

Calculation and Analysis of Multi‑scale Earth Gravity Field Parameters Based on Self‑developed EIGEN‑5C Model Software

Bolong Yu¹ · Yanyi Li2 · Qiwen Sun3 · Jian Shi[4](http://orcid.org/0000-0002-5456-7868)

Accepted: 16 October 2022 / Published online: 2 November 2022© The Author(s), under exclusive licence to Springer Nature Switzerland AG 2022

Abstract

There are many calculation model schemes with practical value for calculating the critical parameters of the Earth's gravity feld. However, the research work of systematically using the EIGEN-5C model for multi-scale inversion of the earth's gravity feld needs to be supplemented. To solve the above problems, this study uses the 360-order EIGEN-5C model to calculate the four parameters of the Earth's gravity feld: disturbance potential, geoid height, gravity anomaly, and gravity disturbance. This study first calculates the normalized-associated Legendre function, reads the coefficients of the EIGEN-5C gravity field model, then calculates the normal gravity and converts the geodetic coordinate system into the spherical coordinate system. On this basis, the spherical harmonic series expansion of the gravity feld parameters is carried out. Through experiments, it is found that the calculation process introduced in this study can better calculate and express the critical parameters of the multi-scale Earth's gravity feld. For the local scale, the researchers calculated from the experimental areas of low, medium, and high latitudes. For the global scale, the researchers also calculated the above indicators. The calculation results can sufficiently express the geophysical situation of the study area. Additionally, to better realize the above process, we creatively developed a set of software for calculating the parameters of the Earth's gravity feld based on the EIGEN-5C model. The software can calculate all the above key parameters of gravity feld, and realize the visual expression of the results. All in all, this study systematically explains using the EIGEN-5C model to carry out the inversion and result in the expression of multi-scale Earth's gravity feld parameters. And our self-developed software has a certain reference value for the research feld of geographic information mapping technology.

Keywords Earth gravity feld · Disturbance potential · Geoid height · Gravity anomaly · Gravitational disturbance · EIGEN-5C · Geographic information

Introduction

Since 2000, the launch missions of CHAMP, GRACE, GOCE, and other Earth gravity satellites have been successfully implemented internationally. Chinese and foreign scholars have researched the Earth's gravity feld model by

 \boxtimes Jian Shi 201801020123@sdust.edu.cn

- ¹ School of Geodesy and Geomatics, Wuhan University, Wuhan 430070, China
- ² College of Surveying and Geo-Informatics, Tongji University, Shanghai 200092, China
- College of Geoscience and Surveying Engineering, China University of Mining & Technology (Beijing), Beijing 100083, China
- ⁴ Shanghai Dianji University, Shanghai 201306, China

gravity satellites. Relevant scholars believe that satellite gravity measurement is the basis of subsequent inversion of high-precision, high-spatial resolution Earth's gravity feld, inversion of the global geomagnetic feld, and prediction of global climate change (Zheng et al. [2010](#page-15-0); Tingtao et al. [2020](#page-15-1); Junyong [2006\)](#page-15-2). Signifcant progress has been made in retrieving the earth's gravity feld, and various institutions have released their global gravity feld models. For example, the EIGEN series Champ-only model developed by the German Research Center for Geosciences Potsdam (GFZ) in Germany, GGM series GRACE-only models developed by the Center for Space Research (CSR) of the University of Texas in the USA (Tingtao et al. [2020\)](#page-15-1). The early EIGEN-1, EIGEN-2, EIGEN-CHAMP03s, and other models are completely based on the observation data of the CHAMP satellite, with the lower highest order and rapid error accumulation speed. Because of the shortcomings of CHAMP's slight improvement in the accuracy and spatial resolution of gravity feld models, the later EIGEN-GRACE01S and EIGEN-GRACE02S models were completely based on the observation data of the GRACE satellite. The maximum inversion order was increased to 150, which improved the accuracy of the model in the short wave stage. In contrast, the EIGEN-5C model combined the two gravity satellite data of GRACE and LAGEOS and was enhanced by ground observation data. The maximum order was increased to 360. The accuracy of the model has also been greatly improved. It is a good gravity feld model; there are also other good models such as EGM2008, GOCO06, and EIGEN-6C.

The EIGEN-5C model is widely used in earthquake research, land hydrology, glacier ice cover research, and other felds and has achieved good results. The gravity gradient experiment of Johannes Bouman et al. (Bouman et al. [2011](#page-14-0); Arabelos and Tscherning [1998\)](#page-14-1) shows that EIGEN-5C has strong consistency with the current general ultra-high order gravity feld model EGM2008 in the ocean but low consistency on the continent. On this basis, Alexandre Bernardino Lopes et al. (Lopes and Harari [2012](#page-15-3)) made an oceanographic evaluation of EIGEN-5C. Using this model and the EGM2008 model, the geostrophic current at the Brazil Malvinas junction was calculated, respectively. The results were similar to those of the numerical hydrodynamic model, which proved the consistency of the two models in large-scale aspects. Dimitrios Tsoulis et al. (Tsoulis and Papanikolaou [2013\)](#page-15-4) used GOCE satellite orbit analysis to evaluate diferent gravity models. They found that EIGEN-5C has the smoothest root mean square change in the whole spectral range, proving the stability of the model. Christoph Förste et al. (Förste et al. [2011\)](#page-15-5) improved the combination procedure of satellite and ground data because of the shortcomings of EIGEN-5C in the Andes, Africa, and the Himalayas due to the low quality of surface observation data and signifcant model errors.

