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A Methodology for Tracing the Rank Invariance Region in Multi-Criterion Selection Problems: Application to Negative Emission Technologies

Raymond R. Tan [1,2] · Elvin Michael R. Almario [3] · Kathleen B. Aviso [2] · Jose B. Cruz Jr [1] · Michael Angelo B. Promentilla [2]

Abstract

It has been previously established that, for multi-criterion selection problems, over a finite set of options, solved via simple additive weighting, there exists a set of weights for which the optimal alternative remains the best choice. This invariance region in the n-dimensional hyperspace defined by the problem criteria contains the set of all weights for which the optimal solution is robust to preference changes. When the set of options is a continuum, there are no known invariance regions reported in the literature. In this work, we develop a procedure to characterize and trace this invariance region when the number of options is finite. The methodology is demonstrated on a case study involving a selection of negative emission technologies based on carbon sequestration potential, water footprint, energy demand, and cost.

Introduction

Multiple-attribute decision-making (MADM) problems are prevalent in sustainable engineering, where technology options often need to be evaluated based on a number of criteria representing technical, economic, environmental, social, and other relevant considerations. Simple additive weighting (SAW) is an MADM method whose origins can be traced back to the work of MacCrimmon (1968). SAW involves a two-step procedure, where the performance ratings of alternatives are first normalized to a commensurate dimensionless scale, such as the interval [0, 1], where the interval limits include the worst and best possible scores, respectively. The weighted sum of the score of each alternative is then

computed, and the options are finally ranked based on the values of the aggregate scores. This general approach is valuable for identifying optimal technologies from a sustainability perspective (Sikdar 2009). It is assumed that the weights used are exogenously determined, for instance, using the pairwise comparison approach of the analytic hierarchy process (AHP) of Saaty (1980), and that the weights sum up to unity.

For any given MADM problem, when the number of options is finite, the SAW method involves the following equations:

$$\Sigma_j \, w_j \, d_{ij} = s_i \qquad \forall i \tag{1}$$

$$\Sigma_j \, w_j = 1 \tag{2}$$

where w_j is the dimensionless weight of criterion j, d_{ij} is the dimensionless score of alternative i with respect to criterion j, and s_i is the aggregate score of alternative I, where i is a member of a finite set. The optimal solution is the alternative with the highest aggregate score, $s_{i'}$. The choice of weighting factors clearly determines the solution of the problem. These weights reflect the decision-maker's preferences and are thus inherently subjective. As such, elicitation of appropriate weights remains a major issue in the MADM research.

Research on MADM methodology has sought to develop means to capture uncertainties inherent in weighting. For

✉ Raymond R. Tan
Raymond.Tan@dlsu.edu.ph

1 National Academy of Science and Technology, Science Heritage Building, DOST Complex, General Santos Ave, 4044 Taguig, Metro Manila, Philippines

2 Chemical Engineering Department, De La Salle University, 2401 Taft Avenue, 0922 Manila, Philippines

3 Cruz & Associates, 19 Piko Street, 4118 Silang, Cavite, Philippines

example, imprecise weight values can be represented using appropriate mathematical frameworks for representing epistemic uncertainty, such as fuzzy set theory (e.g., Geldermann et al. 2000; Tseng 2013). Alternatively, sensitivity analysis is also widely used to handle uncertainties with respect to decision-maker preferences in MADM problems. The dispersion of aggregate scores can give important insights such as identifying critical attributes (Maliene et al. 2018). A common approach is to vary the weight of one criterion from 0 to 1, while assuming that the other weights make up the balance and retain constant relative proportions (e.g., Tan et al. 2016). Such approaches are able to detect rank reversals among all alternatives, but are limited to the isolated analysis of one criterion at a time. On the other hand, the decision-maker may be concerned only with the optimal alternative and competitors which threaten to overtake it, but may be interested in the interplay among criteria weights. Cruz and Almario (2018) established the existence of a rank invariance region in n-dimensional space defined by the criteria in the MADM problem, provided that the number of options is finite. This region contains the set of all criteria weights for which the optimal alternative in a SAW problem remains optimal. There is no known result on invariance when the number of options is not finite. Cruz and Almario (2018) is the only paper that characterized the invariance region. As no numerical procedure was proposed by Cruz and Almario (2018) to trace this rank invariance region, we propose a methodology for this purpose and illustrate it with an MADM example from the literature. The main novelty of this work lies in providing a specific procedure for performing a special form of sensitivity analysis in MADM problems.

Problem Statement

The problem addressed in this work may be stated as follows:

- Given a set of M alternatives each rated with respect to N criteria on a scale in the interval [0, 1], where 1 corresponds to the most desirable score;
- Given a set of normalized weights w_j by which the aggregate scores of the alternatives s_i can be found via SAW;
- Given an optimal solution identified by inspection of its aggregate score s_i;
- The task is to determine the set of weights for which the optimal solution remains optimal.

