Krishnagopal *Applied Network Science* (2022) 7:62
https://doi.org/10.1007/s41109-022-00503-w

Applied Network Science

RESEARCH

Open Access

The collective vs individual nature of mountaineering: a network and simplicial approach

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Abstract

Mountaineering is a sport of contrary forces: teamwork plays a large role in mental fortitude and skills, but the actual act of climbing, and indeed survival, is largely individualistic. This work studies the effects of the structure and topology of relationships within climbers on the level of cooperation and success. It does so using simplicial complexes, where relationships between climbers are captured through simplices that correspond to joint previous expeditions with dimension given by the number of climbers minus one and weight given by the number of occurrences of the simplex. First, this analysis establishes the importance of relationships in mountaineering and shows that chances of failure to summit reduce drastically when climbing with repeated partners. From a climber-centric perspective, it finds that climbers that belong to simplices with large dimension were more likely to be successful, across all experience levels. Then, the distribution of relationships within a group is explored to categorize collective human behavior in expeditions, on a spectrum from polarized to cooperative. Expeditions containing simplices with large dimension, and usually low weight (weak relationships), implying that a large number of people participated in a small number of joint expeditions, tended to be more cooperative, improving chances of success of all members of the group, not just those that were part of the simplex. On the other hand, the existence of small, usually high weight (i.e., strong relationships) simplices, subgroups lead to a polarized style where climbers that were not a part of the subgroup were less likely to succeed. Lastly, this work examines the effects of individual features (such as age, gender, climber experience etc.) and expedition-wide factors (number of camps, total number of days etc.) that are more important determiners of success in individualistic and cooperative expeditions respectively. Centrality indicates that individual features of youth and oxygen use while ascending are the most important predictors of success. Of expedition-wide factors, the expedition size and number of expedition days are found to be strongly correlated with success rate.

Keywords: Mountaineering data analysis, Success prediction, Social network analysis, Simplicial complex, Group interactions

Introduction

Extreme mountaineering is an increasingly popular sport that straddles the boundary between an individual sport and a group activity. Under extreme settings, success requires not only very high levels of physical fitness (Huey and Eguskitza 2001; Szymczak et al. 2021), but also psychological (Ewert 1985) and sociological state (Savage et al. 2020; Helms 1984). A majority of mountaineering is undertaken as a part of an expedition, with a high level of inter-dependency between climbers, often relying on their fellow climbers in life or death scenarios. The psychology is largely driven by relationships between climbers; for instance climbers that frequently climb together may develop better communication and group dynamics, and consequently lower failure. Simultaneously, certain aspects of extreme mountaineering are well-known to be individualistic, especially as one gets closer to the death zone (8000m altitude) (Crockett et al. 2020). The interplay of these conflicting forces makes extreme mountaineering an interesting setting to study collective human behavior. Such analyses are made possible using data from the meticulously documented Himalayan dataset (Salisbury 2004). This work is a data-driven study of the structure of relationships between climbers within an expedition and its effects on individual and group success.

Relationships between climbers in an expedition may result from a previous joint expedition comprising of a subset of the climbers. Relationships between climbers are naturally captured by a network framework. A conventional network can capture the interactions between two climbers (or 'nodes') as a link (or 'edge') with a weight given by the number of expeditions that they jointly participated in. Network approaches have been used successfully in predictive medicine (Krishnagopal 2020), climate prediction (Steinhaeuser et al. 2011), predictions in group sports (Lusher et al. 2010), disease spreading (y Piontti et al. 2019) etc. However, such pairwise networks are unable to accurately capture scenarios where interactions between more than two climbers occur, and reduce them to multiple pairwise interactions, which is fundamentally misleading. Simplicial complexes (Giusti et al. 2016; Bianconi 2021; Battiston et al. 2020; Torres et al. 2021; Battiston et al. 2020) are an important tool for modeling systems with simultaneous interactions between more than two entities. Indeed, simplicial representations involving simplices (e.g. triangles, tetrahedra etc.) have been successfully used to model a variety of systems such a social communication networks (Wang et al. 2020), complex systems (Salnikov et al. 2018; Benson et al. 2016), disease spreading (Iacopini et al. 2019) etc. and are a rapidly growing field of data analysis. In this work, an expedition involving a subgroup of climbers can, for instance, be represented as a filled triangle, differentiating it from a set of three edges. The weight of this filled triangle, which is a simplex, is given by the number of previous expeditions containing the three climbers.

The composition of a group and the relationships therein influence its effectiveness. Sherman and Chatman (2013) uses the distribution of climbers' nationalities as a meaningful measure of the extent of collaboration and competition, and finds that collective mentality, which may often result from strong relationships, boosted summiting when national diversity was high. Despite wide ranging applications of group dynamics, there is limited investigation of the effects of relationships on collective behavior, especially in mountaineering literature. This work studies how the diversity and structure of relationships, formed through climbing together, influence the cooperation or competition between them and ultimately success. Specifically, it uses topological relationships between climbers as a predictor of expedition style on a spectrum from polarized (where subgroups with strong relationships and without relationships, or weak relationships, are likely to have different outcomes) to globally cooperative (where climbers, regardless of their relationships with others, share the benefit of the existence of high-dimensional simplices in the group).

