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Improvements in methodology to determine feedback reactivity coefficients

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Abstract The reactivity of a nuclear reactor is the most important safety and operating parameter. Due to short reactor period, the Light Water Reactor (LWR) designs require the compensations of rapid unfavorable reactivity increases. The increase in fuel or moderator temperature leads to compensate the reactivity jumps as inherent safety characteristics. The safe and reliable reactor operation requires the accurate assessment of these reactivity changes. This paper highlights the improvements in the methodology to determine the feedback reactivity changes in IAEA MTR benchmark. This method incorporates the reactivity effects of fuel temperature in moderator regions and vice versa. For this purpose, a detailed 3D model of the IAEA 10 MW MTR benchmark reactor is developed employing OpenMC computer code. OpenMC is a probabilistic computer code for neutronic calculations. This work uses temperature-dependent JEFF 3.2 cross-sectional library. The model is validated against the reference results of eigenvalues for control rods (inserted and in fully withdrawn position), control rod reactivity worth, averaged thermal flux in the central flux trap, and power fraction for each fuel element at beginning of life. The validated model is applied to simulate the feedback reactivity coefficients against the conventional reference results. In order to improve the methodology, the effect of the moderator temperature and void on fuel is incorporated to obtain a more realistic value of the fuel temperature coefficient. Similarly, the moderator temperature coefficient and void coefficient are improved by incorporating the coupling effects of fuel temperature on moderator. This methodology can be applied to improve the LWR designs.

Keywords OpenMC · MTR benchmark · Reactivity feedback coefficients · Fuel temperature coefficient · Moderator temperature coefficient · Void coefficient of reactivity

1 Introduction

Research reactors are commonly used to obtain sufficient neutron flux to irradiate materials for scientific and commercial purposes. The safe utilization of such a reactor demands evaluation of reactor physics parameters. The temperature changes in the reactor core affect many of these parameters. For instance, the fluctuation in reactor temperature leads to change in effective multiplication factor which in turn affects core reactivity as a feedback. Since reactivity is degree of off-criticality, a change in reactivity actually shows a change in neutron population [1]. Therefore, temperature change can lead to a change in neutron flux and reactor power. The reactivity change due to the temperature is called temperature coefficient of reactivity. This temperature change can occur in fuel, moderator, coolant, and structural material [2]. Reactivity feedback coefficients are among the most important reactor safety parameters. They quantify the variation of reactivity with core temperature [3]. Reactivity feedback plays a vital role in control of the reactor. Negative reactivity feedback keeps the reactor inherently safe [3], whereas positive

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reactivity feedback can lead to unstable and hence dangerous situations [4].

In a reactor core, there are different materials with different temperatures. An increase in reactor power will increase temperature of fuel first, since power is generated in the fuel and then heat is transferred to the coolant. The time required for temperature change in fuel is shorter than the coolant, so the fuel temperature coefficient (FTC) is the prompt coefficient and moderator temperature coefficient (MTC) is the delayed one [3]. Magnitude and effect of different feedback coefficients are also different and hence the affect they produce on reactivity needs to be quantified for the safe reactor operation.

The temperature coefficient of reactivity (α_T) is defined as a change in the reactivity (ρ) with a change in the temperature [5] as shown in Eq. (1).

$$\alpha_T = \frac{\Delta\rho}{\Delta T}, \tag{1}$$

whereas reactivity is calculated using Eq. 2:

$$\rho = \frac{k_{eff} - 1}{k_{eff}}. \tag{2}$$

Differentiating Eq. (2) with respect to temperature and noting that k_{eff} is close to unity for the practical cases, Eq. (3) is obtained [6].

$$\alpha_T \cong \frac{1}{k_{eff}} \frac{\Delta k_{eff}}{\Delta T} \tag{3}$$

If T in Eq. (3) is temperature of fuel (T_f), then α_T defines the FTC of reactivity. The phenomenon of Doppler broadening is associated with a rise in fuel temperature. For thermal reactors, it causes an increase in fuel capture cross section, and the change in reactivity it incurs per unit fuel temperature is termed as the Doppler coefficient of reactivity [6]. The α_T defines the MTC of reactivity when T represents the moderator temperature (T_M). Since water density decreases with a rise in temperature, the moderation also decreases. This hardens the neutron spectrum which leads to a negative feedback in thermal reactors.

Void coefficient (VC) of reactivity is the rate of change of reactivity in a light water-moderated reactor with an increase in the steam bubble formation. This reduces the moderation and produces negative feedback effect.

Although the reactivity coefficients are calculated independent of each other, in reality, a change in temperature of one region also affects the other regions. To account for this coupling effect, the calculation scheme used for feedback reactivity coefficients needs to be changed [4]. For instance, in determination of moderator coefficient of reactivity, change in moderator temperature also causes spectral changes in the fuel region [7]. Therefore, for accurate representation of feedback reactivity coefficients, the incorporation of coupling effects to determine feedback coefficient is necessary.

This paper presents the effect of changing one parameter on the other. These effects include the effect of changing fuel temperature (T_f) on the MTC, effect of changing moderator temperature (T_m) on VC, effect of changing moderator void (V) on FTC, and vice versa, respectively.

2 Materials and methods

There are many research reactors operating all over the world. Due to proliferation concerns, most of the reactors employing highly enriched uranium (HEU) fuel are redesigned to use low enriched uranium (LEU) fuels. The International Atomic Energy Agency (IAEA) has published a standard MTR benchmark problem to help this reactor redesigning [8]. Several organizations have performed reactor physics calculations which are reported in various documents [8, 9].

The LEU core of 10 MW IAEA MTR benchmark reactor is considered [10] for this study. The core configuration consists of a 6 × 5 grid containing 21 standard fuel elements (SFE) and four control fuel elements (CFE). The core is reflected by graphite reflecting elements (GRE) on two opposite faces and surrounded by light water on all of the faces. Axially, core is reflected on both the faces by a 15.0 cm thickness of mixture of Al and H_2O containing 20% aluminum and 80% water by volume. Each SFE contains 23 fuel plates, while each CFE has 17 fuel plates with two regions for fork-type control absorber blades [11]. To benchmark the feedback reactivity coefficients, the water in central flux trap is replaced with a block of aluminum containing a square hole of 5 cm on each side [10]. The reactor was modeled with exact specifications as given in the IAEA benchmark problem.

The detailed description of the reactor is given in Table 1. Control absorber material considered in this work was Ag–In–Cd alloy in the ratio 80%, 15%, and 5%, respectively.

The OpenMC [12] and WIMS/D4 [13] are used as neutronics simulating tools. For benchmarking analysis, the literature results are reported for fresh fuel, beginning of life (BOL), and EOL LEU cores [8, 10]. However, the present work focuses on the fresh fuel and BOL cores for the purpose of benchmarking.

