

Constraints on the skewness coefficient of symmetric nuclear matter within the nonlinear relativistic mean field model

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Abstract Within the nonlinear relativistic mean field (NL-RMF) model, we show that both the pressure of symmetric nuclear matter at supra-saturation densities and the maximum mass of neutron stars are sensitive to the skewness coefficient, J_0 , of symmetric nuclear matter. Using experimental constraints on the pressure of symmetric nuclear matter at supra-saturation densities from flow data in heavy-ion collisions and the astrophysical observation of a large mass neutron star PSR J0348+0432, with the former favoring a smaller J_0 while the latter favors a larger J_0 , we extract a constraint of $-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$ based on the NL-RMF model. This constraint is compared with the results obtained in other analyses.

Keywords Equation of state of nuclear matter · Heavy-ion collisions · Neutron stars

Dedicated to Joseph B. Natowitz in honour of his 80th birthday.

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1 Introduction

Determination of the equation of state (EOS) of asymmetric nuclear matter (ANM) is one of the fundamental questions in contemporary nuclear physics and astrophysics. The exact knowledge on the EOS of ANM provides important information on the in-medium nuclear effective interactions which play a central role in understanding the structure and decay properties of finite nuclei as well as the related dynamical problems in nuclear reactions ([1–17]). The EOS of ANM also plays a decisive role in understanding a number of important issues in astrophysics including the structure and evolution of neutron stars as well as the mechanism of supernova explosion ([18–26]). Conventionally, the EOS of ANM is given by the binding energy per nucleon as functions of nucleon density, ρ , and isospin asymmetry, δ , i.e., $E(\rho, \delta)$, and some bulk characteristic parameters defined at the saturation density, ρ_0 , of symmetric nuclear matter (SNM) are usually introduced to quantitatively characterize the EOS of ANM. For example, the energy, $E_0(\rho_0)$, and incompressibility, K_0 , of SNM, as well as the symmetry energy, $E_{\text{sym}}(\rho_0)$, and its slope parameter, L , are the four famous lower-order bulk characteristic parameters of EOS of ANM. These bulk parameters defined at ρ_0 provide important information on both sub- and supra-saturation density behaviors of the EOS of ANM ([27, 28]).

Based on the empirical liquid-drop-like model analyses of high precision data about nuclear masses, the $E_0(\rho_0)$ is well known to be about -16 MeV . The incompressibility has been determined to be $K_0 = 240 \pm 40 \text{ MeV}$ from analyzing experimental data of nuclear giant monopole resonances (GMR) ([1, 29–33]). For $E_{\text{sym}}(\rho_0)$ and L , the existing constraints extracted from terrestrial laboratory measurements and astrophysical observations are found to be essentially consistent with $E_{\text{sym}}(\rho_0) = 32.5 \pm 2.5 \text{ MeV}$ and $L = 55 \pm$

25 MeV (see, e.g., Refs. [34, 35]). While these lower-order bulk characteristic parameters have been relatively well determined or in significant progress, our knowledge on the higher-order bulk characteristic parameters remains very limited. Following $E_0(\rho_0)$, K_0 , $E_{\text{sym}}(\rho_0)$, and L , the next bulk characteristic parameter should be the skewness coefficient, J_0 , (also denoted as K' or Q_0 in some literature) of SNM, which is related to the third-order density derivative of the binding energy per nucleon of SNM at ρ_0 . The higher-order bulk characteristic parameter J_0 is expected to be important for the high density behaviors of nuclear matter EOS and thus may play an essential role in heavy-ion collisions (HIC), the structure and evolution of neutron stars, supernova explosion, and gravitational wave radiation from merging of compact stars. To our best knowledge, there is very little experimental information on the J_0 parameter, and it is thus of great interest and critical importance to constrain the J_0 parameter, which is the main motivation of the present work.

Within the nonlinear relativistic mean field (RMF) model, we demonstrate in this work that the pressure of SNM at supra-saturation densities and the maximum mass of neutron stars provide good probes of the skewness coefficient, J_0 . In particular, combining the experimental constraints on the pressure of SNM at supra-saturation densities from flow data in HIC and the recent astrophysical observation of a large mass neutron star, PSR J0348+0432, one can obtain a strong constraint on the J_0 parameter.

2 The skewness coefficient J_0 in nonlinear RMF model

2.1 Nuclear matter characteristic parameters

The EOS of isospin asymmetric nuclear matter, namely $E(\rho, \delta)$, can be expanded as a power series of even-order terms in δ as

$$E(\rho, \delta) \simeq E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + \mathcal{O}(\delta^4), \tag{1}$$

where $E_0(\rho) = E(\rho, \delta = 0)$ is the EOS of symmetric nuclear matter, and the symmetry energy is expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \Big|_{\delta=0}. \tag{2}$$

Around the saturation density, ρ_0 , the $E_0(\rho)$ can be expanded, e.g., up to third order in density, as,