Various other factors, both personal and expedition-wide, have differing effects on success based on expedition style, with the former playing a larger role in individualistic expeditions and the latter in a cooperative. Personal elements such as effective use of proper equipment, climber experience, mental state etc. Schussman et al. (1990) are crucial to maximizing safety and chances of success. Several works have studied the effects of age, sex, nationality (Huey et al. 2007; Weinbruch and Nordby 2013), experience (Huey et al. 2020), commercialization (Westhoff et al. 2012) etc. on success, and highlight the importance of age as a dominant determining factor. Expedition-related factors such as effective use of proper equipment, climber experience, mental strength and self-reliance are all measures (Schussman et al. 1990) to increase safety and chances of success. In, Krishnagopal (2021), the effects of various personal features like age, sex, experience etc. as well as expedition-wide factors like length of expedition, number of sherpas (mountain workers and guides in regions around Nepal) etc. on success are studied using a multiscale network. This work investigates relationships between climbers and personal features such as oxygen use, age, sex, experience etc. through a bipartite network, projected onto a network with features as the nodes for further analysis (Krishnagopal et al. 2020; Larremore et al. 2014). The natural question emerges, which of these features, which can be represented by nodes, are central to maximizing chances of success? An active area of research investigates the importance of nodes (Mo and Deng 2019) through centrality-based measures (Solá et al. 2013; Saito et al. 2016) that serve as reliable indicators of 'important' features. Expedition-wide factors such as ratio of sherpas to paying climbers, number of days to summit, and number of camps, intra-expedition social relationships etc. are also considered as predictors of success. Several works have studied the effect of sex and gender on success and death (Huey et al. 2007, 2020) (for a full list see the references in Salisbury (2004)), however, outside of Krishnagopal (2021), which this work builds on, there is limited literature that studies the effect of such expedition-wide factors.

Data

The data used comes from the open access Himalayan Database (Salisbury 2004), which is a compilation of records for all expeditions from 1905 to 2021 in the Nepal Himalayan ranges. The dataset has records of 468 peaks, over 10,500 expedition records and over 78,400 records of climbers, where a record of any type is associated with an ID. Details of what The data extracted from expedition records are

- Peak climbed (height)
- Days from basecamp to summit
- Number of camps above basecamp
- Total number of paying members and hired personnel
- List of all joint expeditions that contain a set of climbers (calculated)

- Result: (1) success (main peak/foresummit/claimed), (2) no summit, (3) death.

The success rate of an expedition is calculated as the fraction of members that summited. The following data are extracted from climber records:

- Demographics: age, sex, nationality
- Oxygen tank use while ascending, and oxygen use while descending.
- Previous experience above 8000m (calculated)
- Result:

1 Success
2 Altitude related failure: Acute Mountain Sickness (AMS) symptoms, breathing problems, frostbite, snowblindness or coldness,
3 Logistical or planning failure: Lack of supplies, support or equipment problems, O2 system failure, too late in day or too slow, insufficient time left for expedition,
4 Fatigue related failure: exhaustion, fatigue, weakness or lack of motivation
5 Accident related failure: death or injury to self or others

The climber ID of each climber is tracked in the dataset through history to generate a log of their previous expeditions. The number, type, and nature of previous joint expeditions are then extracted by calculating the overlap between climber logs.

The effect of climbing with repeated partners

The first question is, is it advantageous to climb with people one has climbed with before? It is natural to assume that friendship and familiarity with each others' climbing style may improve confidence and the accuracy of calculated risks, thus limiting failure, but may also lead to a false and potentially dangerous sense of comfort. Here, the average

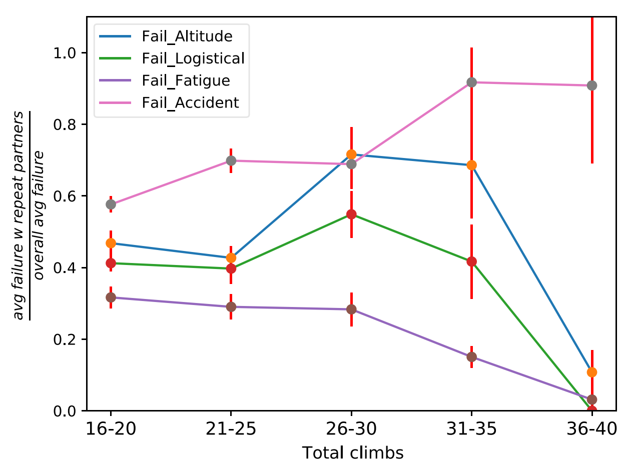

Fig. 1 The fraction of failures (of several types) when climbing in a group with at least one repeated partner (someone they have done a logged Himalaya expedition with before) over the their personal average. The x-axis denotes the total number of previous climbs to normalize for levels of experience

failure rate of a climber is compared with their failure rate when climbing in expeditions with at least one repeated partner (that they have climbed with before). Figure 1 shows the fraction of failures when climbing with repeated partners over the climber average. These failures are classified into altitude related, fatigue related, logistical and planning failures, and accident/illness. The effect of total experience is normalized for by plotting across the total number of climbs on the x-axis starting at least 15 climbs, hence not considering beginner climbers.