The Winfrith Improved Multigroup Scheme (WIMS) is a general purpose multi-group transport theory-based deterministic code [13]. As input, WIMS/D4 requires isotopic nuclear data and the description of the reactor lattice. It then solves the neutron transport equation over a specified region of the lattice. This region is called the lattice cell or unit cell [14]. Isotopic data include number densities

Table 1 Specifications of LEU core of IAEA 10 MW benchmark reactor [8]

Parameter	Value
Active core height	600 mm
Space at the grid plate per fuel element	77 mm × 81 mm
Fuel element cross section (including support plate)	76 mm × 80.5 mm
Fuel meat dimensions	63 mm × 0.51 mm × 600 mm
Density of aluminum	2.7 g/cm^3
Thickness of support plate	4.75 mm
Thickness of fuel plate	1.27 mm
Number of fuel plates per SFE	23 identical
Number of fuel plates per CFE	17 identical
Weight percent of U in UAl_x−Al	72 w/o
Mass of U-235 per SFE	390 g
Total power	10 MW_{th}
Graphite reflecting element dimensions	77 mm × 81 mm

of isotopes and the microscopic cross-sectional data for the isotopes. The main purpose of the code is the generation of homogenized macroscopic cross sections for the representative zones of the reactor core and burn-up of the fuel [14]. In the present work, the main aim in using WIMS/D4 is to obtain the number densities of different isotopes at different burn-up steps, needed in the BOL core. The unit-cell model and the energy group structure in WIMS/D4 are the same as that described in the literature [11].

OpenMC is an open-source and Monte Carlo neutron transport code developed by Computational Reactor Physics Group (CRPG) at Massachusetts Institute of Technology (MIT), starting in 2011. Like MCNP [15], it is capable of simulating 3D models based on Constructive Solid Geometry (CSG). The continuous energy particle interaction data are based on HDF5 format. The OpenMC (version 0.9.0) equipped with temperature-dependent JEFF 3.2 cross sections [16] is used to find the reactivity coefficients and their associated coupling effects as improvements.

3 Methodology

The main objective of this work was to develop the calculation methodology to determine the reactivity feedback coefficients incorporating the coupling effects of one parameter on the others. In order to account for such coupling effects, reactivity is taken under consideration as a function of two variables. The number of variables can be increased once this method is developed.

In the coupling FTC and VC, i.e., to quantify the effect of change in void fraction on FTC, the variables are fuel temperature, T_F, and void percent, V [4, 7]. First, FTC and VC are determined independent of each other, and these are the uncoupled reactivity coefficients. The coupling method consists of considering the uncoupled coefficients as first-order terms in Taylor expansion [7]. The expression for reactivity is expanded up to the third-order term in Taylor expansion, which provides terms for simultaneous changes in two coefficients [4]. Equation (4) shows the Taylor expansion of reactivity by considering it a function of two variables.

$$\begin{aligned}\rho(T_F+\Delta T_F, V+\Delta V) = \rho(T_F,V) &+\left(\frac{\partial}{\partial T_F}\Delta T_F+\frac{\partial}{\partial V}\Delta V\right)\rho(T_F,V)\\ &+\frac{1}{2}\left(\frac{\partial}{\partial T_F}\Delta T_F+\frac{\partial}{\partial V}\Delta V\right)^2\rho(T_F,V)\\ &+\frac{1}{6}\left(\frac{\partial}{\partial T_F}\Delta T_F+\frac{\partial}{\partial V}\Delta V\right)^3\rho(T_F,V)\end{aligned} \tag{4}$$

where T_F is fuel temperature, ΔT_F is change in fuel temperature, V is coolant void, and ΔV is change in coolant void. In the present work, fuel temperature was changed in increments of 100 K starting from 300 K and up to 900 K, while void percent was changed in increments of 3%, starting from 0% and ending at 18%. Therefore, ΔT_F is 100 K and ΔV is 3%. By re-arranging the above, Eq. (5) is obtained:

$$\begin{aligned}\Delta\rho(T_F,V) = &\left(\frac{\partial\rho}{\partial T_F}+\frac{1}{2}\frac{\partial^2\rho}{\partial T_F^2}\Delta T_F+\frac{1}{6}\frac{\partial^3\rho}{\partial T_F^3}(\Delta T_F)^2\right)\Delta T_F\\ &+\left(\frac{\partial\rho}{\partial V}+\frac{1}{2}\frac{\partial^2\rho}{\partial V^2}\Delta V+\frac{1}{6}\frac{\partial^3\rho}{\partial V^3}\Delta V^2\right)\Delta V\\ &+\left(\frac{\partial^2\rho}{\partial T_F\partial V}+\frac{1}{2}\frac{\partial^3\rho}{\partial T_F^2\partial V}\Delta T_F+\frac{1}{2}\frac{\partial^3\rho}{\partial T_F\partial V^2}\Delta V\right)\\ &\Delta T_F\Delta V\end{aligned} \tag{5}$$

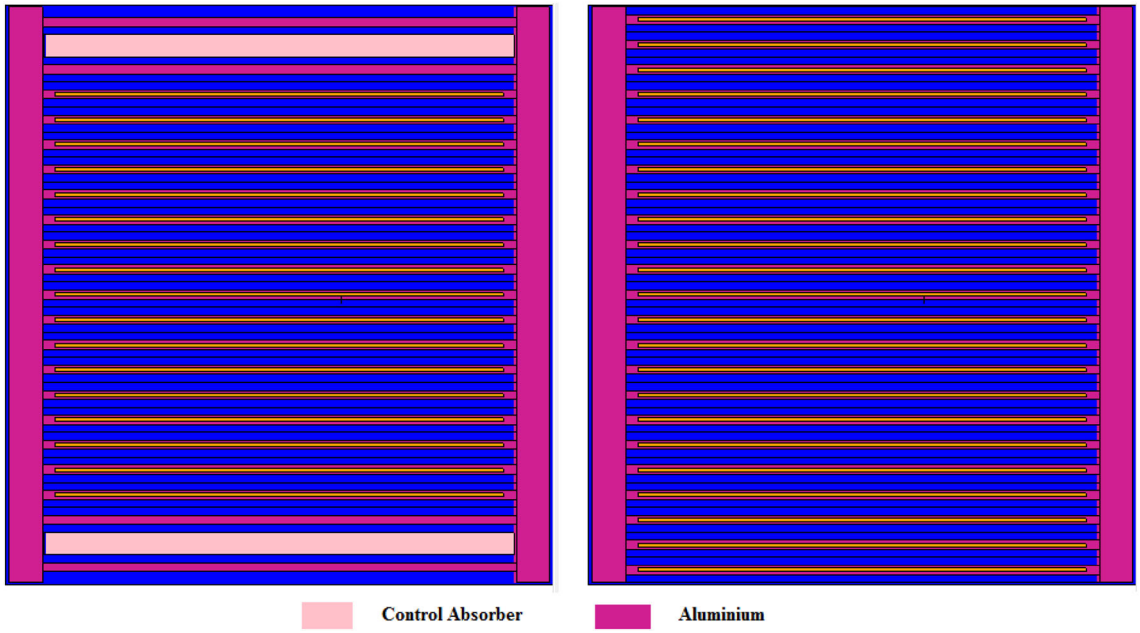

Fig. 1 (Color online) Standard fuel element (right) and control fuel element (left) of IAEA MTR benchmark