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + \mathcal{O}(\chi^4), \tag{3}$$

where $\chi = (\rho - \rho_0)/3\rho_0$ is a dimensionless variable characterizing the deviations of the density from the saturation

density, ρ_0 . The first term $E_0(\rho_0)$ on the right-hand side of Eq. (3) is the binding energy per nucleon in SNM at ρ_0 and the coefficients of other terms are

$$K_0 = 9\rho_0^2 \frac{d^2 E_0(\rho)}{d\rho^2} \Big|_{\rho=\rho_0}, \tag{4}$$

$$J_0 = 27\rho_0^3 \frac{d^3 E_0(\rho)}{d\rho^3} \Big|_{\rho=\rho_0}, \tag{5}$$

where K_0 is the well-known incompressibility coefficient of SNM and J_0 is the skewness coefficient of SNM, i.e., the third-order incompressibility coefficient of SNM ([27, 28]).

Similarly, one can expand the $E_{\text{sym}}(\rho)$ around an arbitrary reference density, ρ_r , as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_r) + L(\rho_r)\chi_r + \mathcal{O}(\chi_r^2), \tag{6}$$

with $\chi_r = (\rho - \rho_r)/3\rho_r$, and the slope parameter of the symmetry energy at ρ_r is expressed as [36]

$$L(\rho_r) = 3\rho_r \frac{dE_{\text{sym}}(\rho)}{d\rho} \Big|_{\rho=\rho_r}. \tag{7}$$

For $\rho_r = \rho_0$, the $L(\rho_r)$ is reduced to the conventional slope parameter $L \equiv 3\rho_0 dE_{\text{sym}}(\rho)/d\rho|_{\rho=\rho_0}$.

If δ and χ are assumed to be small quantities on the same order, nuclear matter bulk characteristic parameters can then be classified accordingly in different orders. For example, L and J_0 are on the same order-3, i.e., $\delta^2\chi$ for L and $\delta^0\chi^3$ for J_0 . In this sense, $E_0(\rho_0)$ is on the order-0, and, K_0 and $E_{\text{sym}}(\rho_0)$ are on the order-2. To see the role of J_0 in the EOS of SNM, one can rewrite Eq. (3) in a slightly different form as

$$E_0(\rho) \simeq E_0(\rho_0) + \frac{1}{2}K_0\chi^2 \left(1 + \frac{\chi J_0}{3K_0}\right). \tag{8}$$

Assuming J_0 has roughly the same magnitude as K_0 , one can see that the contribution from the J_0 term to the EOS of SNM becomes comparable with that from the K_0 term if the baryon density is larger than about $3\rho_0$, corresponding to the typical densities inside a neutron star. On the other hand, the J_0 term plays a minor role for the EOS of SNM at subsaturation densities relevant for nuclear structure properties. As we will see later, the pressure of SNM at supra-saturation densities and the maximum mass of neutron stars indeed display strong sensitivity on the J_0 parameter.

2.2 Nuclear matter characteristic parameters in nonlinear RMF model

The nonlinear RMF model has made great success during the last decades in describing many nuclear

phenomena (see, e.g., [37–50]). In the following, we briefly describe the nonlinear RMF model that we shall adopt in this work and present some useful expressions of nuclear matter characteristic parameters, especially the skewness coefficient, J_0 . The interacting Lagrangian of the nonlinear RMF model supplemented with couplings between the isoscalar and the isovector mesons reads ([51–57])

$$\begin{aligned} \mathcal{L} = & \bar{\psi} [\gamma_\mu (i\partial^\mu - g_\omega \omega^\mu - g_\rho \rho^\mu \cdot \tau) - (M - g_\sigma \sigma)] \psi \\ & - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{4} c_\omega (g_\omega \omega_\mu \omega^\mu)^2 \\ & + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu - \frac{1}{4} \rho_{\mu\nu} \cdot \rho^{\mu\nu} \\ & + \frac{1}{2} g_\rho^2 \rho_\mu \cdot \rho^\mu \Lambda_V g_\omega^2 \omega_\mu \omega^\mu, \end{aligned} \tag{9}$$

where $\omega_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\rho_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$ are strength tensors for ω field and ρ field, respectively. ψ , σ , ω_μ , ρ_μ are nucleon field, isoscalar-scalar field, isoscalar-vector field, and isovector-vector field, respectively, and the arrows denote the vector in isospin space; $U(\sigma) = b_\sigma M (g_\sigma \sigma)^3 / 3 + c_\sigma (g_\sigma \sigma)^4 / 4$ is the self-interaction term for σ field. Λ_V represents the coupling constant between the isovector ρ meson and the isoscalar ω meson, and it is important for the description of the density dependence of the symmetry energy. In addition, M is the nucleon mass and m_σ , m_ω , m_ρ are masses of mesons.