The y-axis in Fig. 1 plots the ratio of failure rate when climbing with at least one repeated partner over average failure rate over all climbs. Since the values of all failure ratios are below 1, the chance of failure is significantly lower when climbing with repeated partners, in every failure category. In particular, the probability of failure due to fatigue-related issues is the most decreased when climbing with repeated partners, followed by failure due to logistical or planning issues. This may be expected since climbers that have participated in previous joint expeditions typically are better at communication and knowing each others' limitations. Note that only climbers with over 15 logged climbs are considered, indicating that complete lack of experience is not a cause of failure. Additionally, the most experienced climbers (that have logged 36-40 climbs) have nearly no failure due to fatigue or logistics, as one may expect. Similarly, failure due to altitude-related and cold-related problems also drastically reduce when climbing with repeated partners. Lastly, the cause of failure due to accident shows an increasing trend as a function of increasing experience, which may be attributed to the fact that more experienced climbers tend to tackle more dangerous mountains.

Methods

Simplicial complexes

This work is interested in analyzing the *structure of relationships* between subgroups of climbers and their effect on expedition style, from an individualistic to cooperative spectrum. Relationships between climbers may result from previous joint expeditions. Such joint expeditions may involve a subgroup of individuals of size two or more. Networks, although natural to model interactions, can only capture pairwise relationships and hence fail to accurately capture higher-order interactions (interaction between more than 2 climbers in a single expedition). However, multi-node interactions can be explained by a higher-order network: a mathematical framework called *simplicial complexes*. For instance, a three-way interaction can be represented as a triangle, four-way as a tetrahedra etc.

Mathematically, given a set of l nodes $n_0, n_1 \ldots, n_{l-1} \in N$ in a network, a $p-$simplex is a subset $\sigma_p = [n_0, n_1, \ldots, n_{p-1}]$ of p nodes and a $q-$face of σ_p is a set of q nodes (for $q < p$) that is a subset of the nodes of σ_p. A *simplicial complex* (Salnikov et al. 2018; Torres and Bianconi 2020) K consists of a set of simplices, that are closed under inclusion:

$$\tau \subseteq \sigma \Rightarrow \tau \in K \text{ for any } \sigma \in K, \tag{1}$$

where '$\subseteq$' denotes the subset relation between σ and τ, two subsets of the simplicial complex. When $\tau \subseteq \sigma$, we say that τ is a *face* of σ, which by the inclusion axiom implies *every face of a simplex is again a simplex*. Figure 2 shows examples of faces of a simplicial complex.

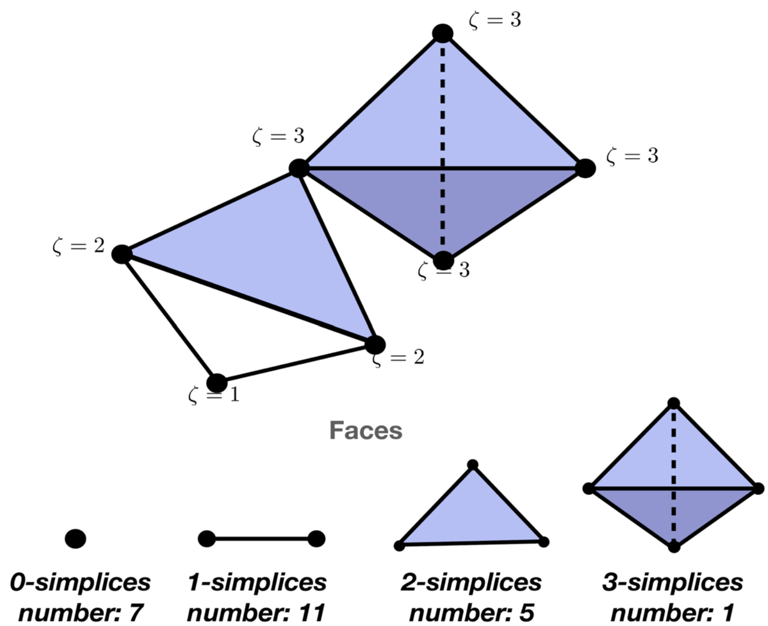

Fig. 2 Simplices that form the faces of the simplicial complex. The number of k-simplices in the top simplicial complex are listed. The influence, or maximal simplicial dimension, η for each node is listed

The *dimension* of a simplex equals the number of vertices in the simplex minus one; for instance 0-dimensional simplices are nodes and 1-dimensional simplices are edges. Each previous joint expedition is represented as a simplex. The dimension of a simplicial complex is the largest dimension of its simplices, where each expedition is modeled as a simplicial complex with the nodes representing climbers. By S_k we will denote the set of all simplices σ with dimension k, i.e. as

$$S_k := \{\sigma \in K : |\sigma| = k + 1\}, \tag{2}$$

We call the simplices in S_k the k-simplices of K and let $|S_k|$ denote the number of k-simplices in the simplicial complex.

