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Multiattribute group decision-making based on weighted correlation coefficient of linguistic q-rung orthopair fuzzy sets and TOPSIS method

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Abstract

Linguistic q-rung orthopair fuzzy number (Lq-ROFN) is a valuable tool for expressing the uncertainty of qualitative information that has received a lot of attention over the last 5 years. In this article, we propose the correlation coefficient to measure the strength of the relationship between two linguistic q-rung orthopair fuzzy sets (Lq-ROFSs). We also provide the various properties of the proposed correlation coefficient of Lq-ROFSs. Moreover, we also propose the weighted correlation coefficient of Lq-ROFSs. Afterward, using the proposed weighted correlation coefficient of Lq-ROFSs and the "technique for order of preference by similarity to ideal solution" (TOPSIS) method, we develop a novel multiattribute group decision-making (MAGDM) method under the Lq-ROFNs environment. We also solve the different MAGDM problems using the proposed MAGDM method and compare the preference order (PO) obtained by the proposed MAGDM method with the POs obtained by the existing MAGDM methods. The comparison analysis shows that the drawbacks of the existing MAGDM methods can be successfully overcome by the proposed MAGDM method, where existing MAGDM methods cannot distinguish the POs of alternatives. In the Lq-ROFNs environment, the proposed MAGDM method provides a useful decision-making method for solving MAGDM problems.

Keywords Fuzzy set · MAGDM · TOPSIS · Correlation coefficient · q-ROFS · Lq-ROFS

1 Introduction

Multiattribute group decision-making (MAGDM) is a crucial component of decision-making theory. It involves selecting the optimal alternative based on quantitative or qualitative evaluations of each possible attribute by a group of decision-making experts (DMExs). Due to the uncertain information, the most challenging job for the DMEXs is to provide the alternative's assessment of a MAGDM problem. Therefore, Zadeh (1965) introduced the theory of fuzzy sets (FS) to manage imprecise concepts in quantitative data analysis. Fuzzy set theory has only explored membership degrees (MDs) for decision-making but ignores non-membership degrees (NMDs), which may result in erroneous results in many realistic evaluations. Several applications utilizing FSs have been introduced in previous studies (Chen and Jian 2017; Chen et al. 2019; Zeng et al. 2019; Chen and Hsu 2008; Chen 1996; Chen and Lee 2010; Lin et al. 2006; Chen and Chen 2002; Savita et all. 2024; Akram and Martino 2023; Noor et al. 2023; Muneeza and Abdullah 2023; Farman et al. 2023). To compensate for the inadequacy of the FS, Atanassov (1986) proposed the extension of the FS known as intuitionistic fuzzy sets (IFS) $\langle \theta, \vartheta \rangle$ that also includes NMD. The IFS satisfies the condition $\theta + \vartheta \leq 1$, where θ is the MD and ϑ is the NMD. But in some cases, IFS is not able to express the assessments of the DMEXs, where $\theta + \vartheta > 1$. To handle these types of problems, Yager (2013) proposed the pythagorean fuzzy set (PFS) $\langle \theta, \vartheta \rangle$ which satisfy the condition $\theta^2 + \vartheta^2 \leq 1$. The PFS provides more space for DMEXs to express their assessment of the alternatives compared to the IFSs. But in some cases, PFS is not also able to express the assessments of the decision-making

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experts (DMEXs) where $\theta^2 + \vartheta^2 > 1$. Therefore, Yager (2017) proposed the generalization of the IFS and PFS known as q-rung orthopair fuzzy set (q-ROFS) $\langle \theta, \vartheta \rangle$ which satisfies the condition: $\theta^q + \vartheta^q \leq 1$ and $q \geq 1$, which provides a more range to express the information. Under these environments, different MAGDM approaches (Liu et al. 2020; Kumar and Chen 2023a; Garg and Chen 2020; Pathak et al. 2024; Alcantud 2023; Kumar and Kumar 2023) have been developed by the researchers. Liu et al. (2020) proposed the partitioned Maclaurin symmetric mean AO for MAGDM under the intuitionistic fuzzy number (IFNs) environment. Kumar and Chen (2023a) defined the entropy measure and arithmetic mean aggregation operator (AO) for MAGDM under the PFSs environment. Garg and Chen (2020) defined the neutrality AOs for MAGDM under the q-rung orthopair fuzzy number (q-ROFNs) environment.

However, using IFSs, PFSs, and q-ROFSs, the DMExs can express the assessment information only in numerical terms. In certain circumstances, DMExs may discover that it is challenging to describe their assessment in numerical terms. For instance, DMExs may have challenges when expressing the weather conditions of any city. In that case, the DMExs can use the linguistic phrases like “freezing”, “cold”, “chilly”, “warm”, “hot”, and “burning” to express the weather condition instead of numerical values. First, Zadeh (1975) proposed the concept of linguistic variables (LVs), where various applications (Herrera and Martínez 2001; Xu 2004; Saha et al. 2024; Akram et al. 2023a, b) based on the LVs environment have been developed. Afterward, Chen et al. (2015) defined the idea of linguistic intuitionistic fuzzy sets (LIFS) by combining the features of IFNs and LVs to express the qualitative assessments more conveniently. Some MAGDM approaches (Malik et al. 2024; Kumar and Chen 2023b; Arora and Garg 2019; Kumar and Chen 2022a, b; Rahim 2023) have been developed under the LIFSs environment. Afterward, Garg (2018) defined the concept of the linguistic PFS (LPFS) by combining the features of the PFS and LVs, which is the extension of LIFS. Han et al. (2019) defined the technique for order of preference by similarity to ideal solution (TOPSIS) method based on the entropy measures and distance measures for the LPFSs. Lin et al. (2019) proposed the TOPSIS method based on the correlation coefficient and entropy measures for the LPFSs. In 2019, Liu and Liu (2019a) extended the idea of LIFSs and LPFSs, and defined the idea of linguistic q-rung orthopair fuzzy (Lq-ROF) set (Lq-ROFS) and Lq-ROF number (Lq-ROFN), where the MD and NMD of the Lq-ROFN are indicated by LVs. The Lq-ROFS allows DMExs to provide assessment information across a wider range. Several decision-making applications utilizing Lq-ROFSs have been introduced in previous studies(Neelam et al. 2024; Liu and Liu 2019a, b; Peng et al. 2019; Akram et al. 2021; Bao and Shi 2022; Li and Zhang 2023; Liu et al. 2022; Jana et al. 2023). Liu and Liu (2019a) proposed the power Bonferroni AO of Lq-ROFNs and MAGDM approach based on the proposed AOs under the Lq-ROFNs environment. Liu and Liu (2019b) introduced the power Muirhead mean AO and entropy measures for the MAGDM approach under the Lq-ROFNs environment. Peng et al. (2019) defined the similarity measures of Lq-ROFSs and MAGDM approach using proposed similarity measures under the Lq-ROFNs environment. Akram et al. (2021) defined the MAGDM approach based on the Einstein model in the Lq-ROFNs context. Bao and Shi (2022) proposed the MAGDM approach under the Lq-ROFNs environment based on the ELECTRE method. Liu et al. (2022) defined the point weighted aggregation operators (AOs) for Lq-ROFNs and MAGDM approach based on the proposed AOs of Lq-ROFNs. Li and Zhang (2023) defined the MAGDM approach in the context of the Lq-ROFNs environment based on fuzzy preference relations. Jana et al. (2023) defined the MAGDM approach for evaluation of sustainable strategies for urban parcel delivery under the Lq-ROFNs environment.

In this paper, we find that the majority of existing MAGDM approaches under the Lq-ROFNs environment are based on the AOs of Lq-ROFNs, and there is limited research on the classical MAGDM approaches under the Lq-ROFNs environment. We also find that there is no study on the correlation coefficient of Lq-ROFNs. Moreover, we find that the Liu and Liu’s MAGDM approach (Liu and Liu 2019a) and Liu et al.’s MAGDM approach (Liu et al. 2022) have the shortcomings that they cannot distinguish the preference orders (POs) of alternatives in certain cases. Hence, it is necessary to develop a new classical MAGDM approach under the Lq-ROFNs environment to overcome the limitations of Liu and Liu’s MAGDM approach (Liu and Liu 2019a) and Liu et al.’s MAGDM approach (Liu et al. 2022).