Chinese scholars have also carried out many studies on the EIGEN-5C model. Zhang Chuanyin et al. (Chuanyin et al. [2009\)](#page-15-6) tested the accuracy of the EIGEN-5C model in the Chinese mainland using GPS-measured elevation anomaly data. They found that the global accuracy of elevation anomaly was more than 50% lower than that of EGM2008, especially in western China. Lu Biao et al. (Lu et al. [2018\)](#page-15-7) combined EIGEN-5C as a priori model and GOCE GGs signal to obtain a new gravity feld model IGGT_R1, which effectively improves the model accuracy in these areas. On this basis, Cai Lin et al. (Cai et al. [2021\)](#page-14-2) developed a gravity feld model called I3GG with EIGEN-5C as a priori model. It was verifed that the average model value afected by noise was reduced by about 20%. The gravity feld model has a smaller root mean square value than IGGT_R1 in all regions outside the Netherlands and has stronger consistency with EIGEN-5C in the wholefrequency range. Moreover, in recent years, other scholars have also explored and applied a series of gravity feld models in diferent experimental areas on diferent gravity feld models to analyze the local geophysical environment further and further elaborate on the research value of parameter inversion of the earth's gravity feld model in many sub-felds (Shu et al. [2011](#page-15-8); Yilmaz and Karaali [2011](#page-15-9); Förste et al. [2014](#page-15-10); Belay et al. [2022](#page-14-3); Förste et al. [2012](#page-15-11); Yuping [2018;](#page-15-12) Zhou et al. [2016](#page-15-13); Meijde et al. [2013](#page-15-14); Fielding and McKenzie [2012;](#page-15-15) Zhou et al. [2014;](#page-15-16) Li et al. [2014\)](#page-15-17).

Although signifcant progress has been made in the inversion of the earth's gravity feld, there is still some research space for understanding the gravity feld and its application at diferent scales. And intelligent supporting software for the calculation of Earth's gravity feld parameters needs to be developed. Therefore, it is necessary to explore the feasibility of the calculation results of the gravity feld model at diferent scales and further use the software to make a geographic information map containing the key earth's gravity feld parameters. Among them, for the EIGEN-5C gravity feld model used in this paper, how to give a complete set of calculation fow and calculation software in diferent scales and diferent research areas so that it can better analyze and calculate the critical parameters of the Earth's gravity feld in the area to be studied, the research on the above content is still insufficient. The critical parameters of the earth's gravity feld mentioned above mainly include disturbance potential, geoid height, gravity anomaly, gravity disturbance, and other vital parameters, which will be discussed in our research. Therefore, this study will discuss the above problems.

The objectives of this study are as follows:

(1) Aiming at the EIGEN-5C gravity feld model, this study introduces the download platform of the model in the second part.

(2) Quantitatively describes the gravity feld inversion algorithm based on the model and gives the detailed implementation process.

(3) Explains several critical parameters of the gravity feld to be calculated and their quantitative expressions.

On this basis, a program that can stably calculate the gravity feld parameters of diferent regional scales at 360 order is written. In the third part, this paper introduces the results of the calculation of the critical parameters of the gravity feld using the method and software proposed in this paper at diferent scales and makes some geographic information maps containing those parameters. The fourth part is the summary and outlook of the full text.

Calculation Method of Gravity Field Inversion Based on EIGEN‑5C Model

Algorithm Flow

The algorithm flow we designed is shown in Fig. [1](#page-2-0). First, download the EIGEN-5C gravity feld model fle in the international Global Earth Model Center (ICGEM, [http://icgem.](http://icgem.gfz-potsdam.de/tom_longtime) [gfz-potsdam.de/tom_longtime](http://icgem.gfz-potsdam.de/tom_longtime)). After spherical harmonic expansion, the key parameters of gravity feld can be expressed by fully normalized coefficients, Legendre series, normal gravity, and other parameters. The EIGEN-5C Earth gravity feld model can provide basic data for the calculation of these parameters, such as normalized coefficient and orders.

In particular, the reason why EIGEN-5C is used instead of other models is that researchers intend to realize the visualization process from the Earth gravity feld fle to the Earth gravity feld parameters through software, while EIGEN-5C is only used as the data basis.

Prepare the data:

(1) Recursively calculate the normalized Legendre function value corresponding to each series expansion.

 (2) Normalize the gravity field coefficient.

(3) Calculate the normal gravity at each latitude (the global-scale resolution is 1°, and the regional scale resolution is 5′).

(4) Convert the geodetic coordinates into the spherical coordinate system.

(5) Substitute these data into the spherical harmonic series expansion of the gravity feld parameters for the calculation.

(6) Finally, map and display the gravity feld parameter data for analysis at each longitude and latitude.

Method and Implementation

Calculation of Normalized‑associated Legendre Function

In order to calculate the spherical harmonic series of the gravity parameters of the Earth's gravity feld, the normalizedassociated Legendre function value corresponding to each series expansion is required frst. In this experiment, the standard forward recursion method is used to calculate the value of Legendre function. The basic calculation formula is as follows:

$$
P_{00}(\cos\theta) = 1
$$

\n
$$
\overline{P}_{10}(\cos\theta) = \sqrt{3}(\cos\theta)
$$

\n
$$
\overline{P}_{11}(\cos\theta) = \sqrt{3}(\sin\theta)
$$
\n(1)

Then, according to the standard, the Legendre function value of the series expansion of any term is recursively derived from the forward column (in formula [\(2](#page-2-1)): " $n \ge 2$ " and " $n > m$ "; in formula [\(3](#page-2-2)): " $m \ge 2$ "):

$$
\overline{P}_{nm}(\cos\theta) = a_{nm}\cos\theta \overline{P}_{n-1,m}(\cos\theta) - b_{nm}\overline{P}_{n-2,m}(\cos\theta)
$$
 (2)

$$
\overline{P}_{mm}(\cos\theta) = c_{nm}\sin\theta \overline{P}_{m-1,m-1}(\cos\theta)
$$
\n(3)

Among them:

$$
a_{nm} = \sqrt{\frac{(2n-1)(2n+1)}{(n+m)(n-m)}}
$$
(4)

$$
b_{nm} = \sqrt{\frac{(2n+1)(n+m)(n-m-1)}{(n-m)(n+m)(2n-3)}}
$$
(5)

Fig. 1 Algorithm flow diagram

$$
c_{nm} = \sqrt{\frac{2m+1}{2m}}\tag{6}
$$

The pseudo-code table of the above algorithm flow is shown in Table [1](#page-3-0).