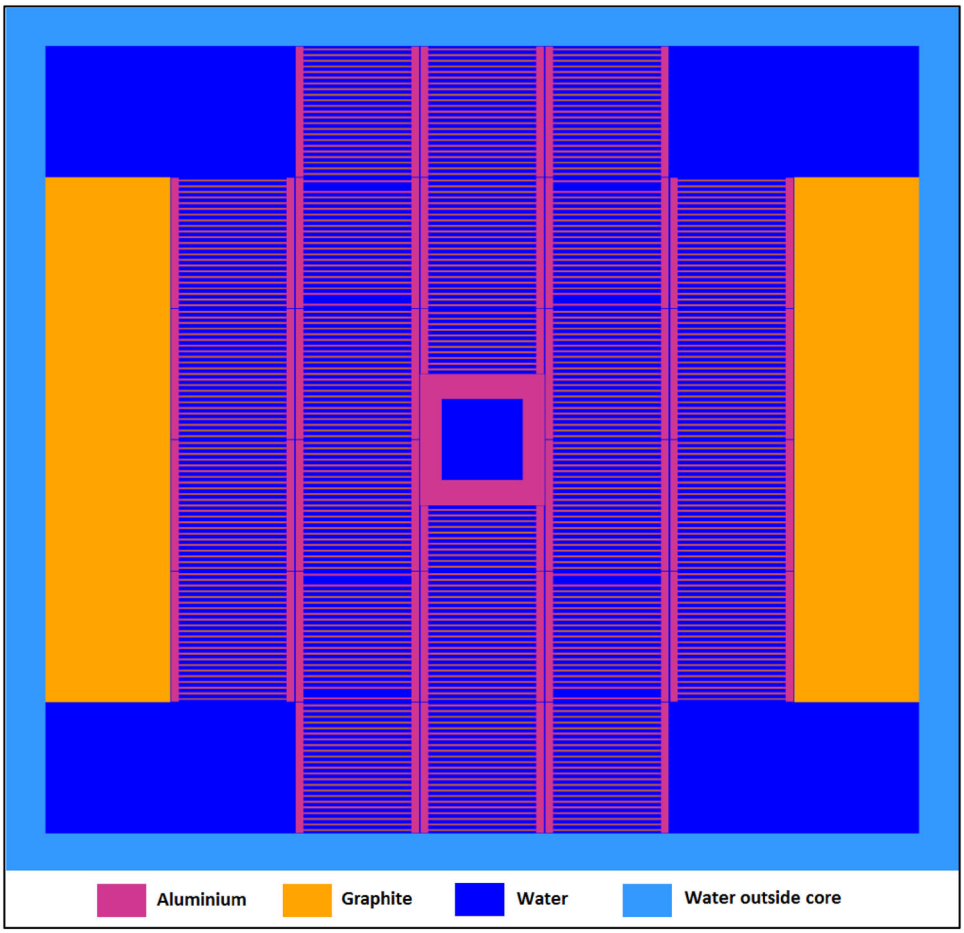

Fig. 2 (Color online) Radial cross-sectional view of the OpenMC model at the mid plane

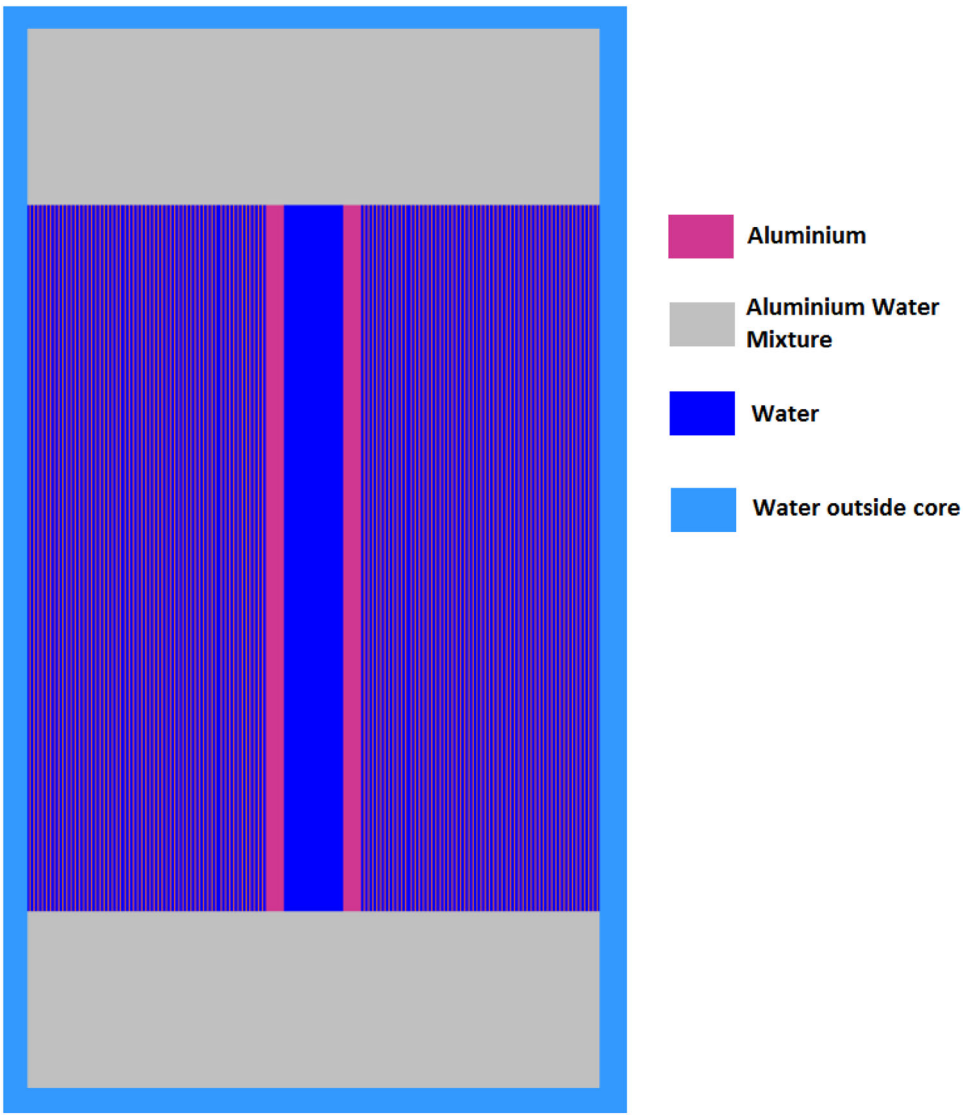

Fig. 3 (Color online) Axial cross-sectional view of the core model at mid plane

Table 2 Comparison of eigenvalues for TECDOC-233 benchmark reactor

	Fresh	BOL
Present work	1.17402 ± 0.00020	1.05478 ± 0.00019
K. S. Chaudri and S. M. Mirza	1.15296 ± 0.00026	1.05916 ± 0.00025
A. Bousbia-Salah et al.	1.17238 ± 0.00033	1.05617 ± 0.00032

In Eq. (5), there are three terms: The first term is for individual (uncoupled) FTC, the second term accounts for uncoupled VC, and the third term accounts for their coupling. Identifying the terms in Eq. (5), the following equations are obtained.

