Granular Computing (2024) 9:71
https://doi.org/10.1007/s41066-024-00487-1

ORIGINAL PAPER

Multi-attribute group decision-making with T-spherical fuzzy Dombi power Heronian mean-based aggregation operators

Mubashar Javed[1] · Shumaila Javeed[1,2] · Tapan Senapati[3,4]

Received: 26 January 2024 / Accepted: 29 May 2024

Abstract

In the realm of expressing fuzzy and vague information, T-spherical fuzzy sets (TSPFSs) emerge as a powerful extension of both picture fuzzy sets (PFSs) and spherical fuzzy sets (SFSs), offering decision-makers a broader spectrum of descriptive capabilities. Within the domain of multi-attribute group decision-making (MAGDM), the significance of T-spherical fuzzy aggregation operators (AOs) under T-spherical fuzzy conditions cannot be overstated. Hence, our manuscript contributes by introducing a collection of ground-breaking T-spherical fuzzy AOs. This paper establishes a set of innovative T-spherical fuzzy operational laws grounded in Dombi t-norm and Dombi t-conorm (DTNCN) principles. Utilizing the strengths of power aggregation operators, which effectively capture the implications of unfavorable information, and Heronian mean (HM) operators, which adeptly assess the collective association among the evaluated arguments. Some aggregation operators are examined, namely T-spherical fuzzy Dombi power Heronian mean (TSPFDPHM) operator, T-spherical fuzzy Dombi weighted power Heronian mean (TSPFDWPHM) operator, T-spherical fuzzy Dombi geometric power Heronian mean (TSPFDGPHM) operator, and T-spherical fuzzy Dombi weighted geometric power Heronian mean (TSPFDWGPHM) operator. Additionally, we present a host of properties exhibited by these proposed AOs, along with specific cases that allow for adjustable parameters. Subsequently, we develop a comprehensive algorithm for MAGDM based on the proposed AOs within the T-spherical fuzzy environment. In conclusion, we apply the devised algorithm to a real-world scenario involving selecting the best road construction company for a post-flood road rehabilitation project in Pakistan. Through comparative analysis with existing methodologies, we demonstrate the validity and superiority of our developed scheme, thereby reinforcing its practical applicability and effectiveness.

Keywords Multiple attribute group decision-making · T-spherical fuzzy sets · Dombi operators Heronian mean operator · Power average operator · Post flood road rehabilitation project

1 Introduction

Mahmood et al. (2019) made a ground-breaking contribution by unveiling the extraordinary and adaptable TSPFS within the domain of fuzzy structures. This remarkable framework is comprehensive and inclusive, embracing many other fuzzy sets, such as the intuitionistic fuzzy set (IFS) and picture fuzzy set (PFS). In the domain of the TSPFS framework, three prominent membership functions take center stage: the membership degree (MD), abstinence degree (AD), and non-membership degree (NMD). These functions are critical in characterizing the framework's degree of belongingness,

✉ Tapan Senapati
math.tapan@gmail.com

Mubashar Javed
mubasharjaved4949@gmail.com

Shumaila Javeed
shumaila_javeed@comsats.edu.pk

1 Department of Mathematics, COMSATS University Islamabad, Islamabad Campus, Park Road, Chak Shahzad, Islamabad 45550, Pakistan

2 Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon

3 School of Mathematics and Statistics, Southwest University, Beibei, Chongqing 400715, China

4 Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Thandalam, Chennai 602105, Tamil Nadu, India

Published online: 20 August 2024

uncertainty, and exclusion. This unique composition offers decision-makers (DMs) unprecedented flexibility in assigning attribute values, guided by the flexible constraint that the sum of the qth power of membership functions must be less than 1. The allure and potential of this fuzzy structure have captivated scholars from across the globe, leading to a wealth of research in diverse fields, such as diabetic retinopathy detection (Kakati et al. 2024a), assessing soil fertility (Hussain et al. 2023), natural agribusinesses (Sarkar et al. 2023a), and solar systems evaluation (Khan et al. 2024).

Recognizing the advantages of aggregation operators over conventional ranking methods, numerous scholars have contributed valuable AOs (Akram et al. 2022; Akram and Martino 2022; Alsalem et al. 2021; Guleria and Bajaj 2021; Sarkar et al. 2023a, b) to enhance decision-making algorithms. Garg et al. (2018) delved into improved operational laws and their corresponding properties, while Mahmood et al. (2021) proposed a generalized MULTIMOORA method based on Dombi-prioritized weighted AOs. Garg et al. (2021) examined attribute relationships using power-averaging AOs. Ullah et al. (2020b) presented an intriguing decision-making problem by employing averaging and geometric AOs within the TSPFS framework. Liu et al. (2019) unveiled T-spherical AOs by ingeniously combining the Muirhead mean operator with the power average operator. Ju et al. (2021) extended the traditional TODIM decision-making scheme to accommodate TSPFSs. Further, Ali et al. (2020) developed a suite of AOs to explore the intricate interrelationship among complex T-spherical fuzzy numbers, unveiling new dimensions of understanding within this fascinating domain. The research conducted by Rong et al. (2022b) on the MARCOS approach, which is based on a cubic Fermatean fuzzy set, and its application provides valuable insights into improving the efficiency of logistics operations. In the study by Rong et al. (2022a), a novel multiple MADM approach was proposed for evaluating emergency management schemes under a picture-fuzzy environment. One valuable contribution to the risk assessment of R&D projects is the FMEA model based on LOPCOW-ARAS methods presented in the article by Rong et al. (2024). Rong and Yu (2023) present a decision support system for prioritizing offshore wind farm sites, offering valuable insights into enhancing the selection process. According to the research conducted by Hussain et al. (2024a), the proposed approach demonstrated improved decision-making capabilities in complex scenarios. Jabeen et al. (2024) present a new methodological framework for MADM using T-spherical fuzzy Aczel Alsina Heronian mean operators. In their study, Saha et al. (2024) proposed a novel approach that utilizes dual probabilistic linguistic consensus to facilitate effective MAGDM processes.

When developing T-spherical fuzzy AOs, two aspects should be focused (1) operational laws and (2) aggregation functions. Dombi (1982) proposed new operational laws with variability and flexibility regarding the adjustable perimeter. Dombi t-norm and t-conorm (DTNCN) have been utilized under uncertainty, and several AOs have been established. Senapati et al. (2024) proposed a novel approach using q-rung orthopair fuzzy Dombi–Archimedean aggregation operators, offering a promising avenue for handling uncertainty and ambiguity in decision processes. Jana et al. (2019a) extended DTNCN to PFS. He (2018) proposed novel operations for hesitant fuzzy sets based on Dombi norms and suggested innovative AOs. Some Dombi-prioritized AOs have been established to address the more complicated decision-making issues (Wei and Wei 2018; Jana et al. 2019b).

Recently, researchers have faced several complications while solving decision-making issues. To cope with such problems, many averaging operators have been utilized to develop more powerful AOs. Yager (2001) introduced a power average operator (PAO) to proffer an AO which permits assessment values to assist each other during the aggregation. Jiang et al. (2018) extended PAO for IFS and discussed entropy among IFSs. Sýkora (2009) extended the classical Heronian mean operator concept in 2009. Wei et al. (2018) utilized the HM operator to study the interrelationship phenomenon among attribute values. Xu et al. (2018) combined the properties of a dual hesitant fuzzy set and q-rung orthopair fuzzy set to develop the HM operators. In recent years, the fusion of AOs has become vital to integrate their characteristics and resolve complex real-life decision-making issues (Kakati et al. 2024b; Hussain et al. 2024b). Many studies have been conducted to demonstrate the combination of PAO and HM operators based on DTNCN. Zhang et al. (2018) extended the idea of the Dombi Heronian mean operator for PFS. Liu et al. (2021) took advantage of Dombi power Heronian mean AOs under 2 tuples linguistic neutrosophic set. Sarkar et al. (2023b) proposed a hybrid approach based on dual hesitant $\boldsymbol{q}$-rung orthopair fuzzy frank power partitioned Heronian mean aggregation operators, offering a comprehensive methodology for estimating sustainable urban transport solutions. Kalsoom et al. (2023) contributed to the field by introducing Schweizer–Sklar power aggregation operators, based on complex interval-valued intuitionistic fuzzy information, further expanding the toolkit available for MADM under uncertain conditions. Senapati et al. (2023) presented an intuitionistic fuzzy power Aczel–Alsina model for prioritizing sustainable transportation-sharing practices, providing insights into decision-making processes to promote environmentally friendly transport solutions. Furthermore, Jabeen et al. (2023) proposed an approach to MADM based on Aczel–Alsina power Bonferroni aggregation operators for q-rung orthopair fuzzy sets, offering a tailored solution for decision-making in complex, uncertain environments.

However, existing literature shows that the combination of Dombi power Heronian means AOs still need to be extended to TSPFSs. Hence, the principle focus of this manuscript is to establish some novel AOs based on the fusion of Dombi, PA, and HM operators under TSPFSs. Additionally, TSPFS features larger decision space and greater freedom than PFS and spherical fuzzy sets (SFS). Further, we develop a novel T-spherical fuzzy MAGDM algorithm by utilizing proposed AOs and exhibit the validity and applicability of the scheme via comparative analysis.

1.1 Motivation of the study

The motivation behind our research paper stems from the dynamic landscape of decision-making under uncertainty, where emerging research frontiers continually push the boundaries of existing methodologies. In this context, we are driven by a commitment to advancing knowledge by exploring the application of advanced aggregation operators within T-spherical fuzzy sets. By venturing into this relatively unexplored territory, our research advances theoretical frameworks and practical methodologies in decision science.

Practical relevance underscores another critical motivation behind our paper. Decision-making processes are integral to various real-world applications, and enhancing these processes in complex and uncertain environments is paramount. Our research addresses this practical need by introducing novel aggregation operators tailored to handle T-spherical fuzzy information, offering practical solutions to decision-makers grappling with ambiguous and imprecise data across diverse application domains.

Methodological innovation is central to our motivation, as innovation in methodology drives progress in decision-making theory and practice. By leveraging the unique properties of T-spherical fuzzy sets and combining them with advanced aggregation operators, our research pioneers a new methodological framework for multi-attribute group decision-making. This innovative approach promises to advance state-of-the-art decision-making methodologies, offering fresh insights into handling complex decision-making challenges.

Interdisciplinary integration is another motivating factor, as decision-making processes often require integration across knowledge domains. Our research reflects a commitment to multidisciplinary integration by bridging concepts from fuzzy set theory, aggregation theory, and decision science. By integrating insights from these diverse disciplines, our research offers a holistic approach to addressing complex decision-making challenges, fostering cross-pollination of ideas and methodologies across disciplinary boundaries.

Contributing to decision support systems (DSS) is also a key motivation behind our research. DSS assists decision-makers by providing analytical tools and information processing capabilities. Our study contributes to developing more robust and effective DSS by introducing novel aggregation operators tailored to handle T-spherical fuzzy information. This lays the foundation for developing advanced DSS capable of handling complex decision-making scenarios with heightened uncertainty and ambiguity.

Addressing societal challenges is a fundamental motivation driving our research. Many decision-making problems have profound societal implications, ranging from resource allocation and environmental management to healthcare and public policy. Our research addresses pressing societal needs by offering innovative solutions to complex decision-making problems. Our research promotes sustainable development and societal well-being by developing a novel MAGDM scheme tailored to a real-world scenario involving road construction company selection for post-flood rehabilitation.

1.2 Contribution of the study

The contributions of our paper are multifaceted and aimed at advancing the field of MAGDM within the framework of TSPFSs, as outlined below:

1. *Enhanced decision-making processes* Our research introduces TSPFSs into the analysis and leverages the mathematical properties of Dombi power Heronian mean aggregation operators to improve decision-making processes. By incorporating TSPFSs and the proposed operators, decision-makers gain access to a broader spectrum of descriptive capabilities, leading to more informed and effective decisions in complex and uncertain environments.
2. *Investigation of desirable properties* The paper meticulously investigates the desirable properties of the introduced aggregation operators. Understanding these properties is paramount for evaluating their effectiveness and applicability in practical decision-making scenarios. Through this investigation, we provide insights into the strengths and limitations of the proposed methodology, contributing to the theoretical underpinnings of MAGDM.
3. *Development of a novel MAGDM scheme* Our research presents a novel MAGDM scheme tailored to address real-world challenges, such as selecting the best road construction company for post-flood rehabilitation projects in Pakistan. By developing a customized approach grounded in TSPFSs and the proposed aggregation operators, we offer a practical solution to complex decision-making problems, thus facilitating more effective and transparent decision processes.

4. *Validation through comparative analysis* To validate the effectiveness of our proposed scheme, we conduct a rigorous comparative analysis against several aggregation operators from pre-existing literature. This comparative analysis highlights the strengths and advantages of our approach, demonstrating its superiority over existing methodologies in terms of practical applicability and effectiveness. Through this validation process, we reinforce the credibility and robustness of our proposed methodology, paving the way for its adoption in diverse decision-making contexts.

1.3 Organization of the study

The other segments are arranged as follows. Section 2 recalls some basic notations about Dombi, HM, PA operations, and TSPFS. We present a family of Dombi power Heronian mean AOs and some valuable characteristics for TSPFS in Sect. 3. A MAGDM approach is discussed in Sect. 4. We develop a novel MAGDM algorithm based on proposed AOs, and a case study is discussed to confirm the validity and supremacy of the suggested method in Sect. 5. We summarize the paper in Sect. 6.

2 Preliminaries

In this section, we review the literature on TSPFSs, DTNCN, HM, and PA, where TSPFSs are related to fuzzy sets (Chen and Wang 1995, 2010; Chen et al. 2009, 2019; Chen and Jian 2017; Horng et al. 2005; Zadeh 1965).

Definition 1 (Mahmood et al. 2019) Let X be a finite ordinary set, while a TSPFS Φ can be defined as follows:

$$\Phi=\left\{\left(x,\left(\dot{s},\underset{.}{i},\underset{\hat{}}{d}\right)\right)|\forall x\in X\right\},\tag{1}$$

where $\dot{s}$, $\underset{.}{i}$, $\underset{\hat{}}{d}$, and r denote the membership degree (MD), non-membership degree (NMD), abstinence degree (AD), and refusal degree (RD), $0\leq(\dot{s}^{q}(x)+\underset{.}{i}^{q}(x)+\underset{\hat{}}{d}^{q}(x)\leq 1$, $q\in\mathbb{Z}^{+}$. Further, $r(x)=\sqrt[q]{1-(\dot{s}^{q}(x)+\underset{.}{i}^{q}(x)+\underset{\hat{}}{d}^{q}(x))}$ is the hesitancy degree.

Definition 2 (Mahmood et al. 2019; Ullah et al. 2020a) For two TSFNs $\Phi_1=(\dot{s}_1,\underset{.}{i}_1,\underset{\hat{}}{d}_1)$ and $\Phi_2=(\dot{s}_2,\underset{.}{i}_2,\underset{\hat{}}{d}_2)$ and a real $\zeta>0$, then characteristics axioms are defined as:

(1) $\Phi_1^{C}=(\underset{\hat{}}{d}_1,\underset{.}{i}_1,\dot{s}_1)$,

(2) $\Phi_1\subseteq\Phi_2$ if $\dot{s}_1\leq\dot{s}_2$, $\underset{.}{i}_1\geq\underset{.}{i}_2$ and $\underset{\hat{}}{d}_1\geq\underset{\hat{}}{d}_2$,

(3) $\Phi_1=\Phi_2$ if $\Phi_1\subseteq\Phi_2$ and $\Phi_2\subseteq\Phi_1$,

(4) $\Phi_1\oplus\Phi_2=\left(\sqrt[q]{\dot{s}_1^{q}+\dot{s}_2^{q}-\dot{s}_1^{q}\dot{s}_2^{q}},\underset{.}{i}_1\underset{.}{i}_2,\underset{\hat{}}{d}_1\underset{\hat{}}{d}_2\right)$,

(5) $\Phi_1\oplus\Phi_2=\left(\dot{s}_1\dot{s}_2,,\underset{.}{i}_1\underset{.}{i}_2,\sqrt[q]{\underset{\hat{}}{d}_1^{q}+\underset{\hat{}}{d}_2^{q}-\underset{\hat{}}{d}_1^{q}\underset{\hat{}}{d}_2^{q}}\right)$,

(6) $\zeta\Phi_1=\left(\sqrt[q]{1-\left(1-\dot{s}_1^{q}\right)^{\zeta}},\underset{.}{i}_1^{\zeta},\underset{\hat{}}{d}_1^{\zeta}\right)$,

(7) $\Phi_1{}^{\zeta}=\left(\dot{s}_1^{\zeta},\underset{.}{i}_1^{\zeta},\sqrt[q]{1-\left(1-\dot{s}_1^{q}\right)^{\zeta}}\right)$.

