency and shipment lot also increase by 233.33% and 6.77%, respectively. Facing an increased demand, the manufacturer increases the batch size, and the retailer increases the ordering lot to ensure that the demand from end customers can be satisfied.

From Fig. 3, we obtain that the emissions generated from retailers are getting higher due to the increase in inventories stored in the retailers' warehouses. In addition, an increase in the frequency of shipments will also impact the number of emissions produced. When the demand increases by 120%, the total emissions from the retailer system rise by 16.38%. The most significant percentage increase in emissions occurs

**Fig. 4** Percentage change in demand vs percentage change in emissions and costs
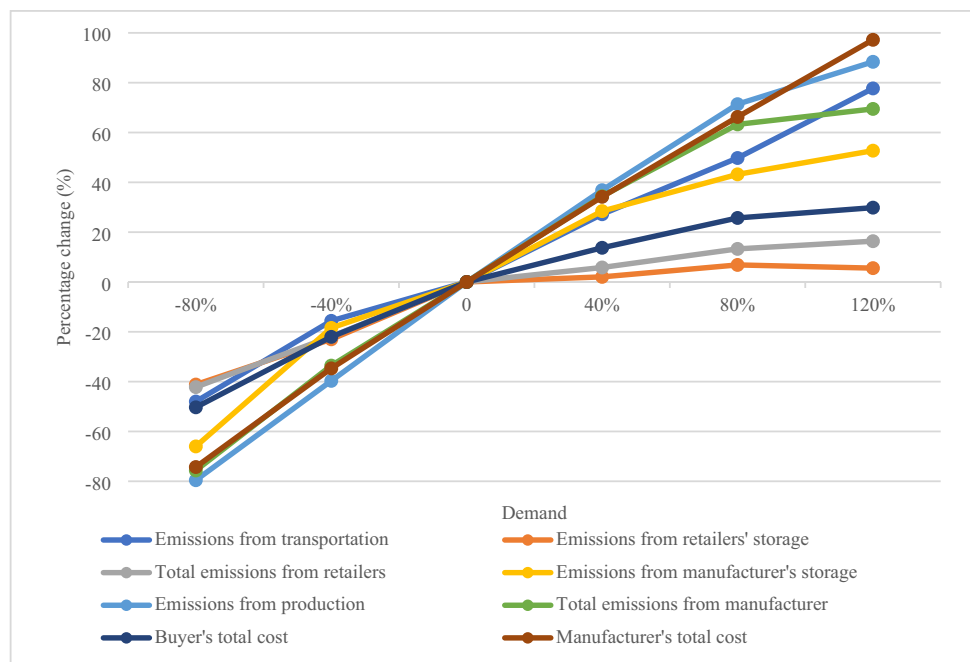


at retailer 1 (24.16%), and the smallest percentage occurs at retailer 2 (10.17%). From Fig. 4, we observe that the changes in demand have a greater effect on the total emissions of the manufacturer than the total emissions of retailers. Compared to other activities, production is the most sensitive activity to the changes in demand. This can be seen from the percentage change in production emissions which range from −79.57 to 69.47%. We further see that the total cost incurred to the retailers, manufacturer, and supply chain remarkably increased due to higher demand. A 120% increase in the demand causes a 29.84% increase in the total cost of retailers. We find that retailer 4 experiences the most significant

increase in cost, which is 34.81%, and the smallest growth in the cost of 25.44% is experienced by retailer 2. Figure 4 shows that fluctuations in demand have a larger influence on the total cost of the manufacturer than on the total cost of the retailer. The manufacturer must bear higher costs due to increased production volumes and emissions. Because the changes in demand have a greater impact on the manufacturing system, the managers need to be careful in making production decisions, especially those related to production allocation and production batch.