Table 3 Comparison of computed and reference results [9, 10]

Parameter	Reference value	Computed value
k_{eff} with control blades fully withdrawn	1.17404 ± 0.00314	1.17948 ± 0.00019
k_{eff} with control blades fully inserted	1.03720 ± 0.00328	1.03722 ± 0.00019
Control blade reactivity worth (pcm)	11,237	11,628
Flux trap thermal flux (n/cm^2 s)	1.93×10^{14}	1.88×10^{14}

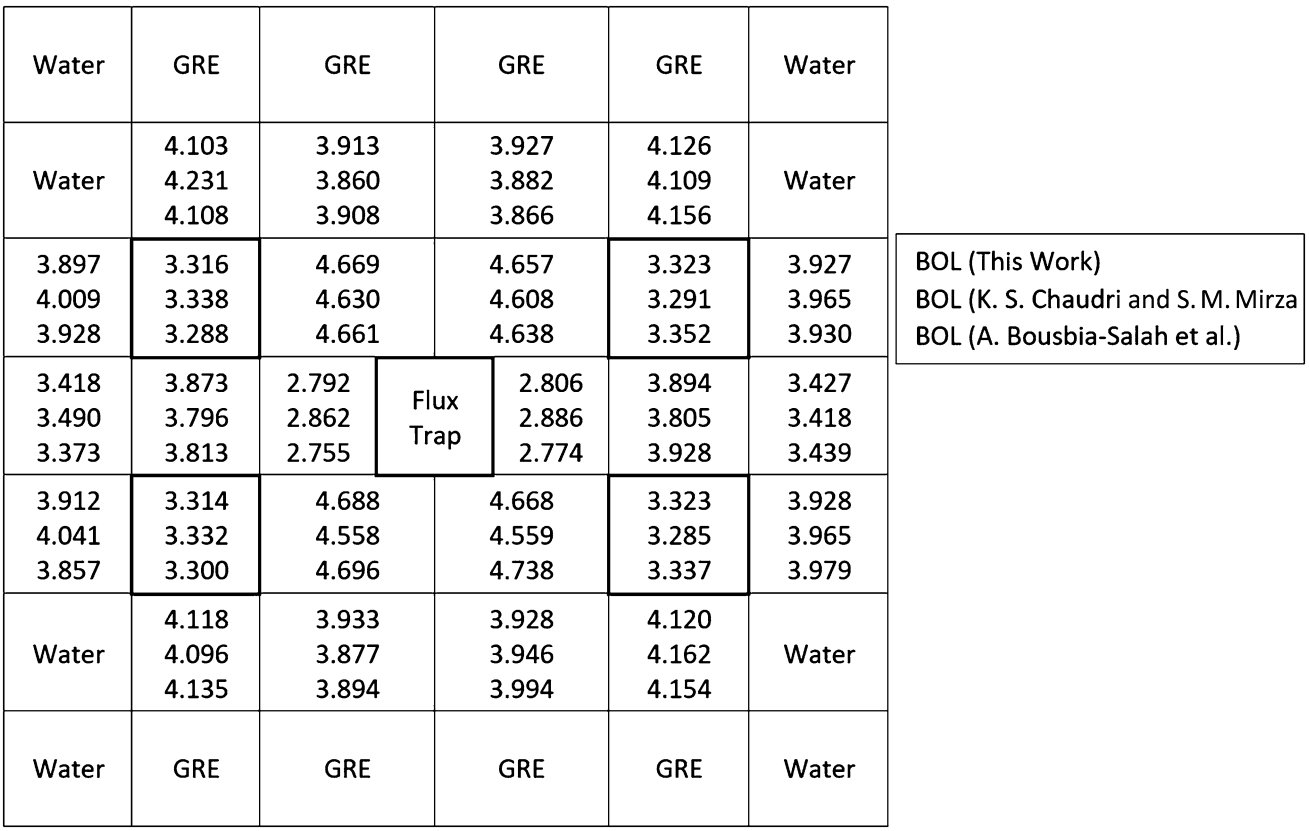

Fig. 4 Power fraction (%) comparison for the BOL core

$$\gamma_{T_F}(T_F) = \frac{\partial \rho}{\partial T_F} + \frac{1}{2}\frac{\partial^2 \rho}{\partial T_F^2}\Delta T_F + \frac{1}{6}\frac{\partial^3 \rho}{\partial T_F^3}(\Delta T_F)^2 \quad (6)$$

$$\gamma_V(V) = \frac{\partial \rho}{\partial V} + \frac{1}{2}\frac{\partial^2 \rho}{\partial V^2}\Delta V + \frac{1}{6}\frac{\partial^3 \rho}{\partial V^3}(\Delta V)^2 \quad (7)$$

$$\gamma_{T_F V}(T_F, V) = \frac{\partial^2 \rho}{\partial T_F \partial V} + \frac{1}{2}\frac{\partial^3 \rho}{\partial T_F^2 \partial V}\Delta T_F + \frac{1}{2}\frac{\partial^3 \rho}{\partial T_F \partial V^2}\Delta V \quad (8)$$

Equation (6) represents independent FTC, $\gamma_{T_F}(T_F)$. Equation (7) represents independent VC, $\gamma_V(V)$ and Eq. (8) represents coupling between FTC and VC. Using these notations, Eq. (9) is obtained [7]:

$$\Delta\rho(T_F, V) = \gamma_{T_F}(T_F)\Delta T_F + \gamma_V(V)\Delta V + \gamma_{T_F V}(T_F, V)\Delta T_F \Delta V. \quad (9)$$

Equation (10) gives the void-fraction-dependent Doppler coefficient of reactivity, and Eq. (11) shows the fuel-temperature-dependent void coefficient of reactivity [4, 7]. These are the improved or modified reactivity coefficients which incorporate the coupling effects.

$$\gamma_{T_F}(T_F, V) = \gamma_{T_F}(T_F) + \gamma_{T_F V}(T_F, V)\Delta V \quad (10)$$

$$\gamma_V(T_F, V) = \gamma_V(V) + \gamma_{T_F V}(T_F, V)\Delta T_F \quad (11)$$

Similarly, equations can be developed to couple MTC with VC, FTC with MTC, and vice versa, respectively.

The partial derivatives in Eqs. (6)–(8) are calculated using the three-point midpoint difference technique since it incurs minimum error in three-point techniques [17, 18].

The independent and modified FTC is calculated by varying T_f and keeping the void fraction constant. A total of 49 simulations are performed to calculate the void-fraction-dependent Doppler coefficient and fuel-temperature-dependent void coefficient of reactivity.

Similarly, void-fraction-dependent MTC, moderator-temperature-dependent void coefficient, fuel-temperature-dependent MTC, and moderator-temperature-dependent FTC of reactivity have been calculated using Eqs. (6)–(10).

4 OpenMC computer model

The IAEA benchmark reactor core is modeled using detailed specifications [8] and OpenMC neutron transport computer code [12] equipped with temperature-dependent nuclear cross-sectional library JEFF 3.2. The MTR benchmark reactor given in IAEA TECDOC-643 is slightly different than the one given in IAEA TECDOC-233 [8].