Definition 3 (Mahmood et al. 2019) For a TSFN $\Phi=\left(\dot{s}(x),\underset{.}{i}(x),\underset{\hat{}}{d}(x)\right)$, the score function and accuracy function for TSFNs is defined as under:

$$SC(\Phi)=\dot{s}^{q}-\underset{\hat{}}{d}^{q},\ \text{and}\ SC(\Phi)\in[-1,1],\tag{2}$$

$$AC(\Phi)=\dot{s}^{q}+\underset{.}{i}^{q}+\underset{\hat{}}{d}^{q},\ \text{and}AC(\Phi)\in[0,1].\tag{3}$$

Definition 4 (Mahmood et al. 2019) Let Φ_1 and Φ_2 be two TSFNs. SC is the "score function", and AC is the "accuracy function", then $\Phi_1>\Phi_2$, where the notation > stands for "preferred to" if either $SC(\Phi_1)>SC(\Phi_2)$ or $SC(\Phi_1)=SC(\Phi_2)$ and $AC(\Phi_1)>AC(\Phi_2)$ holds.

Definition 5 (Liu et al. 2019) Assume $\Phi_1=(\dot{s}_1,\underset{.}{i}_1,\underset{\hat{}}{d}_1)$ and $\Phi_2=(\dot{s}_2,\underset{.}{i}_2,\underset{\hat{}}{d}_2)$ be any two TSFNs. Then, the Hamming distance between Φ_1 and Φ_2 is formulated as

$$\bar{\bar{\mathrm{d}}}(\Phi_1,\Phi_2)=\frac{1}{3}\left(\left|\dot{s}_1-\dot{s}_2\right|+\left|\underset{.}{i}_1-\underset{.}{i}_2\right|+\left|\underset{\hat{}}{d}_1-\underset{\hat{}}{d}_2\right|\right).\tag{4}$$

The following introduces new operation rules for TSFNs based on Dombi t-norm and t-conorm (DTCN). The generator of DTCN is defined as under.

Definition 6 (Dombi 1982) Consider a real number $\zeta>0$ and $y,z\in[0,1]$. Then, DTNCN is defined as

$$T_{D,\zeta}(y, z) = \frac{1}{1 + \left(\left(\frac{1-y}{y}\right)^{\zeta} + \left(\frac{1-z}{z}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}. \tag{5}$$

$$T^{*}{}_{D,\zeta}(y, z) = 1 - \frac{1}{1 + \left(\left(\frac{1-y}{y}\right)^{\zeta} + \left(\frac{1-z}{z}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}. \tag{6}$$

Based on DTNCN, we suggest some new operational laws for TSFNs.

Definition 7 Let $\Phi_1 = (\dot{s}_1, \underset{\cdot}{i}_1, \underset{\hat{}}{d}_1)$ and $\Phi_2 = (\dot{s}_2, \underset{\cdot}{i}_2, \underset{\hat{}}{d}_2)$ be any two TSFNs, and $\zeta > 0, \gamma > 0$ are real numbers, then

1. $$\Phi_1 \oplus \Phi_2 = \left(\sqrt[q]{1 - \frac{1}{1+\left(\left(\frac{\dot{s}_1^{\,q}}{1-\dot{s}_1^{\,q}}\right)^{\zeta} + \left(\frac{\dot{s}_2^{\,q}}{1-\dot{s}_2^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{\frac{1}{1+\left(\left(\frac{1-\underset{\cdot}{i}_1^{\,q}}{\underset{\cdot}{i}_1^{\,q}}\right)^{\zeta} + \left(\frac{1-\underset{\cdot}{i}_2^{\,q}}{\underset{\cdot}{i}_2^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{\frac{1}{1+\left(\left(\frac{1-\underset{\hat{}}{d}_1^{\,q}}{\underset{\hat{}}{d}_1^{\,q}}\right)^{\zeta} + \left(\frac{1-\underset{\hat{}}{d}_2^{\,q}}{\underset{\hat{}}{d}_2^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}} \right),$$
2. $$\Phi_1 \otimes \Phi_2 = \left(\sqrt[q]{\frac{1}{1+\left(\left(\frac{1-\dot{s}_1^{\,q}}{\dot{s}_1^{\,q}}\right)^{\zeta} + \left(\frac{1-\dot{s}_2^{\,q}}{\dot{s}_2^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{1 - \frac{1}{1+\left(\left(\frac{\underset{\cdot}{i}_1^{\,q}}{1-\underset{\cdot}{i}_1^{\,q}}\right)^{\zeta} + \left(\frac{\underset{\cdot}{i}_2^{\,q}}{1-\underset{\cdot}{i}_2^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{1 - \frac{1}{1+\left(\left(\frac{\underset{\hat{}}{d}_1^{\,q}}{1-\underset{\hat{}}{d}_1^{\,q}}\right)^{\zeta} + \left(\frac{\underset{\hat{}}{d}_2^{\,q}}{1-\underset{\hat{}}{d}_2^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}} \right),$$
3. $$\gamma\Phi_1 = \left(\sqrt[q]{1 - \frac{1}{1+\left(\gamma\left(\frac{\dot{s}_1^{\,q}}{1-\dot{s}_1^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{\frac{1}{1+\left(\gamma\left(\frac{1-\underset{\cdot}{i}_1^{\,q}}{\underset{\cdot}{i}_1^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{\frac{1}{1+\left(\gamma\left(\frac{1-\underset{\hat{}}{d}_1^{\,q}}{\underset{\hat{}}{d}_1^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}} \right),$$
4. $$\Phi_1{}^{\gamma} = \left(\sqrt[q]{\frac{1}{1+\left(\gamma\left(\frac{\dot{s}_1^{\,q}}{1-\dot{s}_1^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{1 - \frac{1}{1+\left(\gamma\left(\frac{1-\underset{\cdot}{i}_1^{\,q}}{\underset{\cdot}{i}_1^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}}, \sqrt[q]{1 - \frac{1}{1+\left(\gamma\left(\frac{1-\underset{\hat{}}{d}_1^{\,q}}{\underset{\hat{}}{d}_1^{\,q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}}} \right)$$

Definition 8 (Sýkora 2009) Consider a series of crisp numbers. $x_{ɥ}\left(ɥ = 1, 2, ..., ŋ\right)$, and $s, t \geq 0$, then Heronian mean is defined as follows

$$HM^{s,t}\left(x_1, x_2, \ldots, x_{ɥ}\right) = \left(\frac{2}{ŋ(ŋ+1)} \sum_{ɥ=1}^{ŋ} \sum_{Þ=ɥ}^{ŋ} x_{ɥ}^{s} x_{Þ}^{t}\right)^{\frac{1}{s+t}}. \tag{7}$$

Definition 9 (Sýkora 2009) Consider a series of crisp numbers $x_{ɥ}\left(ɥ = 1, 2, ..., ŋ\right)$, and $s, t \geq 0$, then geometric Heronian mean is defined as

$$GHM^{s,t}\left(x_1, x_2, ..., x_{ɥ}\right) = \left(\frac{1}{s+t} \prod_{ɥ=1}^{ŋ} \prod_{Þ=ɥ}^{ŋ} \left(sx_{ɥ} + tx_{Þ}\right)^{\frac{2}{ŋ(ŋ+1)}}\right). \tag{8}$$

Definition 10 (Yager 2001) Let $\Phi_{ɥ}\left(ɥ = 1, 2, \ldots, ŋ\right)$ be a set of nonnegative real numbers, and the PA operator is defined as follows

$$\mathrm{PA}\left(\Phi_1, \Phi_2, ..., \Phi_{ŋ}\right) = \frac{\sum_{ɥ=1}^{ŋ}\left(1 + T\left(\Phi_{ɥ}\right)\right)\Phi_{ɥ}}{\sum_{Þ=1}^{ŋ}(1 + T(\Phi_{Þ}))}, \tag{9}$$

where

$$T\left(\Phi_{\text{ɥ}}\right)=\sum_{\text{Þ}=1,\,\text{Þ}\neq\text{ɥ}}^{\text{ŋ}}\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right), \tag{10}$$

and $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)=1-\overline{\overline{\mathrm{d}}}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)$, $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)$ is the support for $\Phi_{\text{ɥ}}$ from $\Phi_{\text{Þ}}$, with the following conditions:

1. $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)\in[0,1]$,
2. $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)=\operatorname{Sup}\left(\Phi_{\text{Þ}},\Phi_{\text{ɥ}}\right)$,

$\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)\geq\operatorname{Sup}\left(\Phi_i,\Phi_j\right)$, if $\overline{\overline{\mathrm{d}}}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)\leq\overline{\overline{\mathrm{d}}}\left(\Phi_i,\Phi_j\right)$.

3 T-spherical fuzzy Dombi power Heronian mean aggregation operators

Some novel aggregation operators under TSPFS will be discussed in the given section.

Definition 11 Let $s,t\geq 0$, and Φ_p be a group of "ŋ" TSFNs. Then TSFDHM of $\left(\Phi_1,\Phi_2,\ldots,\Phi_{\text{ŋ}}\right)$ is defined as

$$\mathrm{TSPFDHM}^{s,t}\left(\Phi_1,\Phi_2,\ldots,\Phi_{\text{ŋ}}\right)=\left(\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}\left(\frac{\text{ŋ}\left(1+T\left(\Phi_{\text{ɥ}}\right)\right)}{\sum_{o=1}^{\text{ŋ}}\left(1+T\left(\Phi_o\right)\right)}\Phi_{\text{ɥ}}\right)^{s}\otimes\left(\frac{\text{ŋ}\left(1+T\left(\Phi_p\right)\right)}{\sum_{o=1}^{\text{ŋ}}\left(1+T\left(\Phi_o\right)\right)}\Phi_{\text{Þ}}\right)^{t}\right)^{\frac{1}{s+t}}. \tag{11}$$

where $T\left(\Phi_{\text{ɥ}}\right)=\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)$, $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)=1-\overline{\overline{\mathrm{d}}}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)$, $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)$ is the support for $\Phi_{\text{ɥ}}$ from $\Phi_{\text{Þ}}$, with following conditions: (1) $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)\in[0,1]$, (2) $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)=\operatorname{Sup}\left(\Phi_{\text{Þ}},\Phi_{\text{ɥ}}\right)$, (3) $\operatorname{Sup}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)\geq\operatorname{Sup}\left(\Phi_i,\Phi_j\right)$ if $\overline{\overline{\mathrm{d}}}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)\leq\overline{\overline{\mathrm{d}}}\left(\Phi_i,\Phi_j\right)$ here, $\overline{\overline{\mathrm{d}}}\left(\Phi_{\text{ɥ}},\Phi_{\text{Þ}}\right)$ represents the distance between $\Phi_{\text{Þ}}$ and $\Phi_{\text{ɥ}}$.

Let $\bar{\text{Ṛ}}_{\text{Þ}}=\frac{\left(1+T\left(\Phi_p\right)\right)}{\sum_{o=1}^{\text{ŋ}}\left(1+T\left(\Phi_o\right)\right)}$ and $\bar{\text{Ṛ}}_{\text{ɥ}}=\frac{\text{ŋ}\left(1+T\left(\Phi_{\text{ɥ}}\right)\right)}{\sum_{o=1}^{\text{ŋ}}\left(1+T\left(\Phi_o\right)\right)}$. Then (8) becomes,

$$\mathrm{TSPFDPHM}^{s,t}\left(\Phi_1,\Phi_2,\ldots,\Phi_{\text{ŋ}}\right)=\left(\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\Phi_{\text{ɥ}}\right)^{s}\otimes\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}\right)^{\frac{1}{s+t}}. \tag{12}$$

Theorem 1 Let $s,t\geq 0$ and $\Phi_{\text{ɥ}}=\left(\dot{\text{ṡ}}_{\text{ɥ}},\text{ị}_{\text{ɥ}},\hat{d}_{\text{ɥ}}\right)$ be a group of "ŋ" TSFNs and a real number $\zeta>0$. Then, aggregation of $\Phi_{\text{ɥ}}$ by using Definition 11 is also a TSFN

$$\text{TSPFDPHM}^{s,t}\left(\Phi_1,\ \Phi_2,\ \ldots,\ \Phi_{\text{ŋ}}\right)$$

$$=\left(\begin{array}{c}\sqrt[q]{1/\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(\frac{s}{\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{\dot{\text{ṡ}}_{\text{ɥ}}^{\,q}}{1-\dot{\text{ṡ}}_{\text{ɥ}}^{\,q}}\right)^{\zeta}\right)}+\frac{t}{\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{\dot{\text{ṡ}}_{\text{Þ}}^{\,q}}{1-\dot{\text{ṡ}}_{\text{Þ}}^{\,q}}\right)^{\zeta}\right)}\right)\right)\right)\right)^{1/\zeta}},\\ \sqrt[q]{1-1/\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(\frac{s}{\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{\dot{\text{i}}_{\text{ɥ}}^{\,q}}{1-\dot{\text{i}}_{\text{ɥ}}^{\,q}}\right)^{\zeta}\right)}+\frac{t}{\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{\dot{\text{i}}_{\text{Þ}}^{\,q}}{1-\dot{\text{i}}_{\text{Þ}}^{\,q}}\right)^{\zeta}\right)}\right)\right)\right)\right)^{1/\zeta}},\\ \sqrt[q]{1-1/\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(\frac{s}{\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{\hat{d}_{\text{ɥ}}^{\,q}}{1-\hat{d}_{\text{ɥ}}^{\,q}}\right)^{\zeta}\right)}+\frac{t}{\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{\hat{d}_{\text{Þ}}^{\,q}}{1-\hat{d}_{\text{Þ}}^{\,q}}\right)^{\zeta}\right)}\right)\right)\right)\right)^{1/\zeta}}\end{array}\right)$$

Proof Let $\frac{\dot{\text{ṡ}}_{\text{ɥ}}^{\,q}}{1-\dot{\text{ṡ}}_{\text{ɥ}}^{\,q}}=K_{\text{ɥ}}$, $\frac{\dot{\text{ṡ}}_{\text{Þ}}^{\,q}}{1-\dot{\text{ṡ}}_{\text{Þ}}^{\,q}}=K_{\text{Þ}}$, $\frac{1-\dot{\text{i}}_{\text{ɥ}}^{\,q}}{\dot{\text{i}}_{\text{ɥ}}^{\,q}}=L_{\text{ɥ}}$, $\frac{1-\dot{\text{i}}_{\text{Þ}}^{\,q}}{\dot{\text{i}}_{\text{Þ}}^{\,q}}=L_{\text{Þ}}$, $\frac{1-\hat{d}_{\text{ɥ}}^{\,q}}{\hat{d}_{\text{ɥ}}^{\,q}}=M_{\text{ɥ}}$, $\frac{1-\hat{d}_{\text{Þ}}^{\,q}}{\hat{d}_{\text{Þ}}^{\,q}}=M_{\text{Þ}}$.