If three climbers i, j, k in an expedition have participated in an expedition previously, this is represented as a 2-simplex (triangle) with nodes i, j, k as its 0−faces. Each climber can be a part of multiple previous expeditions, and hence be the face of multiple simplices. Let the simplices (denoted by σ) that contain individual i be given by: $n_i \in \sigma_{k1}, \sigma_{k2}, \ldots, \sigma_k$ where σ_k is a $k-$simplex such that their simplicial dimensions are ordered: $k1 \leq k2 \leq \ldots \leq k$. In other words, we order the simplices that a climber belongs to in order of their simplicial dimension. Explicitly, if a climber belongs to previous expeditions with 2,4 and 6 other people respectively, the node that corresponds to the climber is the face of 3 simplices ordered by degree: 2-simplex, 4-simplex, and 6-simplex respectively, where the dimension of the largest simplex is $k = 6$.

The influence of a climber is a measure of the size of the largest group that they belong to that have climbed together in a joint previous expedition, resulting in the formation of pre-existing group relationship. Specifically, the *influence* ζ_i of the ith climber is defined to be the largest number of climbers, in the current expedition, that have jointly participated in a previous expedition with the ith climber, i.e., dimension of its largest simplex $\zeta_i = k$. Each climber is associated with an influence value. For a climber who has no previous expeditions with any other climbers $\zeta_i = 0$. The co-influence of an expedition E is given by the the average influence of all climbers participating in the expedition.

Topological data analysis

Topological Data Analysis(TDA) is a new and rapidly growing subfield in machine learning and data science for the analysis of simplicial complexes (Wasserman 2018). The central assumption of TDA is that complex and high dimensional data has an underlying shape captured by topological descriptors, which can be exploited for its analysis. Commonly used topological descriptors are simplices as well as Betti numbers, where the k^{th} Betti number gives the number of topological holes or $k-$dimensional cavities (e.g. an unfilled triangle is a 2-dimensional cavity) in the simplicial complex. This work derives inspiration from persistence homology (Aktas et al. 2019), the primary data analysis methodology in TDA, which attempts to extract topological descriptors (e.g. simplices) in the data that persist over various threshold values. Conventionally, topological features are recorded by creating persistence diagrams. These diagrams plot the birth time (on the x-axis), and death time (on the y-axis) of a simplex where time is measured through changing a threshold parameter. Topological features that persist for a large amount of time or across various thresholds are important in the analysis of the simplicial complex.

In a similar vein, this work assigns a weight to each simplex in an expedition, given by the number of previous expeditions of the subgroup represented by the simplex, i.e., the number of times the simplex occurs $w_\sigma = \#\sigma$. For example, if climbers i, j, k have had four previous joint expeditions together, then the simplex $\sigma_2 = [n_i, n_j, n_k]$ is a $2-$simplex with a weight $w_{\sigma_2} = 4$. Naturally the higher the weight of the simplex, the stronger the multi-node relationships between the individuals. One can threshold the weight during generation of the simplicial complex such that only simplices with weights larger than the threshold remain, i.e., only relationships stronger than the threshold are captured. Varying the threshold over a range of values, persistent simplices (that persist across various weight thresholds τ). One can then study properties of the simplicial complex across τ, such as evolution of the number, distribution and dimensionality of simplices, as well as the nature of persistent simplices on outcomes.

Influence as a predictor of climber success: correlating simplicial dimension

Group relationships formed during previous joint expeditions naturally carry forward and influence group dynamics in the current expedition. Here, we test the hypothesis that the influence of a climber is correlated with chances of success, and investigate the precise nature of this relationship.

The boxplot in Fig. 3 (left) shows the average influence, or dimension of the largest simplex, averaged across climbers from three categories: summit success, no summit, and death. As seen from the figure, successful climbers had significantly higher influence than climbers that did not summit, independent of experience (across the x-axis). A Mann-Whitney-U non-parametric statistical test, appropriate for categorical data (e.g. influence), is performed, that tests the null hypothesis that the two sets of samples

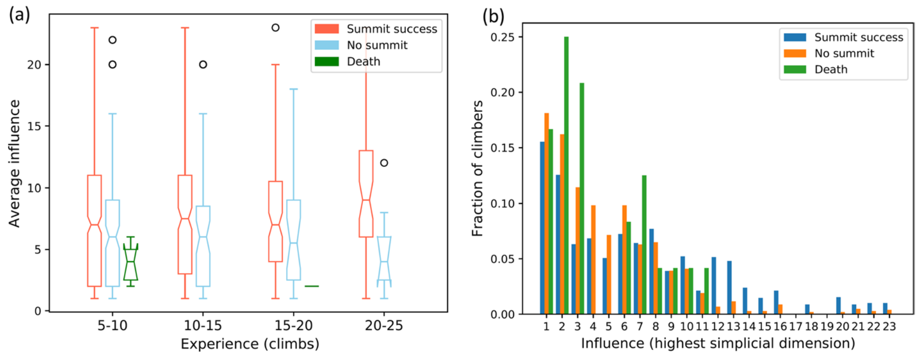

Fig. 3 (left) Comparison of average influence (dimension of largest simplex) of climbers in three categories: summit success, no summit, and death. Statistics are normalized by total previous experience (logged climbs in the database) on the x-axis. (right) Corresponding histogram of the distribution of influence (highest simplicial dimension) for the three categories normalized by the total number of individuals in each category. Climbers with zero influence are excluded to yield 801 successful climbers, 233 climbers that didn't summit, and 8 deaths