In the mean field approximation, after neglecting effects of fluctuation and correlation, meson fields are replaced by their expectation values, i.e., $\bar{\sigma} \rightarrow \sigma$, $\bar{\omega}_0 \rightarrow \omega_\mu$, $\bar{\rho}_0^{(3)} \rightarrow \rho_\mu$, where subscript “0” indicates zeroth component of the four-vector, superscript “(3)” indicates third component of the isospin. Furthermore, we also use in this work the non-sea approximation which neglects the effect due to negative energy states in the Dirac sea. The mean field equations are then expressed as

$$m_\sigma^2 \bar{\sigma} = g_\sigma [\rho_S - b_\sigma M (g_\sigma \bar{\sigma})^2 - c_\sigma (g_\sigma \bar{\sigma})^3], \tag{10}$$

$$m_\omega^2 \bar{\omega}_0 = g_\omega \left[\rho - c_\omega (g_\omega \bar{\omega}_0)^3 - \Lambda_V g_\omega \bar{\omega}_0 (g_\rho \bar{\rho}_0^{(3)})^2 \right], \tag{11}$$

$$m_\rho^2 \bar{\rho}_0^{(3)} = g_\rho [\rho_p - \rho_n - \Lambda_V g_\rho \bar{\rho}_0^{(3)} (g_\omega \bar{\omega}_0)^2], \tag{12}$$

where

$$\rho = \langle \bar{\psi} \gamma^0 \psi \rangle = \rho_n + \rho_p, \rho_S = \langle \bar{\psi} \psi \rangle = \rho_{S,n} + \rho_{S,p}, \tag{13}$$

are the baryon density and scalar density, respectively, with the latter given by

$$\begin{aligned} \rho_{S,J} &= \frac{2}{(2\pi)^3} \int_0^{k_F^J} d\mathbf{k} \frac{M^*}{\sqrt{|\mathbf{k}|^2 + M^{*2}}} \\ &= \frac{M^*}{2\pi^2} \left[k_F^J \mathcal{E}_F^{J*} - M^{*2} \ln \left(\frac{k_F^J + \mathcal{E}_F^{J*}}{M^*} \right) \right], J = p, n. \end{aligned} \tag{14}$$

In the above expression, we have $\mathcal{E}_F^{J*} = \sqrt{k_F^{J2} + M^{*2}}$ and the nucleon Dirac mass is defined as

$$M^* \equiv M_{\text{dirac}}^* = M - g_\sigma \bar{\sigma}. \tag{15}$$

$k_F^J = k_F (1 + \tau_3^J \delta)^{1/3}$ is the Fermi momentum with $\tau_3^n = +1$ for neutrons and $\tau_3^p = -1$ for protons, and $k_F = (3\pi^2 \rho / 2)^{1/3}$ is the Fermi momentum for SNM at ρ .

The energy–momentum density tensor for the interacting Lagrangian density in Eq. (9) can be written as

$$\mathcal{T}^{\mu\nu} = \bar{\psi} i \gamma^\mu \partial^\nu \psi + \partial^\mu \sigma \partial^\nu \sigma - \omega^{\mu\lambda} \partial^\nu \omega_\lambda - \bar{\rho}^{\mu\lambda} \partial^\nu \bar{\rho}_\lambda - \mathcal{L} g^{\mu\nu}, \tag{16}$$

where $g_{\mu\nu} = (+, -, -, -)$ is the Minkowski metric. In the mean field approximation, the mean value of the time (zero) component of the energy–momentum density tensor is the energy density of the nuclear matter system, i.e.,

$$\begin{aligned} \varepsilon &= \langle \mathcal{T}^{00} \rangle \\ &= \varepsilon_{\text{kin}}^n + \varepsilon_{\text{kin}}^p + \frac{1}{2} \left[m_\sigma^2 \bar{\sigma}^2 + m_\omega^2 \bar{\omega}_0^2 + m_\rho^2 (\bar{\rho}_0^{(3)})^2 \right] \\ &+ \frac{1}{3} b_\sigma (g_\sigma \bar{\sigma})^3 + \frac{1}{4} c_\sigma (g_\sigma \bar{\sigma})^4 + \frac{3}{4} c_\omega (g_\omega \bar{\omega}_0)^4 \\ &+ \frac{3}{2} (g_\rho \bar{\rho}_0^{(3)})^2 \Lambda_V (g_\omega \bar{\omega}_0)^2, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \varepsilon_{\text{kin}}^J &= \frac{2}{(2\pi)^3} \int_0^{k_F^J} d\mathbf{k} \sqrt{|\mathbf{k}|^2 + M^{*2}} \\ &= \frac{1}{\pi^2} \int_0^{k_F^J} k^2 dk \sqrt{k^2 + M^{*2}} \\ &= \frac{1}{4} [3\mathcal{E}_F^{J*} \rho_J + M^* \rho_{S,J}], J = p, n, \end{aligned} \tag{18}$$

is the kinetic part of the energy density. Similarly, the mean value of space components of the energy–momentum density tensor corresponds to the pressure of the system, i.e.,

