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Imaging the Subsurface: How Different Visualizations of Cross-sections Affect the Sense of Uncertainty

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Abstract

Geologists struggle to communicate the uncertainty that arise when mapping and interpreting the geological subsurface. Today, open data sharing policies make new value of geological information possible for a broader user group of non-experts. It is crucial to develop standard methods for visualizing uncertainty to increase the usability of geological information. In this study, a web experiment was set up to analyze whether and how different design choices influence the sense of uncertainty. Also, questions about the intuitiveness of symbols were asked. Two-hundred ten participants from different countries completed the experiment, both experts and non-experts in geology. Traditional visualization techniques in geology, like dashed lines, dotted lines and question mark, were tested. In addition, other visualizations were tested, such as hatched area and variations of symbol size, zoom levels and reference information. The results show that design choices have an impact on the participants' assessment of uncertainty. The experts inquire about crucial information if it is not present. The results also suggest that when visualizing uncertainty, all the elements in the representation, and specifically the line and area symbols that delineate and colour the features, must work together to make the right impression.

Keywords Visualization · Uncertainty · Subsurface · Geology · Cross-section

Introduction

How do users evaluate the quality of representations of the physical world? Tversky and Kahneman (1974) discuss information visualization in general and claim that the assessment of probability "resembles the subjective assessment of physical quantities such as distance and size". They list a range of factors that must be present for good judgement of representativeness: Knowledge of prior outcomes, sample size, conception of chance, predictability, validity and conception of regression. Communicating uncertainty in maps can help users make better judgement of the confidence in the representation, and to "avoid ill-informed decisions" (Kinkeldey and Senaratne 2018).

Uncertainty in Geology

Unlike many surficial features on the surface of Earth, the geological subsurface is hard to map. Representations of the intangible and invisible subsurface are therefore more likely to be unprecise and erroneous. When mapping geological features, especially in 3D, interpretations and interpolations are needed to transform raw data, from for example seismic investigations and bore hole logs into 2D and 3D models. These models present the interpreted reality, which can be effectively used by a wider user group. In some areas where bedrock outcrops and data density are high, the seismic may be easy to interpret and verify, while more difficult in areas of low data density. The resulting model is dependent on the geologists' a priori knowledge and experience and therefore subjective (Polson and Curtis 2010). When these models are made, geologists struggle to model and communicate the uncertainty involved (Randle et al. 2018; Pérez-Díaz et al. 2020; Schaaf and Bond 2019).

According to Lark et al. (2015) there are multiple types of uncertainty in geological borders: (1) *Conceptual uncertainty*, which exemplifies whether a border is gradual or not. (2) *Scale-dependent uncertainty* is shown, for example, when a line that may seem continuous at the observed scale is in reality non-

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continuous. (3) *Cartographic uncertainty*, which is uncertainty about errors that were implemented during the map-making process. (4) *Interpretation uncertainty* represents the uncertainty when parts of the border cannot be observed. Uncertain borders lines have been communicated with many different words: "known", "probable", "certain", "uncertain", "accurately located", "approximately located", "inferred", "projected", "concealed" and "queried" (Soller et al. 2002). The different types of interpretation uncertainty are not commonly communicated or, when used, even understood by the user.

Geologists have traditionally used perspective illustrations and cross-sections to portray subsurface geology. 3D models are increasingly used, but a review of 3D web viewers from European geological surveys (Bang-Kittilsen 2019) shows 2D cross-sections are still commonly standard output for end-users (see for example Kessler et al. 2018; Baumberger and Oesterling 2018). Most cross-sections typically have coloured areas symbolizing different geological categories, but no communication of uncertainty.

Standards that have been established for communicating uncertainty within geology are connected 2D maps and cross-sections and are typically used for lines or borders: Uncertain location, invisible border, uncertain type and existence (FGDC 2006). The traditional visualization techniques include changing the appearance of the line or border symbols according to types or degree of uncertainty. Dashed and dotted lines together with question marks are standard techniques geologists apply to communicate uncertainty (Soller et al. 2002; FGDC 2006). Uncertainty about the location is indicated with a range from solid line via dashed line to dotted line. Only borders that were observed in the field can be drawn with confidence on the map as solid lines. Question marks along the line indicate uncertainty about the existence of a border (Soller et al. 2002). It is easy to find research about techniques for assessment and visualization of uncertainty in subsurface geology (see for example Tacher et al. 2006; Schweizer et al. 2017; Zehner 2019), but none of these methods is well established among geologists (Zehner 2019).

To optimize the benefits from geological data, there is a need to simplify and make geological representations that are understood and interpreted adequately by the user. As Häggquist and Söderholm (2015) claim, "the use of geological information implies an initial knowledge threshold, i.e. a basic understanding to appropriate the benefits of this good, and the opportunity cost of learning-by-using will have a significant impact on demand." To lower this threshold, it is important to use a graphical language that is easy to understand for the user. Presenting models, maps and cross-sections that are totally dissociated from the complex data and knowledge they are based on may create bias (McInerny et al. 2014). For the data to be usable for decision-making, it is important that it is correct. Since correctness may be hard to ensure throughout a dataset because, for example, a lack of outcrops or knowledge of the subsurface, the need for locating and quantifying uncertainty is important (Tacher et al. 2006). The practice should be to follow the basic rules of cartographic theory and graphical communication within the limits of standards for data and map exchange in the standardized geographical infrastructure. These standards limit the number of techniques to choose from. The challenge, therefore, remains to communicate complex information, both the interpreted geology and the different dimensions of uncertainties, without increasing the user threshold to an expert level.

Uncertainty Visualization

Bonneau et al. (2014) describe uncertainty to be "the lack of information", while Longley et al. (2005) define uncertainty as the difference between a real geographic phenomenon and the user's understanding of the geographic phenomenon. In Hunter and Goodchild (1993), uncertainty is described as the "degree to which the lack of knowledge about the amount of error is responsible for hesitancy in accepting results and observations without caution". All information contains multiple kinds of uncertainty; for geographical information uncertainty exists across space, time and attribute (MacEachren et al. 2012). Information or data uncertainty is often conceptualized by error, but this is, according to MacEachren et al. (2005), often a too narrow approach to uncertainty: Each category can be split into 9 types: Accuracy/error, precision, completeness, consistency, lineage, currency, credibility, subjectivity and interrelatedness. The INSPIRE directive aims to create a European Union spatial data infrastructure (INSPIRE 2020). INSPIRE (2013) defines 17 categories for data quality: Completeness (commission and omission), logical consistency (conceptual, domain, format and positional), positional accuracy ("absolute or external accuracy", "relative or internal" and "gridded data position"), thematic accuracy ("classification correctness", "non-quantitative attribute correctness", "quantitative attribute correctness", "temporal quality", "temporal consistency" and "temporal validity") and usability. Uncertainty can arise along the whole value chain from data collection, processing, analyses and modelling to final use (Pérez-Díaz et al. 2020).

Visualization of uncertainty, according to Pang et al. (1996) "strives to present data together with auxiliary uncertainty information". The ultimate objective of visualizing uncertainty "is to provide users with visualizations that incorporate and reflect information regarding uncertainty to aid in data analysis and decision making" (Pang et al. 1996).