Using Definition 6, we obtain

$$\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\Phi_{\text{ɥ}}=\left(\sqrt[q]{1-\frac{1}{\left(1+\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\right)^{\frac{1}{\zeta}}K_{\text{ɥ}}\right)}},\sqrt[q]{\frac{1}{\left(1+\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\right)^{\frac{1}{\zeta}}L_{\text{ɥ}}\right)}},\sqrt[q]{\frac{1}{\left(1+\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{ɥ}}\right)^{\frac{1}{\zeta}}M_{\text{ɥ}}\right)}}\right),$$

$$\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\Phi_{\text{Þ}}=\left(\sqrt[q]{1-\frac{1}{\left(1+\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\right)^{\frac{1}{\zeta}}K_{\text{Þ}}\right)}},\sqrt[q]{\frac{1}{\left(1+\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\right)^{\frac{1}{\zeta}}L_{\text{Þ}}\right)}},\sqrt[q]{\frac{1}{\left(1+\left(\text{ŋ}\bar{\text{Ṛ}}_{\text{Þ}}\right)^{\frac{1}{\zeta}}M_{\text{Þ}}\right)}}\right),$$

$$\left(\text{ŋ}\bar{R}_{\text{ɥ}}\Phi_{\text{ɥ}}\right)^{s}=\left(\sqrt[q]{\frac{1}{\left(1+\left(\frac{s}{\left(\text{ŋ}\bar{R}_{\text{ɥ}}K_{\text{ɥ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}},\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{s}{\left(\text{ŋ}\bar{R}_{\text{ɥ}}L_{\text{ɥ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}},\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{s}{\left(\text{ŋ}\bar{R}_{\text{ɥ}}M_{\text{ɥ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}}\right),$$

$$\left(\text{ŋ}\bar{R}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}=\left(\sqrt[q]{\frac{1}{\left(1+\left(\frac{t}{\left(\text{ŋ}\bar{R}_{\text{Þ}}K_{\text{Þ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}},\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{t}{\left(\text{ŋ}\bar{R}_{\text{Þ}}L_{\text{Þ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}},\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{t}{\left(\text{ŋ}\bar{R}_{\text{Þ}}M_{\text{Þ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}}\right),$$

$$\left(\text{ŋ}\bar{R}_{\text{ɥ}}\Phi_{\text{ɥ}}\right)^{s}\otimes\left(\text{ŋ}\bar{R}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}$$

$$=\left(\sqrt[q]{\frac{1}{\left(1+\left(\frac{s}{\left(\text{ŋ}\bar{R}_{\text{ɥ}}K_{\text{ɥ}}{}^{\zeta}\right)}+\frac{t}{\left(\text{ŋ}\bar{R}_{\text{Þ}}K_{\text{Þ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}},\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{s}{\left(\text{ŋ}\bar{R}_{\text{ɥ}}L_{\text{ɥ}}{}^{\zeta}\right)}+\frac{t}{\left(\text{ŋ}\bar{R}_{\text{Þ}}L_{\text{Þ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}},\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{s}{\left(\text{ŋ}\bar{R}_{\text{ɥ}}M_{\text{ɥ}}{}^{\zeta}\right)}+\frac{t}{\left(\text{ŋ}\bar{R}_{\text{Þ}}M_{\text{Þ}}{}^{\zeta}\right)}\right)\right)^{\frac{1}{\zeta}}}}\right),$$

$$\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}\left(\text{ŋ}\bar{R}_{\text{ɥ}}\Phi_{\text{ɥ}}\right)^{s}\otimes\left(\text{ŋ}\bar{R}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}$$

$$=\left(\begin{array}{c}\sqrt[q]{1-\frac{1}{\left(1+\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ɥ}}K_{\text{ɥ}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}K_{\text{Þ}}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{\frac{1}{\left(1+\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ɥ}}L_{\text{ɥ}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}L_{\text{Þ}}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{\frac{1}{\left(1+\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ɥ}}M_{\text{ɥ}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}M_{\text{Þ}}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}}\end{array}\right),$$

$$\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}\left(\text{ŋ}\bar{R}_{\text{ч}}\Phi_{\text{ч}}\right)^{s}\otimes\left(\text{ŋ}\bar{R}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}$$

$$=\left(\begin{array}{c}\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ч}}K_{\text{ч}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}K_{\text{Þ}}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{\frac{1}{\left(1+\left(\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ч}}L_{\text{ч}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}L_{\text{Þ}}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{\frac{1}{\left(1+\left(\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ч}}M_{\text{ч}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}M_{\text{Þ}}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}}\end{array}\right),$$

$$\left(\frac{2}{\text{ŋ}(\text{ŋ}+1)}\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}\left(\text{ŋ}\bar{R}_{\text{ч}}\Phi_{\text{ч}}\right)^{s}\otimes\left(\text{ŋ}\bar{R}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}\right)^{\frac{1}{s+t}}$$

$$=\left(\begin{array}{c}\sqrt[q]{\frac{1}{\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ч}}K_{\text{ч}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}K_{\text{Þ}}{}^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ч}}L_{\text{ч}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}L_{\text{Þ}}{}^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ч}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ч}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\bar{R}_{\text{ч}}M_{\text{ч}}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{R}_{\text{Þ}}M_{\text{Þ}}{}^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}\end{array}\right).$$

We put $\frac{\dot{s}_{\text{ч}}{}^{q}}{1-\dot{s}_{\text{ч}}{}^{q}}=K_{\text{ч}}$, $\frac{\dot{s}_{\text{Þ}}{}^{q}}{1-\dot{s}_{\text{Þ}}{}^{q}}=K_{\text{Þ}}$, $\frac{1-\dot{i}_{\text{ч}}{}^{q}}{\dot{i}_{\text{ч}}{}^{q}}=L_{\text{ч}}$, $\frac{1-\dot{i}_{\text{Þ}}{}^{q}}{\dot{i}_{\text{Þ}}{}^{q}}=L_{\text{Þ}}$, $\frac{1-\hat{d}_{\text{ч}}{}^{q}}{\hat{d}_{\text{ч}}{}^{q}}=M_{\text{ч}}$, $\frac{1-\hat{d}_{\text{Þ}}{}^{q}}{\hat{d}_{\text{Þ}}{}^{q}}=M_{\text{Þ}}$.

$$\left(\frac{2}{\eta(\eta+1)}\sum_{\text{ɥ}=1}^{\eta}\sum_{\text{Þ}=\text{ɥ}}^{\eta}\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\Phi_{\text{ɥ}}\right)^{s}\otimes\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\Phi_{\text{Þ}}\right)^{t}\right)^{\frac{1}{s+t}}$$

$$=\left(\begin{array}{c}\sqrt[q]{\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\eta}\sum_{\text{Þ}=\text{ɥ}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{\dot{s}_{\text{ɥ}}^{q}}{1-\dot{s}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{\dot{s}_{\text{Þ}}^{q}}{1-\dot{s}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥɥ}=1}^{\eta}\sum_{\text{Þ}=\text{ɥ}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{1-\dot{i}_{\text{ɥ}}^{q}}{\dot{i}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{1-\dot{i}_{\text{Þ}}^{q}}{\dot{i}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\eta}\sum_{\text{Þ}=\text{ɥ}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{1-\hat{d}_{\text{ɥ}}^{q}}{\hat{d}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{1-\hat{d}_{\text{Þ}}^{q}}{\hat{d}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}\end{array}\right),$$

$$\text{TSPFDPHM}^{s,t}\left(\Phi_1,\ \Phi_2,\ \ldots,\ \Phi_{\eta}\right)$$

$$=\left(\begin{array}{c}\sqrt[q]{\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\eta}\sum_{\text{Þ}=\text{ɥ}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{\dot{s}_{\text{ɥ}}^{q}}{1-\dot{s}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{\dot{s}_{\text{Þ}}^{q}}{1-\dot{s}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\eta}\sum_{=\text{ɥ}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{1-\dot{i}_{\text{ɥ}}^{q}}{\dot{i}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{1-\dot{i}_{\text{Þ}}^{q}}{\dot{i}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\eta}\sum_{\text{Þ}=\text{ɥ}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ɥ}}\left(\frac{1-\hat{d}_{\text{ɥ}}^{q}}{\hat{d}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{1-\hat{d}_{\text{Þ}}^{q}}{\hat{d}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}\end{array}\right).$$

This fulfills our result.

Now, we discuss some salient properties of the operator.

Example 1 Consider three TSFNs $\Phi_1 = (0.6,0.2,0.3)$ $\Phi_2 = (0.4,0.3,0.7)$ $\Phi_3 = (0.5,0.4,0.4)$,$s = 1t = 2$, $q = 3$ and $\zeta = 3$ we use TSPFDPHM to aggregate the four TSFNs.

$$\sqrt[q]{\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ч}=1}^{\eta}\sum_{\text{Þ}=\text{ч}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ч}}\left(\frac{\dot{\text{ṡ}}_{\text{ч}}^{q}}{1-\dot{\text{ṡ}}_{\text{ч}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{\dot{\text{ṡ}}_{\text{Þ}}^{q}}{1-\dot{\text{ṡ}}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}=0.5326,$$

$$\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ч}=1}^{\eta}\sum_{\text{Þ}=\text{ч}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ч}}\left(\frac{1-\text{ị}_{\text{ч}}^{q}}{\text{ị}_{\text{ч}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{1-\text{ị}_{\text{Þ}}^{q}}{\text{ị}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}=0.2376,$$

$$\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\text{ч}=1}^{\eta}\sum_{\text{Þ}=\text{ч}}^{\eta}1/\left(s/\left(\eta\bar{\text{Ṛ}}_{\text{ч}}\left(\frac{1-\hat{d}_{\text{ч}}^{q}}{\hat{d}_{\text{ч}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\bar{\text{Ṛ}}_{\text{Þ}}\left(\frac{1-\hat{d}_{\text{Þ}}^{q}}{\hat{d}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}$$
$= 0.3513.$

Therefore, we get $\text{TSPFDPHM}^{s,t}(\Phi_1, \Phi_2, \Phi_3) = (0.5326, 0.2376, 0.3513)$.

Furthermore, the TSFDHM operators satisfy the properties given below.

Theorem 2 (Idempotency) Let $s, t \geq 0$ and $\Phi_{\text{ч}} = (\dot{\text{ṡ}}_{\text{ч}}, \text{ị}_{\text{ч}}, \hat{d}_{\text{ч}})$ be a collection of "η" TSFNs, If $\Phi_{\text{ч}}$ is equal $\forall_{\text{ч}}$, ($_{\text{ч}} = 1, 2, \ldots, \eta$) that is, $\Phi_{\text{ч}} = \Phi = \left(\dot{\text{ṡ}}, \text{ị}, \hat{d}\right)$. Then,

$$\text{TSPFDPHM}^{s,t}(\Phi_1, \Phi_2, \ldots, \Phi_\eta) = \Phi. \tag{13}$$

Proof Since, $\Phi_{\text{ч}} = \Phi = \left(\dot{\text{ṡ}}, \text{ị}, \hat{d}\right)$ so we have $\forall \text{Þ}$, $_{\text{ч}} = 1, 2, \ldots, \eta$. Thereby we will prove $\bar{\text{Ṛ}}_{\text{ч}} = \frac{1}{\eta}$, $_{\text{ч}} = 1, 2, \ldots, \eta$.

$$
\begin{aligned}
&\mathrm{TSPFDPHM}^{s,t}\left(\Phi_1,\ \Phi_2,\ \ldots,\ \Phi_{\mathfrak{y}}\right)\\
&=\left(\begin{array}{l}
\sqrt[q]{\dfrac{1}{\left(1+\left(\frac{\mathfrak{y}(\mathfrak{y}+1)}{2(s+t)}\right)\times 1/\left(\sum_{\text{ɥ}=1}^{\mathfrak{y}}\sum_{\text{Þ}=\text{ɥ}}^{\mathfrak{y}} 1/\left(s/\left(\mathfrak{y}\frac{1}{\mathfrak{y}}\left(\frac{\dot{s}^q}{1-\dot{s}^q}\right)^{\zeta}\right)+t/\left(\mathfrak{y}\frac{1}{\mathfrak{y}}\left(\frac{\dot{s}^q}{1-\dot{s}^q}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}}},\\
\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\mathfrak{y}(\mathfrak{y}+1)}{2(s+t)}\right)\times 1/\left(\sum_{\text{ɥ}=1}^{\mathfrak{y}}\sum_{\text{Þ}=\text{ɥ}}^{\mathfrak{y}} 1/\left(s/\left(\mathfrak{y}\frac{1}{\mathfrak{y}}\left(\frac{1-\underset{\cdot}{i}^q}{\underset{\cdot}{i}^q}\right)^{\zeta}\right)+t/\left(\mathfrak{y}\frac{1}{\mathfrak{y}}\left(\frac{1-\underset{\cdot}{i}^q}{\underset{\cdot}{i}^q}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}}},\\
\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\mathfrak{y}(\mathfrak{y}+1)}{2(s+t)}\right)\times 1/\left(\sum_{\text{ɥ}=1}^{\mathfrak{y}}\sum_{\text{Þ}=\text{ɥ}}^{\mathfrak{y}} 1/\left(s/\left(\mathfrak{y}\frac{1}{\mathfrak{y}}\left(\frac{1-\underset{\wedge}{d}^q}{\underset{\wedge}{d}^q}\right)^{\zeta}\right)+t/\left(\mathfrak{y}\frac{1}{\mathfrak{y}}\left(\frac{1-\underset{\wedge}{d}^q}{\underset{\wedge}{d}^q}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}}}
\end{array}\right)
\end{aligned}
$$

$$
=\left(\begin{array}{l}
\sqrt[q]{\dfrac{1}{\left(1+\left(\frac{\mathfrak{y}(\mathfrak{y}+1)}{2(s+t)}\right)\times 1/\left(\sum_{\text{ɥ}=1}^{\mathfrak{y}}\sum_{\text{Þ}=\text{ɥ}}^{\mathfrak{y}} 1/\left((s+t)/\left(\frac{\dot{s}^q}{1-\dot{s}^q}\right)^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}}},\\
\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\mathfrak{y}(\mathfrak{y}+1)}{2(s+t)}\right)\times 1/\left(\sum_{\text{ɥ}=1}^{\mathfrak{y}}\sum_{\text{Þ}=\text{ɥ}}^{\mathfrak{y}} 1/\left((s+t)/\left(\frac{1-\underset{\cdot}{i}^q}{\underset{\cdot}{i}^q}\right)^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}}},\\
\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\mathfrak{y}(\mathfrak{y}+1)}{2(s+t)}\right)\times 1/\left(\sum_{\text{ɥ}=1}^{\mathfrak{y}}\sum_{\text{Þ}=\text{ɥ}}^{\mathfrak{y}} 1/\left((s+t)/\left(\frac{1-\underset{\wedge}{d}^q}{\underset{\wedge}{d}^q}\right)^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}}}
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\sqrt[q]{\dfrac{1}{\left(1+\left(\frac{1}{(s+t)}\right)\times 1/\left(\left(\frac{\dot{s}^q}{1-\dot{s}^q}\right)^{\zeta}/(s+t)\right)\right)^{\frac{1}{\zeta}}}},\\
\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{1}{(s+t)}\right)\times 1/\left(\left(\frac{1-\underset{\cdot}{i}^q}{\underset{\cdot}{i}^q}\right)^{\zeta}/(s+t)\right)\right)^{\frac{1}{\zeta}}}},\\
\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{1}{(s+t)}\right)\times 1/\left(\left(\frac{1-\underset{\wedge}{d}^q}{\underset{\wedge}{d}^q}\right)^{\zeta}/(s+t)\right)\right)^{\frac{1}{\zeta}}}}
\end{array}\right)
$$

$$
=\left(\dot{s},\ \underset{\cdot}{i},\ \underset{\wedge}{d}\right)=\Phi.
$$

Hence, the result is proven.