$$\begin{aligned} P &= \frac{1}{3} \sum_{j=1}^3 \langle \mathcal{T}^{jj} \rangle = P_{\text{kin}}^n + P_{\text{kin}}^p \\ &- \frac{1}{2} \left[m_\sigma^2 \bar{\sigma}^2 - m_\omega^2 \bar{\omega}_0^2 - m_\rho^2 (\bar{\rho}_0^{(3)})^2 \right] \\ &- \frac{1}{3} b_\sigma (g_\sigma \bar{\sigma})^3 - \frac{1}{4} c_\sigma (g_\sigma \bar{\sigma})^4 + \frac{1}{4} c_\omega (g_\omega \bar{\omega}_0)^4 \\ &+ \frac{1}{2} (g_\rho \bar{\rho}_0^{(3)})^2 \Lambda_V (g_\omega \bar{\omega}_0)^2, \end{aligned} \tag{19}$$

where the kinetic part of pressure is given by

$$P_{\text{kin}}^J = \frac{1}{3\pi^2} \int_0^{k_F^J} dk \frac{k^4}{\sqrt{k^2 + M^{*2}}}, \quad J = p, n. \quad (20)$$

The EOS of ANM can be calculated through the energy density, $\varepsilon(\rho, \delta)$, by

$$E(\rho, \delta) = \frac{\varepsilon(\rho, \delta)}{\rho} - M. \quad (21)$$

The EOS of SNM is just $E_0(\rho) \equiv E(\rho, \delta = 0)$, and the characteristic parameters K_0 and J_0 can be obtained from the following expressions

$$K_0(\rho) \equiv 9\rho^2 \frac{d^2 E_0}{d\rho^2} = -\frac{9\rho g_\sigma^2 M^{*2}}{Q_\sigma \mathcal{E}_F^{*2}} + \frac{9\rho g_\omega^2}{Q_\omega} + \frac{3k_F^2}{\mathcal{E}_F^*} - 6L_0(\rho), \quad (22)$$

$$J_0(\rho) \equiv 27\rho^3 \frac{d^3 E_0}{d\rho^3} = -\frac{3k_F^2}{\mathcal{E}_F^*} - \frac{3k_F^4}{\mathcal{E}_F^{*3}} + \frac{27g_\sigma^2 M^{*2} \rho^2}{Q_\sigma \mathcal{E}_F^{*3}} \\ \times \left(\frac{3\pi^2}{2k_F \mathcal{E}_F^*} + \frac{2g_\sigma^2}{Q_\sigma} - \frac{g_\sigma M^* \eta}{Q_\sigma^2} - \frac{2g_\sigma^2 M^{*2}}{Q_\sigma \mathcal{E}_F^{*2}} + \frac{\mathcal{E}_F^* \phi}{2Q_\sigma} \right) \\ - \frac{162c_\omega g_\omega^7 \bar{\omega}_0 \rho^2}{Q_\omega^3} - 9K_0(\rho), \quad (23)$$

with

$$L_0(\rho) \equiv 3\rho \frac{dE_0}{d\rho} = 3 \left[\frac{\mathcal{E}_F^*}{4} - \frac{M^* \rho_S}{4\rho} + g_\omega \bar{\omega}_0 \right. \\ \left. - \frac{1}{\rho} \left(\frac{1}{2} m_\sigma^2 \bar{\sigma}^2 + U(\bar{\sigma}) + \frac{1}{2} m_\omega^2 \bar{\omega}_0^2 + \frac{3}{4} c_\omega g_\omega^4 \bar{\omega}_0^4 \right) \right]. \quad (24)$$

In the above expressions, we have

$$Q_\sigma = m_\sigma^2 + g_\sigma^2 \left(\frac{3\rho_S}{M^*} - \frac{3\rho}{\mathcal{E}_F^*} \right) + 2b_\sigma M g_\sigma^3 \bar{\sigma} + 3c_\sigma g_\sigma^4 \bar{\sigma}^2, \quad (25)$$

$$Q_\omega = m_\omega^2 + 3c_\omega g_\omega^4 \bar{\omega}_0^2, \quad (26)$$

and

$$\eta = 3g_\sigma^3 \left(\frac{2\rho_S}{M^{*2}} - \frac{3\rho}{M^* \mathcal{E}_F^*} + \frac{M^* \rho}{\mathcal{E}_F^{*3}} \right) - 2b_\sigma M g_\sigma^3 - 6c_\sigma g_\sigma^4 \bar{\sigma}, \quad (27)$$

$$\phi = \frac{2g_\sigma^2 k_F^2}{\mathcal{E}_F^{*3}}, \quad (28)$$

with $\mathcal{E}_F^* = (k_F^2 + M^{*2})^{1/2}$. To our best knowledge, Eq. (23) gives, for the first time [58], the analytical expression of the J_0 parameter in the nonlinear RMF model. In addition, we would like to point out that the general expression for

$E_{\text{sym}}(\rho)$ and $L(\rho)$ in the nonlinear RMF model has been derived in Ref. [59].