There are multiple techniques for uncertainty visualization that by MacEachren et al. (2005) and Kinkeldey et al. (2014) are described as a combination of the following dichotomies: Intrinsic techniques change the appearance of existing objects while exintric techniques add new objects that represent uncertainty. Visually separable or integral techniques refer to whether the signification can be read independently or not. This is often the same as intrinsic or exintric techniques. Adjacent or coincident techniques represent respectively visualization of uncertainty in second representation or in the same. In addition, the representation can be either static or dynamic where the latter can be interactive or an animation. Explicit or implicit techniques refer to direct representation or indirect through a series of possible outcomes.

Bonneau et al. (2014) claim that "difficulties in applying pre-existing methods, escalating visual complexity, and the lack of obvious visualization techniques" are overlooked, and that this leaves uncertainty visualization an "unsolved problem". Kinkeldey et al. (2014) found that most studies within uncertainty visualization focus on developing new methods for visualization, and fewer on user studies. Kinkeldey et al. (2014) have done a review of geospatial uncertainty visualization user studies. In the selected studies, usability of different visual variables is often tested, contributing to a graphical semiology of uncertainty visualization. However, they conclude that comparison and generalization are hard because the usability is dependent on the task in hand and whether the method is static or dynamic, for example. A review of studies concerning the effect uncertainty visualization have on decision-making can be found in Kinkeldey et al. (2015). There is proof that uncertainty visualization affects decision-making (MacEachren et al. 2005; Deitrick and Edsall 2006; Kinkeldey et al. 2015), but not that it necessarily makes decisions better (MacEachren et al. 2005; Kinkeldey et al. 2015).

This study is in the crossing-point between these above-mentioned groups of studies, targeting specific needs within subsurface geology to make the user attentive to uncertainty in the representation.

In a study by Bang-Kittilsen et al. (2019), participants were asked to draw the subsurface geology. Results show most participant prefer to use cross-sections. The study aimed to elicit cognitive maps on subsurface geology using sketch maps (Bang-Kittilsen 2019). The study included results from 84 participants, both experts and non-expert. The conclusion was that participants predominantly draw the subsurface as cross-sections. Geographical context and plain language were commonly used in the drawings. The content elements, their categorization and visual depiction were diverse. The participants' uncertainty about the geological subsurface had a wide range of expressions in the drawings. This included white spaces, absent borders, sketchiness and dashed lines.

These ways of portraying uncertainty are tested in this study. Traditional symbology for uncertainty (see for example FGDC 2006) is tested along with geographical context, zoom level and symbol size. The study focuses on effective cartographic communication, and more specifically on the effect of different design choices have on the assessment on uncertainty. Real geological data is used in the examples in cross-section. The question of how geologists model uncertainty is beyond the scope of this study. The participants are divided in two main groups: Domain experts, who have extensive knowledge of subsurface data acquisition methods and are aware of the possible extent of bias and uncertainties. The other group is the non-experts, who typically lack this knowledge and who may be more inclined to perceive the information as facts.

The goal of this study is ultimately to improve the geological representations aimed for a broader user group than domain experts, such as decision-makers and planners. The research questions for the study are:

(1) How do differences in design choices affect the sense of uncertainty for the participants, experts and non-experts?
(2) Which symbols do the participants think are intuitive for different kinds of uncertainty and does area background affect the choice of symbols?

Method

A two-step web experiment was set up in order to unravel the participants' reactions to the research questions. The purpose of the first part of the study was to analyze how different design choices affected the participants' assessment of some point locations in the physical world whether they are accurately portrayed in the cross-section. The intuitiveness of conventional symbols for uncertainty was tested in addition to zoom level/symbol size and variation in reference information. At this point of the experiment, participants were divided into four groups that were presented with the same cross-sections with indicated point locations, but with different graphical design choices. In the second part, the participants were again divided into four new groups. Now they were asked to select their preferred symbols in different scenarios. This part was also set up to analyze the implications of adding area fill behind the symbol. Participants were, within both parts, divided into groups, in order to make it possible to analyze and discuss the relevance of multiple variables and combination of symbols.

Pilot Study

A pilot experiment was set up and completed by 20 participants. The first three participants were observed and asked to think aloud while doing their choices. In addition, an anonymous link was sent to selected experts in 3D modelling, cartography/GIS/planning and finally to some employees at the Geological Survey of Norway. Five participants were 18–34 years of age, 12 participants 34–54 years of age and three were older than 55. There were 10 women and 10 men, 14 of these hold higher education. There was an even distribution of non-experts, participants of medium knowledge and experts in geology (6-6-7) and slightly fewer non-experts than experts in cartography/GIS (4-7-9).

The results and feedback from the test group were used to adjust symbols and language in the questions to make it easier to understand, and to verify that the results could be used within the planned statistical analyses. The changes included changing distances and sizes of the dotted and dashed lines to a larger degree resemble standard symbology. The language in the questions in part one was adjusted to make it evident that the question was about the "specific point location".

Part 1

In part 1, the participants were presented with four cross-sections with annotated layers of geology. The cross-sections were presented in a random order (after the first), and the participant got 1 of 4 alternative visualizations, randomly assigned for each image group. In each image group, despite different visualizations, the same cross-section data with the same point locations marked was used (see Fig. 1).

The experiment used illustrations based on real geological data from Hansen et al. (2013). The original cross-sections were put in their geographical context in a 3D viewer. After that, 2D images were exported and simplified, both graphically and linguistically. The simplifications were carefully made with guidance from the first author (Hansen).

Within the cross-sections for all groups, the points were placed beneath the ground and inside a geological layer. The participants had to use a slider (visual analogue scale), which represented the participants' certainty of placement in one geological layer to another (Fig. 2). They were asked to position the slider towards the most likely geological layer at the different point locations A, B and in one cross-section also a point C.

The point locations were added at the same depth and distance from a border within each group. The end points represented the values 0 and 100. If the point was close to a border between two geological layers, the expected result was closer to the mid-point. Placing the slider at the middle returned the value 50, which represents the highest level of insecurity in the participants. The research question was to measure the effect of portraying uncertainty in different ways. Participants were therefore presented with different graphical presentations of the same cross-section with the same point locations. The symbols for uncertainty were not explained to the participant or described in a legend.

The first group of images wished to compare the dashed line compared to no line and solid line. In addition, one image used a hatched area to cover an area, which was marked as uncertain in the original cross-section.

The second group of images in the study showed images with differences in how the border was drawn (Fig. 1, 2nd row). Both points A and B were close to the border, but in two of the images, the line close to point B had a dotted line or dotted line with a question mark (intrinsic visualization). The other two images used a solid line or no lines as borders.

In these first two groups of images, we wanted to examine the difference in how experts and non-experts experienced the use of conventional geological symbols.

The third group of images was made to analyze the effect of scale and symbol size on the assessment of uncertainty. Two of the images were "zoomed in" to the two-point locations, while the other two showed a larger area. One of each pair of images had larger symbols than the other (Fig. 1, 3rd row).

In the fourth group, the difference between the images was only the reference information or geographical context (Fig. 1, 4th row). The research question was to see whether reference information above ground influenced the assumption of uncertainty. Here, uncertainty is shown by making the reference information more or less detailed and correct. This was put first in first part of the experiment, so the participants had no prior knowledge of the scale.