Theorem 3 (Monotonicity) Let $\Phi_{ч} = (\dot{s}_{ч}, \dot{i}_{ч}, \hat{d}_{ч})$ and $\overline{\Phi_{ч}} = (\overline{\dot{s}_{ч}}, \overline{\dot{i}_{ч}}, \overline{\hat{d}_{ч}})$ be two groups of "ŋ" TSFNs, if $\Phi_{ч} \le \overline{\Phi_{ч}}$, $\forall_{ч}$, then,

$$\mathrm{TSPFDPHM}^{s,t}(\Phi_1, \Phi_2, \ldots, \Phi_{ŋ}) \le \mathrm{TSPFDPHM}^{s,t}(\overline{\Phi_1}, \overline{\Phi_2}, \ldots, \overline{\Phi_{ŋ}}). \tag{14}$$

Proof Let $\mathrm{TSPFDPHM}^{s,t}(\Phi_1, \Phi_2, \ldots, \Phi_{ŋ}) = (\dot{s}, \dot{i}, \hat{d})$ and $\mathrm{TSPFDPHM}^{s,t}(\overline{\Phi_1}, \overline{\Phi_2}, \ldots, \overline{\Phi_{ŋ}}) = (\overline{\dot{s}}, \overline{\dot{i}}, \overline{\hat{d}})$, since $\dot{s}_{ч} \le \overline{\dot{s}_{ч}}$ and $\dot{s}_{Þ} \le \overline{\dot{s}_{Þ}}$, so we get:

$$\sum_{ч=1}^{ŋ}\sum_{Þ=ч}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ч}\left(\frac{\dot{s}_{ч}^{q}}{1-\dot{s}_{ч}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{\dot{s}_{Þ}^{q}}{1-\dot{s}_{Þ}^{q}}\right)^{\zeta}\right)\right)$$

$$\le \sum_{ч=1}^{ŋ}\sum_{Þ=ч}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ч}\left(\frac{\overline{\dot{s}_{ч}}^{q}}{1-\overline{\dot{s}_{ч}}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{\overline{\dot{s}_{Þ}}^{q}}{1-\overline{\dot{s}_{Þ}}^{q}}\right)^{\zeta}\right)\right)$$

and

$$1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ч=1}^{ŋ}\sum_{Þ=ч}^{ŋ} 1/\left(\frac{s}{\left(ŋ\bar{R}_{ч}\left(\frac{\dot{s}_{ч}^{q}}{1-\dot{s}_{ч}^{q}}\right)^{\zeta}\right)} + \frac{t}{\left(ŋ\bar{R}_{Þ}\left(\frac{\dot{s}_{Þ}^{q}}{1-\dot{s}_{Þ}^{q}}\right)^{\zeta}\right)}\right)\right)\right)^{1/\zeta}$$

$$\le 1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ч=1}^{ŋ}\sum_{Þ=ч}^{ŋ} 1/\left(\frac{s}{\left(ŋ\bar{R}_{ч}\left(\frac{\overline{\dot{s}_{ч}}^{q}}{1-\overline{\dot{s}_{ч}}^{q}}\right)^{\zeta}\right)} + \frac{t}{\left(ŋ\bar{R}_{Þ}\left(\frac{\overline{\dot{s}_{Þ}}^{q}}{1-\overline{\dot{s}_{Þ}}^{q}}\right)^{\zeta}\right)}\right)\right)\right)^{1/\zeta}.$$

Therefore

$$\sqrt[q]{\frac{1}{\left(1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ч=1}^{ŋ}\sum_{Þ=ч}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ч}\left(\frac{\dot{s}_{ч}^{q}}{1-\dot{s}_{ч}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{\dot{s}_{Þ}^{q}}{1-\dot{s}_{Þ}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},$$

$$\le \sqrt[q]{\frac{1}{\left(1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ч=1}^{ŋ}\sum_{Þ=ч}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ч}\left(\frac{\overline{\dot{s}_{ч}}^{q}}{1-\overline{\dot{s}_{ч}}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{\overline{\dot{s}_{Þ}}^{q}}{1-\overline{\dot{s}_{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},$$

it shows that $\dot{s}_{ɥ} \leq \overline{\dot{s}_{ɥ}}$, in the same way, we can prove $i_{ɥ} \geq \overline{i_{ɥ}}$ and $\hat{d}_{ɥ} \geq \overline{\hat{d}_{ɥ}}$. Using Definition 3 we obtain $SC(\Phi) \leq SC(\overline{\Phi})$, that is $\Phi \leq \overline{\Phi}$. Hence, $\text{TSFDPHM}^{s,t}(\Phi_1, \Phi_2, ..., \Phi_{\mathfrak{y}}) \leq \text{TSFDPHM}^{s,t}(\overline{\Phi_1}, \overline{\Phi_2}, ..., \overline{\Phi_{\mathfrak{y}}})$.

Theorem 4 (Boundedness) Let $\Phi_{ɥ} = (\dot{s}_{ɥ}, i_{ɥ}, \hat{d}_{ɥ})$ be a group of "$\mathfrak{y}$" TSFNs, if $\Phi^+ = \left(max\left(\dot{s}_{ɥ}\right), min\left(i_{ɥ}\right), min\left(\hat{d}_{ɥ}\right)\right)$ and $\Phi^- = \left(min\left(\dot{s}_{ɥ}\right), max\left(i_{ɥ}\right), max\left(\hat{d}_{ɥ}\right)\right)$, then,

$$\overline{v}_f = \text{TSFDWPHM}^{s,t}(\overline{v}_{f1}, \overline{v}_{f2}, ..., \overline{v}_{f\mathfrak{z}}). \tag{15}$$

Proof Using Theorem 2, we can have

$$\text{TSPFDPHM}^{s,t}\left(\Phi^-, \Phi^-, ..., \Phi^-\right) = \Phi^-, \text{TSPFDPHM}^{s,t}\left(\Phi^+, \Phi^+, ..., \Phi^+\right) = \Phi^+.$$

Subsequently,

$$\text{TSPFDPHM}^{s,t}\left(\Phi^-, \Phi^-, ..., \Phi^-\right) \leq \text{TSPFDPHM}^{s,t}\left(\Phi_1, \Phi_2, ..., \Phi_{\mathfrak{y}}\right) \leq \text{TSPFDPHM}^{s,t}\left(\Phi^+, \Phi^+, ..., \Phi^+\right).$$

Hence, Theorem 4 is proved.

Definition 12 Let $s, t \geq 0$, and $\Phi_{ɥ}$ be a set of "$\mathfrak{y}$" TSFNs. Then, TSPFDWPHM of $\left(\Phi_1, \Phi_2, ..., \Phi_{\mathfrak{y}}\right)$ is defined as

$$\text{TSPFDWPHM}^{s,t}(\Phi_1, \Phi_2, ..., \Phi_{\mathfrak{y}}) = \left(\frac{2}{\mathfrak{y}(\mathfrak{y}+1)} \sum_{ɥ=1}^{\mathfrak{y}} \sum_{Þ=ɥ}^{\mathfrak{y}} \left(\frac{\mathfrak{y}\omega_{ɥ}\left(1+T\left(\Phi_{ɥ}\right)\right)}{\sum_{o=1}^{\mathfrak{y}} \omega_o(1+T(\Phi_o))}\Phi_{ɥ}\right)^s \otimes \left(\frac{\mathfrak{y}\omega_Þ(1+T(\Phi_Þ))}{\sum_{o=1}^{\mathfrak{y}} \omega_o(1+T(\Phi_o))}\Phi_Þ\right)^t\right)^{\frac{1}{s+t}}, \tag{16}$$

where $T\left(\Phi_{ɥ}\right) = \sum_{Þ=ɥ}^{\mathfrak{y}} \text{Sup}\left(\Phi_{ɥ}, \Phi_Þ\right)$, $\text{Sup}\left(\Phi_{ɥ}, \Phi_Þ\right) = 1 - \overline{\overline{d}}\left(\Phi_{ɥ}, \Phi_Þ\right)$, $\text{Sup}\left(\Phi_{ɥ}, \Phi_Þ\right)$ is the support for $\Phi_{ɥ}$ from $\Phi_Þ$ and $\omega = (\omega_1, \omega_2, ...\omega_{\mathfrak{y}})^T$ represents the weight of $\Phi_{ɥ}$ and satisfying $\omega_{ɥ} \in [0,1]$, $\sum_{ɥ=1}^{\mathfrak{y}} \omega_{ɥ} = 1$.

$$\psi_{ɥ} = \frac{\omega_{ɥ}\left(1+T\left(\Phi_{ɥ}\right)\right)}{\sum_{o=1}^{\mathfrak{y}} \omega_o(1+T(\Phi_o))}. \tag{17}$$

For the sake of simplicity of (9), let

$$\text{TSPFDWPHM}^{s,t}\left(\Phi_1, \Phi_2, ..., \Phi_{\mathfrak{y}}\right) = \left(\frac{2}{(\mathfrak{y}+1)} \sum_{ɥ=1}^{\mathfrak{y}} \sum_{Þ=ɥ}^{\mathfrak{y}} \left(\mathfrak{y}\psi_{ɥ}\Phi_{ɥ}\right)^s \otimes (\mathfrak{y}\psi_Þ\Phi_Þ)^t\right)^{\frac{1}{s+t}}. \tag{18}$$

Now, (9) will be given as

Theorem 5 Let $s, t \geq 0$ and $\Phi_{ɥ} = (\dot{s}_{ɥ}, i_{ɥ}, \hat{d}_{ɥ})$ be a group of "$\mathfrak{y}$" TSFNs and a real number $\zeta > 0$. Then, aggregation of $\Phi_{ɥ}$ using Definition 12 is also a TSFN.

$$\mathrm{TSPFDWPHM}^{s,t}\left(\Phi_1,\ \Phi_2,\ ...,\ \Phi_{\eta}\right) = \left(\begin{array}{c} \sqrt[q]{\dfrac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\mathrm{q}=1}^{\eta}\sum_{\mathrm{Þ}=\mathrm{q}}^{\eta} 1/\left(s/\left(\eta\psi_{\mathrm{q}}\left(\frac{\dot{s}_{\mathrm{q}}^{q}}{1-\dot{s}_{\mathrm{q}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\psi_{\mathrm{Þ}}\left(\frac{\dot{s}_{\mathrm{Þ}}^{q}}{1-\dot{s}_{\mathrm{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}, \\ \sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\mathrm{q}=1}^{\eta}\sum_{\mathrm{Þ}=\mathrm{q}}^{\eta} 1/\left(s/\left(\eta\psi_{\mathrm{q}}\left(\frac{1-\dot{i}_{\mathrm{q}}^{q}}{\dot{i}_{\mathrm{q}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\psi_{\mathrm{Þ}}\left(\frac{1-\dot{i}_{\mathrm{Þ}}^{q}}{\dot{i}_{\mathrm{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}, \\ \sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\mathrm{q}=1}^{\eta}\sum_{\mathrm{Þ}=\mathrm{q}}^{\eta} 1/\left(s/\left(\eta\psi_{\mathrm{q}}\left(\frac{1-\hat{d}_{\mathrm{q}}^{q}}{\hat{d}_{\mathrm{q}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\psi_{\mathrm{Þ}}\left(\frac{1-\hat{d}_{\mathrm{Þ}}^{q}}{\hat{d}_{\mathrm{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}} \end{array} \right).$$

Proof. The proof is same as the Theorem 1.

Example 2 Consider four TSFNs $\Phi_1 = (0.7,0.3,0.6)$ $\Phi_2 = (0.8,0.6,0.5)$ $\Phi_3 = (0.5,0.4,0.7)$, $\omega = (0.4,\ 0.25,\ 0.35)$ $s = 1$, $t = 2$, $q = 3$ and $\zeta = 3$, we use TSFWDPHM to aggregate the three TSFNs.

$$\sqrt[q]{\dfrac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\mathrm{q}=1}^{\eta}\sum_{\mathrm{Þ}=\mathrm{q}}^{\eta} 1/\left(s/\left(\eta\psi_{\mathrm{q}}\left(\frac{\dot{s}_{\mathrm{q}}^{q}}{1-\dot{s}_{\mathrm{q}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\psi_{\mathrm{Þ}}\left(\frac{\dot{s}_{\mathrm{Þ}}^{q}}{1-\dot{s}_{\mathrm{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}} = 0.7252,$$

$$\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\mathrm{q}=1}^{\eta}\sum_{\mathrm{Þ}=\mathrm{q}}^{\eta} 1/\left(s/\left(\eta\psi_{\mathrm{q}}\left(\frac{1-\dot{i}_{\mathrm{q}}^{q}}{\dot{i}_{\mathrm{q}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\psi_{\mathrm{Þ}}\left(\frac{1-\dot{i}_{\mathrm{Þ}}^{q}}{\dot{i}_{\mathrm{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}} = 0.3506,$$

$$\sqrt[q]{1-\dfrac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{\mathrm{q}=1}^{\eta}\sum_{\mathrm{Þ}=\mathrm{q}}^{\eta} 1/\left(s/\left(\eta\psi_{\mathrm{q}}\left(\frac{1-\hat{d}_{\mathrm{q}}^{q}}{\hat{d}_{\mathrm{q}}^{q}}\right)^{\zeta}\right)+t/\left(\eta\psi_{\mathrm{Þ}}\left(\frac{1-\hat{d}_{\mathrm{Þ}}^{q}}{\hat{d}_{\mathrm{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}} = 0.5755.$$

Therefore, we get $TSPFWPDHM^{s,t}(\Phi_1, \Phi_2, \Phi_3) = (0.7252, 0.3506, 0.5755)$.

The TSFDWPHM operator satisfies boundedness only, idempotency, and monotonicity are not satisfied.

Definition 13. Let $s, t \geq 0$, and $\Phi_{ꝗ}$ be a group of "ŋ" TSFNs. Then TSFDPGHM of $(\Phi_1, \Phi_2, ..., \Phi_ŋ)$ is defined as

$$\mathrm{TSPFDPGHM}^{s,t}\left(\Phi_1, \Phi_2, ..., \Phi_{ꝗ}\right) = \frac{1}{s+t}\left(\prod_{ꝗ=1}^{ŋ}\prod_{Þ=ꝗ}^{ŋ}\left(s\Phi_{ꝗ}^{\frac{ŋ\left(1+T\left(\Phi_{ꝗ}\right)\right)}{\sum_{o=1}^{ŋ}(1+T(\Phi_o))}} \oplus t\Phi_{Þ}^{\frac{ŋ(1+T(\Phi_{Þ}))}{\sum_{o=1}^{ŋ}(1+T(\Phi_o))}}\right)\right)^{\frac{2}{ŋ(ŋ+1)}}. \tag{19}$$

Based on operation laws defined in Definition 7, the result shown in Theorem 8 can easily be proven.