Read EIGEN-5C Gravity Field Model Coefficient

In order to obtain the parameters of the gravity feld through spherical harmonic series, we also need to know and normalize the gravity field model coefficients \overline{c}_{nm} , \overline{s}_{nm} . Each line of the EIGEN-5C model file indicates in turn: *n*, *m*, $\frac{c_{nm}}{c_{nm}}$, $\frac{c_{nm}}{s_{nm}}$, $\delta \frac{c_{nm}}{s_{nm}}$. In addition, it is necessary to correct the moment of inertia of the coefficient according to the ellipsoid parameters. GRS80 ellipsoid is used in this experiment, and the frst four terms are just enough to meet the calculation accuracy. The correction formula is as follows:

$$
\overline{C}_{nm}^T = -\overline{C}_{nm} - \overline{C}_{nm}^U \tag{7}
$$

$$
\overline{S}_{nm}^T = -\overline{S}_{nm} \tag{8}
$$

$$
\overline{C}_{2n,0}^U = \overline{C}_{2n}^U = -\frac{J_{2n}}{\sqrt{(2(2n)+1)}}
$$
\n(9)

The pseudo-code table of the above algorithm flow is shown in Table [2](#page-3-1).

Calculate Normal Gravity γ _O

To calculate the gravity anomaly ∆*g*(*r*, *θ*, *λ*), the normal gravity at each latitude is also required, and the calculation formula is as follows:

Table 2 Read EIGEN-5C gravity field model coefficient

Step number	Step description	
Step1	Create a class to read the EIGEN-5C txt file read.ReadFile_CS(file_path);	
Step2	Read the coefficients of lines 2, 4, 6, and 8, respec- tively string line = reader. ReadLine(); $Cnm = double. Parse(line. Substring());$	
Step3	Read the normal gravitational potential coefficient of the GRS80 ellipsoid float $J2, J4, J6, J8$; read.ReadEllipsoid(out J2,out J4,out J6,out J8);	
Step4	Perform the following step to the first four line $C20 = C20 + J2/sqrt(5);$ $C80 = C80 + J8/sqrt(17);$	

$$
\begin{cases}\n\gamma_a = \frac{GM}{ab} \left(1 - m - \frac{m}{6} \frac{e' q'}{q_0} \right) \\
q_0 = \frac{1}{2} \left(\left(1 + 3 \frac{b^2}{E^2} \right) \tan^{-1} \frac{E}{b} - 3 \frac{b}{E} \right)\n\end{cases}
$$
\n(10)

$$
\begin{cases}\n\gamma_b = \frac{GM}{a^2} \left(1 + \frac{me'q'_0}{3q_0} \right) \\
q'_0 = 3 \left(1 + \frac{b^2}{E^2} \right) \left(1 - \frac{u}{E} \tan^{-1} \frac{b}{E} \right) - 1\n\end{cases}
$$
\n(11)

Normal gravity at any point on the ellipsoid:

$$
\gamma_Q = \frac{a\gamma_a \cos^2 \varphi + b\gamma_b \sin^2 \varphi}{\sqrt{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi}}
$$
(12)

Among them, φ is the geodetic latitude.

Table 3 Calculation of the normal gravity

Step number	Step description	
Step1	Read GRS80 ellipsoid parameters float omega, $a, b, GM, \gamma a, \gamma b$; read.ReadEllipsoid(out omega, out a, out b, out GM, out γa , out γb);	
Step2	Cycle the latitude according to the resolution and calculate the normal gravity for (double $B = B1$; $B \le B2$; $B + = \text{resl}$) $\gamma Q = \text{cal.}(\gamma)$ B , omega, GM, a, b, $\gamma a, \gamma b$);	

The pseudo-code table of the above algorithm flow is shown in Table [3](#page-4-0).

Convert Geodetic Coordinate System BLH to Spherical Coordinate System r

The spherical harmonic series expansion of gravity feld parameters is completed in the spherical coordinate system, while the data of EIGEN-5C is given in the geodetic coordinate system, so it is necessary to convert the geodetic coordinate system into the spherical coordinate system, and the conversion formula is as follows:

$$
W = \sqrt{1 - e^2 \sin \varphi^2} \tag{13}
$$

$$
N = \frac{a}{W} \tag{14}
$$

$$
\begin{cases}\nX = (N + H)\cos\varphi\cos L \\
Y = (N + H)\cos\varphi\sin L \\
Z = (N(1 - e^2) + H)\sin\varphi\n\end{cases}
$$
\n(15)

$$
\begin{cases}\nr = \sqrt{X^2 + Y^2 + Z^2} \\
\theta = \arctan \frac{\sqrt{X^2 + Y^2}}{Z} \\
\lambda = \arctan \frac{y}{X}\n\end{cases}
$$
\n(16)

The pseudo-code table of the above algorithm flow is shown in Table [4](#page-4-1).

Spherical Harmonic Series Expansion Calculation of Gravity Field Parameters

In order to simplify the calculation in geodesy, we need to change the parameters of the Earth's gravity feld into the form of spherical harmonic function. The spherical harmonic expansion of each gravity feld parameter is as follows:

$$
T(r,\theta,\lambda) = \frac{GM}{r} \left\{ \sum_{n=2}^{N} \left(\frac{a}{r} \right)^n \sum_{m=0}^{n} \left[\left(\overline{C}_{nm}^T \cos(m\lambda) + \overline{S}_{nm}^T \sin(m\lambda) \right) \overline{P}_{nm}(\cos\theta) \right] \right\}
$$
(17)

Table 4 Convert geodetic coordinate system to spherical coordinate system

Step number	Step description	
Step1	Read ellipsoid parameters and geodetic coordinates float $a,e2$; read.ReadEllipsoid(a, e2);	
Step2	Read out the coefficients of lines 2, 4, 6 and 8 respectively string $line = reader.ReadLine();$ $Cnm = double. Parse(line. Substring());$	
Step3	Read the normal gravitational potential coefficient of the GRS80 ellipsoid float $J2, J4, J6, J8$; read.ReadEllipsoid(out J2, out J4, out J6, out J8);	
Step4	Perform the following step to the first four line $C20 = C20 + J2/sqrt(5);$. $C80 = C80 + J8/sqrt(17);$	

$$
N(r,\theta,\lambda) = \frac{GM}{r\gamma_Q} \left\{ \sum_{n=2}^{N} \left(\frac{a}{r} \right)^n \sum_{m=0}^{n} \left[\left(\overline{C}_{nm}^T \cos\left(m\lambda\right) + \overline{S}_{nm}^T \sin\left(m\lambda\right) \right) \overline{P}_{nm}(\cos\theta) \right] \right\}
$$
(18)

$$
\Delta g(r,\theta,\lambda)=\frac{GM}{r^2}\left\{\sum_{n=2}^N(n-1)\Big(\frac{a}{r}\Big)^n\sum_{m=0}^n\Big[\Big(\overline{C}_{nm}^T\cos\left(m\lambda\right)+\overline{S}_{nm}^T\sin(m\lambda)\Big)\overline{P}_{nm}(\cos\theta)\Big]\right\} \eqno{(19)}
$$

$$
\delta q(r,\theta,\lambda) = \frac{GM}{r^2} \left\{ \sum_{n=2}^{N} (n-1) \left(\frac{a}{r} \right)^n \sum_{m=0}^{n} \left[\left(\overline{C}_{nm}^T \cos(m\lambda) + \overline{S}_{nm}^T \sin(m\lambda) \right) \overline{P}_{nm}(\cos\theta) \right] \right\}
$$
(20)