The questions resulted in bipolar scale data from 0 to 100 for each question. For visual analogue scale (VAS) data, used for example in medicine, parametric tests like analysis of variance (ANOVA) and *t* tests are suitable (Philip 1990). ANOVA is a powerful tool that tolerates violations to the normality assumption if the group sizes are not too small (Philip 1990; Laerd Statistics 2020b). In this study, ANOVA was used to compare the results between the groups and across expert levels. The ANOVA tests whether the variance between groups is larger than the variation within groups. In this case, this was used to analyze whether the differences in graphical representation made statistically significant differences in the answers. If the ANOVA test resulted in statistically significant results, post hoc tests (Bonferroni, least significant difference (LSD)) were used for multiple comparison. Means were compared to see whether the difference indicated a higher degree of uncertainty. To compare groups pairwise, the independent *t* test was used. Box plots were also used to explore the results.

Part 2

In the second part of the study, the participants were asked to select suitable symbols for different categories. They were asked which symbol they thought were the most intuitive of four different categories: Certain and well-defined transition between two layers, uncertainty of location, gradual transition and uncertainty whether there was a border at all (Fig. 3). The study tested whether the conventional symbols were selected equally by experts and non-experts. The experiment included a limited set of

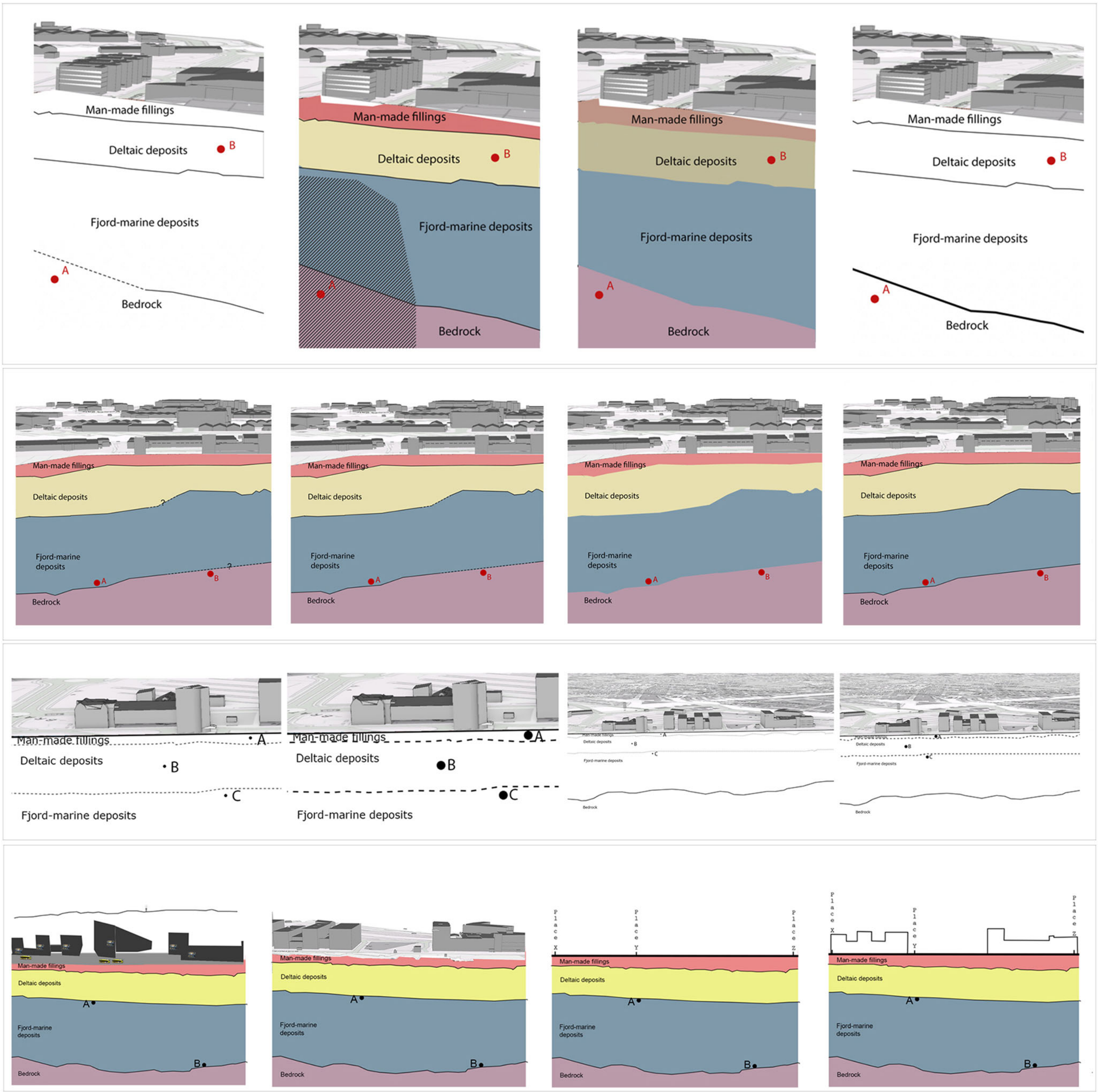
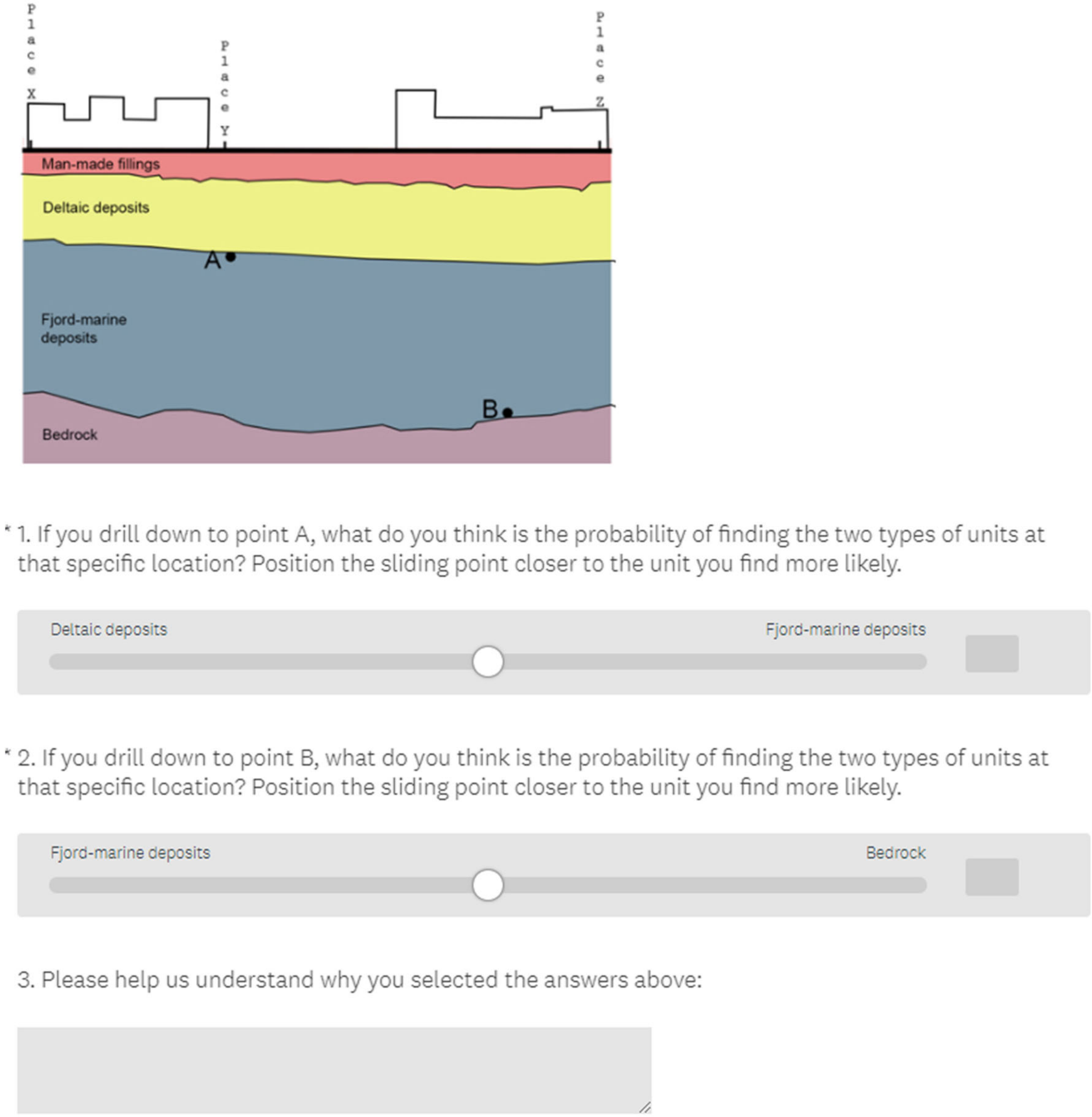