Theorem 8 Let $s, t \geq 0$ and $\Phi_{ꝗ} = (\dot{s}_{ꝗ}, \dot{i}_{ꝗ}, \hat{d}_{ꝗ})$ be a group of "ŋ" TSFNs and a real number $\zeta > 0$. Then, aggregation of $\Phi_{ꝗ}$ using Definition 13 is also a TSFN.

$$\mathrm{TSFDPHM}^{s,t}(\Phi_1, \Phi_2, ..., \Phi_ŋ) = \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{\left(1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ꝗ=1}^{ŋ}\sum_{Þ=ꝗ}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ꝗ}\left(\frac{1-\dot{s}_{ꝗ}^{q}}{\dot{s}_{ꝗ}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{1-\dot{s}_{Þ}^{q}}{\dot{s}_{Þ}^{q}}\right)^{\zeta}\right)\right)\right)\right)\right)^{\frac{1}{\zeta}}}}, \\ \sqrt[q]{\frac{1}{\left(1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ꝗ=1}^{ŋ}\sum_{Þ=ꝗ}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ꝗ}\left(\frac{\dot{i}_{ꝗ}^{q}}{1-\dot{i}_{ꝗ}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{\dot{i}_{Þ}^{q}}{1-\dot{i}_{Þ}^{q}}\right)^{\zeta}\right)\right)\right)\right)\right)^{\frac{1}{\zeta}}}}, \\ \sqrt[q]{1 - \frac{1}{\left(1 + \left(\frac{ŋ(ŋ+1)}{2(s+t)} \times 1/\left(\sum_{ꝗ=1}^{ŋ}\sum_{Þ=ꝗ}^{ŋ} 1/\left(s/\left(ŋ\bar{R}_{ꝗ}\left(\frac{\hat{d}_{ꝗ}^{q}}{1-\hat{d}_{ꝗ}^{q}}\right)^{\zeta}\right) + t/\left(ŋ\bar{R}_{Þ}\left(\frac{\hat{d}_{Þ}^{q}}{1-\hat{d}_{Þ}^{q}}\right)^{\zeta}\right)\right)\right)\right)\right)^{\frac{1}{\zeta}}}} \end{array}\right).$$

Proof

$$\left(\Phi_{ꝗ}\right)^{ŋ\bar{R}_{ꝗ}} = \left(\sqrt[q]{1/\left(1 + \left(ŋ\bar{R}_{ꝗ}\left(\frac{1-\dot{s}_{ꝗ}^{q}}{\dot{s}_{ꝗ}^{q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1 - 1/\left(1 + \left(ŋ\bar{R}_{ꝗ}\left(\frac{\dot{i}_{ꝗ}^{q}}{1-\dot{i}_{ꝗ}^{q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1 - 1/\left(1 + \left(ŋ\bar{R}_{ꝗ}\left(\frac{\hat{d}_{Þ}^{q}}{1-\hat{d}_{Þ}^{q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}\right)}\right),$$

$$(\Phi_{Þ})^{\text{ŋ}\bar{\text{R}}_{Þ}} = \left(\sqrt[q]{1/\left(1+\left(\text{ŋ}\bar{\text{R}}_{Þ}\left(\frac{1-\dot{s}_{Þ}{}^{q}}{\dot{s}_{Þ}{}^{q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1-1/\left(1+\left(\text{ŋ}\bar{\text{R}}_{Ч}\left(\frac{\text{i}_{Þ}{}^{q}}{1-\text{i}_{Þ}{}^{q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1-1/\left(1+\left(\text{ŋ}\bar{\text{R}}_{Ч}\left(\frac{\hat{d}_{Þ}{}^{q}}{1-\hat{d}_{Þ}{}^{q}}\right)^{\zeta}\right)^{\frac{1}{\zeta}}\right)} \right).$$

We put $\frac{\dot{s}_{Ч}{}^{q}}{1-\dot{s}_{Ч}{}^{q}} = K_{Ч}, \frac{\dot{s}_{Þ}{}^{q}}{1-\dot{s}_{Þ}{}^{q}} = K_{Þ}, \frac{1-\text{i}_{Ч}{}^{q}}{\text{i}_{Ч}{}^{q}} = L_{Ч}, \frac{1-\text{i}_{Þ}{}^{q}}{\text{i}_{Þ}{}^{q}} = L_{Þ}, \frac{1-\hat{d}_{Ч}{}^{q}}{\hat{d}_{Ч}{}^{q}} = M_{Ч}, \frac{1-\hat{d}_{Þ}{}^{q}}{\hat{d}_{Þ}{}^{q}} = M_{Þ}.$

$$\left(\Phi_{Ч}\right)^{\text{ŋ}\bar{\text{R}}_{Ч}} = \left(\sqrt[q]{1/\left(1+\left(\text{ŋ}\bar{\text{R}}_{Ч}\right)^{\frac{1}{\zeta}} K_{Ч}\right)}, \sqrt[q]{1-1/\left(1+\left(\text{ŋ}\bar{\text{R}}_{Ч}\right)^{\frac{1}{\zeta}} L_{Ч}\right)}, \sqrt[q]{1-1/\left(1+\left(\text{ŋ}\bar{\text{R}}_{Ч}\right)^{\frac{1}{\zeta}} M_{Ч}\right)}\right),$$

$$(\Phi_{Þ})^{\text{ŋ}\bar{\text{R}}_{Þ}} = \left(\sqrt[q]{1/\left(1+(\text{ŋ}\bar{\text{R}}_{Þ})^{\frac{1}{\zeta}} K_{Þ}\right)}, \sqrt[q]{1-1/\left(1+(\text{ŋ}\bar{\text{R}}_{Þ})^{\frac{1}{\zeta}} L_{Þ}\right)}, \sqrt[q]{1-1/\left(1+(\text{ŋ}\bar{\text{R}}_{Þ})^{\frac{1}{\zeta}} M_{Þ}\right)}\right),$$

$$s\left(\Phi_{Ч}\right)^{\text{ŋ}\bar{\text{R}}_{Ч}} = \left(\sqrt[q]{1-1/\left(1+\left(s/\left(\text{ŋ}\bar{\text{R}}_{Ч} K_{Ч}{}^{\zeta}\right)\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1/\left(1+\left(s/\left(\text{ŋ}\bar{\text{R}}_{Ч} L_{Ч}{}^{\zeta}\right)\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1/\left(1+\left(s/\left(\text{ŋ}\bar{\text{R}}_{Ч} M_{Ч}{}^{\zeta}\right)\right)^{\frac{1}{\zeta}}\right)}\right),$$

$$t\left(\Phi_{Ч}\right)^{\text{ŋ}\bar{\text{R}}_{Ч}} = \left(\sqrt[q]{1-1/\left(1+\left(t/(\text{ŋ}\bar{\text{R}}_{Þ} K_{Þ}{}^{\zeta})\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1/\left(1+\left(t/(\text{ŋ}\bar{\text{R}}_{Þ} L_{Þ}{}^{\zeta})\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1/\left(1+\left(t/(\text{ŋ}\bar{\text{R}}_{Þ} M_{Þ}{}^{\zeta})\right)^{\frac{1}{\zeta}}\right)}\right).$$

Thereafter,

$$s\left(\Phi_{Ч}\right)^{\text{ŋ}\bar{\text{R}}_{Ч}} \oplus t\left(\Phi_{Ч}\right)^{\text{ŋ}\bar{\text{R}}_{Ч}} = \left(\begin{array}{c} \sqrt[q]{1-1/\left(1+\left(s/\left(\text{ŋ}\bar{\text{R}}_{Ч} K_{Ч}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{\text{R}}_{Þ} K_{Þ}{}^{\zeta}\right)\right)^{\frac{1}{\zeta}}\right)}, \sqrt[q]{1/\left(1+\left(s/\left(\text{ŋ}\bar{\text{R}}_{Ч} L_{Ч}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{\text{R}}_{Þ} L_{Þ}{}^{\zeta}\right)\right)^{\frac{1}{\zeta}}\right)}, \\ \sqrt[q]{1/\left(1+\left(s/\left(\text{ŋ}\bar{\text{R}}_{Ч} M_{Ч}{}^{\zeta}\right)+t/\left(\text{ŋ}\bar{\text{R}}_{Þ} M_{Þ}{}^{\zeta}\right)\right)^{\frac{1}{\zeta}}\right)} \end{array}\right)$$

and

$$\prod_{ч=1}^{\eta}\prod_{Þ=ч}^{\eta}\left(s\left(\Phi_{ч}\right)^{\eta\dot{\bar{R}}_{ч}}\oplus t\left(\Phi_{ч}\right)^{\eta\dot{\bar{R}}_{ч}}\right)==\left(\begin{array}{c}\sqrt[q]{1/\left(1+\left(\sum_{ч=1}^{\eta}\sum_{Þ=ч}^{\eta}1/\left(s/\left(\eta\dot{\bar{R}}_{ч}K_{ч}{}^{\zeta}\right)+t/\left(\eta\dot{\bar{R}}_{Þ}K_{Þ}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)},\\ \sqrt[q]{1-1/\left(1+\left(\sum_{ч=1}^{\eta}\sum_{Þ=ч}^{\eta}1/\left(s/\left(\eta\dot{\bar{R}}_{ч}L_{ч}{}^{\zeta}\right)+t/\left(\eta\dot{\bar{R}}_{Þ}L_{Þ}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)},\\ \sqrt[q]{1-1/\left(1+\left(\sum_{ч=1}^{\eta}\sum_{Þ=ч}^{\eta}1/\left(s/\left(\eta\dot{\bar{R}}_{ч}M_{ч}{}^{\zeta}\right)+t/\left(\eta\dot{\bar{R}}_{Þ}M_{Þ}{}^{\zeta}\right)\right)\right)^{\frac{1}{\zeta}}\right)}\end{array}\right),$$

$$\frac{1}{s+t}\left(\prod_{ч=1}^{\eta}\prod_{Þ=ч}^{\eta}\left(s\left(\Phi_{ч}\right)^{\eta\dot{\bar{R}}_{ч}}\oplus t\left(\Phi_{ч}\right)^{\eta\dot{\bar{R}}_{ч}}\right)\right)^{\frac{2}{\eta(\eta+1)}}=$$

$$=\left(\begin{array}{c}\sqrt[q]{1-1/\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{ч=1}^{\eta}\sum_{Þ=ч}^{\eta}1/\left(s/\left(\eta\dot{\bar{R}}_{ч}K_{ч}{}^{\zeta}\right)+t/\left(\eta\dot{\bar{R}}_{Þ}K_{Þ}{}^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)},\\ \sqrt[q]{1/\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{ч=1}^{\eta}\sum_{Þ=ч}^{\eta}1/\left(s/\left(\eta\dot{\bar{R}}_{ч}K_{ч}{}^{\zeta}\right)+t/\left(\eta\dot{\bar{R}}_{Þ}K_{Þ}{}^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)},\\ \sqrt[q]{1/\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{ч=1}^{\eta}\sum_{Þ=ч}^{\eta}1/\left(s/\left(\eta\dot{\bar{R}}_{ч}K_{ч}{}^{\zeta}\right)+t/\left(\eta\dot{\bar{R}}_{Þ}K_{Þ}{}^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}\end{array}\right).$$

We put $\frac{\dot{s}_{\text{ɥ}}^{q}}{1-\dot{s}_{\text{ɥ}}^{q}} = K_{\text{ɥ}}$, $\frac{\dot{s}_{\text{Þ}}^{q}}{1-\dot{s}_{\text{Þ}}^{q}} = K_{\text{Þ}}$, $\frac{1-\dot{i}_{\text{ɥ}}^{q}}{\dot{i}_{\text{ɥ}}^{q}} = L_{\text{ɥ}}$, $\frac{1-\dot{i}_{\text{Þ}}^{q}}{\dot{i}_{\text{Þ}}^{q}} = L_{\text{Þ}}$, $\frac{1-\hat{d}_{\text{ɥ}}^{q}}{\hat{d}_{\text{ɥ}}^{q}} = M_{\text{ɥ}}$, $\frac{1-\hat{d}_{\text{Þ}}^{q}}{\hat{d}_{\text{Þ}}^{q}} = M_{\text{Þ}}$,

$$\frac{1}{s+t}\left(\prod_{\text{ɥ}=1}^{\text{ŋ}}\prod_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}\left(s\left(\Phi_{\text{ɥ}}\right)^{\text{ŋ}\dot{\bar{R}}_{\text{ɥ}}}\oplus t\left(\Phi_{\text{ɥ}}\right)^{\text{ŋ}\dot{\bar{R}}_{\text{ɥ}}}\right)\right)^{\frac{2}{\text{ŋ}(\text{ŋ}+1)}}$$

$$=\left(\begin{array}{c}\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\dot{\bar{R}}_{\text{ɥ}}\left(\frac{1-\dot{s}_{\text{ɥ}}^{q}}{\dot{s}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\text{ŋ}\dot{\bar{R}}_{\text{Þ}}\left(\frac{1-\dot{s}_{\text{Þ}}^{q}}{\dot{s}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{\frac{1}{\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\dot{\bar{R}}_{\text{ɥ}}\left(\frac{\dot{i}_{\text{ɥ}}^{q}}{1-\dot{i}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\text{ŋ}\dot{\bar{R}}_{\text{Þ}}\left(\frac{\dot{i}_{\text{Þ}}^{q}}{1-\dot{i}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}},\\ \sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\text{ŋ}(\text{ŋ}+1)}{2(s+t)}\times 1/\left(\sum_{\text{ɥ}=1}^{\text{ŋ}}\sum_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}1/\left(s/\left(\text{ŋ}\dot{\bar{R}}_{\text{ɥ}}\left(\frac{\hat{d}_{\text{ɥ}}^{q}}{1-\hat{d}_{\text{ɥ}}^{q}}\right)^{\zeta}\right)+t/\left(\text{ŋ}\dot{\bar{R}}_{\text{Þ}}\left(\frac{\hat{d}_{\text{Þ}}^{q}}{1-\hat{d}_{\text{Þ}}^{q}}\right)^{\zeta}\right)\right)\right)\right)^{\frac{1}{\zeta}}\right)}}\end{array}\right).$$

Definition 14. Let$s$, $t \geq 0$, and $\Phi_{\text{ɥ}}$ be a group of "ŋ" TSFNs. Then TSFDWGHM of $\left(\Phi_1, \Phi_2, ..., \Phi_{\text{ŋ}}\right)$ is defined as

$$\text{TSFDWPGHM}^{s,t}\left(\Phi_1, \Phi_2, ..., \Phi_{\text{ɥ}}\right) = \frac{1}{s+t}\left(\prod_{\text{ɥ}=1}^{\text{ŋ}}\prod_{\text{Þ}=\text{ɥ}}^{\text{ŋ}}\left(s\Phi_{\text{ɥ}}^{\frac{\text{ŋ}\omega_{\text{ɥ}}\left(1+T\left(\Phi_{\text{ɥ}}\right)\right)}{\sum_{o=1}^{\text{ŋ}}\omega_o(1+T(\Phi_o))}}\oplus t\Phi_{\text{Þ}}^{\frac{\text{ŋ}\omega_{\text{Þ}}(1+T(\Phi_{\text{Þ}}))}{\sum_{o=1}^{\text{ŋ}}\omega_o(1+T(\Phi_o))}}\right)\right)^{\frac{2}{\text{ŋ}(\text{ŋ}+1)}}, \tag{20}$$

where $\text{T}\left(\Phi_{\text{ɥ}}\right) = \sum_{\substack{p=1\\ u\neq p}}^{\text{ŋ}}\text{Sup}\left(\Phi_{\text{ɥ}}, \Phi_{\text{Þ}}\right)$, $\text{Sup}\left(\Phi_{\text{ɥ}}, \Phi_{\text{Þ}}\right) = 1 - d\left(\Phi_{\text{ɥ}}, \Phi_{\text{Þ}}\right)$, $\text{Sup}\left(\Phi_{\text{ɥ}}, \Phi_{\text{Þ}}\right)$ is the support for $\Phi_{\text{ɥ}}$ from $\Phi_{\text{Þ}}$ and $\omega = \left(\omega_1, \omega_2, ...\omega_{\text{ŋ}}\right)^T$ represents the weight of $\Phi_{\text{ɥ}}$ and satisfying$\omega_{\text{ɥ}} \in [0,1]$, $\sum_{\text{ɥ}=1}^{\text{ŋ}}\omega_{\text{ɥ}} = 1$.

For the sake of simplicity of (8), let.

$$\psi_{\text{ɥ}} = \frac{\omega_{\text{ɥ}}\left(1+\text{T}\left(\Phi_{\text{ɥ}}\right)\right)}{\sum_{o=1}^{\text{ŋ}}\omega_o(1+\text{T}(\Phi_o))}.$$

Now, (19) will be given as

$$\text{TSFDWPGHM}^{s,t}(\Phi_1, \Phi_2, ..., \Phi_\eta) = \frac{1}{s+t}\left(\prod_{ɥ=1}^{\eta}\prod_{Þ=ɥ}^{\eta}\left(s\left(\Phi_ɥ\right)^{\eta\psi_ɥ}\oplus t(\Phi_Þ)^{\eta\psi_Þ}\right)\right)^{\frac{2}{\eta(\eta+1)}}. \quad (21)$$

Based on operation laws defined in Definition 7, the result shown in Theorem 12 can easily be proven.