Table 8 presents the sensitivity analysis results of the buyer's holding cost on the model's solution. As one can

**Table 8** The impact of buyer's holding cost on the model's behavior

| % change in retailer holding cost | − 80% | − 40% | 0% | 40% | 80% | 120% |
|---|---|---|---|---|---|---|
| Allocation factor | 0.55 | 0.55 | 0.55 | 0.55 | 0.55 | 0.55 |
| Number of shipments | 2 | 3 | 3 | 4 | 4 | 4 |
| Shipment lot | 980.74 | 672.89 | 629.55 | 501.37 | 479.55 | 460.40 |
| Production rate | 3504.79 | 3386.16 | 3393.75 | 3327.58 | 3335.41 | 3342.27 |
| Production rate of green prod | 1577.16 | 1523.77 | 1527.19 | 1497.41 | 1500.94 | 1504.02 |
| Production rate of regular prod | 1927.64 | 1862.39 | 1866.56 | 1830.17 | 1834.48 | 1838.25 |
| Safety factor of the 1st shipment | 1.8164 | 1.5981 | 1.4783 | 1.3218 | 1.2403 | 1.1716 |
| Total cost of retailer 1 | 180.29 | 207.07 | 240.38 | 257.55 | 282.72 | 306.35 |
| Total cost of retailer 2 | 200.54 | 223.81 | 258.21 | 273.87 | 300.05 | 324.67 |
| Total cost of retailer 3 | 177.55 | 209.38 | 239.26 | 261.29 | 284.09 | 305.54 |
| Total cost of retailer 4 | 198.61 | 228.40 | 260.71 | 282.71 | 307.59 | 331.02 |
| Total cost of retailer 5 | 162.46 | 181.58 | 207.06 | 220.34 | 239.78 | 258.07 |
| Total cost of green prod | 1739.17 | 1774.93 | 1771.81 | 1791.54 | 1788.64 | 1786.81 |
| Total cost of regular prod | 1851.51 | 1888.54 | 1882.21 | 1904.41 | 1899.42 | 1895.74 |
| Joint total cost | 4510.13 | 4713.70 | 4859.64 | 4991.70 | 5102.30 | 5208.21 |

**Fig. 5** The effects of the change in retailers' holding cost on the inventory level and emissions
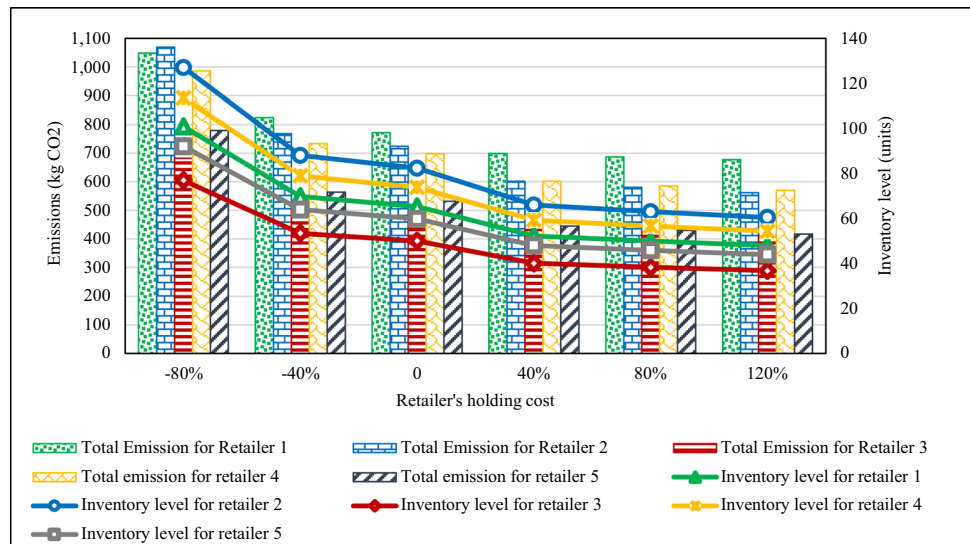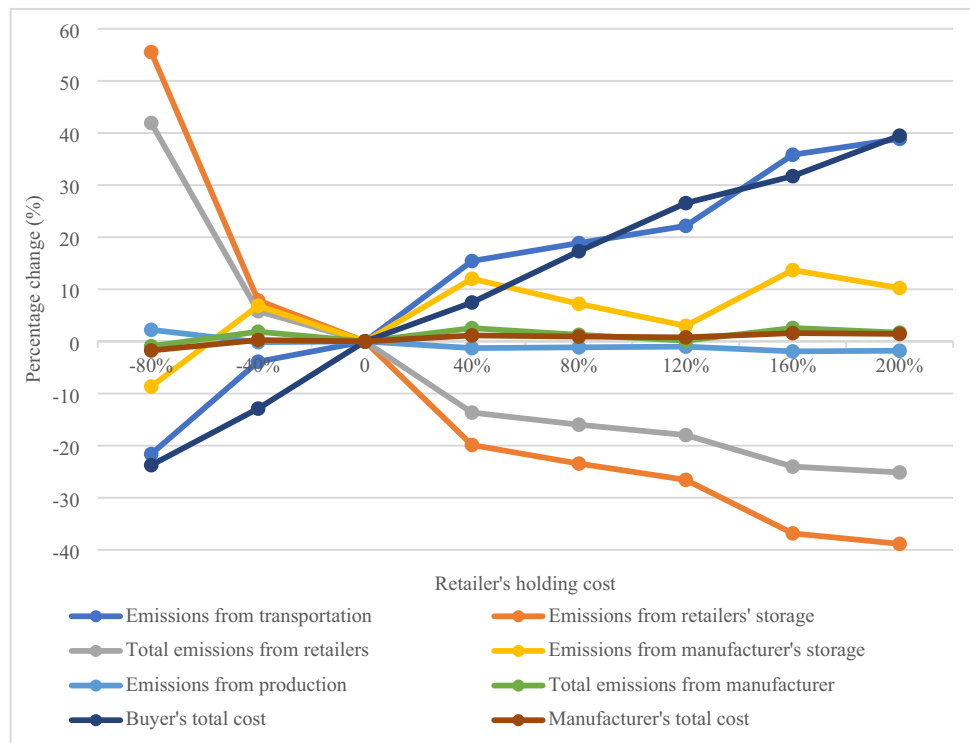