3 Results and discussions

For the Lagrangian in Eq. (9), the properties of infinite nuclear matter is uniquely determined by $f_\sigma = g_\sigma/m_\sigma, b_\sigma, c_\sigma, f_\omega = g_\omega/m_\omega, c_\omega, f_\rho = g_\rho/m_\rho, \Lambda_V$, and M . If the nucleon mass in vacuum is set to be $M = 939$ MeV, one then has seven total parameters to determine the properties of infinite nuclear matter in the nonlinear RMF model. Following the correlation analysis method proposed in Ref. [60] within the Skyrme–Hartree–Fock (SHF) approach, instead of directly using the seven microscopic parameters, i.e., $f_\sigma, b_\sigma, c_\sigma, f_\omega, c_\omega, f_\rho$, and Λ_V , one can determine their values explicitly in terms of seven macroscopic quantities, i.e., $\rho_0, E_0(\rho_0), K_0, J_0, M_{\text{dirac}}^{*0} \equiv M_{\text{dirac}}^*(\rho_0), E_{\text{sym}}(\rho_c)$, and $L(\rho_c)$ where ρ_c is the cross-density whose value is fixed in this work at 0.11 fm^{-3} ([36]). Then, by varying individually these macroscopic quantities within their known ranges, one can examine transparently the correlation of nuclear matter properties with each individual macroscopic quantity. Recently, this simple correlation analysis method has been successfully applied to study the neutron skin ([36, 60]) and giant resonances of finite nuclei ([33, 61]), the higher-order bulk characteristic parameters of ANM ([28]), and the relationship between the nuclear matter symmetry energy and the symmetry energy coefficient in the mass formula ([62]). We would like to point out although the seven macroscopic quantities defined above coherently act on the maximum mass of neutron stars and the pressure of the SNM, they are independent with each other in our analysis since we vary one quantity by keeping other six quantities fixed. This is one of the main advantages of our approach since the physics of these macroscopic quantities is different.

To examine the correlation of pressure of SNM at supra-saturation densities with each macroscopic quantity, we show in Fig. 1 the pressure of SNM $P(\rho)$ at $\rho = 3\rho_0$ from the nonlinear RMF model based on the FSUGold interaction ([55]) by varying individually $\rho_0, E_0(\rho_0), M_{\text{dirac}}^{*0}, K_0$, and J_0 within their empirical uncertain ranges, namely varying one quantity at a time while keeping all others at their default values in FSUGold for which we have $\rho_0 = 0.148 \text{ fm}^{-3}, E_0(\rho_0) = -16.3 \text{ MeV}, M_{\text{dirac}}^{*0} = 0.61M, K_0 = 230 \text{ MeV}, J_0 = -522.6 \text{ MeV}, E_{\text{sym}}(\rho_c) = 27.11 \text{ MeV}$, and $L(\rho_c) = 49.97 \text{ MeV}$. It should be mentioned that the pressure of SNM is independent of the values of $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$. It is seen from Fig. 1 that the pressure of SNM $P(\rho)$ at $\rho = 3\rho_0$ increases with $\rho_0, M_{\text{dirac}}^{*0}, K_0$, and J_0 while it decreases with $E_0(\rho_0)$. In particular, the pressure of SNM $P(\rho)$ at $\rho = 3\rho_0$ displays a specially strong correlation with J_0 . We note that

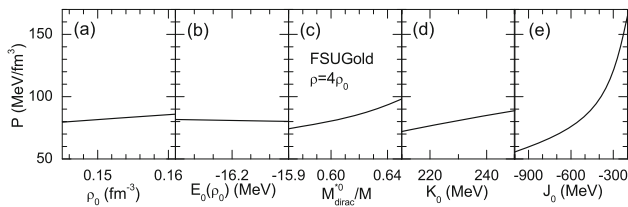


Fig. 1 Pressure of SNM at $\rho = 3\rho_0$ from the nonlinear RMF model based on the FSUGold interaction by varying individually ρ_0 (a), $E_0(\rho_0)$ (b), M_{dirac}^{*0} (c), K_0 (d), and J_0 (e)

the pressure of SNM $P(\rho)$ at other supra-saturation densities exhibits similar correlations with ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , K_0 , and J_0 . These features indicate that the pressure of SNM at supra-saturation densities is sensitive to the J_0 value, and thus the experimental constraints on the pressure of SNM at supra-saturation densities may provide important information on the J_0 value.

Since the pressure of SNM at supra-saturation densities is sensitive to the J_0 value, the maximum mass, M_{max} , of static neutron stars is also expected to be sensitive to the J_0 value. The mass and radius of static neutron stars can be obtained from solving the Tolman–Oppenheimer–Volkoff (TOV) equations with a given neutron star matter EOS. A neutron star generally contains core, inner crust, and outer crust from the center to surface. In this work, for the core where the baryon density is larger than the core–crust transition density, ρ_t , we use the EOS of β -stable and charge neutral, npe μ matter obtained from the nonlinear RMF model. In the inner crust with densities between ρ_{out} and ρ_t where the nuclear pastas may exist, we construct its EOS (pressure, P , as a function of energy density, \mathcal{E}) according to $P = a + b\mathcal{E}^{4/3}$ because of our poor knowledge about its EOS from first principle ([63–65]). The $\rho_{\text{out}} = 2.46 \times 10^{-4} \text{ fm}^{-3}$ is the density separating the inner from the outer crust. The constants a and b are then easily determined by the pressure and energy density at ρ_t and ρ_{out} [64, 65]. In this work, the ρ_t is determined self-consistently within the nonlinear RMF model using the thermodynamical method (see, e.g., [57] for the details). In the outer crust with $6.93 \times 10^{-13} \text{ fm}^{-3} < \rho < \rho_{\text{out}}$, we use the EOS of BPS ([66, 67]), and in the region of 4.73×10^{-15}