Theorem 12 Let $s, t \geq 0$ and $\Phi_ɥ = (\dot{s}_ɥ, \dot{i}_ɥ, \hat{d}_ɥ)$ be a group of "η" TSFNs and a real number $\zeta > 0$. Then, aggregation of $\Phi_ɥ$ by using Definition 13 is also a TSFN.

$$\text{TSPFDWPHM}^{s,t}(\Phi_1, \Phi_2, ..., \Phi_\eta) = \left(\begin{array}{c}\sqrt[q]{1-\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{ɥ=1}^{\eta}\sum_{Þ=ɥ}^{\eta}1/\left(s/\left(\eta\psi_ɥ\left(\frac{1-\dot{s}_ɥ^{\,q}}{\dot{s}_ɥ^{\,q}}\right)^{\zeta}\right)+t/\left(\eta\psi_Þ\left(\frac{1-\dot{s}_Þ^{\,q}}{\dot{s}_Þ^{\,q}}\right)^{\zeta}\right)\right)\right)\right)\right)^{\frac{1}{\zeta}}}}, \\ \sqrt[q]{\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{ɥ=1}^{\eta}\sum_{Þ=ɥ}^{\eta}1/\left(s/\left(\eta\psi_ɥ\left(\frac{\dot{i}_ɥ^{\,q}}{1-\dot{i}_ɥ^{\,q}}\right)^{\zeta}\right)+t/\left(\eta\psi_Þ\left(\frac{\dot{i}_Þ^{\,q}}{1-\dot{i}_Þ^{\,q}}\right)^{\zeta}\right)\right)\right)\right)\right)^{\frac{1}{\zeta}}}}, \\ \sqrt[q]{\frac{1}{\left(1+\left(\frac{\eta(\eta+1)}{2(s+t)}\times 1/\left(\sum_{ɥ=1}^{\eta}\sum_{Þ=ɥ}^{\eta}1/\left(s/\left(\eta\psi_ɥ\left(\frac{\hat{d}_ɥ^{\,q}}{1-\hat{d}_ɥ^{\,q}}\right)^{\zeta}\right)+t/\left(\eta\psi_Þ\left(\frac{\hat{d}_Þ^{\,q}}{1-\hat{d}_Þ^{\,q}}\right)^{\zeta}\right)\right)\right)\right)\right)^{\frac{1}{\zeta}}}}\end{array}\right).$$

This result can be followed using Theorem 5.

Next, we will propose a novel scheme to solve the MAGDM problem.

4 The proposed MAGDM method based on T-spherical fuzzy Dombi power Heronian mean-based aggregation operators

In this section, we present Dombi power Heronian mean AOs to introduce an algorithm for MAGDM under the TSPFS environment. We pick the best alternative among the possible options during the DM activity. Interestingly, we have fuzzy information regarding alternatives in the form of TSPFS.

Consider $\ddot{S} = \{\ddot{S}_1, \ddot{S}_2, ..., \ddot{S}_\eta\}$ be a collection of alternatives and $\breve{Z} = \{\breve{Z}_1, \breve{Z}_2, ..., \breve{Z}_ʒ\}$ are the attributes with $\omega = (\omega_1, \omega_2, ...\omega_ʒ)^T$ is the weight vector of $\Phi_Þ (Þ = 1, 2, ..., ʒ)$, satisfying $\omega_Þ \in [0,1]$, $\sum_{Þ=1}^{ʒ}\omega_Þ = 1$. A panel of decision-makers (DMs) is denoted as $\check{N} = \{\check{N}_1, \check{N}_2, \check{N}_3, ..., \check{N}_r\}$ stands for r DMs where $\varpi = (\varpi_1, \varpi_2, ...\varpi_r)^T$ satisfying $\varpi_P \in [0,1]$, $\sum_{P=1}^{r}\varpi_p = 1$.

Assume that $\Phi^P = (\Phi^P{}_{fg})_{\eta\times ʒ}$ represents the decision matrix, where $\Phi^P{}_{fg} = (\dot{s}_{fg}, \dot{i}_{fg}, \hat{d}_{fg})$ $(P = 1, 2, ..., r)$ denotes the TSPFN of the Pth DM, where f and g denote the variation of attributes and alternatives, respectively, containing the data in the form of TSPFS. The following steps of the new scheme are presented to handle such MAGDM issues.

Step 1: In this step, we aim to normalize the decision matrix Φ^P $(P = 1, 2, ..., r)$, where $V^P = (v^P{}_{fg})_{\eta\times ʒ}$, $f = 1, 2, ..., \eta$; $g = 1, 2, ..., ʒ$; $P = 1, 2, ..., r$. Normalization of the matrix converts every attribute into a benefit type if there is some cost type attribute. For this purpose, Mahmood et al. (2019) provide us with basic characteristics.

$$v^P{}_{fg} = \Phi^P{}_{fg} = \begin{cases}\left(\dot{s}^P{}_{fg}, \dot{i}^P{}_{fg}, \hat{d}^P{}_{fg}\right), & for\ benefit\ type\\ \left(\hat{d}^P{}_{fg}, \dot{i}^P{}_{fg}, \hat{d}^P{}_{fg}\right), & for\ cost\ type,\end{cases}$$

where F_1 represents the benefit type and F_2 represents cost type attributes.

Step 2: Determine the support by (22)

$$Sup\left(v^P{}_{fg}, v^{\overline{P}}{}_{fg}\right) = 1 - d\left(v^P{}_{fg}, v^{\overline{P}}{}_{fg}\right), f = 1, 2, ..., \eta;\ g = 1, 2, ..., ʒ;\ P = 1, 2, ..., r. \quad (22)$$

Step 3: Compute the support $T\left(v^{P}{}_{fg}\right)$ of the TSFN $v^{P}{}_{fg}$ to other TSFNs $v^{\overline{P}}{}_{fg}$ $\left(P,\, \overline{P}=1,\, 2,\, ...,\, r,\; P\neq\overline{P}\right)$

$$T\left(v^{P}{}_{fg}\right)=\sum_{P=1,\,\overline{P}\neq P}^{r}\mathrm{Sup}\left(v^{P}{}_{fg},\, v^{\overline{P}}{}_{fg}\right). \tag{23}$$

Step 4: Compute weights $\nabla^{P}{}_{fg}$ by using weights $\varpi_{P}(P=1,\,2,\,...,\,r)$ for decision-makers associated with the TSFNs $v^{P}{}_{fg}$ given under

$$\nabla^{P}{}_{fg}=\frac{\varpi_{P}\left(1+T\left(v^{P}{}_{fg}\right)\right)}{\sum_{P=1}^{r}\varpi_{P}\left(1+T\left(v^{P}{}_{fg}\right)\right)}(P=1,\,2,\,...,\,r), \tag{24}$$

where $\nabla^{P}{}_{fg}\geq 0,\ \sum_{P=1}^{r}\nabla^{P}{}_{fg}=1$.

Step 5: Aggregate all the individual decision matrices $V^{P}=\left(v^{P}{}_{fg}\right)_{\mathfrak{y}\times\mathfrak{z}}$ to obtain group decision matrix $\overline{V}^{P}=\left(\overline{v}^{P}{}_{fg}\right)_{\mathfrak{y}\times\mathfrak{z}}$ by utilizing TSFDWPHM or TSFDWPGHM aggregation operators,

$$\overline{v}^{P}{}_{fg}=TSFDWPHM^{s,t}\left(\overline{v}^{1}{}_{fg},\,\overline{v}^{2}{}_{fg},\,...,\,\overline{v}^{P}{}_{fg}\right) \tag{25}$$

or

$$\overline{v}^{P}{}_{fg}=TSFDWPGHM^{s,t}\left(\overline{v}^{1}{}_{fg},\,\overline{v}^{2}{}_{fg},\,...,\,\overline{v}^{P}{}_{fg}\right). \tag{26}$$

Calculate the supports $Sup\left(\overline{v}_{fg},\,\overline{v}_{fw}\right)$.

$$Sup\left(\overline{v}_{fg},\,\overline{v}_{fw}\right)=1-d\left(\overline{v}_{fg},\,\overline{v}_{fw}\right)(f=1,\,2,\,...,\,\mathfrak{y};\;g,\,w=1,\,2,\,...,\,\mathfrak{z};\;g\neq w). \tag{27}$$

Step 6: Determine the support $T\left(\overline{v}_{fg}\right)$ of the TSFN $\overline{v}_{fg}$ to the other TSFNs $\overline{v}_{fw}(w=1,\,2,\,...,\,\mathfrak{z};\; f\neq w)$

$$T\left(\overline{v}_{fg}\right)=\sum_{w=1,\,w\neq g}^{\mathfrak{z}}Sup\left(\overline{v}_{fg},\,\overline{v}_{fw}\right). \tag{28}$$

Step 7: Obtain the weights $\overline{k}_{fg}(f=1,\,2,\,...,\,\mathfrak{y};\,g,=1,\,2,\,...,\,\mathfrak{z})$ associated with TSFN $\overline{v}_{fg}$ by attribute weightsω_{g}, and

$$\overline{k}_{fg}=\frac{\omega_{g}\left(1+T\left(\overline{v}_{fg}\right)\right)}{\sum_{g=1}^{\mathfrak{z}}\omega_{g}\left(1+T\left(\overline{v}_{fg}\right)\right)}. \tag{29}$$

Step 8: Aggregate all TSFNs $\overline{v}_{fg}$ to obtain the total assessment values $\overline{v}_{f}(f=1,\,2,\,...,\,\mathfrak{y})$ using the TSFDWPHM or TSFDWPGHM aggregation operator.

$$\overline{v}_{f}=\mathrm{TSFDWPGHM}^{s,t}\left(\overline{v}_{f1},\,\overline{v}_{f2},\,...,\,\overline{v}_{f\mathfrak{z}}\right) \tag{30}$$

or

$$\overline{v}_{f}=\mathrm{TSFDWPHM}^{s,t}\left(\overline{v}_{f1},\,\overline{v}_{f2},\,...,\,\overline{v}_{f\mathfrak{z}}\right). \tag{31}$$

Step 9: Determine the score values $SC\left(\overline{v}_{f}\right)(f=1,\,2,\,...,\,\mathfrak{y})$ using Definition 5.

Step 10: Rank the alternatives $\ddot{S}\,(f=1,\,2,\,...,\,\mathfrak{y})$ according to $SC\left(\overline{v}_{f}\right)$ in ascending or descending order. The higher the SC, the better the alternative.

5 Numerical illustration

In this section, we will present a numerical example.

5.1 Case study

2022 has proved a year of irremediable calamity for Pakistan. The republic that occupies the heartland of ancient South Asian civilization in the Indus River valley first exhibited severe weather extremes at the beginning of the year. Now, the floods have deserted a third of the country's provinces underwater, bringing a new level of human misery besides infrastructure loss. The record monsoon, rainy season in southern Asia when the southwestern monsoon blows, has torn through villages, sweeping away thousands of houses, schools, roads, and bridges and destroying 18,000 km^2 of agricultural land. Sindh, one of the southern provinces of Pakistan, whose capital is Karachi, has suffered the most irrecoverable devastation and destruction as 90% of crops have been ruined after the Indus River burst its banks. It would be worth noting that the province in question produces half the country's food.

Almost every few years, it has been noticed that the roads in the flood-prone region turn dysfunctional after the flood. Many resources are needed to revamp the roads. Further, the revamping process demands valuable time; meanwhile, the broken roads must be fixed or only partly functional. Over the rehabilitation time, obstruction, vehicle operating costs, and aggravation of the drivers rise immensely. It is especially crucial for the state's main highways. As highways bear the provinces' enormous percentage of passenger and carriage movement, any interruption in the road network would introduce a huge deficit to the country's economy. To overcome these conditions, it is imperative to reconstruct or repair the main highways as early as possible.

Khyber Pakhtunkhwa (KP) is a northern province of Pakistan that was severely hit by a recent disaster. KP government wants to repair the road network as its priority. For this, the KP government advertised a global tender in newspapers to invite well-reputed construction companies and considered four attributes: company background $\left(\breve{Z}_{1}\right)$,

Table 1 TSF decision matrix

Experts		$\breve{Z}_1$	$\breve{Z}_2$	$\breve{Z}_3$	$\breve{Z}_4$
$\check{N}^{(1)}$	$\ddot{S}_1$	(0.6, 0.21, 0.33)	(0.54, 0.19, 0.43)	(0.7, 0.45, 0.29)	(0.66, 0.21, 0.76)
	$\ddot{S}_2$	(0.76, 0.45, 0.44)	(0.44, 0.29, 0.49)	(0.83, 0.19, 0.23)	(0.76, 0.13, 0.45)
	$\ddot{S}_3$	(0.66, 0.54, 0.32)	(0.54, 0.43, 0.44)	(0.76, 0.43, 0.22)	(0.43, 0.33, 0.54)
	$\ddot{S}_4$	(0.56, 0.31, 0.73)	(0.87, 0.12, 0.65)	(0.91, 0.13, 0.51)	(0.85, 0.19, 0.54)
$\check{N}^{(2)}$	$\ddot{S}_1$	(0.43, 0.31, 0.77)	(0.24, 0.44, 0.65)	(0.77, 0.65, 0.44)	(0.54, 0.34, 0.76)
	$\ddot{S}_2$	(0.82, 0.23, 0.29)	(0.65, 0.45, 0.36)	(0.65, 0.34, 0.72)	(0.72, 0.32, 0.65)
	$\ddot{S}_3$	(0.59, 0.43, 0.44)	(0.56, 0.54, 0.78)	(0.65, 0.66, 0.32)	(0.43, 0.54, 0.65)
	$\ddot{S}_4$	(0.65, 0.16, 0.64)	(0.54, 0.49, 0.65)	(0.76, 0.34, 0.65)	(0.86, 0.43, 0.65)
$\check{N}^{(3)}$	$\ddot{S}_1$	(0.54, 0.43, 0.45)	(0.69, 0.32, 0.79)	(0.83, 0.34, 0.69)	(0.93, 0.12, 0.22)
	$\ddot{S}_2$	(0.66, 0.54, 0.61)	(0.32, 0.75, 0.81)	(0.76, 0.55, 0.43)	(0.87, 0.43, 0.44)
	$\ddot{S}_3$	(0.77, 0.45, 0.54)	(0.82, 0.15, 0.66)	(0.65, 0.43, 0.44)	(0.65, 0.54, 0.65)
	$\ddot{S}_4$	(0.84, 0.41, 0.67)	(0.55, 0.51, 0.67)	(0.77, 0.37, 0.62)	(0.84, 0.41, 0.67)

mechanical capacity $\left(\breve{Z}_2\right)$, financial standing $\left(\breve{Z}_3\right)$, and the number of projects completed within time $\left(\breve{Z}_4\right)$. The KP government had deputed a panel of three officers as a decision-maker committee. The four construction companies are Khattak Allied Construction Company $\left(\ddot{S}_1\right)$, Karcon Pvt. Ltd $\left(\ddot{S}_2\right)$, Umerjan & Co. $\left(\ddot{S}_3\right)$, Ghulam Rasool & Company Pvt. Ltd. $\left(\ddot{S}_4\right)$ bid for road network rehabilitation scheme. Subsequently, the KP government aims to recognize the best construction company among the possible options. The weight of the attributes is given as $\omega = (0.21, 0.29, 0.19, 0.31)$. The decision panel used TSFNs to represent its assessment. Data are shown in Table 1.

Step 1: The TSF decision matrix does not need to be normalized because all attributes are of benefit types. As a result, we can proceed with further analysis using the TSF decision matrix shown in Table 1.