In this paper, we propose the correlation coefficient for the Lq-ROFSs. The proposed correlation coefficient measures the strength of the relationship between two Lq-ROFSs. We also present proofs of the different properties of the proposed correlation coefficient of Lq-ROFSs. We also propose the weighted correlation coefficient of Lq-ROFSs. Afterward, based on the proposed weighted correlation coefficient of Lq-ROFSs and the TOPSIS method, we propose a new classical MAGDM approach to solve the MAGDM problems in the Lq-ROFNs environment. The proposed MAGDM approach can overcome the drawbacks of Liu and Liu's MAGDM approach (Liu and Liu 2019a) and Liu et al.'s MAGDM approach (Liu et al. 2022), where they cannot distinguish the POs of alternatives in certain cases.

The remaining part of this paper is organized as follows: Sect. 2 provides the fundamental definitions related to this article. In Sect. 3, we develop the correlation coefficient and weighted correlation coefficient for Lq-ROFSs. In Sect. 4, we propose a novel MAGDM approach based on proposed weighted correlation coefficient of Lq-ROFSs and TOPSIS method under the Lq-ROFNs environment. Finally, Sect. 5 gives the conclusion of the paper.

2 Preliminaries

Definition 1 (Herrera and Martínez 2001; Neelam et al. 2023a) A finite linguistic term (LT) set (LTS) $\Upsilon = \left\{s_0, s_1, \ldots, s_h\right\}$ of odd cardinality, where LT s_t reflects a suitable value for a LV. For example, to express the weather condition, we can consider the LTs as $s_0 =$ ''freezing'', $s_1 =$ ''cold'', $s_2 =$ ''chilly'', $s_3 =$ ''warm'' and $s_4 =$ ''hot''.

The LT s_k meets the following criteria:

(i) $s_k \leq s_t \Leftrightarrow k \leq t$;
(ii) $\mathrm{Neg}(s_k) = s_{h-k}$;
(iii) $\max(s_k, s_t) = s_k \Leftrightarrow s_k \geq s_t$;
(iv) $\min(s_k, s_t) = s_t \Leftrightarrow s_k \geq s_t$.

Later on, the continuous LTS (CLTS) $\Upsilon_{[0,h]}$ is developed by extending the discrete LTS Υ as follows (Xu 2004; Neelam et al. 2023b):

$$\Upsilon_{[0,h]} = \left\{s_k \mid s_0 \leq s_k \leq s_h\right\}.$$

Definition 2 (Liu and Liu 2019a) A Lq-ROFS ζ in a finite universal set G is defined as:

$$\zeta = \left\{\langle x, s_{\theta(g)}, s_{\vartheta(g)}\rangle \mid g \in G\right\}, \tag{1}$$

where $s_{\theta(g)}$ and $s_{\vartheta(g)}$ indicate the membership degree (MD) and non-MD (NMD) of g to ζ, respectively, where $s_\theta(g) \in \Upsilon_{[0,h]}$, $s_{\vartheta(g)} \in \Upsilon_{[0,h]}$, $0 \leq (\theta(g))^q + (\vartheta(g))^q \leq h^q$ and $q \geq 1$. The hesitancy degree of g to ζ is defined as $s_{\pi(g)} = s_{(h^q-(\theta(g))^q-(\vartheta(g))^q)^{1/q}}$.

In Liu and Liu (2019a), Liu and Liu called the pair $\langle s_\theta, s_\vartheta\rangle$ in the Lq-ROFS $\zeta = \{\langle x, s_{\theta(g)}, s_{\vartheta(g)}\rangle \mid x \in X\}$ a Lq-ROFN.

Let $\Omega_{[0,h]}$ be the set of all Lq-ROFNs in the CLTS $\Upsilon_{[0,h]}$.

Definition 3 (Liu and Liu 2019a) The score function $S(\varrho)$ of the Lq-ROFN $\varrho = \langle s_\theta, s_\vartheta\rangle$, where $\varrho \in \Omega_{[0,h]}$, is defined as follows:

$$S(\varrho) = \left(\frac{h^q + \theta^q - \vartheta^q}{2}\right)^{1/q}, \tag{2}$$

where $S(\varrho) \in [0, h]$.

Definition 4 (Liu and Liu 2019a) The accuracy function $H(\varrho)$ of the Lq-ROFN $\varrho = \langle s_\theta, s_\vartheta\rangle$, where $\varrho \in \Omega_{[0,h]}$, is defined as follows:

$$H(\varrho) = (\theta^q + \vartheta^q)^{1/q}, \tag{3}$$

where $H(\varrho) \in [0, h]$.

Definition 5 (Liu and Liu 2019a) Let $\varrho_1 = \langle s_{\theta_1}, s_{\vartheta_1}\rangle$ and $\varrho_2 = \langle s_{\theta_2}, s_{\vartheta_2}\rangle$ be two Lq-ROFNs, then the following rules are defined:

(a) if $S(\varrho_1) > S(\varrho_2)$ then $\varrho_1 \succ \varrho_2$.
(b) if $S(\varrho_1) < S(\varrho_2)$ then $\varrho_1 \prec \varrho_2$.
(c) if $S(\varrho_1) = S(\varrho_2)$ then

(i) if $H(\varrho_1) > H(\varrho_2)$ then $\varrho_1 \succ \varrho_2$.
(ii) if $H(\varrho_1) < H(\varrho_2)$ then $\varrho_1 \prec \varrho_2$.
(iii) if $H(\varrho_1) = H(\varrho_2)$ then $\varrho_1 = \varrho_2$.

3 The proposed correlation coefficient of Lq-ROFSs

In this section, we propose the correlation coefficient of Lq-ROFSs. Let $\varepsilon(G)_{[0,h]}$ be the set of all Lq-ROFSs over the universal set $X = \{g_1, g_2, \ldots, g_n\}$, where MD and NMD of each element $g_i \in G$ belong to the CLTS $\Upsilon_{[0,h]}$.

Definition 6 Let $\zeta_1 = \left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i \in G\right\}$ be a Lq-ROFS, where $\zeta_1 \in \varepsilon(G)_{[0,h]}$. The proposed information energy $T(\zeta_1)$ of the Lq-ROFS ζ_1 is defined as:

$$T(\zeta_1) = \frac{1}{nh^{2q}} \sum_{i=1}^{n} \left(\left(\theta_{\zeta_1}(g_i)\right)^{2q} + \left(\vartheta_{\zeta_1}(g_i)\right)^{2q} + \left(\pi_{\zeta_1}(g_i)\right)^{2q}\right), \tag{4}$$

where $0 \leq T(\zeta_1) \leq 1$, $\pi_{\zeta_1}(g_i) = (h^q - (\theta_{\zeta_1}(g_i))^q - (\vartheta_{\zeta_1}(g_i))^q)^{1/q}$ and $q \geq 1$.

Definition 7 Let $\zeta_1 = \left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i \in G\right\}$ and $\zeta_2 = \left\{\langle g_i, s_{\theta_{\zeta_2}(g_i)}, s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i \in G\right\}$ be two Lq-ROFSs, where $\zeta_1 \in \varepsilon(G)_{[0,h]}$ and $\zeta_2 \in \varepsilon(G)_{[0,h]}$. The proposed correlation $C(\zeta_1, \zeta_2)$ between the Lq-ROFNs ζ_1 and ζ_2 is defined as follows:

$$C(\zeta_1,\zeta_2) = \frac{1}{nh^{2q}}\sum_{i=1}^{n}\left((\theta_{\zeta_1}(g_i))^q(\theta_{\zeta_2}(g_i))^q + (\vartheta_{\zeta_1}(g_i))^q(\vartheta_{\zeta_2}(g_i))^q + (\pi_{\zeta_1}(g_i))^q(\pi_{\zeta_2}(g_i))^q\right), \tag{5}$$

where $0 \leq C(\zeta_1, \zeta_2) \leq 1$, $\pi_{\zeta_1}(g_i) = (h^q - (\theta_{\zeta_1}(g_i))^q - (\vartheta_{\zeta_1}(g_i))^q)^{1/q}$, $\pi_{\zeta_2}(g_i) = (h^q - (\theta_{\zeta_2}(g_i))^q - (\vartheta_{\zeta_2}(g_i))^q)^{1/q}$ and $q \geq 1$.