(influence scores of successful climbers and no summit climbers) are derived from the same distribution. The corresponding $p-$values for the different experience levels are $p = 0.084$ for 5-10 climbs, $p = 0.066$ for 10-15 climbs, $p = 0.078$ for 15-20 climbs, and $p = 0.016$ for 20-25 climbs. Assuming a significance level of $\alpha - 0.1$, accepting a 10% risk of concluding that a difference exists when there is no actual difference, since $p < \alpha$ for all experience levels, we conclude that there exists some evidence that the null hypothesis can be rejected, i.e., there is statistically significant differences in the medians of the influence scores for 'summit success' and 'no summit' climbers. Only climbers belonging to a simplex with dimension greater than or equal to 1 are considered. The total number of successful climbers considered are $n_s = 801$, and the number of climbers that did not summit are $n_u = 233$. The total number of deaths are low at $n_d = 8$ and hence statistically insignificant. Next, we consider all climbers from each of the three categories (summit success, no summit, and death) and plot the distribution of their influence. The histogram in Fig. 3 (right) shows the distribution of influence scores as fractions of climbers in each category. The histogram values are normalized by the total number of individuals in that category for comparison across categories such that the sum of all values in a given category is one. The histogram explicitly allows us to look at how the relative statistics of success (success vs summit fail) scale with influence scores. For low influence values (1-6) on the x axis, the fraction of no summit (orange) is higher than summit success (blue), whereas this trend switches as we move into the high influence region (10-20), where the fraction of summit success (blue) is consistently higher than that of no summit (orange). Thus, individuals that don't summit are more densely concentrated, by percentage, around low influence scores, whereas successful climbers typically concentrate, by percentage, around high summit scores. Lastly, even though the deaths were few in number, they were overwhelmingly in climbers with low influence. Note that all data for this section is from Mount Everest expeditions and hence a certain level of expertise is assumed.

In Appendix 9.1, we investigate the distribution of influence scores across the three categories while considering only strong relationships, i.e., increasing the simplicial threshold τ where only simplices with a weight equal to or greater than the threshold (stronger relationships) persist. Simplices with large dimension are particularly infrequent and hence less likely to persist as τ increases. Thus, we find that influence is positively correlated with climber success, i.e., large values of the highest simplicial dimension are more likely to be successful.

Classification of expedition style through persistent simplices

As seen in "Influence as a predictor of climber success: correlating simplicial dimension" section, climbers with higher influence tended to be more successful. This section investigates how the distribution of influence affects expedition strategies. How can the topology and strength of relationships within an expedition be used to predict the strategy of the group, i.e., the spectrum between polarized vs globally cooperative? Polarized expeditions have sub-groups within the expedition that tend to perform differently, indicating a more individualistic strategy. Globally cooperative expeditions tend to display a more cooperative strategy with uniform outcomes largely independent of climber influence.

Figure 4a shows the success rate of members of the expedition that are not part of the largest simplex as a function of the simplicial dimension of the largest simplex in the expedition. Success rate of the 'outside' members is defined as the fraction of individuals that did not belong to the largest simplex that reached the summit and back. This provides insights into the expedition style and its effect on climbers that did not belong to the largest subgroup.

As seen in the figure, outsider success rate is positively correlated with maximum simplicial dimension, indicating that expeditions with large maximal dimension tended to be more cooperative. The expedition is considered to be in the cooperative regime when not only climbers belonging to the largest simplex are benefited, but also those that do not belong to the largest simplex, with the assumption that cooperation and mutual-support between climbers improves chances of success irrespective of influence. Whereas, expeditions where the largest simplex had relatively small dimension resulted in low success rates between non-simplex members, despite having high likelihood of success, irrespective of simplicial dimension, of simplex-members as seen in Fig. 3. Figure 4b shows that the maximum simplicial dimension in the expedition is inversely correlated

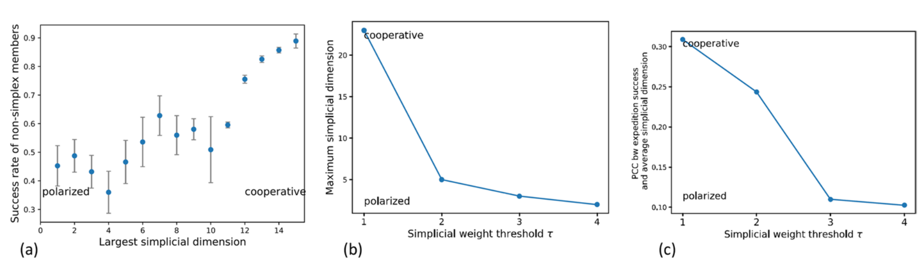

Fig. 4 **a** Average success rate of members of expeditions that were not part of the largest simplex as a function of the dimension of the largest simplex. Error bars are plotted in red. **b** Maximum simplicial dimension of all simplices in the expedition as a function of simplicial weight threshold τ. **c** Pearson's Correlation Coefficient (PCC) between expedition success rate calculated over all members and average simplicial dimension of all simplices in the expedition as a function of τ. Expeditions with no simplex of dimension greater than zero are excluded to yield a total number of expeditions of $n = 273$ for $\tau = 1$, $n = 273$ for $\tau = 2$, $n = 173$ for $\tau = 3$, and $n = 118$ for $\tau = 4$