Among them, *T* stands for disturbance potential, *N* stands for geoid height, Δ*g* stands for gravity anomaly, *δg* stands for gravity anomaly. GM, *a*, and other parameters in the formula can be obtained from the reference ellipsoid GRS80. And their common parameters are shown in Table [5.](#page-5-0)

The pseudo-code table of the above algorithm flow is shown in Table [6.](#page-5-1)

Key parameters of Gravity Field

Disturbance Potential

To accurately obtain the Earth's gravitational potential *W*, we need to know the shape of the Earth's surface and the internal density distribution. The shape of the Earth's surface is what we want to study, and its internal density distribution is also irregular, so we cannot directly obtain the earth's gravitational potential.

In order to study the gravity potential, an approximate gravity potential can be introduced. Its functional relationship is straightforward and close to the real geopotential, which is called the normal geopotential and represented by the symbol *U*. We call the slight diference between the real gravity potential *W* and the normal gravity potential *U* the disturbance potential, which is represented by the symbol *T*, and its defnition formula is as follows:

Table 5 GRS80 ellipsoid parameters

GRS 1980-derived constants	Values	
a	6,378,137 m	
b	6,356,752.3141 m	
GM	$3,986,005 \times 10^8 \text{m}^3 \text{s}^{-2}$	
$_{\rm J2}$	$1082,63 \times 10^8$	
J4	-2.371×10^{-6}	
J6	6.083×10^{-9}	
J8	1.427×10^{-11}	
e^2	6.694×10^{-3}	

Table 6 Spherical harmonic series expansion calculation of gravity feld parameters

Since the disturbance potential is a small quantity, the existence of its solution can be guaranteed. Therefore, the solution of the Earth's gravity feld can be simplifed as the solution of the disturbance feld.

Geoid Height

The positional relationship between the geoid and the reference ellipsoid is shown in Fig. [2](#page-5-2). Suppose a point *P* on the geoid is projected along the normal *n*′ of the reference ellipsoid to point *Q* on the ellipsoid. In that case, the distance of *PQ* is called the geoid height, also known as geoid fuctuation, and is represented by the symbol *N*.

According to Bruns formula, there is also a relationship between geoid height, disturbance potential, and normal gravity:

$$
N = \frac{T}{\gamma} \tag{22}
$$

Gravity Anomaly

The difference between gravity vector g_p of point *P* on geoid and the normal gravity vector γ ⁰ of point *Q* on the reference ellipsoid is called gravity anomaly vector, and the size of this vector is called gravity anomaly, which is defned as follows:

$$
\Delta g = g_p - \gamma_Q \tag{23}
$$

According to the diferent objects considered, the gravity anomaly can be divided into simple-Bouguer gravity anomaly, Bouguer gravity anomaly, and Faye's gravity anomaly. In practical operation, the gravity anomaly can be used to reduce the measured gravity observation value on the ground to the geoid through gravity reduction, and then the shape of the geoid can be determined by using Stokes integral model.

Gravity Disturbance

On the same point *P* of geoid, the diference between gravity vector g_p and normal gravity vector γ_p is called gravity disturbance vector, and the size of this vector is called gravity disturbance, which is defned as follows:

$$
\Delta g = g_p - \gamma_p \tag{24}
$$

The gravitational disturbance can also be expressed as a function of the disturbance potential:

$$
\Delta g = g_p - \gamma_p = -\left(\frac{\partial W}{\partial n} - \frac{\partial U}{\partial n'}\right) \approx -\left(\frac{\partial W}{\partial n} - \frac{\partial UU}{\partial n}\right) = -\frac{\partial T}{\partial n}
$$
\n(25)

Fig. 2 Geoid and reference ellipsoid

Experiment and Verifcation

Self‑developed Software Introduction

In this study, we use the self-developed software to calculate the parameters of the Earth's gravity feld in the following areas (shown in Fig. [5](#page-8-0)). The program chart is shown in Fig. [3](#page-7-0).

The program mainly includes four function classes:

- (1) *Read.cs* Class: read EIGEN-5C data fle, and return the coefficients Cnm, Snm of the spherical harmonic function.
- (2) *Calculator.cs* Class: it includes functions for calculating the parameters of the earth's gravity feld and functions for coordinate system transformation. In this part, *T* function realizes the calculation of disturbance potential; *N* function realizes the calculation of geoid height; Δ*g* function realizes the calculation of gravity anomaly; δ *g* function realizes the calculation of gravity disturbance.
- (3) *Legendre.cs* Class: this function contains functions for calculating Legendre series. P_{nm} function is the standard forward recursion method to realize Legendre function.
- (4) *Form.cs* Class: realize the initialization of interactive forms and the functions of various controls.

The calculation fow of the program is as follows. First, input the EIGEN-5C data fle and GRS80 ellipsoid parameters, call *Read.cs* Class to read the coefficients of spherical harmonics Cnm, Snm, and use GRS80 ellipsoid parameters to correct the coefficients, and then check whether the accuracy of the obtained coefficients meets the requirements.

At the same time, input the GRS80 ellipsoid parameters, the latitude and longitude range to be calculated and the resolution, and call class *Calculator.cs* to convert the latitude and longitude into coordinates in the spherical coordinate system. Call class *Legendr.cs*, input spherical coordinates, and get the Legendre series of 360 orders Pnm. Then, input the corrected spherical harmonic coefficients Cnm, Snm, and Legendre series Pnm, call class *Calculator.cs* to calculate each gravity feld parameter, and save it as TXT fle. Then, the calculation results are displayed on the interactive interface, and TXT is imported into Matlab to draw the twodimensional contour map of each gravity feld parameter.