The proposed correlation of Lq-ROFS satisfies the following properties:

(i) $C(\zeta_1, \zeta_2) = T(\zeta_1)$.
(ii) $C(\zeta_1, \zeta_2) = C(\zeta_2, \zeta_1)$.

Definition 8 Let $\zeta_1 = \left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i \in G\right\}$ and $\zeta_2 = \left\{\langle g_i, s_{\theta_{\zeta_2}(g_i)}, s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i \in G\right\}$ be two Lq-ROFSs, where $\zeta_1 \in \varepsilon(G)_{[0,h]}$ and $\zeta_2 \in \varepsilon(G)_{[0,h]}$. The proposed CC $K(\zeta_1, \zeta_2)$ between the Lq-ROFNs ζ_1 and ζ_2 is defined as follows:

$$K(\zeta_1,\zeta_2) = \frac{C(\zeta_1,\zeta_2)}{\max\{T(\zeta_1), T(\zeta_2)\}} = \frac{\sum_{i=1}^{n}\left((\theta_{\zeta_1}(g_i))^q \cdot (\theta_{\zeta_2}(g_i))^q + (\vartheta_{\zeta_1}(g_i))^q \cdot (\vartheta_{\zeta_2}(g_i))^q + (\pi_{\zeta_1}(g_i))^q \cdot (\pi_{\zeta_2}(g_i))^q\right)}{\max\left\{\sum_{i=1}^{n}\left((\theta_{\zeta_1}(g_i))^{2q} + (\vartheta_{\zeta_1}(g_i))^{2q} + (\pi_{\zeta_1}(g_i))^{2q}\right), \sum_{i=1}^{n}\left((\theta_{\zeta_2}(g_i))^{2q} + (\vartheta_{\zeta_2}(g_i))^{2q} + (\pi_{\zeta_2}(g_i))^{2q}\right)\right\}}, \tag{6}$$

where $0 \leq K(\zeta_1, \zeta_2) \leq 1$, $\pi_{\zeta_1}(g_i) = (h^q - (\theta_{\zeta_1}(g_i))^q - (\vartheta_{\zeta_1}(g_i))^q)^{1/q}$, $\pi_{\zeta_2}(g_i) = (h^q - (\theta_{\zeta_2}(g_i))^q - (\vartheta_{\zeta_2}(g_i))^q)^{1/q}$ and $q \geq 1$.

Example 1 Let $\zeta_1 = \{\langle g_1, s_5, s_3\rangle, \langle g_2, s_4, s_4\rangle, \langle g_3, s_6, s_1\rangle\}$ and $\zeta_2 = \{\langle g_1, s_4, s_2\rangle, \langle g_2, s_5, s_2\rangle, \langle g_3, s_7, s_1\rangle\}$ be two Lq-ROFSs, where $\zeta_1 \in \varepsilon(G)_{[0,8]}$ and $\zeta_2 \in \varepsilon(G)_{[0,8]}$.

First, using Eq. (4), we obtain the information energies $T(\zeta_1)$ and $T(\zeta_2)$ of the Lq-ROFSs ζ_1 and ζ_2, respectively, where $q = 3$,

$$\begin{aligned}
T(\zeta_1) &= \frac{1}{3h^{2q}}\sum_{i=1}^{3}\left((\theta_{\zeta_1}(g_i))^{2q} + (\vartheta_{\zeta_1}(g_i))^{2q} + (\pi_{\zeta_1}(g_i))^{2q}\right) \\
&= \frac{1}{3 \times 8^6} \\
&\left(5^6 + 3^6 + \left(8^3 - 5^3 - 3^3\right)^2 + 4^6 + 4^6 + \left(8^3 - 4^3 - 4^3\right)^2 \right.\\
&\left. + 6^6 + 1^6 + \left(8^3 - 6^3 - 1^3\right)^2\right) \\
&= 0.5535,
\end{aligned}$$

$$\begin{aligned}
T(\zeta_2) &= \frac{1}{3h^{2q}}\sum_{i=1}^{3}\left((\theta_{\zeta_2}(g_i))^{2q} + (\vartheta_{\zeta_2}(g_i))^{2q} + (\pi_{\zeta_2}(g_i))^{2q}\right) \\
&= \frac{1}{3 \times 8^6}\left(4^6 + 2^6 + \left(8^3 - 4^3 - 2^3\right)^2 + 5^6 + 2^6 \right.\\
&\left. + \left(8^3 - 5^3 - 2^3\right)^2 + 7^6 + 1^6 + \left(8^3 - 7^3 - 1^3\right)^2\right) \\
&= 0.6396.
\end{aligned}$$

Now, using Eq. (5), we calculate the correlation $C(\zeta_1, \zeta_2)$ between the Lq-ROFSs ζ_1 and ζ_2, where $q = 3$,

$$C(\zeta_1,\zeta_2)=\frac{1}{3h^{2q}}\sum_{i=1}^{3}((\theta_{\zeta_1}(g_i))^q \cdot(\theta_{\zeta_2}(g_i))^q+(\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_2}(g_i))^q)$$
$$=\frac{1}{3\times 8^6}(5^3\cdot 4^3+3^3\cdot 2^3 +(8^3-5^3-3^3).(8^3-4^3-2^3)+4^3\cdot 5^3+4^3\cdot 2^3 +(8^3-4^3-4^3).(8^3-5^3-2^3)+6^3\cdot 7^3+1^3\cdot 1^3 +(8^3-6^3-1^3).(8^3-7^3-1^3))$$
$$=0.5650.$$

Hence, using Eq. (6), we get the proposed correlation coefficient $K(\zeta_1,\zeta_2)$ between the Lq-ROFSs ζ_1 and ζ_2, where $q=3$,

$$K(\zeta_1,\zeta_2)=\frac{C(\zeta_1,\zeta_2)}{\max\{T(\zeta_1),T(\zeta_2)\}} =0.8834.$$

Theorem 1 *Let* $\zeta_1=\left\{\langle g_i,s_{\theta_{\zeta_1}(g_i)},s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i\in G\right\}$ *and* $\zeta_2=\left\{\langle g_i,s_{\theta_{\zeta_2}(g_i)},s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i\in G\right\}$ *be two Lq-ROFSs, where* $\zeta_1\in\varepsilon(G)_{[0,h]}$ *and* $\zeta_2\in\varepsilon(G)_{[0,h]}$. *The proposed correlation coefficient* $K(\zeta_1,\zeta_2)$ *between the Lq-ROFNs* ζ_1 *and* ζ_2, *defined in Eq.* (6), *satisfies the following conditions*:

(P1) $K(\zeta_1,\zeta_2)=K(\zeta_2,\zeta_1)$.
(P2) $0\le K(\zeta_1,\zeta_2)\le 1$.
(P3) $\zeta_1\zeta_2\Rightarrow K(\zeta_1,\zeta_2)=1$.

Proof Let $\zeta_1=\left\{\langle g_i,s_{\theta_{\zeta_1}(g_i)},s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i\in G\right\}$ and $\zeta_2=\left\{\langle g_i,s_{\theta_{\zeta_2}(g_i)},s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i\in G\right\}$ be two Lq-ROFSs, where $\zeta_1\in\varepsilon(G)_{[0,h]}$ and $\zeta_2\in\varepsilon(G)_{[0,h]}$.