Step 2: We use the Eq. (22) to find the support values. Let's assume that the support between $\upsilon^{P}{}_{fg}$ and $\upsilon^{\overline{P}}{}_{fg}$ is represented as $\varsigma^{P\overline{P}}{}_{fg}$. This means we are calculating how well one set of values $\upsilon^{P}{}_{fg}$ supports another set of values $\upsilon^{\overline{P}}{}_{fg}$. The support values are crucial in understanding the relationships and dependencies between different data sets in our model. By doing this, we can evaluate the degree to which the observed data supports the hypothesized or expected values:

$$\varsigma_{11}{}^{12} = \varsigma_{11}{}^{21} = 0.8074,\ \varsigma_{12}{}^{12} = \varsigma_{12}{}^{21} = 0.8609,\ \varsigma_{13}{}^{12} = \varsigma_{13}{}^{21} = 0.8807,\ \varsigma_{14}{}^{12} = \varsigma_{14}{}^{21} = 0.9466,$$

$$\varsigma_{21}{}^{12} = \varsigma_{21}{}^{21} = 0.9159,\ \varsigma_{22}{}^{12} = \varsigma_{22}{}^{21} = 0.8909,\ \varsigma_{23}{}^{12} = \varsigma_{23}{}^{21} = 0.7698,\ \varsigma_{24}{}^{12} = \varsigma_{24}{}^{21} = 0.9067,$$

$$\varsigma_{31}{}^{12} = \varsigma_{31}{}^{21} = 0.9291,\ \varsigma_{32}{}^{12} = \varsigma_{32}{}^{21} = 0.8382,\ \varsigma_{33}{}^{12} = \varsigma_{33}{}^{21} = 0.8685,\ \varsigma_{34}{}^{12} = \varsigma_{34}{}^{21} = 0.8657,$$

$$\varsigma_{41}{}^{12} = \varsigma_{41}{}^{21} = 0.9161,\ \varsigma_{42}{}^{12} = \varsigma_{42}{}^{21} = 0.7943,\ \varsigma_{43}{}^{12} = \varsigma_{43}{}^{21} = 0.8354,\ \varsigma_{44}{}^{12} = \varsigma_{44}{}^{21} = 0.9294,$$

$$\varsigma_{11}{}^{13} = \varsigma_{11}{}^{31} = 0.9387,\ \varsigma_{12}{}^{13} = \varsigma_{12}{}^{31} = 0.7965,\ \varsigma_{13}{}^{13} = \varsigma_{13}{}^{31} = 0.8051,\ \varsigma_{14}{}^{13} = \varsigma_{14}{}^{31} = 0.6824,$$

$$\varsigma_{21}{}^{13} = \varsigma_{21}{}^{31} = 0.8801,\ \varsigma_{22}{}^{13} = \varsigma_{22}{}^{31} = 0.7121,\ \varsigma_{23}{}^{13} = \varsigma_{23}{}^{31} = 0.8801,\ \varsigma_{24}{}^{13} = \varsigma_{24}{}^{31} = 0.8991,$$

$$\varsigma_{31}{}^{13} = \varsigma_{31}{}^{31} = 0.8710,\ \varsigma_{32}{}^{13} = \varsigma_{32}{}^{31} = 0.7759,\ \varsigma_{33}{}^{13} = \varsigma_{33}{}^{31} = 0.9204,\ \varsigma_{34}{}^{13} = \varsigma_{34}{}^{31} = 0.8554,$$

$$\varsigma_{41}{}^{13} = \varsigma_{41}{}^{31} = 0.8185,\ \varsigma_{42}{}^{13} = \varsigma_{42}{}^{31} = 0.7836,\ \varsigma_{43}{}^{13} = \varsigma_{43}{}^{31} = 0.8496,\ \varsigma_{44}{}^{13} = \varsigma_{44}{}^{31} = 0.9244,$$

$$\varsigma_{11}{}^{23} = \varsigma_{11}{}^{32} = 0.8356,\ \varsigma_{12}{}^{23} = \varsigma_{12}{}^{32} = 0.8048,\ \varsigma_{13}{}^{23} = \varsigma_{13}{}^{32} = 0.8020,\ \varsigma_{14}{}^{23} = \varsigma_{14}{}^{32} = 0.6291,$$

$$\varsigma_{21}{}^{23} = \varsigma_{21}{}^{32} = 0.7960,\ \varsigma_{22}{}^{23} = \varsigma_{22}{}^{32} = 0.6475,\ \varsigma_{23}{}^{23} = \varsigma_{23}{}^{32} = 0.8049,\ \varsigma_{24}{}^{23} = \varsigma_{24}{}^{32}0.8262,$$

$$\varsigma_{31}{}^{23} = \varsigma_{31}{}^{32} = 0.8883,\ \varsigma_{32}{}^{23} = \varsigma_{32}{}^{32} = 0.7610,\ \varsigma_{33}{}^{23} = \varsigma_{33}{}^{32} = 0.9132,\ \varsigma_{34}{}^{23} = \varsigma_{34}{}^{32} = 0.8802,$$

$$\varsigma_{41}{}^{23} = \varsigma_{41}{}^{32} = 0.8595,\ \varsigma_{42}{}^{23} = \varsigma_{42}{}^{32} = 0.9833,\ \varsigma_{43}{}^{23} = \varsigma_{43}{}^{32} = 0.9783,\ \varsigma_{44}{}^{23} = \varsigma_{44}{}^{32} = 0.9733.$$

Step 3: Using Eq. (23), we determine $T\left(\upsilon^{P}{}_{fg}\right)$,

$$T_{11}{}^{1} = 1.7461,\ T_{12}{}^{1} = 1.6431,\ T_{13}{}^{1} = 1.7743,\ T_{14}{}^{1} = 1.6575,$$

$$T_{21}{}^{1} = 1.6658,\ T_{22}{}^{1} = 1.6013,\ T_{23}{}^{1} = 1.6858,\ T_{24}{}^{1} = 1.6827,$$

$$T_{31}{}^{1} = 1.6071,\ T_{32}{}^{1} = 1.6290,\ T_{33}{}^{1} = 1.5757,\ T_{34}{}^{1} = 1.3114,$$

$$T_{41}{}^{1} = 1.7960,\ T_{42}{}^{1} = 1.7120,\ T_{43}{}^{1} = 1.6762,\ T_{44}{}^{1} = 1.6030,$$

$$T_{11}{}^{2} = 1.5385,\ T_{12}{}^{2} = 1.3596,\ T_{13}{}^{2} = 1.6498,\ T_{14}{}^{2} = 1.5747,$$

$$T_{21}{}^{2} = 1.6850,\ T_{22}{}^{2} = 1.8058,\ T_{23}{}^{2} = 1.7329,\ T_{24}{}^{2} = 1.7252,$$

$$T_{31}{}^{2} = 1.8091,\ T_{32}{}^{2} = 1.8175,\ T_{33}{}^{2} = 1.7682,\ T_{34}{}^{2} = 1.6141,$$

$$T_{41}{}^{2} = 1.5992,\ T_{42}{}^{2} = 1.5369,\ T_{43}{}^{2} = 1.7888,\ T_{44}{}^{2} = 1.7817,$$

$$T_{11}{}^{3} = 1.8335,\ T_{12}{}^{3} = 1.7210,\ T_{13}{}^{3} = 1.7458,\ T_{14}{}^{3} = 1.7355,$$

$$T_{21}{}^{3} = 1.7346,\ T_{22}{}^{3} = 1.7756,\ T_{23}{}^{3} = 1.1.6781,\ T_{24}{}^{3} = 1.5779,$$

$$T_{31}{}^{3} = 1.7776,\ T_{32}{}^{3} = 1.7669,\ T_{33}{}^{3} = 1.6851,\ T_{34}{}^{3} = 1.8137,$$

$$T_{41}{}^{3} = 1.8278,\ T_{42}{}^{3} = 1.8538,\ T_{43}{}^{3} = 1.9027,\ T_{44}{}^{3} = 1.8977.$$

Step 4: Using Eq. (24), we calculate the weight $\nabla^{P}{}_{fg}$,

$$\nabla_{11}{}^{1} = 0.4023,\ \nabla_{12}{}^{1} = 0.2420,\ \nabla_{13}{}^{1} = 0.3556,\ \nabla_{14}{}^{1} = 0.4026,$$

$$\nabla_{21}{}^{1} = 0.2524,\ \nabla_{22}{}^{1} = 0.3459,\ \nabla_{23}{}^{1} = 4043,\ \nabla_{24}{}^{1} = 0.2523,$$

$$\nabla_{31}{}^{1} = 0.3434,\ \nabla_{32}{}^{1} = 0.4198,\ \nabla_{33}{}^{1} = 0.2571,\ \nabla_{34}{}^{1} = 0.3230,$$

$$\nabla_{41}{}^{1} = 0.4092,\ \nabla_{42}{}^{1} = 0.2480,\ \nabla_{43}{}^{1} = 0.3427,\ \nabla_{44}{}^{1} = 0.4162,$$

$$\nabla_{11}{}^{2} = 0.2536,\ \nabla_{12}{}^{2} = 0.3301,\ \nabla_{13}{}^{2} = 0.4009,\ \nabla_{14}{}^{2} = 0.2435,$$

$$\nabla_{21}{}^{2} = 0.3555,\ \nabla_{22}{}^{2} = 0.4067,\ \nabla_{23}{}^{2} = 0.2476,\ \nabla_{24}{}^{2} = 0.3456,$$

$$\nabla_{31}{}^{2} = 0.4017,\ \nabla_{32}{}^{2} = 0.2518,\ \nabla_{33}{}^{2} = 0.3464,\ \nabla_{34}{}^{2} = 0.4047,$$

$$\nabla_{41}{}^{2} = 0.2515,\ \nabla_{42}{}^{2} = 0.3437,\ \nabla_{43}{}^{2} = 0.3980,\ \nabla_{44}{}^{2} = 0.2481,$$

$$\nabla_{11}{}^{3} = 0.3538,\ \nabla_{12}{}^{3} =,\ 0.3983\nabla_{13}{}^{3} = 0.2512,\ \nabla_{14}{}^{3} = 0.3504,$$

$$\nabla_{21}{}^{3} = 0.4014,\ \nabla_{22}{}^{3} = 0.2526,\ \nabla_{23}{}^{3} = 0.3439,\ \nabla_{24}{}^{3} = 0.3827,$$

$$\nabla_{31}{}^{3} = 0.2577,\ \nabla_{32}{}^{3} = 0.3594,\ \nabla_{33}{}^{3} = 0.3881,\ \nabla_{34}{}^{3} = 0.2542,$$

$$\nabla_{41}{}^{3} = 0.3576,\ \nabla_{42}{}^{3} = 0.3961,\ \nabla_{43}{}^{3} = 0.2518,\ \nabla_{44}{}^{3} = 0.3519.$$

Step 5: Using TSFDWPHM or TSFDWPGHM aggregation operators, as represented in Eq. (25) or (26), we aggregate all the individu**al** decision matrices to obtain a group decision matrix, as mentioned in Tables 2 and 3. Then, using Eq. (27), we calculate the supports as.

$$Sup\left(\overline{v}_{fg},\ \overline{v}_{fw}\right) = 1 - d\left(\overline{v}_{fg},\ \overline{v}_{fw}\right)\Big(f = 1,\ 2,\ \ldots,\ \mathfrak{y};\ g,\ w = 1,\ 2,\ \ldots,\ \mathfrak{z};\ g \neq w\Big). \tag{27}$$

$$\varsigma_1{}^{12} = \varsigma_1{}^{21} = 0.9542,\ \varsigma_1{}^{13} = \varsigma_1{}^{31} = 0.8719,\ \varsigma_1{}^{14} = \varsigma_1{}^{41} = 0.8066,$$

$$\varsigma_1{}^{23} = \varsigma_1{}^{32} = 0.8686,\ \varsigma_1{}^{24} = \varsigma_1{}^{42} = 0.8059,\ \varsigma_1{}^{34} = \varsigma_1{}^{43} = 0.9029,$$

$$\varsigma_2{}^{12} = \varsigma_2{}^{21} = 0.8881,\ \varsigma_2{}^{13} = \varsigma_2{}^{31} = 0.9792,\ \varsigma_2{}^{14} = \varsigma_2{}^{41} = 0.9412,$$

$$\varsigma_2{}^{23} = \varsigma_2{}^{32} = 0.8673,\ \varsigma_2{}^{24} = \varsigma_2{}^{42} =,\ 0.8534\varsigma_2{}^{34} = \varsigma_2{}^{43} = 0.9451,$$

$$\varsigma_3{}^{12} = \varsigma_3{}^{21} = 0.9218,\ \varsigma_3{}^{13} = \varsigma_3{}^{31} = 0.9836,\ \varsigma_3{}^{14} = \varsigma_3{}^{41} = 0.8710,$$

$$\varsigma_3{}^{23} = \varsigma_3{}^{32} = 0.9095,\ \varsigma_3{}^{24} = \varsigma_3{}^{42} = 0.8754,\ \varsigma_3{}^{34} = \varsigma_3{}^{43} = 0.8638,$$

$$\varsigma_4{}^{12} = \varsigma_4{}^{21} = 0.9655,\ \varsigma_4{}^{13} = \varsigma_4{}^{31} = 0.8942,\ \varsigma_4{}^{14} = \varsigma_4{}^{41} = 0.9162,$$

$$\varsigma_4{}^{23} = \varsigma_4{}^{32} = 0.9281,\ \varsigma_4{}^{24} = \varsigma_4{}^{42} = 0.9474,\ \varsigma_4{}^{34} = \varsigma_4{}^{43} = 0.9755,$$

Table 2 Aggregated values by utilizing TSPFDWPHM

	$\ddot{S}_1$	$\ddot{S}_2$	$\ddot{S}_3$	$\ddot{S}_4$
$\breve{Z}_1$	(0.5456,0.2492,0.3867)	(0.6093,0.2262,0.5035)	(0.7838,0.3986,0.3433)	(0.8844,0.1463,0.2686)
$\breve{Z}_2$	(0.7648,0.2884,0.3582)	(0.5474,0.3424,0.4346)	(0.7785,0.2266,0.2740)	(0.8170,0.1551,0.4704)
$\breve{Z}_3$	(0.7056,0.4651,0.3760)	(0.7421,0.1822,0.5140)	(0.7004,0.4554,0.2610)	(0.5639,0.3901,0.6098)
$\breve{Z}_4$	(0.7681,0.2001,0.6769)	(0.8076,0.1441,0.6584)	(0.8643,0.1559,0.5745)	(0.8488,0.2270,0.6047)

$$\varsigma_1{}^{12} = \varsigma_1{}^{21} = 0.9425,\ \varsigma_1{}^{13} = \varsigma_1{}^{31} = 0.8329,\ \varsigma_1{}^{14} = \varsigma_1{}^{41} = 0.9239,$$

$$\varsigma_1{}^{23} = \varsigma_1{}^{32} = 0.7754,\ \varsigma_1{}^{24} = \varsigma_1{}^{42} = 0.9133,\ \varsigma_1{}^{34} = \varsigma_1{}^{43} = 0.8328,$$

$$\varsigma_2{}^{12} = \varsigma_2{}^{21} = 0.7526,\ \varsigma_2{}^{13} = \varsigma_2{}^{31} = 0.9654,\ \varsigma_2{}^{14} = \varsigma_2{}^{41} = 0.9464,$$

$$\varsigma_2{}^{23} = \varsigma_2{}^{32} = 0.7732,\ \varsigma_2{}^{24} = \varsigma_2{}^{42} = 0.7130,\ \varsigma_2{}^{34} = \varsigma_2{}^{43} = 0.9398,$$

$$\varsigma_3{}^{12} = \varsigma_3{}^{21} = 0.8926,\ \varsigma_3{}^{13} = \varsigma_3{}^{31} = 0.9529,\ \varsigma_3{}^{14} = \varsigma_3{}^{41} = 0.8657,$$

$$\varsigma_3{}^{23} = \varsigma_3{}^{32} = 0.8455,\ \varsigma_3{}^{24} = \varsigma_3{}^{42} = 0.9488,\ \varsigma_3{}^{34} = \varsigma_3{}^{43} = 0.8277,$$

$$\varsigma_4{}^{12} = \varsigma_4{}^{21} = 0.9513,\ \varsigma_4{}^{13} = \varsigma_4{}^{31} = 0.8769,\ \varsigma_4{}^{14} = \varsigma_4{}^{41} = 0.8513,$$

$$\varsigma_4{}^{23} = \varsigma_4{}^{32} = 0.8538,\ \varsigma_4{}^{24} = \varsigma_4{}^{42} = 0.8371,\ \varsigma_4{}^{34} = \varsigma_4{}^{43} = 0.9470.$$

Step 6: Using (28), we obtain support $T\left(\overline{v}_{fg}\right)$,

$$T\left(\overline{v}_{11}\right) = 2.6327,\ T\left(\overline{v}_{12}\right) = 2.6287,\ T\left(\overline{v}_{13}\right) = 2.6434,\ T\left(\overline{v}_{14}\right) = 2.5155,$$

$$T\left(\overline{v}_{11}\right) = 2.8086,\ T\left(\overline{v}_{12}\right) = 2.6089,\ T\left(\overline{v}_{13}\right) = 2.7916,\ T\left(\overline{v}_{14}\right) = 2.7397,$$

$$T\left(\overline{v}_{11}\right) = 2.7765,\ T\left(\overline{v}_{12}\right) = 2.7067,\ T\left(\overline{v}_{13}\right) = 2.7570,\ T\left(\overline{v}_{14}\right) = 2.6103,$$

$$T\left(\overline{v}_{11}\right) = 2.7759,\ T\left(\overline{v}_{12}\right) = 2.8409,\ T\left(\overline{v}_{13}\right) = 2.7978,\ T\left(\overline{v}_{14}\right) = 2.8391,$$

or determine support.