**Fig. 6** Percentage change in retailer's holding cost vs percentage change in emissions and costs



see, the higher holding cost will result in a higher number of shipments, smaller shipment lots, and lower safety factors. We observe that if the cost increases by 80%, the number of shipments increases by 33.33%, and the shipment lot and safety factor decrease by 23.83% and 16.09%, respectively. This finding suggests that retailers keep fewer products to prevent the system from rising storage costs. Although the number of emissions from transportation increases due to the rise in the shipment frequency, the emissions resulting from retailer's storage drastically increase, leading to the rise in the total emissions (see

Fig. 5). As clearly seen in the figure, higher holding cost results in lower total emissions. It is observed that if the cost increases by 80%, the emissions from retailer 1, retailer 2, retailer 3, retailer 4, and retailer 5 decrease by 10.93%, 19.89%, 14.26%, 16.22%, and 19.25%, respectively. The percentage change in total emissions resulting from retailers' storage appears to be greater than the percentage change in total emissions resulting from other activities (see Fig. 6). This is very reasonable because retailers will adjust their inventory level flexibly to maintain the holding cost. We further observe that retailers'

**Table 9** The impact of carbon tax on model's solution

| % change in carbon tax | − 80% | − 40% | 0% | 40% | 80% | 120% |
|---|---|---|---|---|---|---|
| Allocation factor | 0.64 | 0.59 | 0.55 | 0.52 | 0.49 | 0.47 |
| Number of shipments | 3 | 3 | 3 | 3 | 3 | 3 |