(P1) We have

$$K(\zeta_1,\zeta_2)=\frac{\sum_{i=1}^n((\theta_{\zeta_1}(g_i))^q\cdot(\theta_{\zeta_2}(g_i))^q+(\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_2}(g_i))^q)}{\max\left\{\sum_{i=1}^n\left((\theta_{\zeta_1}(g_i))^{2q}+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q}\right),\sum_{i=1}^n\left((\theta_{\zeta_2}(g_i))^{2q}+(\vartheta_{\zeta_2}(g_i))^{2q}+(\pi_{\zeta_2}(g_i))^{2q}\right)\right\}}$$
$$=\frac{\sum_{i=1}^n((\theta_{\zeta_2}(g_i))^q\cdot(\theta_{\zeta_1}(g_i))^q+(\vartheta_{\zeta_2}(g_i))^q\cdot(\vartheta_{\zeta_1}(g_i))^q+(\pi_{\zeta_2}(g_i))^q\cdot(\pi_{\zeta_1}(g_i))^q)}{max\left\{\sum_{i=1}^n\left((\theta_{\zeta_2}(g_i))^{2q}+(\vartheta_{\zeta_2}(g_i))^{2q}+(\pi_{\zeta_2}(g_i))^{2q}\right),\sum_{i=1}^n\left((\theta_{\zeta_1}(g_i))^{2q}+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q}\right)\right\}}$$
$$=K(\zeta_2,\zeta_1).$$

(P2) It is obvious $K(\zeta_1,\zeta_2)\ge 0$. Then we will prove $K(\zeta_1,\zeta_2)\le 1$.

$$C(\zeta_1,\zeta_2)=\sum_{i=1}^{n}((\theta_{\zeta_1}(g_i))^q\cdot(\theta_{\zeta_2}(g_i))^q+(\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_2}(g_i))^q)$$
$$=((\theta_{\zeta_1}(g_1))^q\cdot(\theta_{\zeta_2}(g_1))^q+(\vartheta_{\zeta_1}(g_1))^q\cdot(\vartheta_{\zeta_2}(g_1))^q+(\pi_{\zeta_1}(g_1))^q\cdot(\pi_{\zeta_2}(g_1))^q)$$
$$+((\theta_{\zeta_1}(g_2))^q\cdot(\theta_{\zeta_2}(g_2))^q+(\vartheta_{\zeta_1}(g_2))^q\cdot(\vartheta_{\zeta_2}(g_2))^q+(\pi_{\zeta_1}(g_2))^q\cdot(\pi_{\zeta_2}(g_2))^q$$
$$+\ldots+((\theta_{\zeta_1}(g_n))^q\cdot(\theta_{\zeta_2}(g_n))^q+(\vartheta_{\zeta_1}(g_n))^q\cdot(\vartheta_{\zeta_2}(g_n))^q+(\pi_{\zeta_1}(g_n))^q\cdot(\pi_{\zeta_2}(g_n))^q).$$

According to Cauchy–Schwarz inequality, we have

$$(g_1y_1+g_2y_2+\cdots+g_ny_n)^2\le(g_1^2+g_2^2+\ldots+g_n^2)\cdot(y_1^2+y_2^2+\cdots+y_n^2).$$

Therefore

$$(C(\zeta_1,\zeta_2))^2=\left(\sum_{i=1}^{n}((\theta_{\zeta_1}(g_i))^q\cdot(\theta_{\zeta_2}(g_i))^q +\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_2}(g_i))^q)\right)^2$$
$$\le\left(\sum_{i=1}^{n}(\theta_{\zeta_1}(g_i))^q+(\vartheta_{\zeta_1}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\right)^2 \cdot\left(\sum_{i=1}^{n}(\theta_{\zeta_2}(g_i))^q+(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_2}(g_i))^q\right)^2$$
$$\le\sum_{i=1}^{n}(\theta_{\zeta_1}(g_i))^{2q}+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q} \cdot\sum_{i=1}^{n}(\theta_{\zeta_2}(g_i))^{2q}+(\vartheta_{\zeta_2}(g_i))^{2q}+(\pi_{\zeta_2}(g_i))^{2q}$$
$$=T(\zeta_1)\cdot T(\zeta_2).$$

Therefore, $C(\zeta_1,\zeta_2)\le\max\{T(\zeta_1),T(\zeta_2)\}$. Thus, $K(\zeta_1,\zeta_2)\le 1$.

(P3) Let the Lq-ROFSs $\zeta_1=\zeta_2$ then $\theta_{\zeta_1}(g_i)=\theta_{\zeta_2}(g_i)$, $\vartheta_{\zeta_1}(g_i)=\vartheta_{\zeta_2}(g_i)$ and $\pi_{\zeta_1}(g_i)=\pi_{\zeta_2}(g_i)$, $\forall g_i\in G$. By using Eq. (6), we have

$$
\begin{aligned}
K(\zeta_1,\zeta_2) &= \frac{C(\zeta_1,\zeta_2)}{\max\{T(\zeta_1),T(\zeta_1)\}}\\
&= \frac{\sum_{i=1}^{n}\left((\theta_{\zeta_1}(g_i))^q\cdot(\theta_{\zeta_1}(g_i))^q+(\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_1}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_1}(g_i))^q\right)}{\max\left\{\sum_{i=1}^{n}\left((\theta_{\zeta_1}(g_i))^{2q}+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q}\right),\sum_{i=1}^{n}\left((\theta_{\zeta_1}(g_i))^{2q}+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q}\right)\right\}}\\
&=1.
\end{aligned}
$$

□

In many practical scenarios, distinct elements $g_1, g_2, \ldots, g_n$ may have varying weights. Therefore, we consider the weights $w_1, w_2, \ldots, w_n$, of the elements $g_1, g_2, \ldots, g_n$, respectively, where $w_i \geq 0, i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i = 1$. In the following, we propose the weighted correlation coefficient $K_w(\zeta_1, \zeta_2)$ between the Lq-ROFNs ζ_1 and ζ_2 as follows:

Definition 9 Let $\zeta_1 = \left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i \in G\right\}$ be a Lq-ROFS, where $\zeta_1 \in \varepsilon(G)_{[0,h]}$. The proposed weighted information energy $T_w(\zeta_1)$ of the Lq-ROFS ζ_1 is defined as:

$$
\begin{aligned}
T_w(\zeta_1) = \frac{1}{h^{2q}}\sum_{i=1}^{n} w_i\Big((\theta_{\zeta_1}(g_i))^{2q}\\
+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q}\Big),
\end{aligned}
\tag{7}
$$

where $0 \leq T_w(\zeta_1) \leq 1$, w_i is the weight of the element g_i, $w_i \geq 0, i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} w_i = 1$, $\pi_{\zeta_1}(g_i) = (h^q - (\theta_{\zeta_1}(g_i))^q - (\vartheta_{\zeta_1}(g_i))^q)^{1/q}$ and $q \geq 1$.

Definition 10 Let $\zeta_1 = \left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i \in G\right\}$ and $\zeta_2 = \left\{\langle g_i, s_{\theta_{\zeta_2}(g_i)}, s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i \in G\right\}$ be two Lq-ROFSs, where $\zeta_1 \in \varepsilon(G)_{[0,h]}$ and $\zeta_2 \in \varepsilon(G)_{[0,h]}$. The proposed weighted correlation $C_w(\zeta_1, \zeta_2)$ between the Lq-ROFNs ζ_1 and ζ_2 is defined as follows:

$$
\begin{aligned}
C_w(\zeta_1,\zeta_2) = \frac{1}{h^{2q}}\sum_{i=1}^{n} w_i\big((\theta_{\zeta_1}(g_i))^q(\theta_{\zeta_2}(g_i))^q\\
+(\vartheta_{\zeta_1}(g_i))^q(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_1}(g_i))^q(\pi_{\zeta_2}(g_i))^q\big),
\end{aligned}
\tag{8}
$$

where $0 \leq C_w(\zeta_1, \zeta_2) \leq 1$, w_i is the weight of the element g_i, $w_i \geq 0, i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} w_i = 1$, $\pi_{\zeta_1}(g_i) = (h^q - (\theta_{\zeta_1}(g_i))^q - (\vartheta_{\zeta_1}(g_i))^q)^{1/q}$, $\pi_{\zeta_2}(g_i) = (h^q - (\theta_{\zeta_2}(g_i))^q - (\vartheta_{\zeta_2}(g_i))^q)^{1/q}$ and $q \geq 1$.