with the simplicial weight threshold τ. Simplices with large dimension were less persistent. In other words, subgroups within an expedition comprising of a large number of individuals tended to have weaker relationships (existed for low τ) and be more cooperative, benefiting all members of the group. In contrast, the existence of small subgroups, which are usually strong (persist for a range of τ), that had previous climbed together several times lead to a polarity in the group dynamics. A polarized style is characterized by relatively lower success rate of climbers that are not a part of the largest simplex. Statistics are averaged over all levels of experience. Lastly, Fig. 4c validates our results with the insight that the correlation between the total expedition success rate (calculated across all members) and average simplicial dimension is stronger for low τ, i.e., in the cooperative regime, whereas in the polarized regime average influence becomes a poor predictor of success.

Our analysis reveals that strong previous relationships within an expedition are found when the size of the subgroup is small and largely benefit only members of the subgroup leading to subgroups with different likelihoods of success. However the existence of weak relationships between a larger fraction of the expedition tends to unite the team and result in higher homogeneity in their success.

Polarized vs cooperative expeditions: other factors determining success

The relationships within an expedition serve as a predictor of the extent of cooperation and competition. However several other factors, both personal and expeditional, play a role in success to different degrees based on the expedition style (Krishnagopal 2021). For instance, climber-specific factors may play a larger role in expeditions that fall on the individualistic side of the style spectrum, whereas expedition-wide factors that are shared across the expedition may play a larger role in expeditions that fall on the cooperative side of the style spectrum.

Individual features

Various aspects of mountaineering, such as physical fitness and skill, are indeed unconditionally personal. Here, we study the importance of the following $d = 6$ personal features: age, sex, nationality, experience above 8000m, oxygen use while ascending and oxygen use while descending. To avoid biases originating from differences in the mountain, data from only one mountain (Mount Everest) is considered. Expeditions with less than 12 climbers are excluded. An individual is only considered successful is they satisfy both the following criteria: summit and safe descent without requiring rescue. To generate the intra-expedition network, we start with a bipartite network, with adjacency matrix P, between climbers and features, where a climber is connected (using binary weights) to the features that they are affiliated with. A climber is connected to the 'sex' node if they are male, and age is binarized into above and below median age (40).

We then generate an intra-expedition network, with adjacency matrix I, of size $d \times d$ by projecting the bipartite network into feature space as follows: $I = P^T P$. The edge weight between two nodes (features) is given by the number of people that are connected to both the features. Since the feature graph I is a direct projection of the bipartite graph, it encodes similarities between two features calculated through their simultaneous co-expression in the climbers. We do this independently for all successful and unsuccessful individuals, generating two different networks that encode the properties of the successful and unsuccessful sets of climbers in their network structure.

To explore such properties, measures such as centrality provide insights into the importance of different features (Mo and Deng 2019). For instance, if the group were comprised of mostly high-age individuals, the centrality of the 'age' node would be relatively high. The eigenvector centrality (Solá et al. 2013) is a commonly used measure of how central each feature is in a given graph. Naturally, *differences* in feature centrality between successful vs unsuccessful sets of climbers is a measure of how important the feature may be for success.

As seen in Fig. 5a, where the x-axis is ordered in increasing order of difference in centrality, the least central feature in determining success on summit was the use of oxygen while descending, which is expected since descent features have no effect on summit prospects, except for indicating that oxygen was available on descent meaning there wasn't excessive use during ascent. It is worth noting that most fatalities on Everest happen during the descent. The next features that were slightly more central in successful summits were previous experience above 8000m (for reference, Everest is at 8849m), followed by use of oxygen while ascending. Importantly, there are few studies on the role of oxygen, and it is an often underemphasized aid, which the results suggest is fairly important. Surprisingly, summit centrality for sex (indicating male) was relatively low compared to no-summit centrality indicating that being male had low importance in the chances of success at summit. Lastly, the largest differences in summit vs no summit were from identity (sherpa were much more likely to succeed), and age ($<$ 40 year olds were much more likely to succeed), both of which are intuitive and also seen in previous studies (Huey et al. 2020).

Expedition-wide factors

While the intra-expedition network provides insight into personal features that determine success, expedition-wide factors also play an important role, particularly in