| Shipment lot | 768.03 | 687.74 | 629.55 | 584.95 | 549.43 | 520.30 |
| Production rate | 3800.00 | 3558.74 | 3393.75 | 3266.83 | 3159.50 | 3068.29 |
| Production rate of green prod | 1368.00 | 1459.08 | 1527.19 | 1568.08 | 1611.34 | 1626.19 |
| Production rate of regular prod | 2432.00 | 2099.66 | 1866.56 | 1698.75 | 1548.15 | 1442.09 |
| Safety factor of 1st shipment | 1.5383 | 1.5077 | 1.4783 | 1.4504 | 1.424 | 1.3991 |
| Total cost of retailer 1 | 198.50 | 219.79 | 240.38 | 260.39 | 279.90 | 299.02 |
| Total cost of retailer 2 | 210.64 | 235.21 | 258.21 | 279.99 | 300.81 | 320.82 |
| Total cost of retailer 3 | 198.79 | 219.59 | 239.26 | 258.01 | 276.00 | 293.35 |
| Total cost of retailer 4 | 210.99 | 236.69 | 260.71 | 283.44 | 305.16 | 326.05 |
| Total cost of retailer 5 | 168.56 | 188.58 | 207.06 | 224.38 | 240.75 | 256.36 |
| Total cost of green prod | 1374.03 | 1585.34 | 1771.81 | 1930.32 | 2092.04 | 2220.68 |
| Total cost of regular prod | 1685.35 | 1791.98 | 1882.21 | 1973.19 | 2041.23 | 2126.71 |
| Joint total cost | 4046.87 | 4477.18 | 4859.64 | 5209.73 | 5535.88 | 5842.98 |

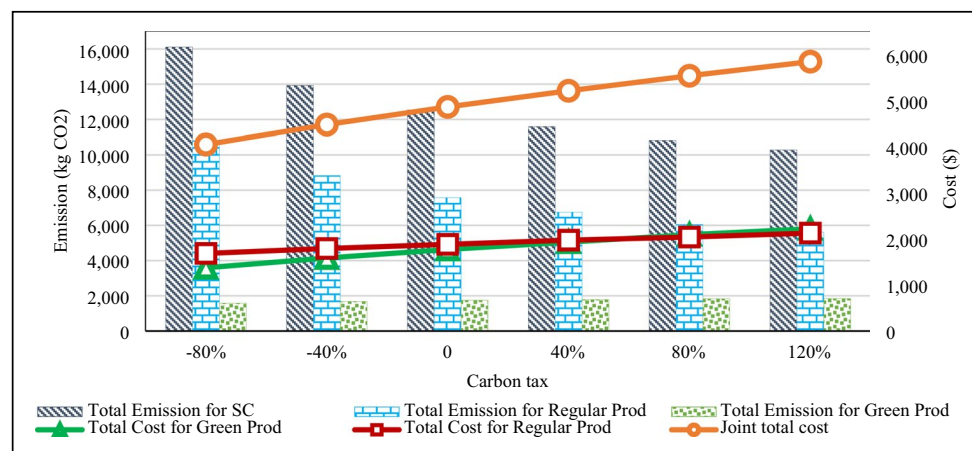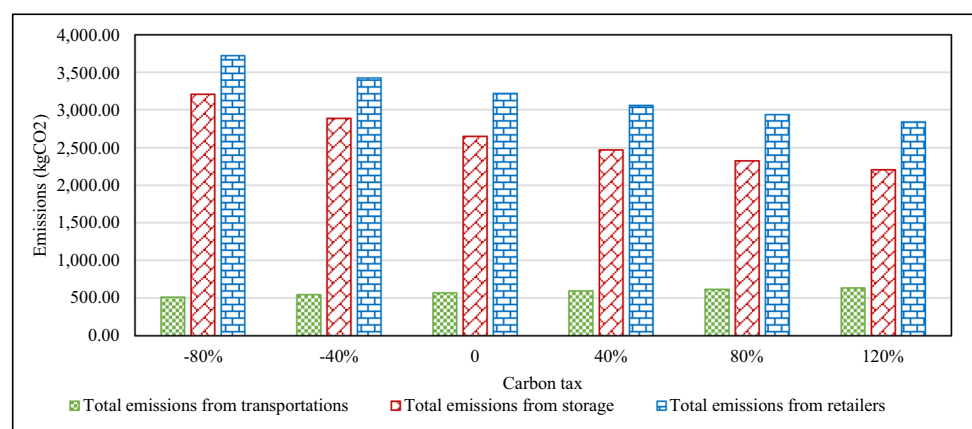**Fig. 7** The influence of the carbon tax on the emissions and cost



**Fig. 8** The influence of the carbon tax on the emissions generated from retailers



total cost is more sensitive to holding cost changes than the manufacturer's total cost. Since the retailer receives a greater impact, the inventory manager needs to carefully determine the optimal inventory level on the retailer's side so that system performance can be maintained.