Definition 11 Let $\zeta_1 = \left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i \in G\right\}$ and $\zeta_2 = \left\{\langle g_i, s_{\theta_{\zeta_2}(g_i)}, s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i \in G\right\}$ be two Lq-ROFSs, where $\zeta_1 \in \varepsilon(G)_{[0,h]}$ and $\zeta_2 \in \varepsilon(G)_{[0,h]}$. The proposed weighted correlation coefficient $K_w(\zeta_1, \zeta_2)$ between the Lq-ROFNs ζ_1 and ζ_2 is defined as follows:

$$
\begin{aligned}
K_w(\zeta_1,\zeta_2) &= \frac{C_w(\zeta_1,\zeta_2)}{\max\{T_w(\zeta_1),T_w(\zeta_2)\}}\\
&= \frac{\sum_{i=1}^{n} w_i\left((\theta_{\zeta_1}(g_i))^q\cdot(\theta_{\zeta_2}(g_i))^q+(\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_2}(g_i))^q+(\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_2}(g_i))^q\right)}{\max\left\{\sum_{i=1}^{n} w_i\left((\theta_{\zeta_1}(g_i))^{2q}+(\vartheta_{\zeta_1}(g_i))^{2q}+(\pi_{\zeta_1}(g_i))^{2q}\right),\sum_{i=1}^{n} w_i\left((\theta_{\zeta_2}(g_i))^{2q}+(\vartheta_{\zeta_2}(g_i))^{2q}+(\pi_{\zeta_2}(g_i))^{2q}\right)\right\}},
\end{aligned}
\tag{9}
$$

where $0 \leq K_w(\zeta_1, \zeta_2) \leq 1$, w_i is the weight of the element g_i, $w_i \geq 0, i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} w_i = 1$, $\pi_{\zeta_1}(g_i) = (h^q - (\theta_{\zeta_1}(g_i))^q - (\vartheta_{\zeta_1}(g_i))^q)^{1/q}$, $\pi_{\zeta_2}(g_i) = (h^q - (\theta_{\zeta_2}(g_i))^q - (\vartheta_{\zeta_2}(g_i))^q)^{1/q}$ and $q \geq 1$.

Example 2 Let $\zeta_1 = \{\langle g_1, s_4, s_3\rangle, \langle g_2, s_7, s_1\rangle, \langle g_3, s_3, s_5\rangle\}$ and $\zeta_2 = \left\{\langle g_1, s_1, s_5\rangle, \langle g_2, s_4, s_3\rangle, \langle g_3, s_3, s_2\rangle\right\}$ be two Lq-ROFSs, where $\zeta_1 \in \varepsilon(G)_{[0,8]}$ and $\zeta_2 \in \varepsilon(G)_{[0,8]}$, with weights $w_1 = 0.3$, $w_2 = 0.4$ and $w_3 = 0.3$, respectively.

First, using Eq. (7), we obtain the weighted information energies $T_w(\zeta_1)$ and $T_w(\zeta_2)$ of the Lq-ROFSs ζ_1 and ζ_2, respectively, where $q = 3$,

$$\begin{aligned}T_w(\zeta_1) &= \frac{1}{h^{2q}}\sum_{i=1}^{3} w_i\Big((\theta_{\zeta_1}(g_i))^{2q} + (\vartheta_{\zeta_1}(g_i))^{2q} + (\pi_{\zeta_1}(g_i))^{2q}\Big)\\ &= \frac{1}{8^6}\Big(0.3\Big(4^6+3^6+(8^3-4^3-3^3)^2\Big)\\ &\quad +0.4\Big(7^6+1^6+(8^3-7^3-1^3)^2\Big)\\ &\quad +0.3\Big(3^6+5^6+(8^3-3^3-5^3)^2\Big)\Big)\\ &= 0.5980,\end{aligned}$$

$$\begin{aligned}T_w(\zeta_2) &= \frac{1}{h^{2q}}\sum_{i=1}^{3} w_i\Big((\theta_{\zeta_2}(g_i))^{2q} + (\vartheta_{\zeta_2}(g_i))^{2q} + (\pi_{\zeta_2}(g_i))^{2q}\Big)\\ &= \frac{1}{8^6}\Big(0.3\Big(1^6+5^6+(8^3-1^3-5^3)^2\Big)\\ &\quad +0.4\Big(4^6+3^6+(8^3-4^3-3^3)^2\Big)\\ &\quad +0.3\Big(3^6+2^6+(8^3-3^3-2^3)^2\Big)\Big)\\ &=0.7275.\end{aligned}$$

Now, using Eq. (8), we calculate the weighted correlation $C_w(\zeta_1,\zeta_2)$ between the Lq-ROFSs ζ_1 and ζ_2, where $q=3$,

$$\begin{aligned}C_w(\zeta_1,\zeta_2) &= \frac{1}{h^{2q}}\sum_{i=1}^{3} w_i\big((\theta_{\zeta_1}(g_i))^q\\ &\quad\cdot(\theta_{\zeta_2}(g_i))^q + (\vartheta_{\zeta_1}(g_i))^q\cdot(\vartheta_{\zeta_2}(g_i))^q + (\pi_{\zeta_1}(g_i))^q\cdot(\pi_{\zeta_2}(g_i))^q\big)\\ &= \frac{1}{8^6}\big(0.3\big(4^3\cdot1^3+3^3\cdot5^3+(8^3-4^3-3^3).(8^3-1^3-5^3)\big)\\ &\quad +0.4\big(7^3\cdot4^3+1^3\cdot3^3+(8^3-7^3-1^3).(8^3-4^3-3^3)\big)\\ &\quad +0.3\big(3^3\cdot3^3+5^3\cdot2^3+(8^3-3^3-5^3).(8^3-3^3-2^3)\big)\big)\\ &=0.5299.\end{aligned}$$

Hence, using Eq. (9), we get the proposed weighted correlation coefficient $K_w(\zeta_1,\zeta_2)$ between the Lq-ROFSs ζ_1 and ζ_2, where $q=3$,

$$\begin{aligned}K_w(\zeta_1,\zeta_2) &= \frac{C(\zeta_1,\zeta_2)}{\max\{T(\zeta_1),T(\zeta_2)\}}\\ &=0.7283.\end{aligned}$$

Theorem 2 *Let* $\zeta_1=\left\{\langle g_i, s_{\theta_{\zeta_1}(g_i)}, s_{\vartheta_{\zeta_1}(g_i)}\rangle \mid g_i\in G\right\}$ *and* $\zeta_2=\left\{\langle g_i, s_{\theta_{\zeta_2}(g_i)}, s_{\vartheta_{\zeta_2}(g_i)}\rangle \mid g_i\in G\right\}$ *be two Lq-ROFSs, where* $\zeta_1\in\varepsilon(G)_{[0,h]}$ *and* $\zeta_2\in\varepsilon(G)_{[0,h]}$. *The proposed weighted correlation coefficient* $K_w(\zeta_1,\zeta_2)$ *between the Lq-ROFNs* ζ_1 *and* ζ_2, *defined in Eq. (9), satisfies the following conditions*:

(P1) $K_w(\zeta_1,\zeta_2)=K_w(\zeta_2,\zeta_1)$.
(P2) $0\le K_w(\zeta_1,\zeta_2)\le 1$.
(P3) $\zeta_1=\zeta_2 \Rightarrow K_w(\zeta_1,\zeta_2)=1$.

Proof The proof is similar to the proof of Theorem 1. □

4 The proposed MAGDM approach based on the proposed weighted correlation coefficient of Lq-ROFSs and the TOPSIS method

In this section, we propose a new MAGDM approach under the Lq-ROFNs environment based on the proposed weighted correlation coefficient of Lq-ROFSs and the TOPSIS method.