Fig. 1 A collection of all cross-sections used in the survey. Each row shows the four cross-sections with points and their various visualizations that were presented to the four groups. The participants were randomly assigned to one image inn each row, with questions about the point locations. 1st row: Uncertainty visualized with dashed line and hatched area compared to no line and solid line. 2nd row: Uncertainty visualized with dotted line and question mark compared to no line and solid line. 3rd row: Cross-sections with different symbol size and zoom levels. 4th row: Cross-sections with different reference information

types of uncertainty in addition to gradual transitions. This was done to present the complexity of geological visualization, where lines in a representation are used to illustrate different forms of transition, from faults to mixed materials.

The nine different symbols to choose from were conventional line symbols for uncertainty (i.e. FDGC 2006) as well as alternative ones. Variables differed in resolution and crispiness (MacEachren 1995). In addition, random symbols of parallel lines were used. The symbols were presented in small images inspired by legend graphics (Fig. 3).

For each question, the participants were asked to select one or more symbols from the image map that for them the best represented the category (Figure). The questions came in random order, but at the same page. The image map had identical alternatives for line symbols, but these were presented in a random order for each question.



* 1. If you drill down to point A, what do you think is the probability of finding the two types of units at that specific location? Position the sliding point closer to the unit you find more likely.

* 2. If you drill down to point B, what do you think is the probability of finding the two types of units at that specific location? Position the sliding point closer to the unit you find more likely.

3. Please help us understand why you selected the answers above:

Fig. 2 Screenshot from the survey. Moving the slider to the left close to 0 reflects the participant is confident that this category is found in the physical world. Moving the slider to the right gives a value closer to 100 and confidence about the second category. A value close to 50 and the mid-point reflects high uncertainty

In addition, another dimension was added to provide for an open-ended analysis. The participants were divided into four groups and shown different area backgrounds (Fig. 4); no background or different variations of visual variables: form (pattern), value (grey tones) and colour (hue). Bertin (2010) provided a comprehensive theory of graphical semiology, which was used as a source of inspiration. The system represented a method to fit information variables to visually variables of the same organization levels to make effective graphics. By selecting the right type of graphic and visual variables, loss of information is prevented, and inherent spatial patterns can be identified if they exist. The shape and colour variable are nominally ordered visual variables and are to be used with nominally ordered information, like marine or river deposits. The value variable is ordered and used to portray ordered information, like thick and thin marine deposits. In typical geological maps, there is often a mix or hierarchy of nominal and ordered categories in the same legend, with nominal variables represented by different colour hue, and subgroups that are ordered for example because of age (bedrock) or thickness (surficial deposits) with difference in value (lightness).

This was an open-ended analysis, to measure and discuss potential effects of the area background on the choices of symbols. Presenting lines on the top of area fill is closer to its practical use. Adding 4 variants to 4 groups made it possible to analyze whether symbols present a uniform understanding regardless of area fill background. Also, the different

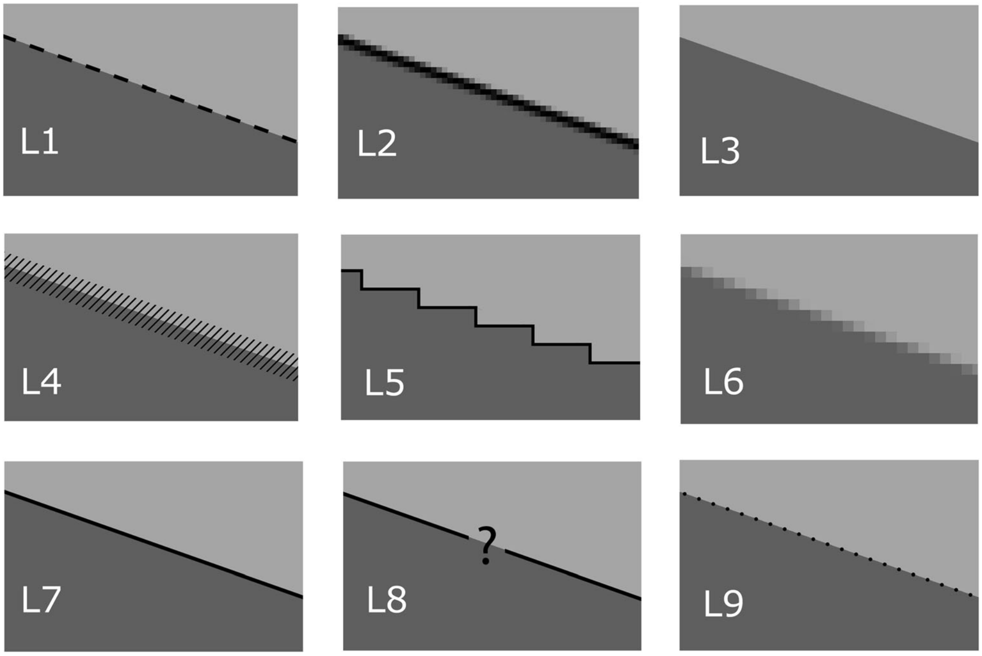


Fig. 3 The questions about preferred symbol type used an image map where the participant could select multiple images by clicking on them

backgrounds would make the results more valuable when it comes to practical use (see Fig. 4).

Part 2 returned a dataset with nominal values of 0 or 1 (chosen or not chosen).

Participants

A wide group of participants was needed to make a statistically viable analysis possible; therefore, a web experiment was selected to collect data. The goal was to reach participants with both limited, medium and expert knowledge of geology. In order to preclude single-country conventions, the experiment was set up for both Norwegian and English-speaking participants. Conducting a web experiment could mean a risk of weakened control over the test, but gave possibilities of more participants, and more experts specializing in 3D geology. The link was sent to employees of the Geological Survey of Norway and collaborating units and contacts, a group for 3D geological modelling experts in Europe as well as Facebook and LinkedIn groups for professionals within planning and maps. To ensure non-expert participants and a high number of participants completing the test, the test was made simple and short, but still using real geological data.