The impact of the carbon tax is now examined, and the results are shown in Table 9. It can be observed that the change in the carbon tax has a pronounced impact on all decision variables except the number of deliveries. The production allocation to the cleaner production system is

**Fig. 9** Percentage change in emission tax vs percentage change in emissions and costs
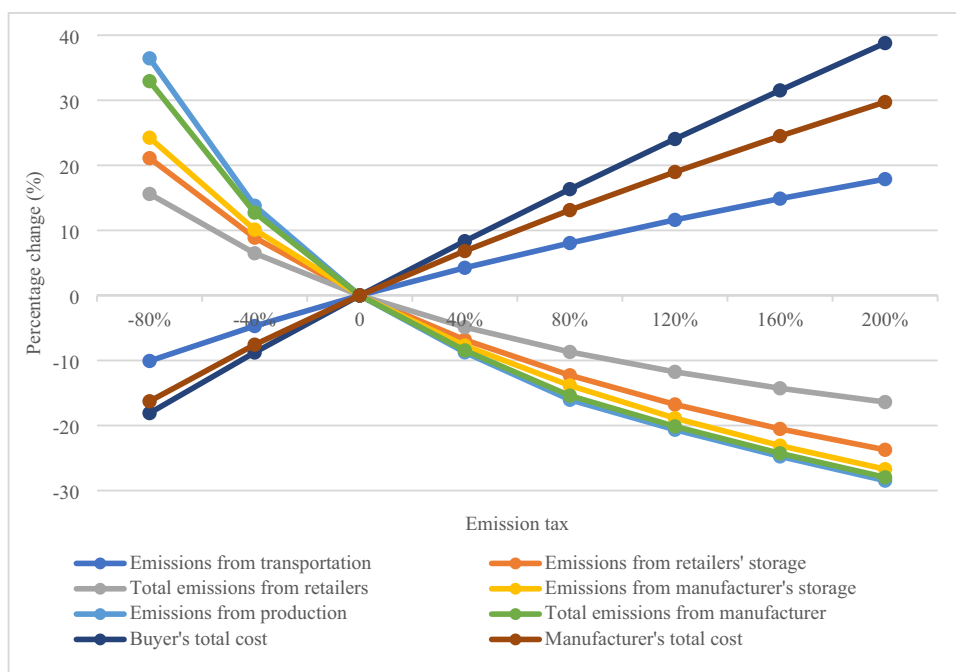


getting higher due to the gradual increase in the carbon tax. This policy makes sense since allocating more productions to the cleaner system will significantly reduce the emissions generated. Consequently, the manufacturer should adjust the production rate at both systems. We observe that if the carbon tax increases by 120%, the production rate for green production increases by 6.48%, and the production rate for regular production decreases by 22.74%.

Figure 7 clearly shows how the carbon tax has a significant impact on emissions and total cost. When the tax imposed by the regulator is increased by 120%, the emissions from green production rise by 5.45%, and the emissions from regular production decrease by 26.04%, which leads to the reduction of the total emissions resulting from the manufacturer's operations. With a higher carbon tax, the retailers must reduce the products stored in the warehouse to maintain the resulting emissions. As shown in Fig. 8, the emissions from storage and the total emissions generated from all retailers remarkably decrease due to the increase in the carbon tax. We further examine the percentage increase in emissions and costs, which is presented in Fig. 9. It is observed that the percentage change in retailers' total emissions and manufacturers' total emissions are within the ranges of $-16.39$ to 15.6% and $-27.96$ to 32.93%, respectively. This suggests that tax changes have a greater impact on the amount of emissions produced by producers, most of which result from production activities. We also observe that the percentage change in retailers' total cost and manufacturer's total cost are within the ranges of $-16.27$ to

29.73% and $-18.09$ to 38.79%, respectively. We note that both parties involved in the supply chain received a significant impact caused by the changes in the carbon tax.

