The Journal of Analysis (2020) 28:867–877
https://doi.org/10.1007/s41478-019-00216-x

ORIGINAL RESEARCH PAPER

On the local convergence of modified Weerakoon's method in Banach spaces

Debasis Sharma[1] · **Sanjaya Kumar Parhi**[1]

Received: 16 September 2019 / Accepted: 17 November 2019 / Published online: 27 November 2019

Abstract
Based on Lipschitz continuity condition, we study the local convergence analysis for the fifth-order convergent modified Weerakoon's method for solving nonlinear equations in Banach spaces. Lipschitz continuity condition on the first derivative is assumed to extend the applicability of the scheme. This analysis enables us to solve such problems for which previous studies based on higher-order derivatives unable to find the solution. A theorem showing the existence and uniqueness of the solution along with computable error bounds is established. Standard numerical examples like nonlinear integral equation and system of nonlinear equations are solved to demonstrate the productiveness of our theoretical outcomes.

Keywords Banach space · Local convergence · Iterative methods · Lipschitz continuity condition

Mathematics Subject Classification 47H99 · 65D10 · 65D99 · 65G49

1 Introduction

The study presented in this paper is based on the problem of finding a locally unique solution x^* of the equation

$$F(x) = 0, \tag{1}$$

where $F : \Omega \subseteq X \to Y$ is a Fréchet differentiable function and Ω is a convex subset of X. X and Y are Banach spaces. In the field of applied science and engineering, a

✉ Debasis Sharma
iiit.debasis@gmail.com

Sanjaya Kumar Parhi
sanjaya@iiit-bh.ac.in

[1] Department of Basic Sciences, International Institute of Information Technology, Bhubaneswar, Odisha 751003, India

large number of problems can be solved by transforming them into nonlinear equations of the form (1). For instance, the boundary value problems occur in Kinetic theory of gases, the integral equations related to radiative transfer theory, problems in optimization and many others can be reduced to the problem of solving nonlinear equations. Usually, the solutions of these nonlinear equations can be found in closed form. So, the most frequently used solution techniques are iterative in nature.

A commonly used iterative technique for solving (1) is Newton's scheme, which can be expressed as:

$$x_{n+1} = x_n - [F'(x_n)]^{-1} F(x_n), \;\; n \geq 0. \tag{2}$$

Evaluation of second and more order derivatives is a major drawback of higher-order iterative schemes and are not appropriate for practical use. Due to the calculation of F'' in each iteration, the cubically convergent classical schemes are not suitable in terms of computational cost. Some classical third-order algorithms include Chebyshev's, the Halley's and Super-Halley's schemes are produced by putting ($\alpha = 0$), ($\alpha = \frac{1}{2}$) and ($\alpha = 1$) respectively in

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}(1 - \alpha H_F(x_n))^{-1} H_F(x_n)\right)[F'(x_n)]^{-1} F(x_n), \tag{3}$$

where $H_F(x_n) = F'(x_n)^{-1} F''(x_n) F'(x_n)^{-1} F(x_n)$.

The local convergence analysis of many varieties of the methods defined in (3) has been studied by numerous authors in Refs. [1–6]. Also, the local convergence analysis for various iterative algorithms is studied in Banach spaces in Refs. [7–12]. In this paper, we use the Lipschitz continuity condition on the first derivative only to enhance the applicability of modified Weerakoon's method in Banach spaces.

In Ref. [13], the authors studied the modification of Weerakoon's method [14] with fifth-order convergence to solve systems of nonlinear equations in $\mathbb{R}^n$. The method is given as:

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1} F(x_n) \\ z_n &= x_n - 2[F'(x_n) + F'(y_n)]^{-1} F(x_n) \\ x_{n+1} &= z_n - F'(y_n)^{-1} F(z_n) \end{aligned} \tag{4}$$

In this method, only the first-order derivative occurs in the iteration function but the convergence is proved with the assumption on higher-order derivatives for which the applicability of the method is restricted. For instance, consider a function F defined on $\Omega = \left[-\frac{1}{2}, \frac{5}{2}\right]$ by

$$F(x) = \begin{cases} x^3 \log(x^2) + x^5 - x^4, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}.$$