The software interface is shown in Fig. [4.](#page-8-1) After testing, the software can stably calculate the gravity feld parameters of each region or the world, taking into account the calculation efficiency and accuracy, and can provide data support for the study of the Earth's gravity feld.

As one of the important contributions of this paper, this paper presents the calculation software of the key parameters of the Earth's gravity feld based on the EIGEN-5C model. Our software can quickly calculate the key parameter information and better realize the inversion of the Earth's gravity feld in the study area. In order to better verify the computing power of the software, the key parameters of the earth's gravity feld are calculated and verifed at the local scale and the global scale in the subsequent two parts of this paper.

Calculation of Gravity Field Parameters at Regional Scale

In order to verify the feasibility of the calculation process of this method, we selected three experimental sample areas (resolution 5′) of ABC on a local scale, and their longitude and latitude information is shown in Table [7.](#page-8-2) Among them, region A is the representative of high latitude samples, region B is the representative of mid latitude samples, and region C is the representative of low latitude samples. The above regions are distributed at the junction of the Earth plates, and the geophysical parameters in these regions have certain complexity in the inversion process, which is suitable for verifying the stability of the algorithm in this experiment.

Representative of High Latitude Sample—Region A

Iceland is located on the ridge of the North Atlantic Ocean, which is formed by volcanic rocks. Most of the islands are plateau areas, and the volcanic activity nearby is intense. Researchers chose Iceland as a high latitude region to calculate the gravity feld parameters. Figure [6](#page-9-0) reflects the geophysical situation of Iceland.

On the island and the oceanic ridge in the southwest region, the values of disturbance potential, geoid height, gravity anomaly, and gravity disturbance are large, and the maximum values of the four occur in the central plateau of Iceland, which are $673.7781 \text{m}^2/\text{s}^2$, 68.5943 m, 94.0109mgal, and 115.1332mgal respectively. In the northeast, southeast, and northwest regions, the terrain fuctuates greatly, and the values of these parameters decrease sharply, and the minimum values appear in the southeast sea area, which are $546.7848 \text{ m}^2/\text{s}^2$, 55.6854 m,−38.5189mgal,−21.3301mgal respectively.

The numerical changes of disturbance potential and geoid height are highly consistent with the local terrain distribution, which shows the accuracy of the calculation method in this experiment from another point of view.

Representative of Mid Latitude Sample—Region B

Taiwan province is located in the southeast sea area of China, facing the Pacifc Ocean in the east and facing Fujian province across the Taiwan Strait in the west. In this study, Taiwan island and its adjacent sea areas are selected as the mid latitude sample area to calculate the gravity feld parameters.

Fig. 3 Program chart

It can be seen from Fig. [7](#page-10-0) that the values of disturbance potential and geoid height are large in the central mountain range of the island and the eastern sea area, in which the maximum disturbance potential can reach $361.7042 \text{m}^2/\text{s}^2$, the maximum geoid height can reach 36.96 m, and the value in the western sea area is low, in which the minimum disturbance potential is $106.4693 \text{ m}^2/\text{s}^2$, and the minimum geoid height is 10.8734 m. There is an obvious dividing line in the east of the island. The overall value of gravity anomaly and gravity disturbance is large, in which the maximum value of gravity anomaly can reach 161.3363mgal, and the maximum value of gravity disturbance can reach 169.5144mgal. There are negative values in some areas in the east and south of

the island, in which the minimum value of gravity anomaly can reach−162.067mgal, and the maximum value of gravity disturbance can reach−157.7213mgal.

Judging from the topographic distribution of Taiwan island, the central and eastern part of Taiwan island is the central mountain range of Taiwan, and there is a maximum physical altitude in this area, which is completely consistent with the calculation results of disturbance potential and geoid height.

Representative of Low Latitude Sample—Region C

Southeast Asia is located at the junction of the Eurasian Plate and the Indian plate. It is located in the Pacifc Rim

Fig. 4 Software interface

Table 7 Longitude and latitude range of the study area

Study area	Longitude range	Latitude range
A area in Fig. 5	60° N ~ 68° N	10° W ~ 26° W
B area in Fig. 5	20° N ~ 28° N	120° E ~ 128° E
C area in Fig. 5	5° S ~ 5° N	100° E ~ 120° E

seismic belt and has solid crustal activity. In this experiment, Indonesia and the sea area near Malaysia are selected as low latitude areas for gravity feld parameter calculation.

It can be seen from Fig. [8](#page-11-0) that there is an obvious dividing line between the disturbance potential and the geoid height. In the west of the dividing line, the value is negative, the minimum disturbance potential is $-182.1987 \text{m}^2/\text{s}^2$, and the minimum geoid height is−18.6284 m. In the east of the dividing line, the value is positive, the maximum disturbance potential can reach $637.5512 \text{m}^2/\text{s}^2$, and the maximum geoid height can reach 65.1865 m. The value changes sharply on the dividing line. The values of gravity anomaly and gravity disturbance are relatively small. The lowest value of gravity anomaly is−79.4894mgal, and the lowest value of gravity disturbance is−85.0148mgal. There are large values in the Iban

Fig. 5 Selected study area (region A stands for the high latitude area sample, region B stands for the mid latitude area sample, region C stands for the low latitude area sample)

Journal of Geovisualization and Spatial Analysis (2022) 6:32

Fig. 6 Gravity feld parameters in Iceland region. (**a**) Disturbance potential in Iceland area; (**b**) geoid height in Iceland area; (**c**) gravity anomaly in Iceland; (**d**) gravity disturbance in Iceland

mountains of Kalimantan island in the East and the Barisal mountains of Sumatra Island in the west, of which the maximum gravity anomaly can reach 195.104mgal, and the maximum value of gravity disturbance can reach 214.7673mgal.

The distribution of the dividing line refects the junction of the Eurasian and Indian plates, which is consistent with our inversion results in geophysical space.

Calculation of Gravity Field Parameters at Global Scale

EIGEN-5C model cannot only get good calculation results of gravity feld parameters on a local scale but also get competitive results on a global scale (resolution of 1°), as shown in Fig. [9](#page-12-0) below.