We also examine the impact of the changes in production cost ($Xg_2$) for green production, and the results are shown in Table 10. The results suggest that the manager should allocate more output to the regular production. When the production cost at the green facility is higher, it is reasonable to allocate some of the production to the regular facility. As a result, the production rate of the regular output needs to be adjusted to a higher level to speed up the production. We observe that the number of shipments, shipment lot, and safety factors keep unchanged due to the increase in $Xg_2$. This means that retailers do not need to change their policies when there are changes in manufacturer production costs.

As for the carbon emissions at the manufacturer, we observe that the emissions resulting from the regular production significantly increase, and the emissions generated from the green production enormously decrease. When $Xg_2$ increases by 120%, the emissions from regular production increase by 40.02%, whereas green production generates a lower emission of 43.65%. As a result, the total emissions released from the supply chain rises by 17.74%. Compared to other activities, production is the activity most affected by changes in production cost components (see Fig. 11). This can be seen from changes in production emissions which are within the range of $-21.24$ to 46.21%. The percentage change in retailers' total emissions is much smaller than that of manufacturers' total emissions. Faced with

**Table 10** The impact of $Xg_2$ on model's behavior

| % change in $X_{g2}$ | − 80% | − 40% | 0% | 40% | 80% | 120% |
|---|---|---|---|---|---|---|
| Allocation factor | 0.38 | 0.45 | 0.55 | 0.62 | 0.68 | 0.72 |
| Number of shipments | 3 | 3 | 3 | 3 | 3 | 3 |
| Shipment lot | 626.63 | 626.98 | 629.55 | 631.29 | 632.75 | 633.77 |
| Production rate | 3800.00 | 3746.86 | 3393.75 | 3191.93 | 3041.53 | 2944.97 |
| Production rate of green prod | 2356.00 | 2060.77 | 1527.19 | 1212.93 | 973.29 | 824.59 |
| Production rate of regular prod | 1444.00 | 1686.09 | 1866.56 | 1979.00 | 2068.24 | 2120.38 |
| Safety factor of 1st shipment | 1.4807 | 1.4804 | 1.4783 | 1.4769 | 1.4757 | 1.4749 |
| Total cost of retailer 1 | 240.41 | 240.41 | 240.38 | 240.37 | 240.36 | 240.35 |
| Total cost of retailer 2 | 258.19 | 258.19 | 258.21 | 258.22 | 258.24 | 258.25 |
| Total cost of retailer 3 | 239.53 | 239.50 | 239.26 | 239.11 | 238.98 | 238.89 |
| Total cost of retailer 4 | 260.85 | 260.84 | 260.71 | 260.63 | 260.56 | 260.51 |
| Total cost of retailer 5 | 207.17 | 207.16 | 207.06 | 207.00 | 206.96 | 206.92 |
| Total cost of green prod | 1534.08 | 1785.65 | 1771.81 | 1725.50 | 1667.12 | 1627.49 |
| Total cost of regular prod | 1393.11 | 1584.02 | 1882.21 | 2098.09 | 2287.55 | 2415.25 |
| Joint total cost | 4133.35 | 4575.77 | 4859.64 | 5028.92 | 5159.76 | 5247.66 |