Notice that F''' is unbounded on Ω. Therefore, the previous studies [13–15] based on higher-order derivatives fail to solve this problem. Also, no information is mentioned regarding the radius of convergence ball in these studies. The local convergence analysis of iterative algorithms provides essential information about the radius of convergence ball. In this paper, we provide the local convergence analysis of the method (4) using the hypotheses only on F' to avoid the use of higher-order derivatives. Particularly, it is assumed that the first derivative is Lipschitz continuous. This study extends the applicability of the method (4) and helps in obtaining the solution of such problems for which previous studies fail.

The rest portion of this paper is arranged as follows: The local convergence analysis of the method (4) is placed in Sect. 2. Section 3 is devoted to demonstrating the applications of our theoretical outcomes on some numerical examples. Conclusions are discussed in the last section.

2 Local convergence analysis

The local convergence analysis of modified Weerakoon's method (4) is studied in this section. Let the open and closed balls in X are denoted as $B(c,\rho)$ and $\bar{B}(c,\rho)$ respectively with center c and radius $\rho > 0$. Suppose the parameters $k_0 > 0$ and $k > 0$ be given with $k_0 \leq k$. To study the local convergence of the scheme (4), we introduce the function J_1 on the interval $\left[0, \frac{1}{k_0}\right)$ by

$$J_1(s) = \frac{ks}{2(1 - k_0 s)} \tag{5}$$

and the parameter

$$R_1 = \frac{2}{2k_0 + k} < \frac{1}{k_0}.$$

Observe that $J_1(R_1) = 1$. Again, we define functions J_2 and K_2 on $\left[0, \frac{1}{k_0}\right)$ by

$$J_2(s) = \frac{k_0}{2}(1 + J_1(s))s \tag{6}$$

and

$$K_2(s) = J_2(s) - 1.$$

Now, $K_2(0) = -1 < 0$ and $\lim_{s \to \left(\frac{1}{k_0}\right)^-} K_2(s) = +\infty$. According to the intermediate value theorem, the interval $(0, \frac{1}{k_0})$ contains the zeros of the function $K_2(s)$. Let the smallest zero of $K_2(s)$ in $\left(0, \frac{1}{k_0}\right)$ is R_2. Also, we introduce functions J_3 and K_3 on $[0, R_2)$ by

$$J_3(s) = \frac{k[1 + J_1(s)]s}{2(1 - J_2(s))} \tag{7}$$

and

$$K_3(s) = J_3(s) - 1.$$

Now, $K_3(0) = -1 < 0$ and $\lim_{s \to R_2^-} K_3(s) = +\infty$. The intermediate value theorem confirms that the interval $(0, R_2)$ contains the zeros of the function $K_3(s)$. Let the smallest zero of $K_3(s)$ in $(0, R_2)$ is R_3. Again, we define J_4 and K_4 on $\left[0, \frac{1}{k_0}\right)$ by

$$J_4(s) = k_0 J_1(s) s \tag{8}$$

and

$$K_4(s) = J_4(s) - 1.$$

Now, $K_4(0) = -1 < 0$ and $\lim_{s \to \left(\frac{1}{k_0}\right)^-} K_4(s) = +\infty$. According to the intermediate value theorem, the interval $\left(0, \frac{1}{k_0}\right)$ contains the zeros of the function $K_4(s)$. Let the smallest zero of $K_4(s)$ in $\left(0, \frac{1}{k_0}\right)$ is R_4. Finally, let us define J_5 and K_5 on $[0, R_4)$ by

$$J_5(s) = \left(1 + \frac{1 + k_0 J_3(s) s}{1 - J_4(s)}\right) J_3(s) \tag{9}$$

and

$$K_5(s) = J_5(s) - 1.$$

Now, $K_5(0) = -1 < 0$ and $\lim_{s \to R_4^-} K_5(s) = +\infty$. The intermediate value theorem confirms that the interval $(0, R_4)$ contains the zeros of the function $K_5(s)$. Let the smallest zero of $K_5(s)$ in $(0, R_4)$ is R_5. Consider