Let $\zeta_1,\zeta_2,\ldots,\zeta_p$ be p alternatives and let $C_1,C_2,\ldots,C_r$ be r attributes. Let $w_1,w_2,\ldots,w_r$ represent the weights of $C_1,C_2,\ldots,C_r$, respectively, where $w_i\ge 0, i=1,2,\ldots,r$ and $\sum_{i=1}^{r} w_i=1$. Let $\xi_1,\xi_2,\ldots,\xi_m$ be the decision-making experts (DMExs) with weights ς_1, ς_2, …, ς_m, respectively $\varsigma_j\ge 0$, $j=1,2,\ldots,m$ and $\sum_{j=1}^{m}\varsigma_j=1$. Every DMEx ξ_j evaluates the attributes C_i of the alternatives ζ_k by utilizing a Lq-ROFN $\tilde{\varrho}^j_{ki}=\langle s_{\tilde{\theta}^j_{ki}}, s_{\tilde{\vartheta}^j_{ki}}\rangle$ to construct the decision matrix (DMx) $\tilde{R^j}=(\tilde{\varrho}^j_{ki})_{p\times r}$, shown as follows:

$$\widetilde{R^j} = \begin{array}{c} \\ \zeta_1\\ \zeta_2\\ \vdots\\ \zeta_p\end{array}\begin{array}{c}\begin{array}{cccc} C_1 & C_2 & \ldots & C_r\end{array}\\ \begin{pmatrix}\tilde{\varrho}^j_{11} & \tilde{\varrho}^j_{12} & \cdots & \tilde{\varrho}^j_{1r}\\ \tilde{\varrho}^j_{21} & \tilde{\varrho}^j_{22} & \cdots & \tilde{\varrho}^j_{2r}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\varrho}^j_{p1} & \tilde{\varrho}^j_{p2} & \cdots & \tilde{\varrho}^j_{pr}\end{pmatrix}\end{array}.$$

Step 1: Convert the DMxs $\tilde{R^1}=(\tilde{\varrho}^1_{ki})_{p\times r}=(\langle s_{\tilde{\theta}^1_{ki}}, s_{\tilde{\vartheta}^1_{ki}}\rangle)_{p\times r}$, $\tilde{R^2}=(\tilde{\varrho}^2_{ki})_{p\times r}=(\langle s_{\tilde{\theta}^2_{ki}}, s_{\tilde{\vartheta}^2_{ki}}\rangle)_{p\times r}$, …, $\tilde{R^m}=(\tilde{\varrho}^m_{ki})_{p\times r}=(\langle s_{\tilde{\theta}^m_{ki}}, s_{\tilde{\vartheta}^m_{ki}}\rangle)_{p\times r}$ into the normalize DMx (NDMxs) $R^1=(\varrho^1_{ki})_{p\times r}=(\langle s_{\theta^1_{ki}}, s_{\vartheta^1_{ki}}\rangle)_{p\times r}$, $R^2=(\varrho^2_{ki})_{p\times r}=(\langle s_{\theta^2_{ki}}, s_{\vartheta^2_{ki}}\rangle)_{p\times r}$, …, $R^m=(\varrho^m_{ki})_{p\times r}=(\langle s_{\theta^m_{ki}}, s_{\vartheta^m_{ki}}\rangle)_{p\times r}$ as follows:

$$\varrho^j_{ki}=\begin{cases}\langle s_{\tilde{\theta}^j_{ki}}, s_{\tilde{\vartheta}^j_{ki}}\rangle: & \text{for benefit type attribute}\\ \langle s_{\tilde{\vartheta}^j_{ki}}, s_{\tilde{\theta}^j_{ki}}\rangle: & \text{for cost type attribute}\end{cases}, \tag{10}$$

where $k=1,2,\ldots,p$, $i=1,2,\ldots r$ and $j=1,2,\ldots,m$.

Step 2: For each NDMx R^j, obtain the positive ideal alternative (PIA) $(\zeta^+)^j$ and the negative ideal alternative (NIA) $(\zeta^-)^j$, where $j=1,2,\ldots,m$, shown as follows:

$$(\zeta^{+})^{j} = \left\{\langle C_i, s_{(\theta_i^{+})^{j}}, s_{(\vartheta_i^{+})^{j}}\rangle, i = 1,2,\ldots,r\right\}, \tag{11}$$

$$(\zeta^{-})^{j} = \left\{\langle C_i, s_{(\theta_i^{-})^{j}}, s_{(\vartheta_i^{-})^{j}}\rangle, i = 1,2,\ldots,r\right\}, \tag{12}$$

where $(\theta_i^{+})^{j} = \max_k\{\theta_{ki}^{j}\}$, $(\theta_i^{-})^{j} = \min_k\{\theta_{ki}^{j}\}, (\vartheta_i^{+})^{j} = \min_k\{\theta_{ki}^{j}\}, (\vartheta_i^{-})^{j} = \max_k\{\theta_{ki}^{j}\}$, $(\pi_i^{+})^{j} = (h^q - ((\theta_i^{+})^{j})^q - ((\vartheta_i^{+})^{j})^q)^{1/q}$, $(\pi_i^{-})^{j} = (h^q - ((\theta_i^{-})^{j})^q - ((\vartheta_i^{-})^{j})^q)^{1/q}$, $k = 1,2,\ldots,p, i = 1,2,\ldots r$ and $j = 1,2,\ldots,m$.

Step 3: Based on Eq. (9), obtain the weighted correlation coefficient $(K_k^{+})^{j}$ between the alternatives ζ_k and the PIA $(\zeta^{+})^{j}$ and obtain the weighted correlation coefficient $(K_k^{-})^{j}$ between the alternatives ζ_k and the NIA $(\zeta^{-})^{j}$ for each DMEx ξ_j, where $j = 1,2,\ldots,m, k = 1,2,\ldots,p$, shown as follows:

$$\begin{aligned}(K_k^{+})^{j} =& K_w^{j}(\zeta_k, (\zeta^{+})^{j})\\ =& \frac{\sum_{i=1}^{r} w_i\left((\theta_{ki}^{j})^q \cdot ((\theta_i^{+})^{j})^q + (\vartheta_{ki}^{j})^q \cdot ((\vartheta_i^{+})^{j})^q + (\pi_{ki}^{j})^q \cdot ((\pi_i^{+})^{j})^q\right)}{\max\left\{\sum_{i=1}^{r} w_i\left((\theta_{ki}^{j})^{2q} + (\vartheta_{ki}^{j})^{2q} + (\pi_{ki}^{j})^{2q}\right), \sum_{i=1}^{r} w_i\left(((\theta_i^{+})^{j})^{2q} + ((\vartheta_i^{+})^{j})^{2q} + ((\pi_i^{+})^{j})^{2q}\right)\right\}},\end{aligned} \tag{13}$$

$$\begin{aligned}(K_k^{-})^{j} =& K_w^{j}(\zeta_k, (\zeta^{-})^{j})\\ =& \frac{\sum_{i=1}^{r} w_i\left((\theta_{ki}^{j})^q \cdot ((\theta_i^{-})^{j})^q + (\vartheta_{ki}^{j})^q \cdot ((\vartheta_i^{-})^{j})^q + (\pi_{ki}^{j})^q \cdot ((\pi_i^{-})^{j})^q\right)}{\max\left\{\sum_{i=1}^{r} w_i\left((\theta_{ki}^{j})^{2q} + (\vartheta_{ki}^{j})^{2q} + (\pi_{ki}^{j})^{2q}\right), \sum_{i=1}^{r} w_i\left(((\theta_i^{-})^{j})^{2q} + ((\vartheta_i^{-})^{j})^{2q} + ((\pi_i^{-})^{j})^{2q}\right)\right\}},\end{aligned} \tag{14}$$

where w_i is the weight of attribute $C_i, w_i \geq 0$, and $\sum_{i=1}^{r} w_i = 1$.

Step 4: Calculate the aggregated positive weighted correlation coefficient (PWCC) (K_k^{+}) and negative weighted correlation coefficient (NWCC) (K_k^{-}) for each alternative ζ_k, where $k = 1,2,\ldots,p$, shown as follows:

$$K_k^{+} = \sum_{j=1}^{m} \varsigma_j (K_k^{+})^{j}, \tag{15}$$

$$K_k^{-} = \sum_{j=1}^{m} \varsigma_j (K_k^{-})^{j}, \tag{16}$$

where ς_j is the weight of DMEx $\xi_j, \varsigma_j \geq 0$, and $\sum_{j=1}^{m} \varsigma_j = 1$.

Step 5: Calculate the closeness coefficient ϕ_k of the alternative ζ_k based on the aggregated PWCC K_k^{+} and aggregated NWCC K_k^{-} of alternatives ζ_k, where

$$\phi_k = \frac{K_k^{+}}{K_k^{+} + K_k^{-}} \tag{17}$$

where $k = 1,2,\ldots,p$.

Step 6: Rank the alternatives $\zeta_1, \zeta_2, \ldots, \zeta_p$ based on the obtained closeness coefficients $\phi_1, \phi_2, \ldots, \phi_p$.