The participants were asked about their age, country, level of education and knowledge levels in geology and cartography/GIS. The questions about knowledge levels were included as this is expected to be a factor (Kinkeldey et al. 2014:384).

Two hundred ten participants completed the experiment. From these, 150 were included in part 1 (elimination explained in detail below) and 206 included in part 2. Four participants were excluded from part 2 as they marked the effort they put into the survey as < 5 (on a scale from 0 to 100). For an overview of the 150 participants in part 1 (see Table 1).

Thirty-eight percent of the participants said they worked within the field of geology, 24.7% in GIS/cartography. There was a dominance of men (62%) and the participants were with few exceptions highly educated.

Participants who marked their participation effort to lower than 15 (on a scale from 0 to 100) were excluded. For part 1, two misunderstandings were revealed through comments and feedback. First, some participants answered about the whole stratigraphy (or drill-log) from the surface down to the position, and not just at the specific position. Second, looking at the answers and comments for one participant, it seemed like

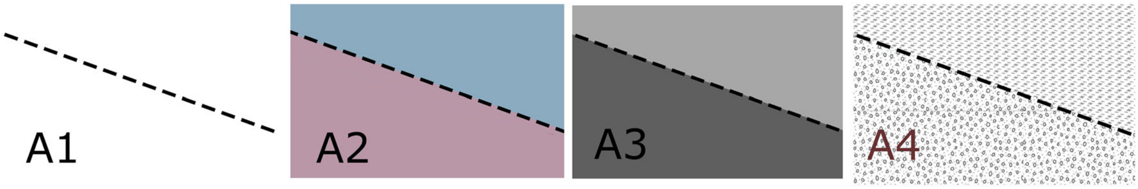


Fig. 4 Each group of participants was presented with different area background for the symbols, representing the visual variables colour, value and pattern

Table 1 Overview of participants, number of participants and percentage of total

Participants (included in part 1)	150				
Gender			Knowledge level geology		
Women	57	38%	Low	58	38.7%
Men	93	62%	Medium	37	24.7%
Education			High	55	36.7%
No higher education	2	1.3%	Knowledge level cartography and GIS		
Some higher education	16	10.7%	Low	31	20.7%
Bachelor or higher	132	88%	Medium	62	41.3%
Country (of work)			High	57	38%
Norway	101	67.3%	Age		
Germany	12	8%	< 18		0.7%
Slovenia	6	4%	18–24	17	11.3%
Switzerland	5	3.3%	25–34	27	18%
USA	4	2.7%	35–44	46	30.7%
Poland	3	2%	45–54	36	24%
China	3	2%	55–64	19	12.7%
Other (Finland, Austria, Belgium, Czech Republic, UK, Denmark, Ireland, Sweden)	13	8.7%	> 65	4	2.7%

the scale bar was misunderstood to represent probability/certainty from 0 to 100%, and not as a bimodal scale from one category to the other. Mapping the probable answers of each type of misunderstanding, a general rule was set for exclusion. Participants who put the slider towards the wrong category on both questions in image group two, where the point locations were furthest from the borders, were excluded. This left 150 participants for analysis. For part two, only participants with an effort lower than 5 were excluded from the analysis.

Results and Discussion

Part 1

Table 2 provides an overview of the results. There were four different cross-sections, each with questions about 2–3 point locations (A, B, C). Each of the cross-sections had four different visualizations, where each participant was randomly assigned to one of these (Fig. 1). One-way ANOVA tests were performed to compare the results between these four groups for each question. In addition, ANOVA tests were performed with expert levels as factor. The tests that returned statistically significant results were tested with post hoc tests. For knowledge levels in geology, there were no significant results from the ANOVA tests. This might be because the different visualization techniques make it difficult to detect differences between participants of different knowledge levels, compared to a more focused study with less variables. It is likely that different knowledge levels have different effect dependent on the visualization used. When discussing the results below, expert levels are therefor also explored in more detail.

Comparison Between Uncertainty Visualized with Dashed Line and Hatched Area

In the first assignment, it was investigated whether two types of uncertainty visualization gave a significant difference in the answers (Fig. 5). Both intrinsic and extrinsic visualization techniques were used. One cross-section had uncertainty marked as a dashed line on a white background, the other as a hatched area. These were compared to no lines and thick line on a white background. The one-way ANOVA test returns a significant difference between all groups for point A (0.001), which was the point in the uncertain area. For point B, the difference is not significant (0.201). When comparing the individual groups with the Bonferroni post hoc test, the results show that the dashed line (L1) returns significant values when compared to all the other groups, while the other groups have no significant difference (Fig. 5). The solid line (L2) gave a statistically significant difference between groups for point A (0.003). The hatched area did not give any significant difference in answers, only compared to

Table 2 The results from the ANOVA test from groups with different visualization technique and with different knowledge levels. There are statistically significant differences ($p < 0.05$) (marked with "*"), for three of the four cross-sections based on visualization technique but not on expert levels

		Vis. technique				Knowledge levels		
Cross-section	Visualization techniques used	Point	df	F-ratio	p value	df	F-ratio	p value
1	Dashed line/solid lines and no colour background, hatched area/no lines with colour background	A	3	6.064	0.001*	2	0.368	0.693
		B	3	2.185	0.092	2	0.816	0.444
2	Dotted line/question mark/no line/solid line	A	3	0.201	0.896	2	0.726	0.486
		B	3	0.916	0.435	2	2.365	0.098
3	Different zoom levels/ symbol size	A	3	2.602	0.054	2	0.673	0.512
		B	3	5.650	0.001*	2	0.488	0.615
		C	3	2.959	0.034*	2	0.681	0.508
4	Different reference information	A	3	0.827	0.481	2	0.386	0.681
		B	3	3.144	0.027*	2	0.154	0.858

the dashed line. It did not give a higher mean on uncertainty assessment compared to the examples with no uncertainty visualization. A hatched area has the advantage that it can better show the area extent of high uncertainty, while the line only describes the uncertainty connected to the line.

The results from this study suggest that this technique requires a legend and explanation, and therefore more time for the user to read and perceive the information. Results from this study therefore supports the conclusion from Slocum et al. (2003) that extrinsic visualization is better for in-depth studies of uncertainty, while intrinsic visualization gives a better overview. According to Harrower (2002), there is “growing evidence that integrated uncertainty symbolization (e.g., bivariate symbols) is superior to separate displays, at least in static maps.” The answers show that the level of uncertainty was much higher for the participants who were shown the dashed line compared to the thick, solid line (see Table 3). The difference was also significant between dashed line and hatched area (0.008) and close to significant for dashed line and no line (0.059). The uncertainty for point B was also a bit higher for the alternative with the dashed line. This may suggest an “out of sight, out of mind” effect for uncertainty.

When comparing experts and non-experts, there is a higher significance for experts than non-experts for the comparison between the dashed line and no line. This suggests the experts know the dashed line usually means uncertainty. The results give no answers to how effective these symbols would be when the non-experts become familiar with them. As Harrower (2002) concludes: “Knowing how users react in a test setting to maps they have likely not seen before (“cold” test subjects) makes it difficult to know how these maps could become integrated into their everyday intellectual activities.”