$\text{fm}^{-3} < \rho < 6.93 \times 10^{-13} \text{ fm}^{-3}$, we use the EOS of FMT ([66]).

Similarly, as in Fig. 1, we plot in Fig. 2 the maximum mass, M_{max} , of static neutron stars from the nonlinear RMF model based on the FSUGold interaction by varying individually ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , K_0 , J_0 , $E_{\text{sym}}(\rho_c)$, and $L(\rho_c)$ within their empirical uncertain ranges. Indeed, one can see that the M_{max} displays a very strong positive correlation with the J_0 parameter. In addition, the M_{max} exhibits weak positive correlation with the M_{dirac}^{*0} and K_0 , and weak negative correlation with the ρ_0 and $E_0(\rho_0)$. It is interesting to see that the M_{max} is essentially independent of the values of $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$, implying that, in the nonlinear RMF model, the M_{max} is basically determined by the isoscalar part of the nuclear matter EOS. Since the seven microscopic parameters change with the macroscopic quantities, it is thus not surprising to see that the maximum mass of a neutron star based on the FSUGold interaction by varying macroscopic quantities may be totally different from the default one from FSUGold, which is about $1.74M_{\odot}$. These features indicate that the observed largest mass of neutron stars may put important constraint on the J_0 value. We would like to point out that the interaction FSUGold is only used in Figs. 1 and 2 as a reference for the correlation analyses and using other RMF interactions will not change our conclusion.

Experimentally, the pressure of SNM at supra-saturation densities (from $2\rho_0$ to about $5\rho_0$) has been constrained by measurements of collective flows in HIC ([3]), which is shown as a band in the left window of Fig. 3. In the nonlinear RMF model, if one only changes the J_0 value while the other six macroscopic quantities are kept at their values in the FSUGold interaction, one can find that the J_0 value should be in the range of $-985 \text{ MeV} \leq J_0 \leq -327 \text{ MeV}$ to be consistent with the flow data in HIC ([3]). However, keeping the other six macroscopic quantities at their values in the FSUGold interaction is obviously a strong assumption because the extraction of the J_0 value from the flow data in HIC will also depend on the values of ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , and K_0 , which can be varied within their empirical uncertain ranges. For the nonlinear RMF model, we use, in this work, the following empirical uncertain ranges for

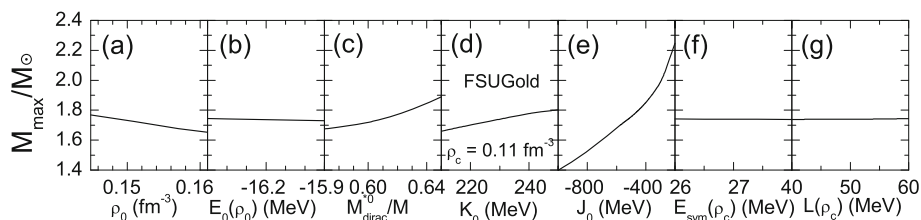


Fig. 2 (Color online) Maximum mass of static neutron stars from the nonlinear RMF model based on the FSUGold interaction by varying individually ρ_0 (a), $E_0(\rho_0)$ (b), M_{dirac}^{*0} (c), K_0 (d), J_0 (e), $E_{\text{sym}}(\rho_c)$ (f), and $L(\rho_c)$ (g)

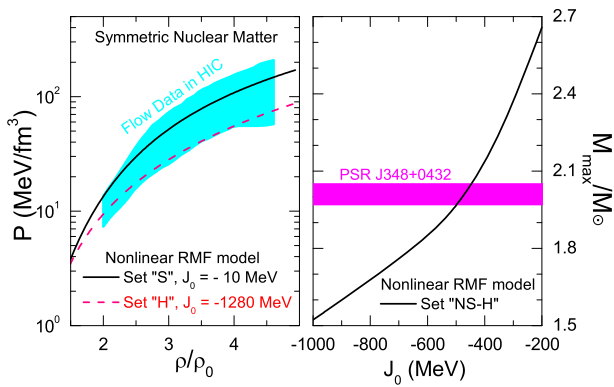


Fig. 3 (Color online) Left window: Pressure of SNM as a function of baryon density. The solid (dashed) line is the prediction from the nonlinear RMF model with $J_0 = -10$ (-1280) MeV and the set “S (H)” for ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , and K_0 . The band represents the constraints from flow data in HIC [3]. Right window: The maximum mass of static neutron stars as a function of J_0 in the nonlinear RMF model with the set “NS-H” for ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , K_0 , $E_{\text{sym}}(\rho_c)$, and $L(\rho_c)$. The band represents mass $2.01 \pm 0.04M_\odot$ for PSR J0348+0432 [68]