It can be seen from Fig. [9](#page-12-0) that the distribution of disturbance potential and geoid height is complex. It changes violently in the Pacific Rim seismic belt, Mediterranean Himalayan seismic belt, and seismic ridge belt. The maximum disturbance level can reach $819.3976 \text{m}^2/\text{s}^2$, and the maximum geoid height can reach 83.7761 m. The lowest value appears in the sea area near the Maldives, of which

the lowest value of disturbance potential is -1040.7 m^2 $s²$, and the lowest value of geoid height is -106.3993 m. The distribution of gravity anomaly and gravity disturbance is relatively stable, and the value is relatively low. There are high values in the Qinghai Tibet region of China, Greenland, and the Andes mountains of South America. Among them, the maximum value of gravity anomaly is 289.7696mgal, the maximum value of gravity disturbance is 285.9281mgal, and the value around the seismic ridge belt is relatively low, in which the minimum value of gravity anomaly is−338.6184mgal, and the minimum value of gravity disturbance is−360.0623mgal.

Among them, the larger values of gravity anomaly and gravity disturbance appear in the plate boundary zone, which clearly outlines the distribution of the eight plates, and further explains the correctness of the calculation results in this experiment.

Discussion

In this study, the calculation of the Earth's gravity feld parameters at the regional scale is analyzed, and three

sample areas of ABC are selected to represent the high latitude area, the middle latitude area, and the low latitude area for analysis. From the analysis results, the results of the regional analysis are consistent with the local topographic relief in the expression of two key parameters, disturbance potential and geoid height, which further shows the correctness of our calculation results. From the two indicators of regional gravity anomaly and gravity disturbance, they can better refect the specifc changes in the local gravity feld.

In order to verify the correctness of the experimental results, the researchers used the calculation function provided by ICGEM website to compare the calculation results of EGM2008 model with the calculation results of this study. It is evaluated by RMS and STD. As the ICGEM website cannot calculate the grid disturbance potential data at present, the verifcation of experimental results only involves geoid height, gravity anomaly, and gravity disturbance.

We selected Iceland for experimental analysis for the high latitude region at the regional scale. The longitude and latitude range of the sample region is shown in Table [7.](#page-8-2) Among them, the maximum values of the four gravity feld parameters in Iceland appear in the plateau area in the center of the island, with values of $673.7781 \text{ m}^2/\text{s}^2$, 68.5943 m , 94.0109mgal, and 115.1332mgal, respectively. The lowest values appear in the southeast sea area, with $546.7848 \text{ m}^2/\text{s}^2$, 55.6854 m,−38.5189mgal, and−21.3301mgal, respectively. As shown in Table [8,](#page-12-1) the maximum and minimum values of geoid height calculated in this study do not deviate more than 2 m from the calculation results of ICGEM website, and the errors of RMS and STD are within 1 m. The deviation of the maximum value of gravity anomaly is 3.2768mgal, the deviation of the minimum value of gravity anomaly is 26.5824mgal, the STD deviation is 1.3994mgal, and the RMS deviation is 0.5199mgal. The deviation of the maximum value of gravity disturbance is 3.7729mgal, the deviation of the minimum value of gravity disturbance is 26.5017mgal, the STD deviation is 1.3945mgal, and the RMS deviation is 0.6135mGal.

We selected Taiwan for experimental analysis for the mid latitude region at the regional scale. The longitude and latitude range of the sample region is shown in Table [7.](#page-8-2) Among

Fig. 8 Gravity field parameters
in equatorial region. (a) Dis-
turbance potential in equato-
rial region; (b) geoid height in
equatorial region; (c) gravity disturbance in equa-
anomaly in equatorial region;
(d) gravity di in equatorial region. (**a**) Disturbance potential in equatorial region; (**b**) geoid height in equatorial region; (**c**) gravity anomaly in equatorial region; (**d**) gravity disturbance in equa-**32** Page 12 of 16
 Fig. 8 Gravity fin

in equatorial regium

turbance potentia

rial region; (**b**) g

equatorial region

(**d**) gravity distur

torial region

 -50

Fig. 9 Global gravity feld parameters. (**a**) Global disturbance potential; (**b**) global geoid height; (**c**) global gravity anomaly; (**d**) global gravity disturbance

them, the maximum values of Taiwan's disturbance potential and geoid height appear in the central mountain range of Taiwan island, with values of $361.7042 \text{m}^2/\text{s}^2$ and 36.96 m , respectively, and the minimum values appear in the western sea area of Taiwan island, with values of $106.4693 \text{m}^2/\text{s}^2$ and 10.8734 m respectively. The maximum values of gravity anomaly and gravity disturbance appear on Taiwan island and the northern sea area, with values of 161.3363mgal

Table 8 Verifcation of regional scale results (region A)

and 169.5144mgal, respectively, and the minimum values appear in the eastern part of the island and−162.067mga and−157.7213mgal respectively. As shown in Table [9,](#page-13-0) the maximum and minimum values of geoid height calculated in this study do not deviate more than 1 m from the calculation results of ICGEM website, and the errors of RMS and STD are within 1 m. The deviation of the maximum value of gravity anomaly is 19.808mgal, the deviation of the minimum

value of gravity anomaly is 16.3518mgal, the STD deviation is 21.6666mgal, and the RMS deviation is 15.6953mgal. The deviation of the maximum value of gravity disturbance is 19.4901mgal, the deviation of the minimum value of gravity disturbance is 16.5329mgal, the STD deviation is 2.5799mgal, and the RMS deviation is 2.5645mgal.

For low latitudes at the regional scale, we selected Indonesia around the equator and the sea area near Malaysia for experimental analysis. The longitude and latitude of the sample area are shown in Table [7.](#page-8-2) Among them, the maximum values of disturbance potential and geoid height in the equatorial region appear in Kalimantan island and nearby sea areas, with values of $637.5512 \text{m}^2/\text{s}^2$ and 65.1865 m respectively, and the minimum values appear in Sumatra islands and nearby sea areas, with values of $-182.1987 \text{m}^2/\text{s}^2$ and -18.6284 m respectively. Gravity anomaly and gravity disturbance have maximum values in the Iban mountain range of Kalimantan island in the east and Barisal mountain range of Sumatra island in the west, which are 195.104mgal and 214.7673mgal respectively. In contrast, the values in other regions are low, and the minimum values are−79.4894mgal and−85.0148mgal, respectively. As shown in Table [10](#page-13-1), the maximum and minimum values of geoid height calculated in this study do not deviate more than 2 m from the calculation results of ICGEM website, and the errors of RMS and STD are within 1 m. The deviation of the maximum value of gravity anomaly is 26.7863mgal, the deviation of the minimum value of gravity anomaly is 4.6153mgal, the STD deviation is 14.3041mgal, and the RMS deviation is 1.3398mgal. The deviation of the maximum value of gravity disturbance is 27.0078mgal, the deviation of the minimum value of gravity disturbance is 3.6761mgal, the STD deviation is 1.9981mgal, and the RMS deviation is 1.3336mgal.