Table 4 Comparison of reactivity coefficients [21]

	ANL	INTERATOM	JAERI	EIR	JEN	Batan	Current work
Temperature range 20–38 °C $\left(-\Delta\rho/_{^\circ C} \times 10^5\right)$							
α_{T_M}	8.2	7.9	9.6	8.5	7.1	8.2	7.3
	(−)	(− 3.7)[a]	(+17.1)	(+3.7)	(− 13.4)	(0)	(− 11)
α_D	8.3	7.9	6.3	8.5	13.6	8	7.9
	(−)	(− 4.8)	(− 24.1)	(+2.4)	(+63.9)	(− 3.6)	(− 4.8)
α_{T_F}	2.63	2.19	1.94	2.37	3.15	2.73	2.18
	(−)	(− 16.7)	(− 26.2)	(− 9.9)	(+19.8)	(+3.8)	(− 17.1)
Temperature range 38–50 °C $\left(-\Delta\rho/_{^\circ C} \times 10^5\right)$							
α_{T_M}	8.1	7.7	9.2	8.2	6.8	7.8	7.3
	(−)	(− 4.9)	(13.6)	(1.2)	(− 16)	(− 3.7)	(− 9.9)
α_D	12.3	11.2	9.7	11.7	19.6	11.7	12.9
	(−)	(− 8.9)	(− 21.1)	(− 4.9)	(59.3)	(− 4.9)	(4.9)
α_{T_F}	2.58	2.17	1.92	2.16	3.08	2.68	2.47
	(−)	(− 15.9)	(− 25.6)	(− 16.3)	(19.4)	(3.9)	(− 4.3)
Temperature range 50–100 °C $\left(-\Delta\rho/_{^\circ C} \times 10^5\right)$							
α_{T_M}	7.8	7.5	8.2	7.8	6.2	7.2	8.4
	(−)	(− 3.8)	(5.1)	(0)	(− 20.5)	(− 7.7)	(7.7)
α_D	18.6	17.1	14.3	18.1	29.8	17.5	17.4
	(−)	(− 8.1)	(− 23.1)	(− 2.7)	(60.2)	(− 5.9)	(− 6.5)
α_{T_F}	2.52	2.12	1.89	2.19	2.94	2.55	2.45
	(−)	(− 15.9)	(− 25)	(− 13.1)	(16.7)	(1.2)	(− 2.8)
Water density range 0.998–0.958 g/cc $\left(-\Delta\rho/_{\Delta\rho_w}\right)$							
α_V	0.344	0.316	0.232	0.337	0.513	0.322	0.29
	(−)	(− 8.1)	(− 32.6)	(− 2)	(49.1)	(− 6.4)	(− 15.7)
Water density range 0.958–0.900 g/cc $\left(-\Delta\rho/_{\Delta\rho_w}\right)$							
α_V	0.305	0.28	0.237	0.299	0.49	0.289	0.319
	(−)	(− 8.2)	(− 22.3)	(− 2)	(60.7)	(− 5.2)	(4.6)

[a]Relative difference from ANL (%)

α_{T_F} Reactivity coefficient of fuel temperature

α_V Reactivity coefficient of void in terms of water density

α_D Reactivity coefficient of water density

α_{T_M} Reactivity coefficient of water temperature

The difference lies in the central flux trap. This work simulates both cores for the validation purpose. For feedback reactivity coefficients and their coupling, the MTR benchmark given in TECDOC-643 is considered. The model includes SFE, CFE (Fig. 1), flux trap, graphite, and water inside and outside the core as shown in Fig. 2.

Figure 3 shows the axial view of the core. Axially, the core is reflected by a 15 cm region containing a homogenized mixture of 80% water and 20% aluminum (Al) as volume fractions [11]. Al is added to compensate the non-fuel length of fuel plates, side plates, and the grid support structure [19, 20].

5 Model validation

The developed model is validated against reference results [9, 11] of: eigenvalues with all absorber rods in the fully withdrawn position and fully inserted position, control rod reactivity worth, power fractions, average thermal neutron flux in the central flux trap, and feedback reactivity coefficients.

The comparison of the effective multiplication factor when control absorber blades are fully out is shown in Table 2 [9, 11]. A small difference between the simulated and reference results may be attributed to the different cross-sectional libraries and uncertainties in cross sections

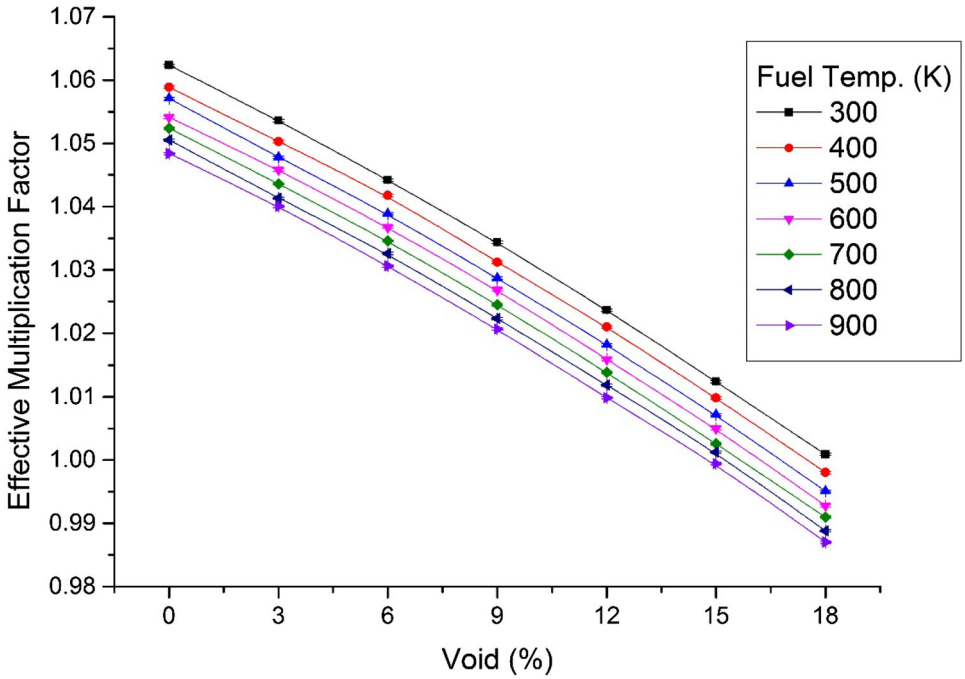

Fig. 5 Effect of fuel temperature on void coefficient of reactivity

Table 5 Calculation of modified void coefficient of reactivity

Void (%)	VC without coupling (pcm/%void)	VC with coupling (pcm/%void)	Improvement (%)
T_f = 500 K			
6	− 313.0378	− 315.3755	0.75
9	− 334.6119	− 335.9792	0.41
12	− 359.1378	− 363.2013	1.13
T_f = 600 K			
6	− 298.5897	− 301.3766	0.93
9	− 348.8928	− 353.2155	1.24
12	− 348.7683	− 351.8242	0.88
T_f = 700 K			
6	− 318.3464	− 319.7300	0.43
9	− 340.1575	− 343.9492	1.11
12	− 366.1114	− 367.8799	0.48

at different temperatures. For this work, JEFF 3.2 cross-sectional library at 311 K was employed.

The comparison of other parameters (fresh core) used for the model validation, with the reference results [9, 10] is given in Table 3. The comparison involving control blades is based on IAEA benchmark reactor as given in TECDOC-643 [10], whereas the value for thermal flux is compared with that of K. S. Chaudri and S. M. Mirza [9].