$$T\left(\overline{v}_{11}\right) = 2.6994,\ T\left(\overline{v}_{12}\right) = 2.6313,\ T\left(\overline{v}_{13}\right) = 2.4413,\ T\left(\overline{v}_{14}\right) = 2.6701,$$

$$T\left(\overline{v}_{11}\right) = 2.6646,\ T\left(\overline{v}_{12}\right) = 2.2389,\ T\left(\overline{v}_{13}\right) = 2.6785,\ T\left(\overline{v}_{14}\right) = 2.5993,$$

$$T\left(\overline{v}_{11}\right) = 2.7112,\ T\left(\overline{v}_{12}\right) = 2.6870,\ T\left(\overline{v}_{13}\right) = 2.6262,\ T\left(\overline{v}_{14}\right) = 2.6423,$$

$$T\left(\overline{v}_{11}\right) = 2.6803,\ T\left(\overline{v}_{12}\right) = 2.6422,\ T\left(\overline{v}_{13}\right) = 2.6786,\ T\left(\overline{v}_{14}\right) = 2.6352.$$

Step 7: Using (29), we calculate weight as

$$\overline{k}_{11} = 0.2121,\ \overline{k}_{12} = 2925,\ \overline{k}_{13} = 0.1924,\ \overline{k}_{14} = 0.3030,$$

$$\overline{k}_{21} = 0.2146,\ \overline{k}_{22} = 0.2808,\ \overline{k}_{23} = 0.1933,\ \overline{k}_{24} = 0.3111,$$

$$\overline{k}_{31} = 0.2143,\ \overline{k}_{32} = 0.2904,\ \overline{k}_{33} = 0.1929,\ \overline{k}_{34} = 0.3024,$$

$$\overline{k}_{41} = 0.2076,\ \overline{k}_{42} = 0.2917,\ \overline{k}_{43} = 0.1889,\ \overline{k}_{44} = 0.3167,$$

or

$$\overline{k}_{11} = 0.2145,\ \overline{k}_{12} = 0.2907,\ \overline{k}_{13} = 0.1805,\ \overline{k}_{14} = 0.3142,$$

$$\overline{k}_{21} = 0.2184,\ \overline{k}_{22} = 0.2666,\ \overline{k}_{23} = 0.1983,\ \overline{k}_{24} = 0.3167,$$

$$\overline{k}_{31} = 0.2125,\ \overline{k}_{32} = 0.2916,\ \overline{k}_{33} = 0.1879,\ \overline{k}_{34} = 0.3079,$$

$$\overline{k}_{41} = 0.2145,\ \overline{k}_{42} = 0.2890,\ \overline{k}_{43} = 0.1912,\ \overline{k}_{44} = 0.3083.$$

Step 8: We obtain the comprehensive assessment value of each alternative as

$$\overline{v}_1 = (0.8262,\ 0.1774,\ 0.3158),\ \overline{v}_2 = (0.7822,\ 0.1872,\ 0.3358),\ \overline{v}_3 = (0.7059,\ 0.4069,\ 0.3287),\ \overline{v}_4 = (0.8434,\ 0.1762,\ 0.6061)$$

or

$$\overline{v}_1 = (0.6364,\ 0.4414,\ 0.6599),\ \overline{v}_2 = (0.5225,\ 0.5221,\ 0.6468),\ \overline{v}_3 = (0.4529,\ 0.6167,\ 0.6463),\ \overline{v}_4 = (0.5225,\ 0.5221,\ 0.6468).$$

Step 9: We calculate the score values as

$$SC\left(\overline{v}_1\right) = 0.5325,\ SC\left(\overline{v}_2\right) = 0.4406,\ SC\left(\overline{v}_3\right) = 0.3163,\ SC\left(\overline{v}_4\right) = 0.3771$$

Table 3 Aggregated values by utilizing TSPFDWPGHM

	$\ddot{S}_1$	$\ddot{S}_2$	$\ddot{S}_3$	$\ddot{S}_4$
$\breve{Z}_1$	(0.5056, 0.3637, 0.6650)	(0.3006, 0.3605, 0.7134)	(0.7574, 0.5482, 0.5990)	(0.6242, 0.2747, 0.7249)
$\breve{Z}_2$	(0.7242, 0.4763, 0.5263)	(0.3791, 0.6599, 0.7270)	(0.7333, 0.4666, 0.6114)	(0.7717, 0.3638, 0.5504)
$\breve{Z}_3$	(0.6558, 0.4832, 0.4696)	(0.5877, 0.4598, 0.6917)	(0.6782, 0.5566, 0.3727)	(0.4635, 0.5021, 0.6769)
$\breve{Z}_4$	(0.6306, 0.3503, 0.6853)	(0.5847, 0.4657, 0.6577)	(0.7982, 0.3299, 0.6035)	(0.8495, 0.3860, 0.6321)

Table 4 Impact of parameters $\dot{s}$ and t on score values

Parameters	Score values				Ranking orders
	$\ddot{S}_1$	$\ddot{S}_2$	$\ddot{S}_3$	$\ddot{S}_4$	
$s=1, t=2, \zeta=3$	0.5325	0.4406	0.3163	0.3772	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=5, t=9, \zeta=3$	0.4514	0.3576	0.2435	0.2536	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=9, t=15, \zeta=3$	0.4379	0.3436	0.2323	0.2324	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=19, t=31, \zeta=3$	0.4253	0.3310	0.221	0.2127	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_3 > \ddot{S}_4$
$s=2, t=130, \zeta=3$	0.2532	0.1779	0.0634	−0.1236	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_3 > \ddot{S}_4$
$s=35, t=3, \zeta=3$	0.4969	0.4050	0.3124	0.3390	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=150, t=7, \zeta=3$	0.4945	0.4076	0.3175	0.3451	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=250, t=11, \zeta=3$	0.4940	0.4071	0.3178	0.3454	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$

Table 5 Influence of parameter ζ on ranking results

Parameters	Score values				Ranking orders
	$\ddot{S}_1$	$\ddot{S}_2$	$\ddot{S}_3$	$\ddot{S}_4$	
$s=1, t=2, \zeta=3$	0.5325	0.4406	0.3163	0.3772	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=7$	0.7133	0.5536	0.4400	0.4923	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=15$	0.7672	0.6112	0.5018	0.5687	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=25$	0.7815	0.6300	0.5224	0.5955	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=45$	0.7892	0.6404	0.5338	0.6108	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=75$	0.7921	0.6443	0.5383	0.6169	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=100$	0.7930	0.6455	0.5397	0.6188	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s=1, t=2, \zeta=110$	0.7932	0.6458	0.5400	0.6192	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$

or

$$SC(\overline{v}_1) = -0.0295,\ SC(\overline{v}_2) = -0.1280,\ SC(\overline{v}_3) = -0.3013,\ SC(\overline{v}_4) = -0.1771.$$

Step 10: Finally, we obtain the final ranking pattern as

$$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$$

or

$$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3.$$

5.2 Sensitivity analysis

In the subsequent analysis, we delve into the impact of three independent parameters, namely s, t, and ζ, on decision-making. We examine how altering these parameters affects the ranking results derived from applying TSPFPDWHM and TSPFPDWGHM operators.

To begin with, we explore the influence of different values of s and t, while keeping ζ fixed, as part of Step 7. Subsequently, we investigate the impact of parameters ζ on the ranking results while s and t remain constant. The findings of these investigations are presented in Tables 4, 5, and 6.

Upon careful examination of Tables 4, 5, and 6, it becomes apparent that variations in the ranking of alternatives emerge

Table 6 Impact of higher values of q

Parameters	Score values				Ranking orders
	$\ddot{S}_1$	$\ddot{S}_2$	$\ddot{S}_3$	$\ddot{S}_4$	
$s = 1, t = 2, \zeta = 3, q = 3$	0.5325	0.4406	0.3163	0.3772	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
$s = 1, t = 2, \zeta = 3, q = 5$	0.4291	0.3178	0.2013	0.3809	$\ddot{S}_1 > \ddot{S}_4 > \ddot{S}_2 > \ddot{S}_3$
$s = 1, t = 25, \zeta = 3, q = 8$	0.2931	0.1734	0.0913	0.2777	$\ddot{S}_1 > \ddot{S}_4 > \ddot{S}_2 > \ddot{S}_3$
$s = 1, t = 2, \zeta = 3, q = 11$	0.2104	0.0991	0.0445	0.1869	$\ddot{S}_1 > \ddot{S}_4 > \ddot{S}_2 > \ddot{S}_3$
$s = 1, t = 2, \zeta = 3, q = 15$	0.1420	0.0500	0.0182	0.1093	$\ddot{S}_1 > \ddot{S}_4 > \ddot{S}_2 > \ddot{S}_3$
$s = 1, t = 2, \zeta = 3, q = 23$	0.0707	0.0144	0.0034	0.0407	$\ddot{S}_1 > \ddot{S}_4 > \ddot{S}_2 > \ddot{S}_3$

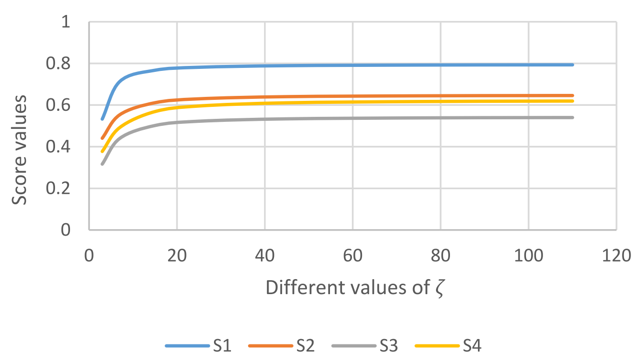

Fig. 1 Geometrical interpretation of scores when "ζ" varies, where $s = 1, t = 2$

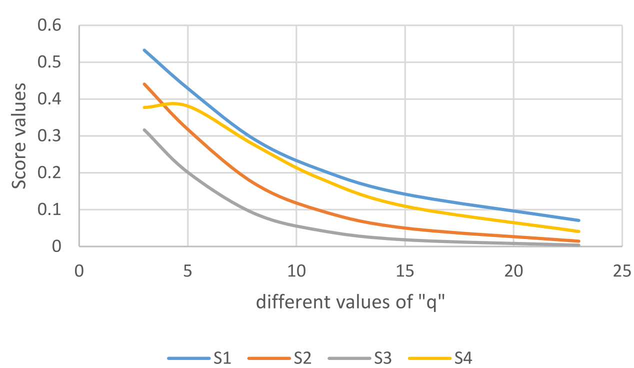

Fig. 2 Geometrical interpretation of scores when "q" varies, where s = 1, t = 2

for different combinations of s and t. However, despite these variations, the optimal and suboptimal construction companies consistently maintain their positions throughout the analysis. Furthermore, Table 5 reveals that, as the value of ζ increases, the ranking rule remains stable, while the score function exhibits a consistent upward trend.

Figure 1 visually represents this trend, demonstrating a uniform increase in score values up to $\zeta = 20$, beyond which the changes in score values become negligible. Figure 2, on the other hand, highlights a decreasing behavior in the score values of the four alternatives as the parameter q increases. Although slight fluctuations in ranking patterns are observed, the best and worst options remain unchanged.

5.3 Comparative analysis

In this subsection, we aim to validate the proposed approach's stability and advantages by comparing it to existing techniques in the realm of T-spherical fuzzy sets (TSPFSs).

To provide a comprehensive assessment of the novel method's validity, we will contrast it with the T-spherical fuzzy Hamacher weighted averaging (TSFHWA) operator (Ullah et al. 2020a), the T-spherical fuzzy weighted averaging (TSFWA) operator (Ullah et al. 2020b), and the T-spherical fuzzy DTNCN aggregation operator (Ullah et al. 2021). Each of these operators will be employed to evaluate the above-mentioned real-life problem, and the resulting scores and ranking outcomes will be presented in Table 7.

When comparing our method with TSFHWA operator (Ullah et al. 2020a), which utilizes the Hamacher t-norm and t-conorm, notable differences in the ranking order emerge. This discrepancy in ranking patterns indicates that the TSFHWA operator overlooks the interrelation among attributes and exhibits less flexibility than our proposed method.

Likewise, the rankings obtained from TSFWA and TSFWG differ due to using a simple averaging aggregation operator, which needs more flexibility to handle unreasonable data and make nuanced evaluations.

Furthermore, a careful analysis of Table 7 reveals that TSFDWA (Ullah et al. 2021), TSFDWG (Ullah et al. 2021), and TSFHWG (Ullah et al. 2020a) consistently identify the optimal construction company. This consistency further strengthens the validity of the newly introduced MAGDM model. It becomes evident that the anticipated operators are more rational and practical for addressing MAGDM issues within the TSPF environment.

Table 7 Ranking patterns by applying different AOs

Methods	$\ddot{S}_1$	$\ddot{S}_2$	$\ddot{S}_3$	$\ddot{S}_4$	Ranking order
TSFHWA (Ullah et al. 2020a)	0.0056	0.1073	− 0.0204	− 0.0198	$\ddot{S}_2 > \ddot{S}_1 > \ddot{S}_4 > \ddot{S}_3$
TSFHWG (Ullah et al. 2020a)	0.4955	0.3787	0.3055	0.2768	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
TSFWA (Ullah et al. 2020b)	0.2426	0.2581	0.1437	0.1969	$\ddot{S}_2 > \ddot{S}_1 > \ddot{S}_4 > \ddot{S}_3$
TSFWG (Ullah et al. 2020b)	0.0112	0.1250	0.0534	0.1517	$\ddot{S}_4 > \ddot{S}_2 > \ddot{S}_3 > \ddot{S}_1$
TSFDWA (Ullah et al. 2021)	0.3380	0.3242	0.1959	0.2959	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
TSFDWG (Ullah et al. 2021)	0.1232	0.0497	0.0083	− 0.0771	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
TSFDWHM	0.5325	0.4406	0.3163	0.3772	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$
TSDWGHM	− 0.0295	− 0.1280	− 0.3013	− 0.1771	$\ddot{S}_1 > \ddot{S}_2 > \ddot{S}_4 > \ddot{S}_3$

6 Conclusions

This article delves into integrating the Dombi operator, HM operator, and PA operator within the framework of TSPFSs, aiming to address MAGDM challenges. A novel set of AOs is introduced, extending the sphere of influence to include the spherical fuzzy Dombi power Heronian mean operator and spherical fuzzy Dombi power in the TSPF environment.

The article commences with a foundation-laying exploration, providing essential definitions of TSPFSs while revisiting the fundamental expressions of HM, PA, and Dombi operators. Subsequently, a family of AOs is established, including the TSPFDPHM operator, TSPFDPHM operator, TSPFDPHM operator, and TSPFDPHM operator, alongside the definition of their desirable properties. Additionally, the numerical problem of MAGDM is addressed by utilizing TSPFS, a task unobtainable by SPFSs.

Building upon the newly introduced AOs, a MAGDM approach is formulated and presented. The proposed approach provides a flexible and comprehensive framework for addressing complex decision problems, particularly in post-disaster scenarios like flood recovery projects.

Through a real-world case study, we demonstrate the effectiveness of our approach in providing a systematic and transparent decision-making process. Considering uncertainties and varying degrees of importance among decision criteria, our method empowers decision-makers to make informed choices that align with project objectives and stakeholder preferences.

Future research directions may include applying this approach to other decision problems and exploring different parameterization schemes for Dombi power aggregation operators to tailor the method further to specific contexts. Our proposed methodology offers a valuable tool for improving decision-making in complex scenarios and can contribute to more resilient and sustainable disaster recovery efforts.

Author contributions All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Funding There is no specific funding for this project.

Data availability No data were used to support this study.

Declarations

Conflict of interest The authors declare no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by the author.

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