Based on the calculation of the critical parameters of the Earth's gravity feld at the local scale with good results, we further extend the application of the EIGEN-5C model in calculating the Earth's gravity feld parameters to the numerical calculation of the global gravity feld. From the calculation results, the disturbance potential and geoid height change violently in the Pacifc Rim seismic belt, the Mediterranean Himalaya seismic belt, and the seismic ridge belt, with the highest values of $819.3976 \text{m}^2/\text{s}^2$ and 83.7761 m, respectively, and the lowest values of−1040.7 m^2/s^2 and -106.3993 m respectively in the sea area near the Maldives. Gravity anomaly and gravity disturbance have the highest values in the Qinghai Tibet region of China, the Greenland rim, and the Andes mountains of South America. Their values are 289.7696mgal and 285.9281mgal, respectively. There are the lowest values around the seismic ridge belt, of which the lowest value of gravity anomaly is−338.6184mgal and the lowest value of gravity disturbance is−360.0623mgal.The calculation results of the following two indicators better refect the distribution and junction of plates, which also shows the accuracy and correctness of the calculation results of this model and related software. As shown in Table [11](#page-14-4), the maximum and minimum values of geoid height calculated in this study do not deviate more than 1 m from the calculation results of ICGEM website, and the errors of RMS and STD are within 0.1 m. The deviation of the maximum value of gravity anomaly is 24.3865mgal, the deviation of the minimum value of gravity anomaly is 68.3720mgal, the STD deviation is 2.3501mgal, and the RMS deviation is 2.3511mgal. The deviation of the maximum value of gravity disturbance is 5.9542mgal, the deviation of the minimum value of gravity disturbance is 88.8712mgal, the STD deviation is 2.082mgal, and the RMS deviation is 2.0754mgal.

From the verification results of regional scale and global scale, the calculation of geoid height in this study

is relatively accurate, the diference between the calculated maximum and minimum values and the ICGEM calculation results is not more than 2 m, and the diference between the STD and RMS indicators and the ICGEM calculation results is not more than 1 m, which indicates that the calculation results of geoid are very close to the ICGEM calculation results. Except that the calculated results of STD and RMS of gravity anomaly in region B are quite diferent from those of ICGEM, STD, and RMS of other scales are in good agreement with the calculated results of ICGEM.

Summary and Prospect

Summary and Conclusions

1. The method of downloading and using the EIGEN-5C gravity feld model fle is given in detail. The calculation formulas of the spherical harmonic series expansion of gravity feld parameters such as disturbance potential, geoid height, gravity anomaly, and gravity disturbance are introduced, and the calculation process is shown in the form of a pseudo-code table.

2. Based on the calculated data at different regional scales, we use self-developed software to draw twodimensional contour maps of different gravity field parameters at global and regional scales and make a quantitative and qualitative analysis of the images.

3. For the versatility and efficiency of the calculation, we have developed software independently, so that the users only need to input the latitude, longitude, and resolution of the area to be calculated in the interactive interface, and then the numerical value and the two-dimensional contour maps of the gravity feld parameters in the area can be obtained.

Prospect

In this study, the EIGEN-5C model is used to carry out the inversion and result in the expression of the Earth's gravity field parameters in different scale regions. This work has a specifc reference value for the feld of gravity feld parameter calculation and research, but limited to equipment factors, the accuracy of the calculation results of this study can be improved, and the calculation efficiency can be further improved.

Acknowledgements This work was supported by the Wuhan University Innovation Training Program for College Students (W2021214026). Thanks for the precise data provided by ICGEM. You can know more information about the program in this research from this link: [https://](https://github.com/pineappleBBL/Earth-gravity-field-model) [github.com/pineappleBBL/Earth-gravity-feld-model](https://github.com/pineappleBBL/Earth-gravity-field-model).

Author Contribution Conceptualization, B.Y., J.S., Q.S.; methodology, B.Y.; software, B.Y.; validation, B.Y.,Y.L.; formal analysis, B.Y., Q.S.; investigation, B.Y.,Y.L; resources, J.S.; data curation, B.Y; writing original draft preparation, B.Y; writing—review and editing, Y.L., J.S.; visualization, B.Y.,Q.S.; supervision, L.Y.; project administration, J.S; funding acquisition, J.S. All authors have read and agreed to the published version of the manuscript.

Funding This research was funded by Frontier Physics Experiment Facility for Extremely Deep Underground Extremely Low Radiation Background of Key National Technology Infrastructure Construction-Subproject of pre-research project (Total No. 2018–000052-73–01- 002125) and Wuhan University undergraduate innovation and entrepreneurship training program (No. W2021214026).

Declarations

Conflict of Interest The authors declare no competing interests.

Ethics Approval We confrm that this work is original and has not been published elsewhere, nor is it currently under consideration for publication elsewhere.

Informed Consent.

All authors know this article, and all authors agree with this publication.