these macroscopic quantities, i.e., $\rho_0 = 0.153 \pm 0.008 \text{ fm}^{-3}$, $E_0(\rho_0) = -16.2 \pm 0.3 \text{ MeV}$, $M_{\text{dirac}}^{*0}/M = 0.61 \pm 0.04$, and $K_0 = 230 \pm 20 \text{ MeV}$, which represent the typical uncertain ranges known or predicted from different interactions in the nonlinear RMF model ([56]).

Based on the pressure of SNM constrained by flow data in HIC ([3]), to extract the upper limit of the J_0 value, one should use the values of ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , and K_0 that make the resulting pressure of SNM as small as possible when J_0 is fixed. This can be obtained by using $\rho_0 = 0.145 \text{ fm}^{-3}$, $E_0(\rho_0) = -15.9 \text{ MeV}$, $M_{\text{dirac}}^{*0}/M = 0.57$, and $K_0 = 210 \text{ MeV}$, denoted as set “S”, since the pressure of SNM $P(\rho/\rho_0)$ at supra-saturation densities increases with ρ_0 , M_{dirac}^{*0} , and K_0 while decreases with $E_0(\rho_0)$, as shown in Fig. 1. With the set “S” for ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , and K_0 , one can find the upper limit of $J_0 = -10 \text{ MeV}$ for the J_0 value, which is indicated by a solid line in the left window of Fig. 3. For $J_0 > -10 \text{ MeV}$, the model would over-predict the pressure of SNM constrained by flow data in HIC ([3]). Similarly, one can obtain the lower limit of the J_0 value by using the values of ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , and K_0 that make the resulting pressure of SNM as large as possible when J_0 is fixed, and this can be obtained with $\rho_0 = 0.161 \text{ fm}^{-3}$, $E_0(\rho_0) = -16.5 \text{ MeV}$, $M_{\text{dirac}}^{*0}/M = 0.65$, and $K_0 = 250 \text{ MeV}$, denoted as set “H”. Using the set “H” for ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , and K_0 , one can extract the lower limit of $J_0 = -1280 \text{ MeV}$, which is indicated by dashed line in the left window of Fig. 3. The model would under-predict the pressure of SNM constrained by flow data in HIC ([3]) if $J_0 < -1280 \text{ MeV}$. Therefore, from the

pressure of SNM constrained by flow data in HIC ([3]), one can extract the constraint of $-1280 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$.

Recently, a new neutron star, PSR J0348+0432, with a mass of $2.01 \pm 0.04M_\odot$ was discovered ([68]), and this neutron star is only the second pulsar with a precisely determined mass around $2M_\odot$ after PSR J1614-2230 ([69]) and sets a new record of the maximum mass of neutron stars. The lower mass limit of $1.97M_\odot$ for PSR J0348+0432 thus may set a lower limit of the J_0 value below which the model cannot predict a neutron star with mass equal or above $1.97M_\odot$. To extract the lower limit of the J_0 value from the observed heaviest neutron star, PSR J0348+0432, one can use the values of ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , K_0 , $E_{\text{sym}}(\rho_c)$, and $L(\rho_c)$ that make the resulting maximum mass of neutron stars as large as possible when J_0 is fixed, and from Fig. 2 this can be obtained with $\rho_0 = 0.145 \text{ fm}^{-3}$, $E_0(\rho_0) = -16.5 \text{ MeV}$, $K_0 = 250 \text{ MeV}$, $M_{\text{dirac}}^{*0}/M = 0.65$, $E_{\text{sym}}(\rho_c) = 26 \text{ MeV}$, and $L(\rho_c) = 60 \text{ MeV}$, denoted as set “NS-H”. This leads to a lower limit of $J_0 = -494 \text{ MeV}$ for the J_0 value as shown in the right window of Fig. 3 where the maximum mass of neutron stars is plotted as a function of J_0 when the other six macroscopic quantities are fixed at their values as in set “NS-H”. For $J_0 < -494 \text{ MeV}$, the maximum mass of static neutron stars predicted in the nonlinear RMF model would be always smaller than $1.97M_\odot$. It should be pointed out that here the interior of neutron stars has been assumed to be $npe\mu$ matter. New degrees of freedom, such as hyperons or/and quark matter that could be present in the interior of neutron stars, usually soften the EOS of neutron star matter and thus a larger J_0 value would be necessary to obtain a neutron star with mass of $1.97M_\odot$. Therefore, including the new degrees of freedom in neutron stars will be consistent with the constraint of $J_0 \geq -494 \text{ MeV}$.