A higher closeness coefficient ϕ_k for alternative ζ_k indicates a superior preference order (PO) for that alternative, where $k = 1,2,\ldots,p$.

Table 1 PIA $(\zeta^{+})^{j}$ and NIA $(\zeta^{-})^{j}$ for each DMEx ξ_j for Example 3, where $j = 1,2,3$.

DMEx	PIA and NIA	C_1	C_2	C_3	C_4
ξ_1	$(\zeta^{+})^{1}$	$\langle s_5, s_1\rangle$	$\langle s_6, s_2\rangle$	$\langle s_6, s_1\rangle$	$\langle s_4, s_2\rangle$
	$(\zeta^{-})^{1}$	$\langle s_4, s_3\rangle$	$\langle s_4, s_3\rangle$	$\langle s_5, s_2\rangle$	$\langle s_3, s_4\rangle$
ξ_2	$(\zeta^{+})^{2}$	$\langle s_5, s_2\rangle$	$\langle s_6, s_2\rangle$	$\langle s_6, s_1\rangle$	$\langle s_5, s_1\rangle$
	$(\zeta^{-})^{2}$	$\langle s_3, s_5\rangle$	$\langle s_4, s_4\rangle$	$\langle s_4, s_3\rangle$	$\langle s_3, s_3\rangle$
ξ_3	$(\zeta^{+})^{3}$	$\langle s_5, s_1\rangle$	$\langle s_7, s_1\rangle$	$\langle s_5, s_3\rangle$	$\langle s_5, s_2\rangle$
	$(\zeta^{-})^{3}$	$\langle s_4, s_2\rangle$	$\langle s_4, s_2\rangle$	$\langle s_3, s_5\rangle$	$\langle s_3, s_3\rangle$

Example 3 (Liu and Liu 2019a) We are analyzing a specific postgraduate entrance requirement at a college. There are four potential students denoted as ζ_1, ζ_2, ζ_3, and ζ_4, but only two enrollment places are open. The college aims to conduct a comprehensive assessment of the four students and ultimately admit the two most suitable applicants. The college has invited three DMExs named ξ_1, ξ_2, and ξ_3 to evaluate the performance of four students ζ_1, ζ_2,

ζ_3, and ζ_4. The DMExs ξ_1, ξ_2, and ξ_3 have their weights $\varsigma_1 = 0.35$, $\varsigma_2 = 0.4$, and $\varsigma_3 = 0.25$, respectively. The DMExs ξ_1, ξ_2 and ξ_3 examine the students ζ_1, ζ_2, ζ_3 and ζ_4 under the four attributes denoted as C_1 ("the score of the written test"), C_2 ("the professional relevance"), C_3 ("the logical ability"), and C_4 ("the learning attitude"), where $w_1 = 0.25$, $w_2 = 0.15$, $w_3 = 0.25$, and $w_4 = 0.35$ are the weights of the attributes C_1, C_2, C_3 and C_4, respectively, using the Lq-ROFNs $\tilde{\varrho}^j_{ki} = \langle s_{\tilde{\theta}^j_{ki}}, s_{\tilde{\vartheta}^j_{ki}} \rangle$, where $\tilde{\varrho}^j_{ki} \in \Omega_{[0,8]}$, $k = 1,2,3,4$; $i = 1,2,3,4$ and $j = 1,2,3$, to construct the DMxs $\widetilde{R}^1 = (\tilde{\varrho}^1_{ki})_{4\times4}$, $\widetilde{R}^2 = (\tilde{\varrho}^2_{ki})_{4\times4}$ and $\widetilde{R}^3 = (\tilde{\varrho}^3_{ki})_{4\times4}$, respectively, shown as follows:

$$\widetilde{R}^1 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle s_5, s_1 \rangle & \langle s_5, s_2 \rangle & \langle s_5, s_1 \rangle & \langle s_4, s_3 \rangle \\ \langle s_4, s_3 \rangle & \langle s_6, s_2 \rangle & \langle s_6, s_1 \rangle & \langle s_3, s_3 \rangle \\ \langle s_4, s_2 \rangle & \langle s_5, s_2 \rangle & \langle s_5, s_2 \rangle & \langle s_4, s_4 \rangle \\ \langle s_5, s_2 \rangle & \langle s_4, s_3 \rangle & \langle s_5, s_2 \rangle & \langle s_3, s_2 \rangle \end{pmatrix} \end{array},$$

$$\widetilde{R}^2 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle s_4, s_2 \rangle & \langle s_5, s_2 \rangle & \langle s_5, s_2 \rangle & \langle s_3, s_3 \rangle \\ \langle s_3, s_5 \rangle & \langle s_6, s_2 \rangle & \langle s_4, s_3 \rangle & \langle s_5, s_2 \rangle \\ \langle s_5, s_3 \rangle & \langle s_4, s_4 \rangle & \langle s_4, s_1 \rangle & \langle s_4, s_1 \rangle \\ \langle s_4, s_3 \rangle & \langle s_5, s_3 \rangle & \langle s_6, s_2 \rangle & \langle s_4, s_3 \rangle \end{pmatrix} \end{array},$$

$$\widetilde{R}^3 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle s_5, s_1 \rangle & \langle s_6, s_2 \rangle & \langle s_3, s_5 \rangle & \langle s_4, s_3 \rangle \\ \langle s_4, s_2 \rangle & \langle s_7, s_1 \rangle & \langle s_4, s_4 \rangle & \langle s_5, s_2 \rangle \\ \langle s_5, s_2 \rangle & \langle s_4, s_2 \rangle & \langle s_4, s_3 \rangle & \langle s_4, s_2 \rangle \\ \langle s_4, s_1 \rangle & \langle s_6, s_2 \rangle & \langle s_5, s_3 \rangle & \langle s_3, s_3 \rangle \end{pmatrix} \end{array}$$

In the following, we use the proposed MAGDM method to solve this MAGDM problem, shown as follows:

Step 1: Since all the attributes C_1, C_2, C_3 and C_4 are of benefit type, by using Eq. (10), we get NDMxs $R^1 = (\tilde{\varrho}^1_{ki})_{4\times4} = (\varrho^1_{ki})_{4\times4} = (\langle s_{\theta^1_{ki}}, s_{\vartheta^1_{ki}} \rangle)_{4\times4}$, $R^2 = (\tilde{\varrho}^2_{ki})_{4\times4} = (\varrho^2_{ki})_{4\times4} = (\langle s_{\theta^2_{ki}}, s_{\vartheta^2_{ki}} \rangle)_{4\times4}$ and $R^3 = (\tilde{\varrho}^3_{ki})_{4\times4} = (\varrho^3_{ki})_{4\times4} = (\langle s_{\theta^3_{ki}}, s_{\vartheta^3_{ki}} \rangle)_{4\times4}$.

Step 2: By using Eqs. (11) and (12), we obtain the PIAs $(\zeta^+)^1$, $(\zeta^+)^2$ amd $(\zeta^+)^3$ and the NIAs $(\zeta^-)^1$, $(\zeta^-)^2$ and $(\zeta^-)^3$ for the DMExs ξ_1, ξ_2 and ξ_3, respectively, as given in Table 1.

Step 3: By utilizing Eqs. (13) and (14), we obtain the weighted correlation coefficient $(K^+_k)^j$ between the alternative ζ_k and the PIA $(\zeta^+)^j$ and the weighted correlation coefficient $(K^-_k)^j$ between the alternatives ζ_k and the NIA $(\zeta^-)^j$, where $q = 4$, $j = 1,2,3$, $k = 1,2,3,4$, $(K^+_k)^j = K^j_w(\zeta_k, (\zeta^+)^j)$, $(K^-_k)^j = K^j_w(\zeta_k, (\zeta^-)^j)$, $(K^+_1)^1 = 0.9480$, $(K^+_1)^2 = 0.8787$, $(K^+_1)^3 = 0.9452$, $(K^+_2)^1 = 0.9688$, $(K^+_2)^2 = 0.9308$, $(K^+_2)^3 = 0.9620$, $(K^+_3)^1 = 0.9405$, $(K^+_3)^2 = 0.8739$, $(K^+_3)^3 = 0.8633$, $(K^+_4)^1 = 0.9137$, $(K^+_4)^2 = 0.9330$, $(K^+_4)^3 = 0.9099$, $(K^-_1)^1 = 0.9705$, $(K^-_1)^2 = 0.9868$, $(K^-_1)^3 = 0.9224$, $(K^-_2)^1 = 0.9448$, $(K^-_2)^2 = 0.9247$, $(K^-_2)^3 = 0.8826$, $(K^-_3)^1 = 0.9764$, $(K^-_3)^2 = 0.9907$, $(K^-_3)^3 = 0.9827$, $(K^-_4)^1 = 0.9907$, $(K^-_4)^2 = 0.9314$ and $(K^-_4)^3 = 0.9568$.