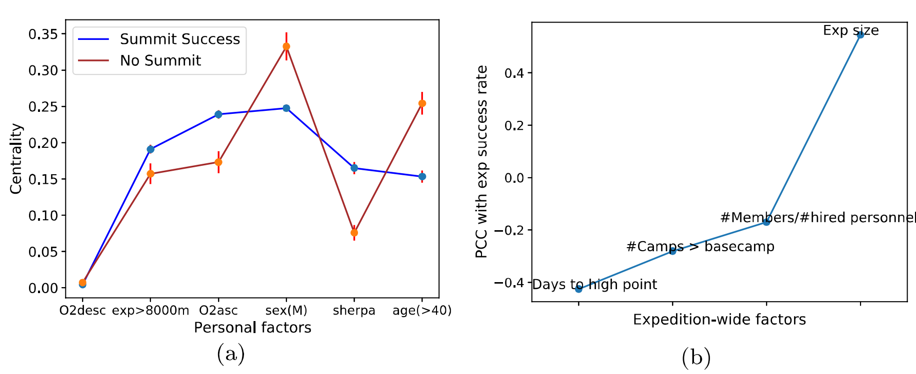

Fig. 5 **a** Mean eigenvector centrality as a function of expedition features for Everest expeditions greater than 12 members plotted for groups of successful vs unsuccessful climbers ordered by increasing difference between success and no-success centralities. Error bars show standard error on the centrality. **b** Pearson's correlation coefficient (PCC) between layer (factor) values and expedition success rate. The exact values across x-axis layers are −0.45, −0.36, −0.12, 0.57, 0.84. The corresponding *p*-values are $5.5 \times 10^{-10}, 1.15 \times 10^{-6}, 0.1, 5.7 \times 10^{-16}, 8.9 \times 10^{-47}$

globally cooperative expeditions. The expedition-wide factors considered here are: (1) number of days to summit from base camp, (2) number of high points/camps, (3) expedition size (including hired personnel), (4) ratio of number of paying climbers to number of hired personnel.

The expedition success rate is defined as the fraction of climbers that succeed at summiting. The importance of an expedition-wide factor can be inferred from the correlation between the values of a given factor across a range of expeditions and their corresponding success rates.

Figure 5b shows the Pearson's correlation coefficient between the expedition-wide factors and the success rate. A higher correlation implies higher importance in determining success. Despite sherpas having a high chance of personal success as seen in "Individual features" section, the ratio of number of paying members to number of hired personnel on the team is only weakly correlated with success, i.e., has a relatively small effect on *expeditional* success. Both number of camps above basecamp and days to summit/high point are negatively correlated with success, as one might expect, with the latter having a larger effect. Also surprisingly, the *expedition size* is found to be relatively important in determining success (with a correlation coefficient of > 0.5). All p-values are extremely low, indicating that the correlation is statistically significant except for the number of members to hired personnel.

Discussion

This work presents the first network-based simplicial analysis of mountaineering data, studying the effect of the structure and strength of relationships on the nature of cooperation and success, both from an individual perspective and across the expedition. Using the Himalayan dataset, it establishes that relationships between climbers play an important role in success and failure, by showing that the chances of summit failure (due to fatigue, logistical failure etc.) drastically reduce when climbing with repeated partners, especially for more experienced climbers. Further, individuals with high influence, i.e., belonging to a simplex with large simplicial dimension (encoding a previous joint expedition with a large number of members) were more likely to be successful in summiting, irrespective of experience level. However, the effects of having subgroup relationships on the collective group behavior varied. Specifically, expeditions with large simplices that tended to have lower weight (indicating weak relationships) had a more cooperative style, with the average simplicial dimension being a good predictor of the expedition success rate. In contrast, expeditions with smaller simplices of typically high weight encoded strong relationships between a small group of people, and tended to be more polarized. In such expeditions, individuals that were a part of the highly weighted simplex had a high likelihood of succeeding, whereas those that weren't had a low likelihood of success. It also studies various other indicators of success, such as personal features that are more important in individualistic expeditions, and expedition-wide factors are more important in cooperative expeditions. A bipartite climber-feature network is projected into feature space to study the relative importance of personal features. The largest difference in centralities amongst successful and unsuccessful groups is found in the 'age' node, indicating that it's the strongest driver of success. The expedition-wide factors that have high correlation with expeditional success are high expedition size and low total number of days.

In conclusion, this work presents novel analyses and new results that demonstrate the importance of different types of inter-personal relationships at high peaks on the extent of cooperation vs individualism, and effect on success, both from intra-group individual as well as expedition-wide perspectives. It extends work from Krishnagopal (2021) studying the effects of both personal and expedition-wide factors on success, and highlights their relative importance in expeditions of different styles. Lastly, it is the first work applying simplicial complexes and topological data analysis to mountaineering data, opening it up for further analysis from the network science community. Code can be found at *https://github.com/chimeraki/mountaineeringsimplicial*.

Appendix

Influence distribution as a function of simplex threshold

Figure 6 shows the distribution of the highest simplicial dimension for increasing thresholds $\tau = 2, 3, 4$ respectively. For $\tau = 2$, The total number of successful climbers are $n_s = 398$, no summit climbers are $n_u = 117$, and climbers that died are $n_d = 7$. similarly, for $\tau = 3$, $n_s = 197$, $n_u = 65$, and $n_d = 1$, and for $\tau = 4$, $n_s = 113$, $n_u = 32$, and $n_d = 1$. As seen in the figure, successful climbers are concentrated in regions of higher influence scores compared to unsuccessful climbers across simplicial thresholds.

Structure of simplices

The intra-expedition graph structure in feature space for both summit and no summit cases is presented in Fig. 7. Here, nodes denote personal factors and edges are the strength of co-occurences between personal features in successful and unsuccessful climbers. The graphs are generated by projecting the bipartite climber-feature network into feature space as outlined in "Individual features" section.