Methodology

This methodology is partly based on a procedure developed by Cortés-Borda et al. (2013) for sensitivity analysis with

respect to weights in the context of life cycle assessment (LCA). However, in the context of LCA, the weights are generally not subject to the constraint defined by Eq. (2) in the SAW method. Instead, LCA weights are independent and thus potentially unbounded, which limits the previously developed method to the analysis of one weight factor at a time. In this work, the convex n-dimensional polytope that defined the rank invariance region can be traced numerically.

Stage 1: Determining the Bounding Polytope

The decision-maker may seek to determine the upper and lower bounds of the weights for which the optimal alternative remains optimal. For any given weight w_j, the upper bound (w^U_j) can be determined by solving the linear programming (LP) model:

$$\max w_j \tag{3a}$$

subject to:

$$s_i \leq s_{i'} \qquad \forall i \neq i' \tag{3b}$$

$$\Sigma_j w_j d_{ij} = s_i \qquad \forall i \tag{3c}$$

$$\Sigma_j w_j = 1 \tag{3d}$$

where constraint (3b) ensures that the optimal alternative score $s_{i'}$ is at least as good as any of its competitors, and constraints (3c) and (3d) are the same as Eqs. (1) and (2) in the basic SAW method. Conversely, for each weight w_j, the lower bound (w^L_j) can be determined by retaining all the LP constraints, and replacing objective function (3a) with

$$\min w_j \tag{4}$$

In Cruz and Almario (2018), the invariance set is defined by (3b), (3c), and (3d). In this paper, we provide a numerical procedure for finding this set.

This procedure is repeated for all the weights, thus determining a convex polytope bounded by pairs of parallel hyperplanes that correspond to these upper and lower bounds. The resulting polytope gives the outer bounds of the rank invariance region, which is itself a convex polytope inscribed within the said limits. For some decision-makers, this information may be sufficient, as any weight deviation beyond the identified limits is guaranteed to result in a new optimum.

Stage 2: Tracing the Rank Invariance Region

The next step is to trace the rank invariance region itself, which is inscribed within the polytope determined by the previous step. The hyperplanes which define the faces of the rank invariance regions signify the limits beyond which the baseline optimal solution is overtaken by other alternatives as the weights shift. The rank invariance region corresponds to the

feasible region of the LP above. Applying the simplex algorithm to the model, however, is not guaranteed to identify all of the vertices of the feasible region, since the procedure will halt as soon as an optimum is reached. Instead, the exact boundaries of this region can be found by applying any of a range of established algorithms for tracing convex polytopes (Mathiess and Rubin 1980). For sufficiently small problems with up to four criteria, the resulting region is easily visualized for decision support purposes. On the other hand, for problems with more than four criteria, direct visualization will not be possible, so the decision-maker can opt to use the bounds determined in Stage 1 as an outer approximation of the polytope.

Case Study

This case study involves the comparison of negative emission technologies (NETs), which are techniques for removing atmospheric carbon. Haszeldine et al. (2018) argue that NETs will need to be deployed commercially in the coming decades in order to stabilize climate change to safe levels. The data used in this example are drawn from the conservative estimates reported by Smith et al. (2016) on selected NETs that can be used in the UK. The available options are listed and described briefly in Table 1, while their normalized scores with respect to carbon, water, and energy footprints are given in Table 2. Note that values of 0 and 1 signify the worst and best alternatives for any given criterion; the normalization of the raw data into this dimensionless form is a trivial step which

Table 2 Normalized scores of NETs with respect to criteria (adapted from Smith et al. 2016)

| Alternative	Sequestration potential	Water requirement	Energy requirement	Specific cost	Aggregate score
BECCS	0.63	0.00	0.64	0.98	0.64
AR*	0.72	0.73	0.79	1.00	0.78*
SCS	0.00	1.00	0.79	1.00	0.38
BA	0.22	1.00	1.00	0.86	0.50
DAC	0.63	0.96	0.00	0.61	0.60
EW	1.00	1.00	0.07	0.00	0.71

*Optimal alternative at baseline state

is omitted here for brevity. The weights of the criteria are initially assumed to be 0.6, 0.1, 0.1, and 0.2, respectively, which leads to alternative AR being selected as the optimal solution based on its aggregate score of 0.78.