Combining the constraint of $-1280 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$ from the pressure of SNM constrained by flow data in HIC ([3]) which favors a smaller J_0 value and the constraint of $J_0 \geq -494 \text{ MeV}$ from the recently discovered heaviest neutron star PSR J0348+0432 ([68]) which favors a larger J_0 value, one can extract the following constraint for the J_0 parameter

$$-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}. \tag{29}$$

It should be emphasized that the constraint $-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$ represents a conservative extraction based on flow data in HIC ([3]) and the recently discovered heaviest neutron star, PSR J0348+0432 ([68]) in the nonlinear RMF model. This is because we have not considered the possible correlations that existed among the ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , K_0 , $E_{\text{sym}}(\rho_c)$, and $L(\rho_c)$, and have simply set simultaneously their values in the boundary of their

empirical uncertain ranges. Considering the correlations possibly existed among the ρ_0 , $E_0(\rho_0)$, M_{dirac}^{*0} , K_0 , $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ should further narrow the constraint $-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$, and it will be interesting to see the quantitative constraint on the J_0 parameter based on the data of finite nuclei, neutron stars, and heavy-ion collisions using the exhaustive statistical analysis method although this is beyond the scope of the present work. The conservative constraint $-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$ obtained in the present work indicates that, if the J_0 value is out of the region $-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$, the nonlinear RMF model either cannot predict the pressure of SNM constrained by flow data in HIC ([3]) or cannot describe the recently discovered heaviest neutron star PSR J0348+0432 ([68]). It is worth mentioning that the constraint on the J_0 depends on our knowledge of the other six quantities. Any improvement on these six macroscopic quantities will make the range for the J_0 constraint narrower. In addition, the extracted constraint on the J_0 could depend on the form of the energy density functional, and it will be interesting to see how the constraint changes if other energy density functionals are used.

In Fig. 4, we show the comparison of the J_0 constraint obtained in our analysis with those obtained with other analyses and/or other methods ([25, 28, 70, 71]), including the constraint of $J_0 = -700 \pm 500 \text{ MeV}$ obtained by Ref. [70] from the analysis of nuclear GMR, the constraint of $J_0 = -280_{-410}^{+72}$ (-500_{-290}^{+170}) MeV obtained by Ref. [71] from analyzing a heterogeneous data set of six neutron stars using a Markov chain Monte Carlo algorithm within a Bayesian framework by assuming $r_{\text{ph}} \gg R$ ($r_{\text{ph}} = R$) where r_{ph} is the photospheric radius at the time the flux is evaluated and R is the stellar radius, the constraint of $J_0 = -355 \pm 95 \text{ MeV}$ deduced by Ref. [28] based on a correlation analysis method within SHF energy density functional, and the constraint of $J_0 = -390 \pm 90 \text{ MeV}$

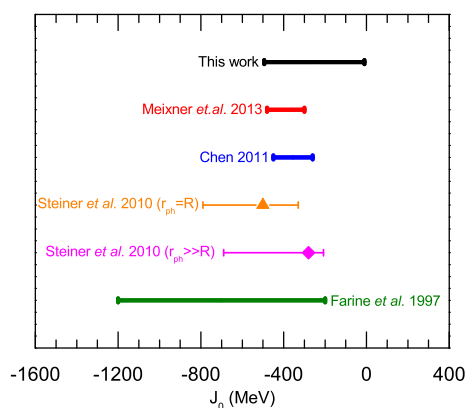


Fig. 4 (Color online) Comparison between the constraint of J_0 extracted in the present work and those from the existing literature [25, 28, 70, 71]

deduced by Ref. [25] who used a similar method as Ref. [28]. It is seen that the constrained region of J_0 obtained in the present work has a remarkable overlap with those existing in the literature. In particular, our present constraint from the relativistic model is nicely consistent with the constraints deduced from the nonrelativistic SHF approach, and they all indicate that the J_0 parameter should be larger than about -500 MeV .

4 Summary

Within the nonlinear relativistic mean field model, using macroscopic nuclear matter characteristic parameters instead of the microscopic coupling constants as direct input quantities, we have demonstrated that the pressure of symmetric nuclear matter at supra-saturation densities and the maximum mass of neutron stars provide useful probes for the skewness coefficient, J_0 , of symmetric nuclear matter. In particular, using the existing experimental constraints on the pressure of symmetric nuclear matter at supra-saturation densities from flow data in heavy-ion collisions and the astrophysical observation of recently discovered heaviest neutron star PSR J0348+0432, with the former requiring a smaller J_0 while the latter requires a larger J_0 , we have extracted a constraint of $-494 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$.

We have compared the present constraint with the results obtained in other analyses and found they are nicely in agreement. In particular, our present constraint from the relativistic model is nicely consistent with the constraints deduced from the nonrelativistic Skyrme–Hartree–Fock approach, and they all indicate that the J_0 parameter cannot be too small, namely it should be larger than about -500 MeV . The present constraint on the J_0 parameter provides important information on the high density behaviors of the EOS of symmetric nuclear matter and also may be potentially useful for the determination of the high density behaviors of the EOS of asymmetric nuclear matter, especially the high density symmetry energy.

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