**Fig. 10** The influence of $Xg_2$ on the emissions and cost
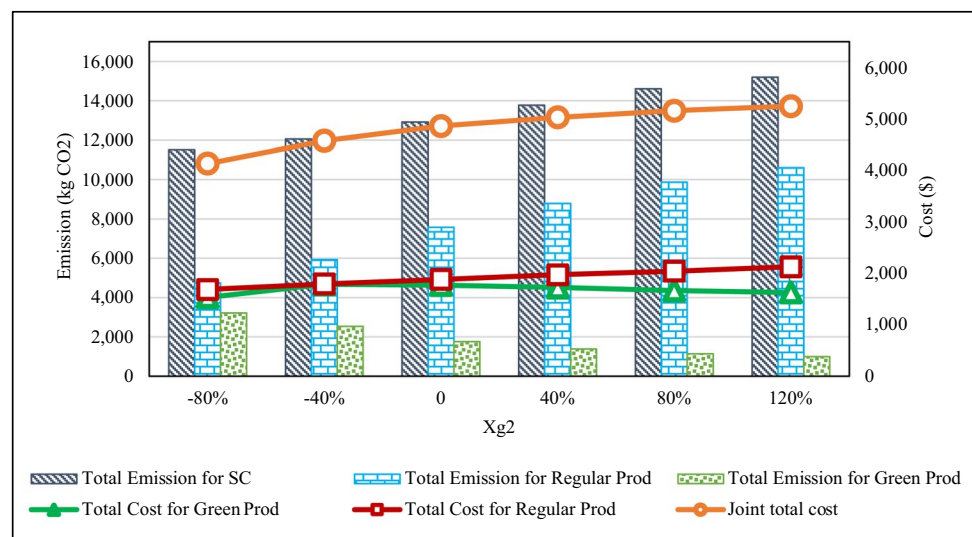


increased $Xg_2$, the manufacturer will adjust the production by setting an appropriate level of production allocation to maintain the performance of the production system. As seen in Fig. 10, the costs incurred by the parties and the supply chain are significantly influenced by $Xg_2$. If $Xg_2$ increases by 120%, the total cost for the manufacturer and supply chain increase by 10.64% and 7.98%, respectively, and the total costs for the retailers remain unchanged. Moreover, the results from Fig. 11 show that the percentage change in the manufacturer's total cost is greater than that of the retailers' total cost. We observe that the total cost will move over a range of − 0.07 to 0.04% for retailers and − 19.89 to 13.88% for the manufacturer. This suggests managers to be more careful in determining production and inventory policies for the manufacturer so that the system can run efficiently.

## Managerial Insights

The proposed model is beneficial for decision-makers to manage production systems that use two different types of production technology. The model can help decision-makers determine the optimal production capacity in both systems and align them with the retailers' decisions. In addition, the information generated is also beneficial to support a green investment project carried out by the manufacturer. For example, information related to the allocation factor and the green production rate can be helpful to determine the number of green production machines that must be purchased in the investment project to cope with an increased carbon tax. Choosing the right technologies in the green investment is not easy since many types of

**Fig. 11** Percentage change in $X_{g2}$ vs percentage change in emissions and costs
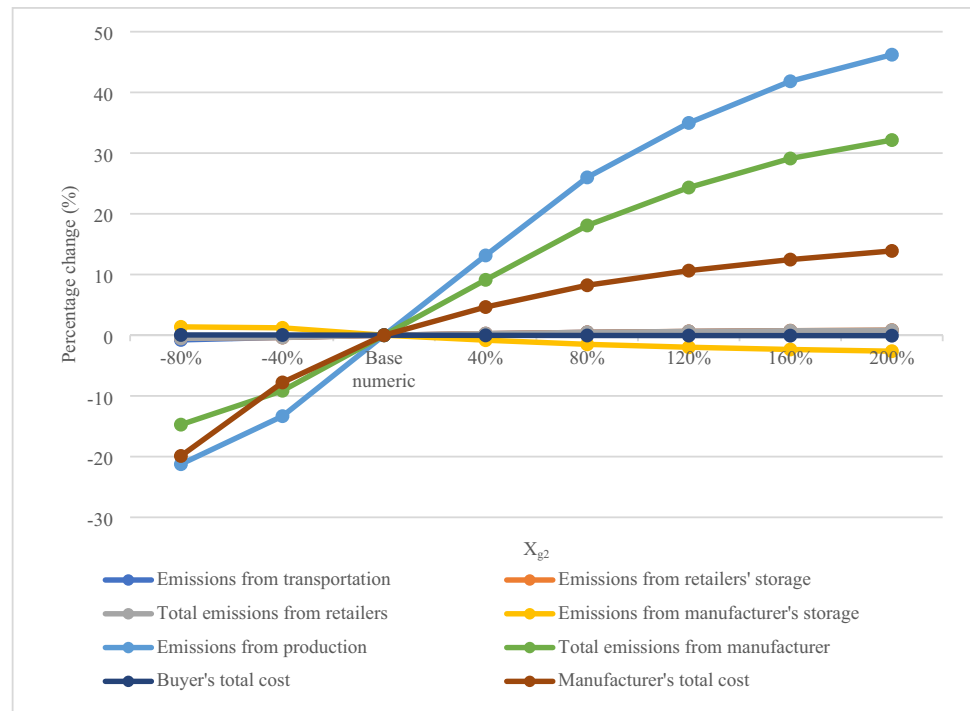