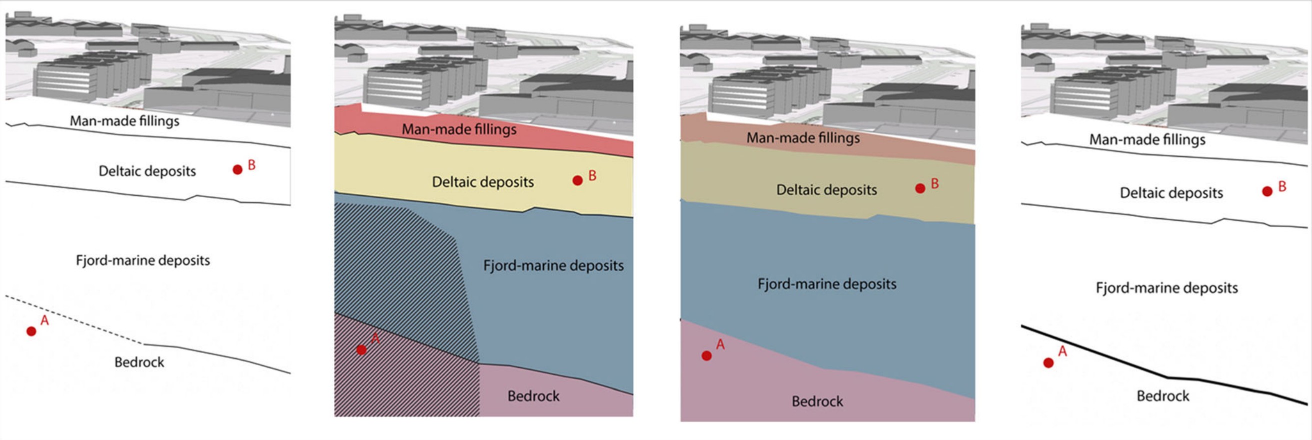


Fig. 5 Results show that participants were more uncertain that point A was situated within bedrock when below the hatched line, than in the other groups. The hatched area gave no increased uncertainty in this study

Table 3 The mean value for the participants for different designs. The dashed line is an effective way for communicating uncertainty (closer to the mid-point of 50). The results also show that the presence of the dashed line increases the uncertainty for the other point

	Dashed line, no colour	Hatched area, colour	No lines, no colour	Solid lines, colour
Mean point A (reversed)	26.98*	15.06	17.93	14.11
Mean point B	12.31	8.38	7.24	9.28

Comparison Between Uncertainty Visualized with Dotted Line and Question Mark

Figure 6 presents the cross-sections evaluated for uncertainty visualized with dotted line and question mark. The ANOVA and *t* tests comparing the groups gave no significant results. When comparing pair of groups (uncertainty visualized or not), the independent *t* test returns a *p* value of 0.136. There is a difference in the mean values (uncertainty visualized in point B is on average higher when uncertainty is visualized), but still not a significant difference. This may be explained by the graphical differences between the images being too small. The colours used are probably too dominant compared to the symbols that varies between images. With more graphically distinctive symbolizing, results may have been different, and therefore, no conclusions can be drawn from these results about the symbols used for uncertainty in this part.

Comparison Between Cross-sections with Different Symbol Size and Zoom Level

The research question for this assignment was whether decreased symbol size would give an impression of detail and correctness that would make the participants' uncertainty decrease (Fig. 7). The different zoom levels were also expected to give a similar effect. It was expected that when the image was easier to read, uncertainty would decrease. The results show when "zoomed in" and symbol size decreases, participants are more certain about the category. Nine outliers were detected in an outlier analysis and were removed as is recommended before running an ANOVA analysis (Laerd statistics 2020a). One-way ANOVA test returned significant differences in mean for point B (0.001) and C (0.034), while the value for point A is close to significant (0.054). This means the 0 hypothesis must be rejected: The results show size of symbols and/or zoom levels do matter when it comes to sense of uncertainty.

The post hoc test revealed there are statistically significant differences between the groups having the zoomed-in image with the small symbols and both zoomed-out images for point B and C. This was also the case for the zoomed-in image with larger symbols compared to the zoomed-in with small symbols for points B and C. This means the 0 hypothesis may be rejected on the counts of zoom level. For symbol size, the LSD post hoc returns close to significant values for point B (0.88 and 0.72) when comparing different symbol size, but the same zoom levels. For points B and C, the group having the zoomed-out images were less certain about the categorization, with the highest difference for point B. For point A, the groups seem to agree that this most certainly are fillings, with a mean close to the endpoint.

These results suggest that it is possible to use zoom and symbol size to give an impression of higher or lower uncertainty with the overall representation.

There was no significant difference found between experts and non-experts in this category.

Comparison Between Cross-sections with Different Reference Information

The inquiry when comparing cross-sections with different reference information above ground was whether this information (base data) has an effect on the overall uncertainty levels (Fig. 8). The groups were shown the exact same representation of the geology, while the reference information above the surface varied from tics with place names to sketches of 2D to 3D building outlines.

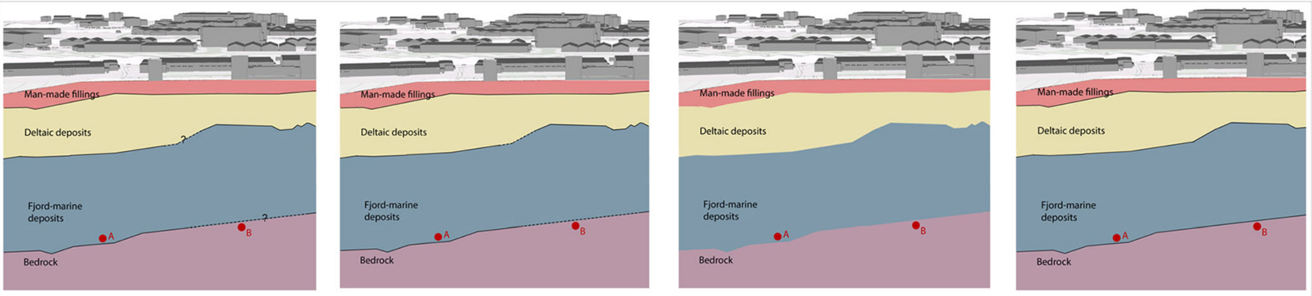


Fig. 6 Uncertainty visualized with dotted line and question mark gave some differences between groups, but they were not statistically significant. This could probably be explained by the colours being too dominant in the images, leaving the differences represented by the uncertainty visualization too small

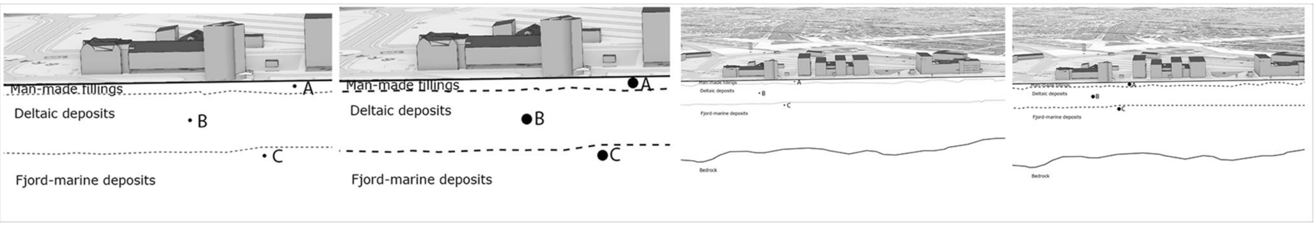


Fig. 7 Zoom levels and symbol sizes make a difference. Smaller symbols in a zoomed-in image decrease the uncertainty

Two outliers were removed. The one-way ANOVA test found a statistically significant result for point B (0.027) while A was not (0.481). Point B is located near the border of bedrock, and some experts comment that the bedrock surface is easy to detect from possible seismic data, which have influenced the different results. The *t* test comparing the two images with less detailed reference information compared with the two with 2D building outline and 3D buildings returned a significant value of 0.021 for B (0.160 for A). This may be explained by the hypothesis that when the reference information looks precise, it can be expected that the user has increased confidence to that the geological borders and categories are correct. A similar hypothesis was discussed by MacEachren (1995:437). He suggested three visual variables "crispiness", "resolution" and "transparency" for uncertainty visualization. Crispiness refers to different degrees of detail and how precise sign vehicles are defined. Resolution, according to MacEachren (1995), refers to "the spatial precision of the map's geographical base, with a coarse base (possibly) suggesting lack of certainty about data depicted on that base.