References

Arabelos D, Tscherning CC (1998) Calibration of satellite gradiometer data aided by ground gravity data. J Geod 72:617–625

- Belay EY Godah W Szelachowska M Tenzer R. (2022). ETH-GQS: an estimation of geoid-to-quasigeoid separation over Ethiopia. Geodesy and Geodynamics 13(1).<https://doi.org/10.1016/j.geog.2021.09.006>
- Bouman J, Fiorot S, Fuchs M, Gruber T, Schrama E, Tscherning C, Veicherts M and Visser P (2011) GOCE gravitational gradients along the orbit. J Geod 85(11). [https://doi.org/10.1007/](https://doi.org/10.1007/s00190-011-0464-0) [s00190-011-0464-0](https://doi.org/10.1007/s00190-011-0464-0)
- Cai L, Wan X, Hsu H, Ran J, Meng X, Luo Z and Zhou Z (2021) The earth's gravity feld recovery using the third invariant of

the gravity gradient tensor from GOCE. Sci Rep 11(1). [https://](https://doi.org/10.1038/s41598-021-81840-1) doi.org/10.1038/s41598-021-81840-1

- Chuanyin Z, Chunxi G, Junyong C, Zhang L and Bin W (2009) EGM 2008 and its application analysis in chinese mainland. Acta Geod et Cartogr Sin 38(4)
- Fielding, E. J., & McKenzie, D. (2012). Lithospheric fexure in the Sichuan Basin and Longmen Shan at the eastern edge of Tibet. Geophysical Research Letters, 39(9). [https://doi.org/10.1029/](https://doi.org/10.1029/2012GL051680) [2012GL051680.](https://doi.org/10.1029/2012GL051680)
- Förste C, Bruinsma SL, Shako R, Abrikosov O, Flechtner F, Marty J-C, Lemoine J-M, Dahle C, Neumeyer H, Barthelmes F, Biancale R, Balmino G, König R (2012) A new release of EIGEN-6: the latest combined global gravity feld model including LAGEOS, GRACE and GOCE data from the collaboration of GFZ Potsdam and GRGS Toulouse. EGU General Assembly 2012:14
- Förste C, Bruinsma S, Shako R, Marty J, Abrikosov O, Dahle C, Lemoine J, Biancale R, Barthelmes F, König R and Balmino G (2011) EIGEN-6 A new combined global gravity feld model including GOCE data from the collaboration of GFZ-Potsdam and GRGS- Toulouse. EGU General Assembly 2011, 3–8 April 2011, Vienna, Austria, April
- Förste C, Bruinsma S, Abrikosov O, Flechtner F, Marty JC, Lemoine JM, Dahle C, Neumayer H, Barthelmes F, König R, Biancale R, Förste C, Bruinsma S, Abrikosov O, Flechtner F and Marty J (2014) EIGEN-6C4 - The latest combined global gravity feld model including GOCE data up to degree and order 1949 of GFZ Potsdam and GRGS Toulouse. EGU General Assembly 16
- Chen Junyong (2006) Progress and Prospect of the fve-year operation of gravity satellites in determining the earth's gravity feld model. Prog Geosci 21(7):661–666, insert 1-insert 2. [https://doi.](https://doi.org/10.3321/j.issn:1001-8166.2006.07.001) [org/10.3321/j.issn:1001-8166.2006.07.001](https://doi.org/10.3321/j.issn:1001-8166.2006.07.001)
- Li J, Jiang W, Zou X, Xu X and Shen W (2014) Evaluation of recent GRACE and GOCE satellite gravity models and combined models using GPS/leveling and gravity data in China. Int Asso Geod Symposia 141. <https://doi.org/10.1007/978-3-319-10837-79>
- Lopes AB and Harari J (2012) Use of recent geoid models to estimate mean dynamic topography and geostrophic currents in South Atlantic and Brazil Malvinas Confuence. Brazilian J Oceanogr 60(1). <https://doi.org/10.1590/S1679-87592012000100005>
- Lu B, Luo Z, Zhong B, Zhou H, Flechtner F, Förste C, Barthelmes F and Zhou R (2018) The gravity feld model IGGT_R1 based on the second invariant of the GOCE gravitational gradient tensor. J Geod 92(5).<https://doi.org/10.1007/s00190-017-1089-8>
- van der Meijde M, Julià J and Assumpção M (2013) Gravity derived Moho for South America. In Tectonophysics. 609. [https://doi.](https://doi.org/10.1016/j.tecto.2013.03.023) [org/10.1016/j.tecto.2013.03.023](https://doi.org/10.1016/j.tecto.2013.03.023)
- Shu C, Li F and Hao W (2011) Evaluation of EGM2008 and its application analysis over a particular region of China. Wuhan Daxue Xuebao (Xinxi Kexue Ban)/Geomatics and Information Science of Wuhan University. 36(8)
- Tingtao Wu, Wei Zheng, Wenjie Yin et al (2020) Research progress on gravity feld model of earth satellite and its application. J Science technology and engineering 20(25):10117–10132. <https://doi.org/10.3969/j.issn.1671-1815.2020.25.001>
- Tsoulis, D., & Papanikolaou, T. (2013). Degree-wise validation of satellite-only and combined Earth gravity models in the frame of an orbit propagation scheme applied to a short GOCE arc. Acta Geodaetica et Geophysica, 48(3). [https://doi.org/10.1007/](https://doi.org/10.1007/s40328-013-0020-x) [s40328-013-0020-x.](https://doi.org/10.1007/s40328-013-0020-x)
- Sun Tengke Visualization of EGM2008 Earth Gravity Field Model [J] Mapping and Spatial Geographic Information, 2013,36 (12): 25–27. <https://doi.org/10.3969/j.issn.1672-5867.2013.12008>.
- Yilmaz N and Karaali C (2011) Investigation of discrepancies of some geoids determined using various methods for Turkey. Int J Phys Sci 6(19)
- Li Yuping 2018 Algorithm and implementation of calculating elevation anomaly with ultra-high order gravity feld model. J Marine surveying and mapping38 (3) 1–4 [https://doi.org/10.3969/j.issn.](https://doi.org/10.3969/j.issn.1671-3044.2018.03.001) [1671-3044.2018.03.001](https://doi.org/10.3969/j.issn.1671-3044.2018.03.001).
- Zheng Wei Xu, Houze Zhong Min et al (2010) Research progress and status quo of earth gravity feld model. J Geodesy and geodynamics 30(4):83–91.<https://doi.org/10.3969/j.issn.1671-5942.2010.04.016>
- Zhou Z, Du Q, Liang Q, Wang J, Zeng Z and Wang X (2014) Relationship among steepest gradient of gravity feld, active fault zone and large earthquake, and its rheological model. Diqiu Kexue - Zhongguo Dizhi Daxue Xuebao/Earth Sci - J China Univ Geosci 39(12). <https://doi.org/10.3799/dqkx.2014.173>
- Zhou B, Luo Z, Wu Y and Cen Y (2016) Far-zone contributions of airborne gravity anomalies' upward/downward continuation. Geod Geodyn 7(6).<https://doi.org/10.1016/j.geog.2016.08.001>

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.