technology are in the market. Different types of green technologies are available in the market, including machines that use renewable, green chemistry, and technologies that improve recycling processes. Because the technology selection decision is crucial, managers must pay attention to several important aspects, including budget constraints, the effectiveness of the technology to reduce emissions, and the suitability of the technology with the existing facilities.

In order to have a competitive advantage, coordination between parties in the supply chain is an essential factor that must be considered in integrated inventory management. The proposed model can assist the managers in improving the coordination between manufacturers and retailers by jointly deciding production and shipment decisions. In the multi-retailer case, the coordination between retailers must also be considered in the decision-making process. The model provides essential decisions, such as production batch, production rate, shipment lot, number of shipments, and safety factors, that can be used to manage inventories across the supply chain. Although the decisions in inventory management can be made, their implementation in the real world is not easy. This is because several parties will have different interests and goals. Thereby, wise leadership is needed to encourage all parties to work together to support mutually agreed decisions. In addition, information system support is also necessary to increase visibility along the supply chain.

# Conclusions

This paper develops a mathematical model for a supply chain inventory system involving a manufacturer who produces products and multiple retailers who sell products to the end customers. The manufacturer employs a hybrid production system that consists of green and regular facilities and has an opportunity to control the production rate adjustably. The model contributes to the current inventory literature by proposing a hybrid production system and synchronizing it with replenishment and shipment decisions on the retailers' side. In the case of multi-retailers, such synchronization is necessary to ensure that the system can run effectively and comply with the imposed carbon policy. Retailers' inventory system is operated with two different reorder points to deal with different lead times. This feature allows retailers the opportunity to manage inventory more efficiently, resulting in minimum holding costs. The emissions are generated from production, storage, and transportation activities regulated through a carbon tax policy. Emissions resulting from production activities are related to production speed to control production rate adjustments. The proposed mathematical model minimizes the joint total cost and determines the optimal allocation factor, shipment frequency, safety factor, shipment lot, and production rate.

The findings of this study are explained as follows. First, the emissions resulting from the supply chain are significantly influenced by the demand, holding cost, carbon tax,

and production cost. We observe that the emissions from the retailers are more affected by the change in demand, holding cost, and carbon tax than the change in production cost. To manage the emissions resulting from the retailers, the manager needs to control the shipment lot, the number of deliveries, and safety factors carefully. Second, the manager should allocate more production to the cleaner system when there are increases in the carbon tax and reduce the allocation if the green production cost is getting more expensive. Third, it is observed that by setting the decision variables optimally, the generated emissions can be significantly reduced, thus minimizing the cost incurred to the supply chain. The manager can control the emissions from production by adjusting the allocation factor and production rate.

The model can be extended in various ways. First, the shipment of the product from the manufacturer to the retailers is carried out without considering the vehicle's route. Future studies may investigate the influence of routes on emissions and costs. Second, with the rapid development of technology, green transporter in product delivery can help the supply chain reduce emissions from transportation. Third, the model can be extended by considering the imperfect production on the manufacturer's side. If an imperfect process is allowed, the defective product may result from the manufacturing system which the production rate can also influence its quantity.

**Data Availability**  My manuscript has no associated data, or the data will not be deposited.

## Declarations

**Conflict of Interest**  The authors declare no competing interests.

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