$$R = min\{R_1, R_3, R_5\} \tag{10}$$

Now, we have

$$0 \leq J_1(s) < 1, \tag{11}$$

$$0 \leq J_2(s) < 1, \tag{12}$$

$$0 \leq J_3(s) < 1, \tag{13}$$

$$0 \leq J_4(s) < 1, \tag{14}$$

and

$$0 \leq J_5(s) < 1 \tag{15}$$

for each $s \in [0, R)$. Furthermore, let us assume the followings hold for the Fréchet differentiable function $F : \Omega \subseteq X \to Y$.

$$F(x^*) = 0,\ F'(x^*)^{-1} \in BL(Y, X),$$

$$||F'(x^*)^{-1}(F'(x) - F'(x^*))|| \le k_0||x - x^*||,\ \forall x \in \Omega \tag{16}$$

and

$$||F'(x^*)^{-1}(F'(x) - F'(y))|| \le k||x - y||,\ \forall x, y \in \Omega, \tag{17}$$

where $BL(Y, X)$ is the set of all bounded linear operators from Y to X.

In several studies [1, 2, 9, 16, 17], a third condition assumed is

$$||F'(x^*)^{-1}F'(x)|| \le M,\ \forall x \in B\left(x^*, \frac{1}{k_0}\right). \tag{18}$$

This assumption is not taken in our study. We use the following results to avoid this extra condition.

Lemma 1 *If F obeys (16) and $\bar{B}(x^*, R) \subseteq \Omega$, then $\forall x \in B(x^*, R)$, we get*

$$||F'(x^*)^{-1}F'(x)|| \le 1 + k_0||x - x^*|| \tag{19}$$

and

$$||F'(x^*)^{-1}F(x)|| \le (1 + k_0||x - x^*||)||x - x^*|| \tag{20}$$

Proof Applying (16), we obtain

$$||F'(x^*)^{-1}F'(x)|| \le 1 + ||F'(x^*)^{-1}(F'(x) - F'(x^*))|| \le 1 + k_0||x - x^*||.$$

For $\theta \in [0, 1]$,

$$||F'(x^*)^{-1}F'(x^* + \theta(x - x^*))|| \le 1 + k_0\theta||x - x^*|| \le 1 + k_0||x - x^*||$$

The mean value theorem is used to obtain

$$\begin{aligned} ||F'(x^*)^{-1}F(x)|| &= ||F'(x^*)^{-1}(F(x) - F(x^*))|| \\ &\le ||F'(x^*)^{-1}F'(x^* + \theta(x - x^*))(x - x^*)|| \\ &\le (1 + k_0||x - x^*||)||x - x^*||. \end{aligned}$$

□

Next, the local convergence analysis of the method (4) is presented in Theorem 1.

Theorem 1 *Let $F : \Omega \subseteq X \to Y$ be a Fréchet differentiable function. Suppose $x^* \in \Omega, F(x^*) = 0, F'(x^*)^{-1} \in BL(Y, X)$, F obeys (16), (17) and*

$$\bar{B}(x^*, R) \subseteq \Omega, \tag{21}$$

where R is defined in (10). Starting from $x_0 \in B(x^*, R)$ the method (4) generates the sequence of iterates $\{x_n\}$ which is well defined, $\{x_n\}_{n\geq 0} \in B(x^*, R)$ and converges to the solution x^* of (1). Moreover, the following estimations hold $\forall n \geq 0$

$$||y_n - x^*|| \leq J_1(||x_n - x^*||)||x_n - x^*|| < ||x_n - x^*|| < R, \tag{22}$$

$$||z_n - x^*|| \leq J_3(||x_n - x^*||)||x_n - x^*|| < ||x_n - x^*|| < R, \tag{23}$$

and

$$||x_{n+1} - x^*|| \leq J_5(||x_n - x^*||)||x_n - x^*|| < ||x_n - x^*|| < R, \tag{24}$$

where the functions J_1, J_3 and J_5 are given in (5), (7) and (9) respectively. Furthermore, the solution x^* of the equation $F(x) = 0$ is unique in $\bar{B}(x^*, \varDelta) \cap \Omega$, where $\varDelta \in [R, \frac{2}{k_0})$.