Table 3 shows that the values of k_{eff} for both cases when absorber blades are inserted and withdrawn are close to the reference values. The relative difference in control blade reactivity worth between computed and reference values is 3.5%. Similarly, the relative difference in thermal flux in the central flux trap between computed and reference results is 2.6%.

Figure 4 presents the comparison of computed power fractions in percentage (%) for the BOL core with the reference results [9, 11]. The computed values of power fractions are quite close to the reference values.

The validation of the developed OpenMC model is extended to the comparison of the simulated feedback reactivity coefficients with the published values using conventional methodology. For this purpose, the following four coefficients are considered [10, 21].

1. Change in water temperature only (α_{T_M})—water temperature is varied. Temperatures considered are 20, 38, 50, 75, and 100 °C.
2. Change in water density only (α_D)—water density is changed according to change in water temperatures of

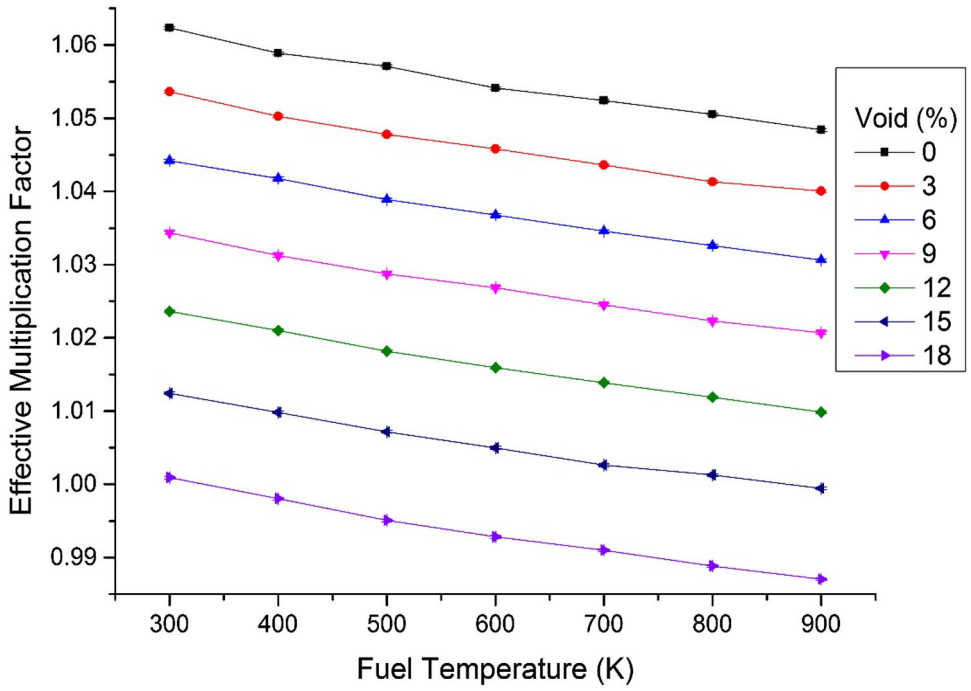

Fig. 6 Effect of void percent on FTC

Table 6 Calculation of FTC after coupling it with VC

Fuel temperature (K)	FTC without coupling (pcm/K)	FTC with coupling (pcm/K)	Improvement (%)
$V = 6\%$			
500	− 2.0447	− 2.1148	3.43
600	− 1.7360	− 1.8196	4.82
700	− 1.9206	− 1.9621	2.16
$V = 9\%$			
500	− 1.8596	− 1.9007	2.21
600	− 2.2597	− 2.3894	5.74
700	− 1.9968	− 2.1105	5.70
$V = 12\%$			
500	− 2.1141	− 2.2360	5.77
600	− 2.0445	− 2.1361	4.48
700	− 1.9681	− 2.0211	2.70

20, 38, 50, 75, and 100 °C. These are 0.998, 0.993, 0.988, 0.975, and 0.958 g/cc.

3. Change in fuel temperature only (α_{T_F})—fuel temperature has been changed as 20, 38, 50, 75, 100, and 200 °C.
4. Core void coefficient (α_V)—for this case, 10% and 20% void is introduced in the core and the reactivity variation is studied with respect to the water density (ρ_w).

Table 4 shows the comparison of the results with the reference values for various temperature ranges. The results of the current work are listed in last column.

The relative difference between the current work and ANL values is given in the table, and results lie in the acceptable range.

6 Results and discussion

Conventionally, the FTC determines the reactivity change due to fuel temperature only, whereas in actuality the change in fuel temperature also causes the change in moderator temperature [22]. This associated (or coupled) effect of moderator temperature with fuel temperature is ignored in conventional FTC calculation. The incorporation of moderator temperature effect associated with fuel

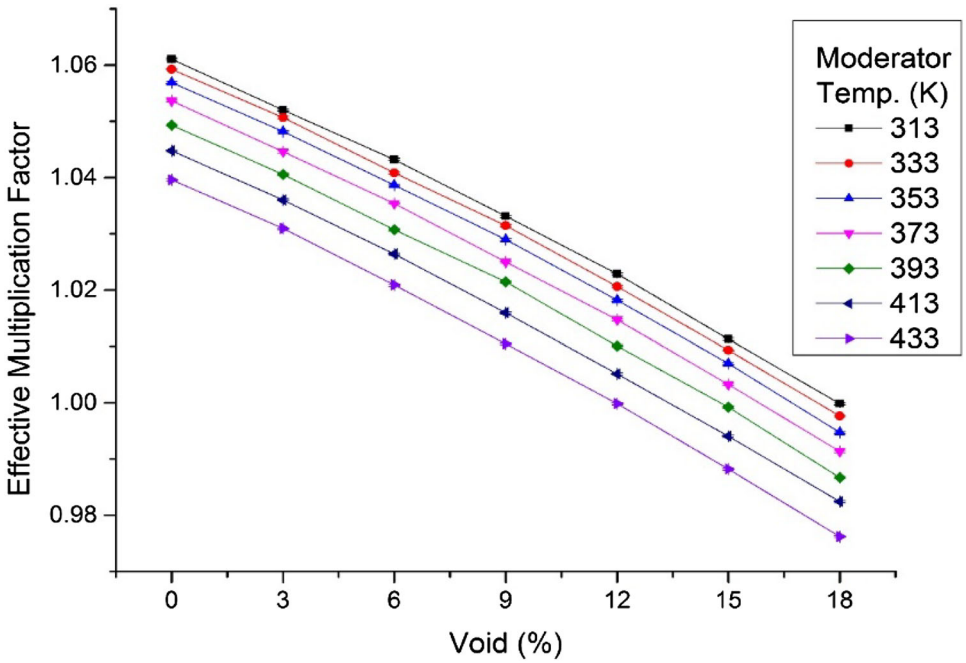

Fig. 7 Effect of moderator temperature on void coefficient

Table 7 Calculation of moderator-temperature-dependent VC

Void (%)	VC without coupling (pcm/%void)	VC with coupling (pcm/%void)	Improvement (%)
T_M = 353 K			
6	− 302.0039	− 307.8439	1.93
9	− 341.8392	− 347.2127	1.57
12	− 367.9747	− 369.9228	0.53
T_M = 373 K			
6	− 326.8422	− 328.3466	0.46
9	− 329.2672	− 335.4181	1.87
12	− 376.8175	− 381.7508	1.31
T_M = 393 K			
6	− 295.7572	− 306.2197	3.54
9	− 368.4422	− 376.3262	2.14
12	− 356.8628	− 359.2572	0.67

temperature improves the overall FTC. Similarly, MTC and VC can also be improved [22].