In this problem, all possible weight combinations can be mapped on a three-dimensional tetrahedral diagram whose vertices correspond to full weighting (i.e., $w_j = 1$) to each of the four criteria. The methodology developed is implemented using the commercial optimization software LINGO. Applying Stage 1 of the procedure gives the upper and lower bounds of the weights, which are shown in Table 3. These results show that the optimal solution AR is most sensitive to the weight of the water requirement (w_2), because rank reversal definitely occurs for $w_2 > 0.53$. On the other hand, the optimal alternative can remain optimal-specific cost weights (w_4) ranging from 0 to 1 and is thus least sensitive to this criterion.

Subsequent application of Stage 2 generates the rank invariance region. This 3-dimensional polytope which can be visualized by parametrically varying the weight of one of the criteria to generate as a series of 2-dimensional polygons. These polygons represent parallel cross-sections or cuts through the polytope. For example, Fig. 1 shows the two-dimensional sections of the rank invariance region with cuts at $w_4 = 0$, 0.3, 0.6, and 0.9, plotted here using the spreadsheet developed by Graham and Midgley (2000). Each of the four sections shown is a rank invariance region at a corresponding value of w_4. The boundary of the rank invariance at $w_4 = 0$ is indicated by the green triangle in Fig. 1. The edges of this

Table 1 Alternative NETs (Smith et al. 2016)

Alternative	Description
Bioenergy with CO_2 capture and storage (BECCS)	Augmentation of inherently carbon-neutral biomass combustion systems with CO_2 capture and storage (CCS) to effect net removal of atmospheric carbon.
Afforestation and reforestation (AR)	Increasing carbon stock in forest through plant growth.
Soil carbon sequestration (SCS)	Modification of agricultural practices to maximize soil carbon stock.
Biochar application (BA)	Application of stable, carbonized biomass to soil to achieve long-term sequestration of recalcitrant carbon.
Direct air capture (DAC)	Direct removal of atmospheric CO_2 using alkaline scrubbing systems coupled with CO_2 storage.
Enhanced weathering (EW)	Acceleration of natural weathering of basic minerals with chemical affinity for atmospheric CO_2 by size reduction and application to soil.

Table 3 Upper and lower bounds of criteria weights

	Sequestration potential	Water requirement	Energy requirement	Specific cost
Upper limit	0.78	0.53	0.70	1
Baseline	0.60	0.10	0.10	0.20
Lower limit	0	0	0	0
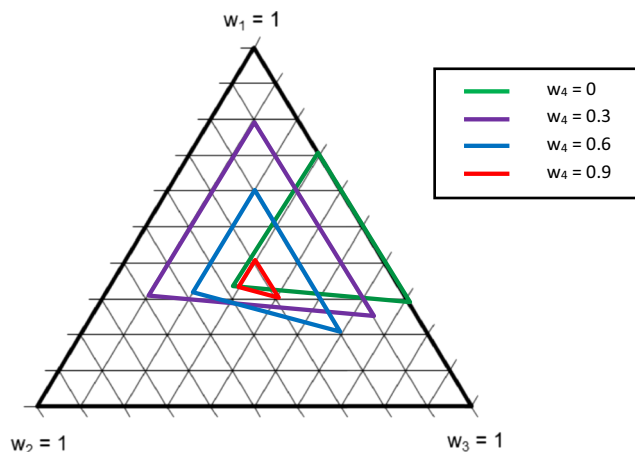
Fig. 1 Rank invariance region of case study showing sections at different values of w_4

region correspond to weight values where the baseline optimal choice, AR, is overtaken by other alternatives. Above and to the left of the rank invariance region, the new optimal choice is EW, while to the left of this region, the best alternative becomes BA. At the vertex with coordinates (0.33, 0.40, 0.27), alternatives AR, EW, and BA all have equal performance. As the importance of cost (i.e., the value of w_4) increases, the optimal choice AR becomes increasingly more robust to further perturbations in w_1, w_2, and w_3. For instance, at $w_4 = 0.6$, the rank invariance region indicated by the blue triangle occupies most of the feasible range of values of w_1, w_2, and w_3 (whose values should sum up to 0.4). Rank reversal occurs below the lower edge of this region at higher values of w_2 (e.g., at $w_2 = 0.4$ and $w_1 = w_3 = 0$), in which case, SCS becomes the optimal solution.

Conclusion

A method has been developed to numerically determine the invariance region of an MADM problem solved using the SAW method, when the number of options is finite. This invariance region contains the set of weights for which the optimal alternative remains optimal. The procedure has been illustrated on a problem involving ranking of NETs based on three footprints (carbon, water, and energy) and implementation cost. The method is easily scalable and can be easily applied to larger problems. The methodology is particularly valuable for assessing novel sustainable technologies using criteria whose priority weights are subject to volatility due to the presence of multiple stakeholders or shifting priorities. In future work, the algorithm can potentially be extended to other MADM methods that use an additive structure, such as AHP.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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