Proof Using the definition of R, the equation (16) and the assumption $x_0 \in B(x^*, R)$, we find

$$||F'(x^*)^{-1}(F'(x_0) - F'(x^*))|| \leq k_0||x_0 - x^*|| < k_0 R < 1. \tag{25}$$

Now, Banach Lemma on invertible functions [18–23] confirms that $F'(x_0)^{-1} \in BL(Y, X)$ and

$$||F'(x_0)^{-1}F'(x^*)|| \leq \frac{1}{1 - k_0||x_0 - x^*||} < \frac{1}{1 - k_0 R}. \tag{26}$$

Hence, it follows from the first step of the method (4) for $n = 0$ that y_0 is well defined. Again,

$$\begin{aligned} y_0 - x^* &= x_0 - x^* - F'(x_0)^{-1}F(x_0) \\ &= -\left[F'(x_0)^{-1}F'(x^*)\right]\left[\int_0^1 F'(x^*)^{-1}(F'(x^* + \theta(x_0 - x^*))\right. \\ &\quad \left. -F'(x_0))(x_0 - x^*)d\theta\right]. \end{aligned} \tag{27}$$

Using (5), (10), (11) and (17), we find

$$\begin{aligned} ||y_0 - x^*|| &\leq \left[\left|\left|F'(x_0)^{-1}F'(x^*)\right|\right|\right]\left[\left|\left|\int_0^1 F'(x^*)^{-1}(F'(x^* + \theta(x_0 - x^*))\right.\right.\right. \\ &\quad \left.\left.\left. -F'(x_0))(x_0 - x^*)\ d\theta\right|\right|\right] \\ &\leq \frac{k||x_0 - x^*||}{2(1 - k_0||x_0 - x^*||)}||x_0 - x^*|| \\ &= J_1(||x_0 - x^*||)||x_0 - x^*|| < ||x_0 - x^*|| < R \end{aligned} \tag{28}$$

and this shows (22) for $n = 0$. Then we show $[F'(x_0) + F'(y_0)]^{-1} \in BL(Y, X)$. The equations (6), (10), (12), (16) and (28) are used to obtain

$$
\begin{aligned}
&||(2F'(x^*))^{-1}(F'(x_0)+F'(y_0)-2F'(x^*))|| \\
&\quad \leq \frac{1}{2}[||F'(x^*)^{-1}(F'(x_0)-F'(x^*))||+||F'(x^*)^{-1}(F'(y_0)-F'(x^*))||] \\
&\quad \leq \frac{k_0}{2}[||x_0-x^*||+||y_0-x^*||] \\
&\quad \leq \frac{k_0}{2}[||x_0-x^*||+J_1(||x_0-x^*||)||x_0-x^*||] \\
&\quad = \frac{k_0}{2}[1+J_1(||x_0-x^*||)]||x_0-x^*|| \\
&\quad = J_2(||x_0-x^*||) < J_2(R) < 1.
\end{aligned}
$$

Now, we obtain $[F'(x_0)+F'(y_0)]^{-1} \in BL(Y,X)$ using Banach Lemma on invertible functions. Also,

$$
||[F'(x_0)+F'(y_0)]^{-1}F'(x^*)|| \leq \frac{1}{2(1-J_2(||x_0-x^*||))}. \tag{29}
$$