The validated model is used to quantify the effect of one parameter (e.g., fuel temperature) over the other (e.g., moderator temp) and vice versa. Following, coupling effects are studied.

6.1 Coupling of FTC and VC

To quantify the coupling effect of fuel temperature on VC and vice versa, the confirmed computational model is executed to simulate the eigenvalues. Figure 5 shows that with increase in the void percentage in the core, the reactivity decreases. The increase in fuel temperature further reduces the reactivity. Using the k_{eff} values and Eqs. (6)–(11), the void-fraction-dependent Doppler coefficient and fuel-temperature-dependent void coefficient of reactivity are calculated.

Table 5 depicts the results of void coefficient of reactivity. This modified VC is calculated by taking into account the effect of the fuel temperature on the reactivity feedback coefficient of the void.

Figure 6 shows effect of void percent on FTC. The decreasing trend is demonstrated since with an increase in fuel temperature, the neutron absorption in the resonance peaks increases which reduces the reactivity. The increase in void fraction in the core drastically reduces the multiplication factor. This is contrary to Fig. 5, where there is only a slight decrease in the multiplication factor with increasing fuel temperature. Thus, the effect of void on

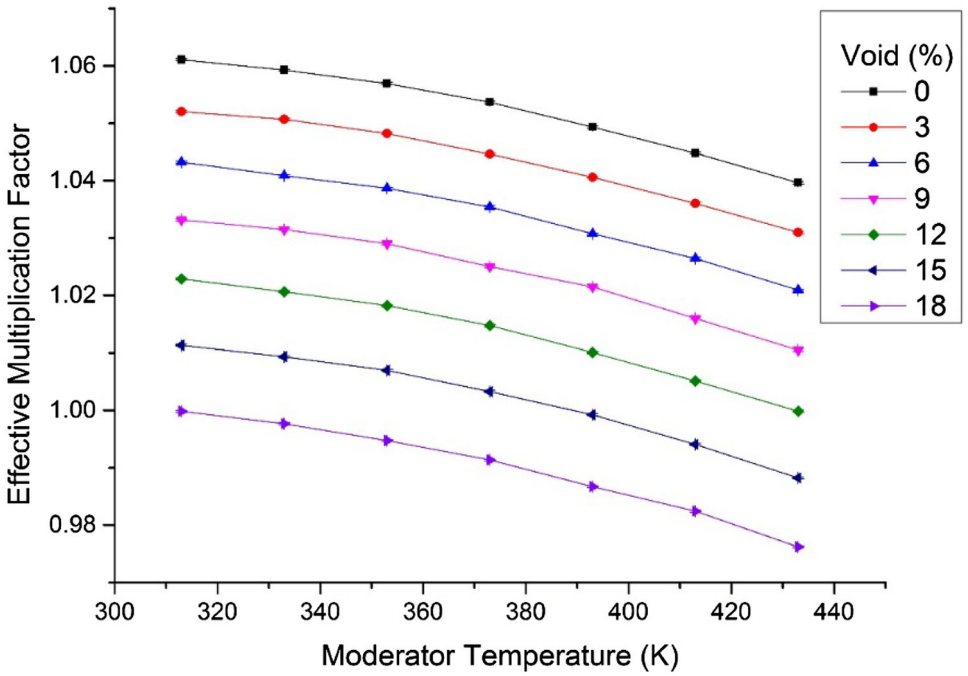

Fig. 8 Effect of void on MTC

Table 8 Calculated MTC after coupling with void coefficient

Moderator temperature (K)	MTC without coupling (pcm/K)	MTC with coupling (pcm/K)	Improvement (%)
$V = 6\%$			
353	− 15.7158	− 16.5918	5.57
373	− 21.4525	− 21.6782	1.05
393	− 20.1717	− 21.7410	7.78
$V = 9\%$			
353	− 18.4481	− 19.2541	4.37
373	− 17.0151	− 17.9377	5.42
393	− 26.6598	− 27.8423	4.44
$V = 12\%$			
353	− 17.6938	− 17.9861	1.65
373	− 22.3021	− 23.0421	3.32
393	− 23.9813	− 24.3405	1.50

FTC is more as compared to the effect of fuel temperature on VC.

Table 6 presents the individual (i.e., conventional) FTC and modified (incorporating the coupling effects of void) FTC. More improvement in FTC is seen than that of Table 5 for modified VC. This may be due to the importance of thermal neutron spectrum in the regions. The neutrons that enter the fuel region for fission are thermalized in the moderator, so thermal neutron spectrum is strongly dependent on moderator conditions as compared to fuel conditions. Thus, the effect of void on FTC is more pronounced than the effect of fuel temperature on void. The value of VC is almost two orders of magnitude larger than FTC. This is due to larger spectral hardening as a result of reduced thermalization per unit volume. This also means that the reactivity is more sensitive to change in void percent as compared to the change in fuel temperature.

6.2 Coupling of MTC and VC

To calculate the coupling effect of void fraction on MTC and vice versa, the respective eigenvalues have been simulated using the OpenMC model of the reactor core.