The results from this experiment confirm the hypothesis that more precise reference information gives a higher confidence in the categorization. When the cross-section has only tags and place names as reference information, the mean value is closer to the mid-point, which reflects higher uncertainty (49.5). When the reference information contains 2D building outlines, the mean value is closer to the endpoint (24.66), which means a clearer certainty about the category. The Bonferroni test returns a value of 0.017 for B. Thus, increasing the group size gives clearer results, all confirming the hypothesis. In the questions regarding this alternative, which was the first in the experiment for all participants, some participants commented that they struggle to understand what they are supposed to do or see and wonder about the intention of the experiment. Some participants commented they answered the first questions wrong. Some of the participants likely did not understand the connection between the subsurface and the reference information above ground or understand its relevance. The amount of misunderstandings may have influenced the results. With a better explained assignment, the patterns may have been more evident.

There were no significant differences when comparing experts with non-experts.

Geographical information systems and databases limit the cartographic language to its objects like lines, areas, voxels and volumes. Out-of-the-box visualization offers simple variation of visual variables. There are unlimited possibilities, though, only limited by development of new methods. Sketchy visualizations are offered today in Geographical Information Systems (GIS), available with some programming (Wood et al. 2012; GISSE 2020).

In geology, in order to make more understandable maps, the whole presentation could be made sketchy. Alternatively, the geographical base or the geological features could be made sketchy. Areas where dense, detailed and certain measurements are available or areas where digging and blasting have revealed "the truth" can have solid, standard cartography, while uncertain areas are made sketchy.

Experts Versus Non-experts

Results from this study suggest that experts understand uncertainty visualized the conventional ways, even when there is no legend. The one-way ANOVA returns statistical significance of 0.019 between expert level groups for point A in the image with dashed line, and 0.136 for point B. This analysis includes

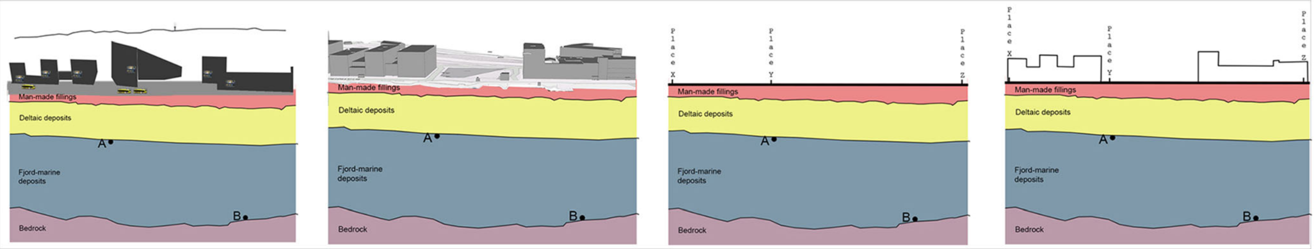


Fig. 8 Results from this comparison confirm the hypothesis that increased detail in the reference information decreases uncertainty, and therefore confidence in the information presented

42 participants. Also, for the question mark and dotted lines, there is a difference in mean values following the same trend. This result suggests uncertainty information is of great guidance for the geologist to evaluate uncertainty. For the non-experts, the uncertainty visualization must be made more obvious, so that also non-experts make the same assessment. Another difference between experts and non-experts is also identified through the comments, where experts think it is difficult to answer the questions because they lack information about drillings and other data the interpretation is based on. Another example from the comments is that the bedrock surface can be easy to map compared to other geological layers. They know it is hard to separate the softer layers from a seismic image. The different answers from experts and non-experts show a potential of communicating more of the elements that requires domain knowledge in the representation more distinctively. This could be shown visually for example by making the bedrock line more distinct than the other lines.

Part 2

For this part of the experiment, the participants were asked to select suitable line symbols for different kind of uncertainty, which is typically found in geological representations. The research question aimed to assess what participants prefer, whether the conventional symbols in geology also are the preferred symbols by the non-experts. In addition, part of the question was to evaluate the impact area background has.

The participants were divided in four groups, where each group had a different background area fill together with the line symbol. The questions and alternatives were randomized, but at the same page. Table 4 presents the distribution of participants across groups and knowledge levels.

The most preferred symbols (independent of area fill) (Fig. 3) for the respective questions are shown in Table 5. As expected, almost all the participants selected the solid line (L7) for certain and well-defined transitions between two geological layers. For gradual transitions, the randomly selected symbol with oblique lines (L4) was the most selected, followed by the stepwise transition with no line present (L6). The dashed line (L1), oblique lines (L4) and solid line with a question mark (L8) were the main alternatives chosen to best represent uncertain location. The alternative when two, separable layers are divided by a solid line with a question mark (L8) was preferred by half of the participants for representing uncertainty if there actually are two separate layers (Figs. 3 and 4 and Tables 5 and 6).

Table 4 The distribution of the 206 participants across groups and knowledge levels in part 2

		Level of knowledge in geology		
	Percent	Low	Medium	Experts
A1	17.5%	12	10	14
A2	32.5%	24	19	24
A3	32.0%	30	14	22
A4	18.0%	11	11	15

Comparing Groups Across Knowledge Levels

For uncertain location, the question was: “Select the line symbols that you think are suitable for representing an uncertain location between two layers”. When comparing across knowledge levels (Table 5), the Pearson chi-square test returned a statistically significant value for the dashed line (L1) and the thick, blurry line (L2). As Table 5 shows, more experts preferred the dashed line, while 25% of the non-experts suggested the thick blurry line.

Symbols selected when the issue was uncertainty if there actually are two separate layers gave statistically significant results from the Pearson chi-square for the dashed line (L1). Half as many non-experts as experts selected this. It should also be noted that the question mark seems effective for all knowledge levels for uncertainty if there is a transition.

Comparing Groups Across Area Fill Behind the Symbols

The results show that area fill makes a difference when choosing line symbols (Table 6 ; Figs. 3 and 4 for images of line symbols (L1–L9) and area background (A1–A4)). The Pearson chi test resulted in multiple significant results, marked with "*" in Table 6. No line (L3) for certain borders should be disregarded for group A1 as a blank symbol marked “no line” probably was too abstract for the participants. It is common to show cross-sections with no border line between the features. The dashed line for gradual transition was chosen by more participants when the background area was different grey tones, but almost by none when there was patterned fill. Together with the patterned background, the dashed line was less distinct.