Now, it follows from the second step of the method (4) for $n=0$ that z_0 is well defined. Using the definition of R, (13), (17), (28) and (29), we get

$$
\begin{aligned}
||z_0-x^*|| &\leq \left(||[F'(x_0)+F'(y_0)]^{-1}F'(x^*)||\right)\Bigg(||\int_0^1 F'(x^*)^{-1}(F'(x_0) \\
&\quad -F'(x^*+\theta(x_0-x^*)))(x_0-x^*)d\theta|| \\
&\quad +||\int_0^1 F'(x^*)^{-1}(F'(y_0)-F'(x^*+\theta(x_0-x^*)))(x_0-x^*)d\theta||\Bigg) \\
&\leq \frac{\frac{k}{2}||x_0-x^*||^2+k\int_0^1(||y_0-x^*-\theta(x_0-x^*)||)d\theta||x_0-x^*||}{2(1-J_2(||x_0-x^*||))} \\
&\leq \frac{\frac{k}{2}||x_0-x^*||^2+k(||y_0-x^*||+\frac{||x_0-x^*||}{2})||x_0-x^*||}{2(1-J_2(||x_0-x^*||))} \\
&\leq \frac{\frac{k}{2}||x_0-x^*||^2+k[J_1(||x_0-x^*||)||x_0-x^*||+\frac{||x_0-x^*||}{2}]||x_0-x^*||}{2(1-J_2(||x_0-x^*||))} \\
&\leq \frac{(k||x_0-x^*||+kJ_1(||x_0-x^*||)||x_0-x^*||)||x_0-x^*||}{2(1-J_2(||x_0-x^*||))} \\
&= \frac{[k||x_0-x^*||+kJ_1(||x_0-x^*||)||x_0-x^*||]||x_0-x^*||}{2(1-J_2(||x_0-x^*||))} \\
&= \frac{k[(1+J_1(||x_0-x^*||))||x_0-x^*||]||x_0-x^*||}{2(1-J_2(||x_0-x^*||))} \\
&= J_3(||x_0-x^*||)||x_0-x^*|| < ||x_0-x^*|| < R.
\end{aligned} \tag{30}
$$

Hence, we establish (23) for $n=0$. Again,

$$||F'(x^*)^{-1}(F'(y_0) - F'(x^*))|| \le k_0||y_0 - x^*|| < k_0 J_1(||x_0 - x^*||)||x_0 - x^*|| \\ = J_4(||x_0 - x^*||) < 1. \tag{31}$$

So, $F'(y_0)^{-1} \in BL(Y, X)$ with

$$||F'(y_0)^{-1}F'(x^*)|| \le \frac{1}{1 - J_4(||x_0 - x^*||)}. \tag{32}$$

Now, it follows from the last step of the method (4) for $n = 0$ that x_1 is well define. Finally, we use (10), (15), (20), (30) and (32) to get

$$\begin{aligned} ||x_1 - x^*|| &\le ||z_0 - x^*|| + ||F'(y_0)^{-1}F(z_0)|| \\ &\le ||z_0 - x^*|| + ||F'(y_0)^{-1}F'(x^*)||\ ||F'(x^*)^{-1}F(z_0)|| \\ &\le ||z_0 - x^*|| + \frac{(1 + k_0||z_0 - x^*||)||z_0 - x^*||}{1 - J_4(||x_0 - x^*||)} \\ &\le \left(1 + \frac{(1 + k_0||z_0 - x^*||)}{1 - J_4(||x_0 - x^*||)}\right)||z_0 - x^*|| \\ &\le \left(1 + \frac{(1 + k_0 J_3(||x_0 - x^*||)||x_0 - x^*||)}{1 - J_4(||x_0 - x^*||)}\right)J_3(||x_0 - x^*||)||x_0 - x^*|| \\ &= J_5(||x_0 - x^*||)||x_0 - x^*|| < ||x_0 - x^*|| < R. \end{aligned} \tag{33}$$