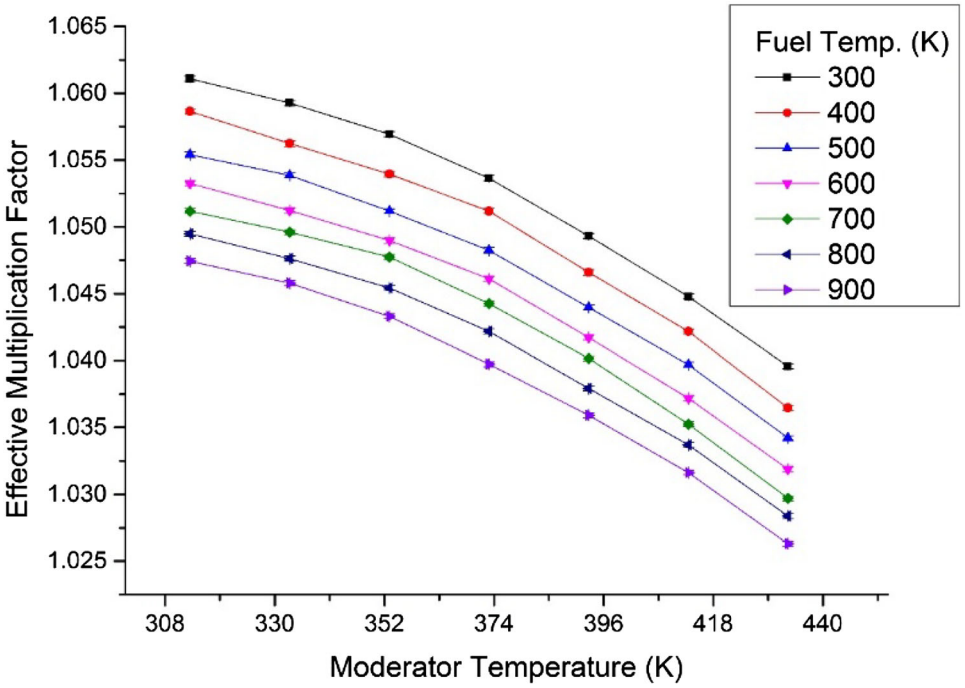

Fig. 9 Effect of fuel temperature on MTC

Table 9 Improved MTC after coupling it with FTC

Moderator temperature (K)	MTC without coupling (pcm/K)	MTC with coupling (pcm/K)	Improvement (%)
T_f = 500 K			
353	− 13.3858	− 13.8488	3.46
373	− 19.4297	− 19.5433	0.58
393	− 19.9039	− 20.2014	1.49
T_f = 600 K			
353	− 12.2769	− 12.4534	1.44
373	− 20.5348	− 20.7707	1.15
393	− 21.0367	− 21.2602	1.06
T_f = 700 K			
353	− 15.1171	− 15.2375	0.80
373	− 18.3985	− 18.5679	0.92
393	− 21.7210	− 21.8299	0.50

Figure 7 demonstrates the variation of k_{eff} with void fraction. The reactivity decrement is more pronounced in the case of high moderator temperature changes.

The calculated results of VC are listed in Table 7. There is an improvement in the value of VC since it gets more negative. Thus, taking into account the effect of the moderator temperature on reactivity feedback incurred by void formation gives an improved value of VC.

Figure 8 shows the variation of k_{eff} with the change in T_M for various void fractions. As the void fraction in the core increases, the reactivity reduces, keeping the moderator temperature constant. The reduction due to increased void formation is more than the reduction due to moderator temperature increase. This again confirms that reactivity is more sensitive to a change in void fraction as compared to the moderator temperature in LWR.

The improved MTC after the incorporating effect of void is shown in Table 8. Tables 7 and 8 show that the improvement in void-dependent MTC is greater than that of moderator-temperature-dependent VC.

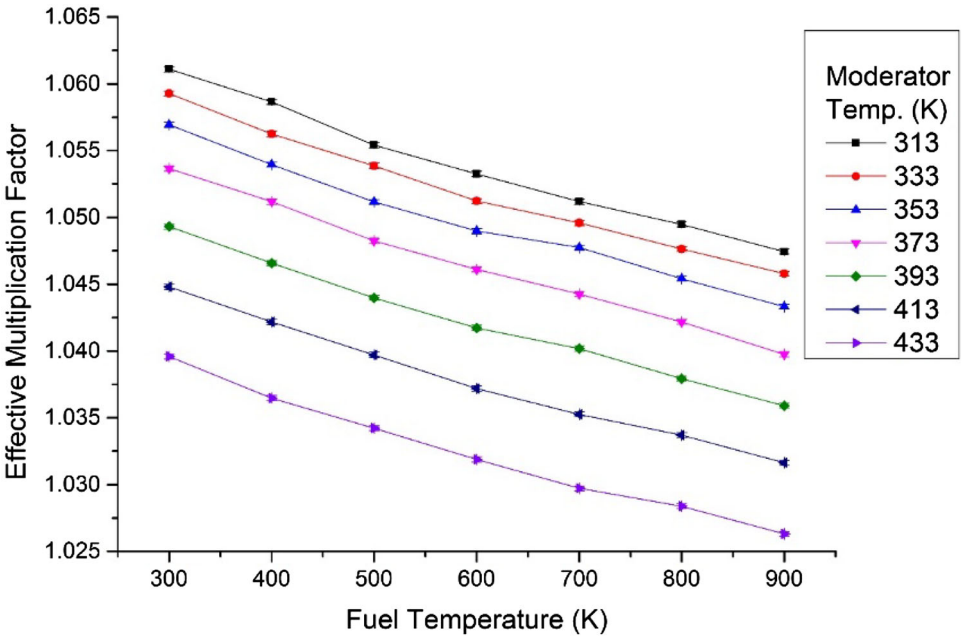

Fig. 10 Effect of moderator temperature on FTC

Table 10 Improved FTC after coupling it with MTC

Fuel temperature (K)	FTC without coupling (pcm/K)	FTC with coupling (pcm/K)	Improvement (%)
T_M = 353 K			
500	− 2.0550	− 2.1476	4.50
600	− 1.5006	− 1.5359	2.35
700	− 2.3265	− 2.3505	1.04
T_M = 373 K			
500	− 1.7517	− 1.7744	1.30
600	− 2.0876	− 2.1348	2.26
700	− 1.4585	− 1.4923	2.32
T_M = 393 K			
500	− 1.9407	− 2.0002	3.07
600	− 1.7357	− 1.7804	2.56
700	− 2.0363	− 2.0581	1.07

6.3 Coupling of MTC and FTC

The modified MTC and FTC are calculated by varying T_M keeping fuel temperature constant. The coupling effect of fuel temperature on the MTC of reactivity and vice versa is calculated. Figure 9 presents the corresponding values of k_{eff} for variations of T_M and T_F.

Table 9 demonstrates the improvements in MTC on incorporating the effect of fuel temperature. The change in moderator temperature affects the fuel temperature which in turn incurs its own feedback. Thus, the coupling of the moderator and fuel temperature is justified.

The effect of moderator temperature variation on reactivity by changing fuel temperature is shown in Fig. 10. The reactivity decrease for change in T_M from 313 to 333 K is smaller as compared to the decrease for change in T_M from 413 to 433 K. Increasing moderator temperature further will increase this decrement further.

This increase in the reactivity decrement means that the effect of MTC on fuel temperature intensifies with an increase in moderator temperature. The effect of moderator temperature on FTC leads to the improved value of FTC as shown in Table 10.

7 Conclusion

The conventional method to determine the feedback coefficient ignores the reactivity coupling effects, i.e., effect of one parameter (fuel temperature) on the others (moderator temperature, etc.) simultaneously. This methodology suggests the incorporation of coupling effects of fuel temperature, moderator temperature, and void fraction to improve the value of the corresponding feedback coefficient. The improvements in feedback reactivity coefficient account for variation in fuel, moderator temperature, and moderator void simultaneously. This work determines the improvements in feedback reactivity coefficients for IAEA 10 MW MTR benchmark research reactor as overall or improved reactivity coefficient.

The state of the art is the combination of the criticality calculations employing OpenMC transport code and application of Taylor series expansion for core reactivity. For improved or coupled FTC and VC, it was found that the effect of void on FTC is more pronounced than the effect of fuel temperature on void. This arises since thermal neutron spectrum is strongly dependent on moderator conditions as compared to fuel conditions. MTC and VC coupling dictates that reactivity is more sensitive to change in void fraction as compared to moderator temperature in LWRs. It is also found that the effect of moderator temperature on FTC is more pronounced than the effect of fuel temperature on MTC. The results conclude that the reactivity coefficient with coupled spectral effects is more accurate than the coefficient without spectral coupling. Moreover, the change in reactivity is more sensitive to void than it is to fuel temperature.

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