The dotted line was chosen by almost half for uncertain location when there was no area background, and only by 17% when the background was grey tones.

To show uncertainty if there actually are two separate layers, the dotted line was more often chosen than when there was no area background. When the background was filled with a pattern, more participants chose the solid line for this category. Also, some participants chose a solid line when the background was grey tones. This can possibly be explained by either, that some of the participants did not read the question right (“line symbol”) and/or that the area and line symbols are being intertwined and perceived as a whole. Regardless of the

Table 5 Participants choosing the symbols L1–L9 for the different categories compared to knowledge levels in geology in percent (%). The Pearson chi-square test shows statistically significant difference ($p < 0.05$) between groups marked with "*"

	Certain			Gradual transition			Uncertain location			Uncertain existence		
	Low	Medium	Expert	Low	Medium	Expert	Low	Medium	Expert	Low	Medium	Expert
L1	13	6	4	8	15	9	35*	52*	53*	21*	33*	44*
L2	14	11	11	19	19	12	25*	17*	9*	18	11	7
L3	22	28	27	4	2	4	4	2	1	9	7	5
L4	3	0	3	69	48	59	47	43	36	35	26	25
L5	31	20	25	18	15	9	6	7	5	6	2	5
L6	6	2	4	42	52	44	23	22	11	26	15	12
L7	90	96	93	0	0	1	3	0	1	4	2	5
L8	1	0	4	3	7	3	35	35	47	45	57	47
L9	8	2	3	9	9	11	23	30	29	22	30	27

reason, it illustrates typical challenges with graphical communication. The challenge increases as the data presented gets more advanced and domain specific.

Other results

Degree of Difficulty, Relevance and Effort

As mentioned earlier, 25% of the participants were excluded for part 1, as they very likely had misunderstood this part of the assignment. All 210 participants that completed the experiment are included in this evaluation part of the analysis.

When asked about the relevance of this type of information privately or professionally, the results show that 44% of the participants convey it as very or extremely relevant (Table 7). This is no surprise when 37.6% of the participants were working in the field of geology, and 54% in cartography and GIS. More surprisingly, 26.6% answer it as not so or not at all relevant with subsurface information. This may be because of the use of domain-specific language and no explanation on what the subsurface information means in practice. Table 8 presents the effort that the participants felt they put into the survey. An average of 9 min and 29 s was used to complete the experiment.

Evaluation of the Method

It proved difficult to get a large number of participants to do the experiment. One-in-three participants did not complete. Some stated that the reason they did not complete the survey was because they wanted to change their answer in the first part when they looked at the possible symbolization of uncertainty in the second part. It was, however, not possible to go back and correct answers. One person stated the language was too difficult ("geological unit"). A participant said the slider

Table 6 Percent of the participants who selected the different line symbols L1–l9 across different area background (A1–9). The areas marked with "*" came out as statistically significant difference ($p < 0.05$) between expert levels in the Pearson chi-square statistics

	Certain line				Gradual transition				Uncertain location				Uncertain existence			
	A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
L1	3	7	9	11	11*	6*	18*	3*	50	40	44	57	39	33	29	32
L2	19	10	6	19	19	13	15	22	17	15	20	16	0*	9*	15*	24*
L3	0*	31*	38*	16*	3	1	3	8	0*	0*	2*	11*	19*	3*	3*	11*
L4	0	3	2	3	69	60	52	65	36	45	39	46	19	21	38	38
L5	36	19	21	38	11	19	12	11	6	7	6	5	3	6	3	8
L6	8	4	2	5	50	43	52	32	17	16	27	8	14	21	18	16
L7	94	93	94	89	0	1	0	0	3	3	0	0	0	4	3	8
L8	0	0	5	3	0	3	6	5	36	39	42	38	53	52	52	35
L9	3	1	9	3	8	9	9	14	44*	24*	17*	35*	42*	30*	12*	27*

Table 7 The relevance of subsurface information privately or professionally, according to the participants

	Relevance Frequency	Percent
Extremely	51	24.3%
Very	42	20%
Somewhat	55	26.2%
Not so relevant	32	15.2%
Not at all relevant	24	11.4%
I do not know	6	2.9%

with numbers was confusing, and there were examples of misunderstandings, as described above.

The slider as a measure of confidence and certainty gave statistically significant results in this study. The challenge, however, is to decrease the number of misunderstandings dealing with real data, unfamiliar for many and domain-specific expert language.

Conclusions

Unused potential of improved graphical communication and ultimately more optimal use of geological information exist when communicating geological representations. Unfortunately, a gap in the degree of understanding exists between experts and non-experts when it comes to the interpretations of maps.

There is a crucial need for communicating uncertainty in geological subsurface representations. Uncertainty visualization gives the geologists and others the means to express different degrees of certainty about locations that are intangible or invisible, but also where the model is influenced by the geologist's subjective interpretations. Without uncertainty visualization, crucial information will always be lacking.

Results from all parts of this study provide evidence that different design choices have a significant effect on the assessment of uncertainty, even though these are not explained in a legend. Design choices that, in this study, proved to be effective are as follows: Changing the appearance of borders between geological layers, making the reference information less detailed and changing scale and symbol size. The dashed line was proven to be a solid choice for experts, and an effective symbol for uncertainty overall. Adding uncertainty into cross-sections could be an excellent tool, which would add understanding both for experts and non-experts. The different answers from experts and non-experts show a potential of communicating more of the elements that requires domain knowledge in the representation more distinctively. A more focused experiment, using a similar method as in part one, but with more guidance in the beginning, could potentially give more knowledge into how users perceive uncertainty visualizations.

The results from this study show that uncertainty visualization, which changes the appearance of the objects, seems effective if the design choices are conventional and/or intuitive. The awareness of possible effects of difference design choices is important and alternative designs should be user-tested before developing new representations. The knowledge of which symbols increase or decrease the sense of uncertainty could be developed and effectively used to improve the usability of geological representations. For expert users, there may be a demand for a comprehensive 3D model of uncertainty for in-depth studies. These cases require more advanced solutions for visualization that the methods tested here.

Subsurface information is different from visible surface information, as it communicates something invisible, intangible and not directly observable, which in many cases is full of uncertainty. The graphical border between geological layers is now used for a lot of information: Type of transition (for example fault or gradual transition), uncertainty and in some cases also as direction of movement. New visualization techniques should be developed for visualizing geology in the same model as the observable and more easily measurable objects above the ground. There is an important difference between measured and interpreted information, and it would be beneficial to the user if this difference became evident with the help of graphical techniques.

In a representation, all elements together influence what the user perceives. The users should be in focus and the time and effort they need to interpret the information and understand the potential uncertainties should be reduced. Testing different designs with the intended user group should be done to ensure information is perceived in the right manner.

Table 8 The effort that was put into the survey and degree of difficulty, according to the participants themselves

Effort	Frequency	Percent	Easy or hard	Frequency	Percent
0–20	34	16.2	Very easy	16	7.6
21–40	46	21.9	Easy	47	22.4
41–60	50	23.8	Neither easy nor hard	110	52.4
61–80	62	29.5	Hard	29	13.8
81–100	18	8.6	Very hard	8	3.8

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Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

Ethical Approval Not applicable.

Informed Consent Yes, voluntary decision to participate in the study.

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