Thus, we show the estimate (24) for $n = 0$. We get the estimates (22)-(24) by substituting x_n, y_n, z_n and x_{n+1} in place of x_0, y_0, z_0 and x_1 respectively in the previous estimations. Using the fact $||x_{n+1} - x^*|| \le J_5(R)||x_n - x^*|| < R$, we derive that $x_{n+1} \in B(x^*, R)$ and $\lim_{n\to\infty} x_n = x^*$. Now, we want to show the uniqueness of the solution x^*. Suppose there exist another solution y^* of $F(x) = 0$ in $B(x^*, \Delta)$. Consider $T = \int_0^1 F'(y^* + \theta(x^* - y^*))d\theta$. From equation (16), we get

$$\begin{aligned} ||F'(x^*)^{-1}(T - F'(x^*))|| &\le \int_0^1 k_0||y^* + \theta(x^* - y^*) - x^*||\ d\theta \\ &\le \frac{k_0}{2}||x^* - y^*|| \\ &\le \frac{k_0\Delta}{2} < 1. \end{aligned}$$

Applying Banach Lemma, we find $T^{-1} \in BL(Y, X)$. Now, Using the identity $0 = F(x^*) - F(y^*) = T(x^* - y^*)$, it is concluded that $x^* = y^*$. This ends the proof. □

3 Numerical examples

Example 1 Define F on $\Omega = [-\frac{1}{2}, \frac{5}{2}]$ by

Table 1 Parameters for example 1

MWM
$R_1 = 0.006896$
$R_3 = 0.006060$
$R_5 = 0.004426$
$R = 0.004426$

$$F(x) = \begin{cases} x^3 \log(x^2) + x^5 - x^4, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}.$$

We have $x^* = 1$. Also, $k_0 = k = 96.6628$. The value of R is determined using the definitions of "J" functions (Table 1).

Example 2 Let us define F on $\bar{B}(0,1)$ for $(x_1, x_2, x_3)^t$ by

$$F(x) = \left(e^{x_1} - 1, \frac{e-1}{2}x_2^2 + x_2, x_3\right)^t$$

We have $x^* = (0,0,0)^t$. Also, we have $k_0 = e - 1$ and $k = e$. We determine the value of R using "J" functions (Table 2).

Example 3 Let us define F on $\Omega = [-1, 1]$ by

$$F(x) = \sin(x)$$

We have $x^* = 0$. Also, we have $k_0 = k = 1$. R is determined using "J" functions (Table 3).

Table 2 Parameters for example 2

MWM
$R_1 = 0.324947$
$R_3 = 0.268633$
$R_5 = 0.184350$
$R = 0.184350$

Table 3 Parameters for example 3

MWM
$R_1 = 0.666667$
$R_3 = 0.585786$
$R_5 = 0.427846$
$R = 0.427846$

Table 4 Parameters for example 4

MWM
$R_1 = 0.066667$
$R_3 = 0.053333$
$R_5 = 0.035647$
$R = 0.035647$

Example 4 Consider the nonlinear Hammerstein type integral equation given by

$$F(x)(s) = x(s) - 5\int_0^1 stx(t)^3\ dt,$$

where $x(s) \in C[0,1]$. We have $x^* = 0$. Also, $k_0 = 7.5$ and $k = 15$. Using the definitions of "J'' functions the value of R is determined (Table 4).

4 Conclusions

We studied the local convergence analysis of the method (4) to find a locally unique solution of a nonlinear equation in Banach spaces. The Lipschitz continuity condition on the first derivative is used to enhance the applicability of these methods. This study helps in solving those problems for which higher-order derivative based previous studies fail. Lastly, the theoretical outcomes are applied on standard numerical examples like Hammerstein equation and system of nonlinear equations.

Acknowledgements This research is supported by University Grants Commission of India (Ref. no.: 988/(CSIR-UGC NET JUNE 2017)).

Compliance with ethical standards

Conflict of interest The authors declare that they do not have conflict of interests.

Ethical standards This research complies with ethical standards.

References

1. Argyros, I.K., and Á.A. Magreñán. 2015. A study on the local convergence and the dynamics of Chebyshev–Halley-type methods free from second derivative. *Numerical Algorithms* 71 (1): 1–23.
2. Argyros, I.K., and S. George. 2015. Local convergence of deformed Halley method in Banach space under Holder continuity conditions. *Journal of Nonlinear Sciences and Applications* 8: 246–254.
3. Argyros, I.K., S. George, and Á.A. Magreñán. 2015. Local convergence for multi-point-parametric Chebyshev–Halley-type methods of higher convergence order. *Journal of Computational and Applied Mathematics* 282: 215–224.
4. Argyros, I.K., and S. George. 2015. Local convergence of modified Halley-like methods with less computation of inversion. *Novi Sad Journal of Mathematics* 45 (2): 47–58.