Success rates of members within and outside largest simplex

The success rates of individuals has been shown to be proportional to their influence, i.e., dimension of the largest simplex that they belong to (see "Influence as a

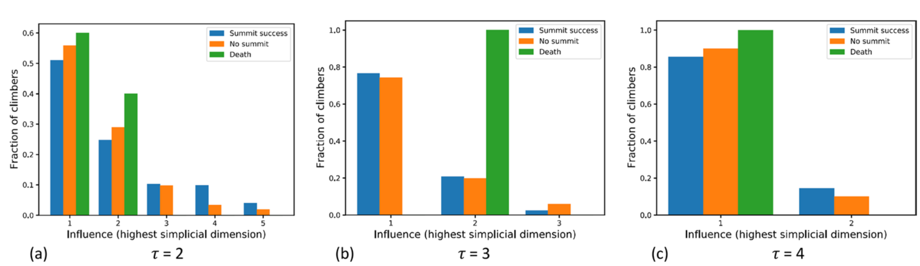

Fig. 6 (Histograms showing the distribution of influence (highest simplicial dimension) for the three categories for varying simplicial weight threshold τ. The fraction in each cateogory is normalized by the total number of members in that category such that the such of fractions across influence is 1 for each category. Climbers with zero influence are excluded

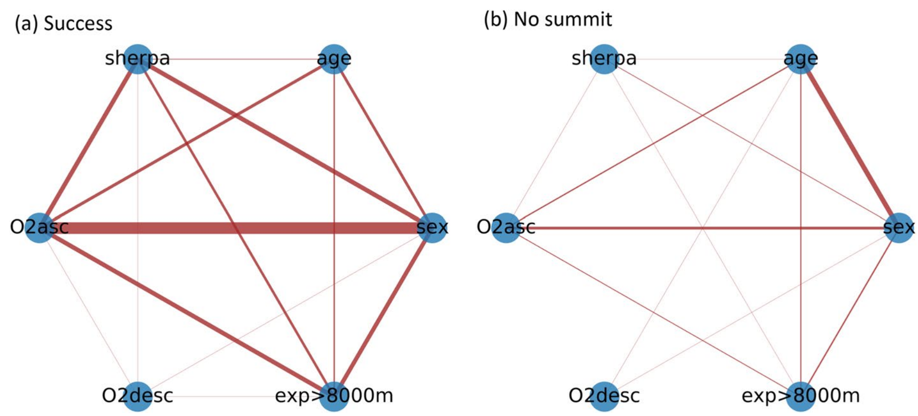

Fig. 7 Intra-expedition graph structure with relative edge weights in summit success, and no-summit groups respectively

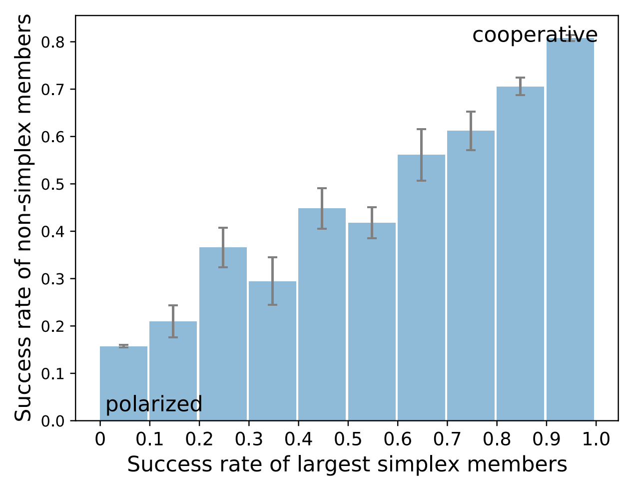

Fig. 8 Success rate of members within the largest simplex as a function of success rate of individuals outside the largest simplex. A total of 146 expeditions are considered

predictor of climber success: correlating simplicial dimension" section). Expeditions are denoted by simplicial complexes, and expeditions with largest dimension of the simplicial complex (high dimension of largest simplex) are associated with a cooperative expedition style, as seen in "Classification of expedition style through persistent simplices" section. Figure 8 studies the success rates of individuals in the largest simplex as a function of success rates of those outside the largest simplex. It is natural that high dimension of largest simplex, leads to high influence of within expedition members and more cooperative strategies, hence, success rates of within-simplex and outside-simplex members are correlated. However, success rates of outside simplex members is slightly lower than that of within simplex members.

Acknowledgements
Not applicable.

Author contributions
SK conceptualized the idea, developed the methods, carried out the analyses, and drafted and edited the manuscript. All authors read and approved the final manuscript.

Funding
This work was supported by the Wellcome Trust [110114/Z/15/Z].

Availability of data and materials
The datasets used in this paper in the Himalayan dataset Salisbury (2004) which is openly available at www.himalayandatabase.com.

Declarations

Ethics approval and consent to participate
No human or animal experiments were conducted in this work. Not applicable.

Consent for publication
This work does not contain any type of personal data that can be used to identify an individual. Not applicable.

Competing interests
The authors declare that they have no competing interests.

Received: 8 February 2022 Accepted: 22 August 2022
Published online: 04 September 2022

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