5. Argyros, I.K., Y.J. Cho, and H.M. Ren. 2013. Convergence of Halley's method for operators with the bounded second derivative in Banach spaces. *Journal of Inequalities and Applications* 2013: 260.
6. Candela, V., and A. Marquina. 1990. Recurrence relations for rational cubic methods I: The Halley method. *Computing* 44 (2): 169–184.
7. Singh, S., D.K. Gupta, R.P. Badoni, E. Martínez, and J.L. Hueso. 2016. Local convergence of a parameter based iteration with Holder continuous derivative in Banach spaces. *Calcolo*. 54 (2): 527–539. https://doi.org/10.1007/s10092-016-0197-9
8. Martínez, E., S. Singh, J.L. Hueso, and D.K. Gupta. 2016. Enlarging the convergence domain in local convergence studies for iterative methods in Banach spaces. *Applied Mathematics and Computation* 281: 252–265.
9. Argyros, I.K., and S. Hilout. 2013. On the local convergence of fast two-step Newton-like methods for solving nonlinear equations. *Journal of Computational and Applied Mathematics* 245: 1–9.
10. Argyros, I.K., and S.K. Khattri. 2015. Local convergence for a family of third order methods in Banach spaces. *Applied Mathematics and Computation* 251: 396–403.
11. Cordero, A., J.A. Ezquerro, and M.A. Hernandez-Veron. 2014. On the local convergence of a fifth-order iterative method in Banach spaces. *Journal of Mathematics* 46: 53–62.
12. Argyros, I.K., Y.J. Cho, and S. George. 2014. On the "Terra incognita" for the Newton–Kantrovich method. *Journal of the Korean Mathematical Society* 51 (2): 251–266.
13. Cordero, A., J.L. Hueso, E. Martínez, and J.R. Toregrossa. 2012. Increasing the convergence order of an iterative method for nonlinear systems. *Applied Mathematics Letters* 25: 2369–2374.
14. Weerakoon, S., and T.G.I. Fernando. 2000. A variant of Newton's method with accelerated third-order convergence. *Applied Mathematics Letters* 13 (8): 87–93.
15. Nishani, H.P.S., S. Weerakoon, T.G.I. Fernando, and M. Liyanag. 2018. Weerakoon–Fernando method with accelerated third-order convergence for systems of nonlinear equations. *International Journal of Mathematical Modelling and Numerical Optimisation* 8 (13): 287–304.
16. Argyros, I.K., Y.J. Cho, and S. George. 2016. Local convergence for some third order iterative methods under weak conditions. *Journal of the Korean Mathematical Society* 53 (4): 781–793.
17. Argyros, I.K., and D. González. 2015. Local convergence for an improved Jarratt-type method in Banach space. *International Journal of Artificial Intelligence* 3: 20–25. (3(Special Issue on Teaching Mathematics Using New and Classic Tools)).
18. Argyros, I.K. 2008. *Convergence and application of Newton-type iterations*. Berlin: Springer.
19. Argyros, I.K., and S. Hilout. 2013. *Computational methods in nonlinear Analysis*. New Jersey: World Scientific Publishing House.
20. Petković, M.S., B. Neta, L. Petković, and D. Dzunić. 2013. *Multipoint methods for solving nonlinear equations*. Amsterdam: Elsevier.
21. Rall, L.B. 1979. *Computational solution of nonlinear operator equations*. New York: Robert E. Krieger.
22. Traub, J.F. 1964. *Iterative methods for solution of equations*. Englewood Cliffs: Prentice-Hall.
23. Argyros, I.K., Y.J. Cho, and S. Hilout. 2012. *Numerical methods for equations and its applications*. New York: Taylor & Francis.