Step 4: By utilizing Eqs. (15) and (16), we get the PWCC (K^+_k) and NWCC (K^-_k) for each alternative ζ_k, where $k = 1,2,3,4$, $K^+_1 = 0.9196$, $K^+_2 = 0.9519$, $K^+_3 = 0.8946$, $K^+_4 = 0.9205$, $K^-_1 = 0.9650$, $K^-_2 = 0.9212$, $K^-_3 = 0.9837$, $K^-_4 = 0.9585$.

Step 5: By using Eq. (17), we get the relative closeness of coefficient ϕ_1, ϕ_2, ϕ_3 and ϕ_4 of the alternative ζ_1, ζ_2, ζ_3 and ζ_4, respectively, where $\phi_1 = 0.4880$, $\phi_2 = 0.5082$, $\phi_3 = 0.4763$ and $\phi_4 = 0.4899$.

Step 6: Because $\phi_2 \succ \phi_4 \succ \phi_1 \succ \phi_3$, where $\phi_1 = 0.4880$, $\phi_2 = 0.5082$, $\phi_3 = 0.4763$ and $\phi_4 = 0.4899$, the PO of the alternatives ζ_1, ζ_2, ζ_3 and ζ_4 is "$\zeta_2 \succ \zeta_4 \succ \zeta_1 \succ \zeta_3$". Therefore, ζ_2 is the best alternative among the alternatives ζ_1, ζ_2, ζ_3 and ζ_4.

Table 2 provides a comparison of the POs of the alternatives ζ_1, ζ_2, ζ_3 and ζ_4 obtained by the different MAGDM approaches for *Example* 3. From Table 2, it is clear that the proposed MAGDM approach, the Liu and Liu's MAGDM approach (Liu and Liu 2019a), and the Liu et al.'s MAGDM approach (Liu et al. 2022) give the same PO "$\zeta_2 \succ \zeta_4 \succ \zeta_1 \succ \zeta_3$" of ζ_1, ζ_2, ζ_3 and ζ_4.

Example 4 Let ζ_1, ζ_2, and ζ_3 be three alternatives, and C_1, C_2, and C_3 be three attributes, where $w_1 = 0.3$, $w_2 = 0.3$, and $w_3 = 0.4$ are the weights of C_1, C_2, and C_3, respectively. The weight of DMExs ξ_1, ξ_2 and ξ_3 are $\varsigma_1 = 0.3$, $\varsigma_2 = 0.4$ and $\varsigma_3 = 0.3$, respectively. The DMExs ξ_1, ξ_2 and

Table 2 A comparison of the POs of the alternatives obtained by different MAGDM approaches for Example 3

MAGDM approaches	POs
Liu and Liu's MAGDM approach (Liu and Liu 2019a)	$\zeta_2 \succ \zeta_4 \succ \zeta_1 \succ \zeta_3$
Liu et al.'s MAGDM approach (Liu et al. 2022)	$\zeta_2 \succ \zeta_4 \succ \zeta_1 \succ \zeta_3$
The proposed MAGDM approach	$\zeta_2 \succ \zeta_4 \succ \zeta_1 \succ \zeta_3$

ξ_3 evaluate the attribute C_i of the alternative ζ_k by using a Lq-ROFN $\tilde{\varrho}_{ki}^j = \langle s_{\tilde{\theta}_{ki}^j}, s_{\tilde{\vartheta}_{ki}^j} \rangle$, where $\tilde{\varrho}_{ki}^j \in \Omega_{[0,8]}$, to construct the DMxs $\tilde{R}^1 = (\tilde{\varrho}_{ki}^1)_{3\times3}$, $\tilde{R}^2 = (\tilde{\varrho}_{ki}^2)_{3\times3}$ and $\tilde{R}^3 = (\tilde{\varrho}_{ki}^3)_{3\times3}$, respectively, shown as follows:

$$\widetilde{R}^1 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle s_3, s_4 \rangle & \langle s_8, s_0 \rangle & \langle s_3, s_5 \rangle \\ \langle s_1, s_4 \rangle & \langle s_4, s_2 \rangle & \langle s_4, s_3 \rangle \\ \langle s_4, s_3 \rangle & \langle s_2, s_5 \rangle & \langle s_1, s_5 \rangle \end{pmatrix} \end{array},$$

$$\widetilde{R}^2 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle s_5, s_4 \rangle & \langle s_4, s_3 \rangle & \langle s_4, s_1 \rangle \\ \langle s_4, s_0 \rangle & \langle s_5, s_1 \rangle & \langle s_4, s_0 \rangle \\ \langle s_4, s_4 \rangle & \langle s_3, s_1 \rangle & \langle s_2, s_1 \rangle \end{pmatrix} \end{array},$$

$$\widetilde{R}^3 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle s_4, s_1 \rangle & \langle s_1, s_6 \rangle & \langle s_4, s_3 \rangle \\ \langle s_3, s_1 \rangle & \langle s_8, s_0 \rangle & \langle s_3, s_5 \rangle \\ \langle s_2, s_5 \rangle & \langle s_6, s_1 \rangle & \langle s_5, s_1 \rangle \end{pmatrix} \end{array}.$$

In the following, we use the proposed MAGDM method to solve this MAGDM problem, shown as follows:

Step 1: Since all the attributes C_1, C_2 and C_3 are of benefit type, using Eq. (10), we get NDMxs $R^1 = (\tilde{\varrho}_{ki}^1)_{3\times3} = (\varrho_{ki}^1)_{3\times3} = (\langle s_{\theta_{ki}^1}, s_{\vartheta_{ki}^1} \rangle)_{3\times3}$, $R^2 = (\tilde{\varrho}_{ki}^2)_{3\times3} = (\varrho_{ki}^2)_{3\times3} = (\langle s_{\theta_{ki}^2}, s_{\vartheta_{ki}^2} \rangle)_{3\times3}$ and $R^3 = (\tilde{\varrho}_{ki}^3)_{3\times3} = (\varrho_{ki}^3)_{3\times3} = (\langle s_{\theta_{ki}^3}, s_{\vartheta_{ki}^3} \rangle)_{3\times3}$.

Step 2: By utilizing Eqs. (11) and (12), we obtain the PIAs $(\zeta^+)^1$, $(\zeta^+)^2$, amd $(\zeta^+)^3$ and the NIAs $(\zeta^-)^1$, $(\zeta^-)^2$, and $(\zeta^-)^3$ for the DMExs ξ_1, ξ_2, and ξ_3, respectively, as given in Table 3.

Step 3: By utilizing Eqs. (13) and (14), we obtain the weighted correlation coefficient $(K_k^+)^j$ between the alternative ζ_k and the PIA $(\zeta^+)^j$ and the weighted correlation coefficient $(K_k^-)^j$ between the alternatives ζ_k and the NIA $(\zeta^-)^j$, where $q = 4$, $j = 1,2,3$, $k = 1,2,3$, $(K_k^+)^j = K_w^j(\zeta_k, (\zeta^+)^j)$, $(K_k^-)^j = K_w^j(\zeta_k, (\zeta^-)^j)$, $(K_1^+)^1 = 0.9623, (K_1^+)^2 = 0.9947, (K_1^+)^3 = 0.6731, (K_2^+)^1 = 0.6899, (K_2^+)^2 = 0.9719, (K_2^+)^3 = 0.9838, (K_3^+)^1 = 0.6356, (K_3^+)^2 = 0.9257, (K_3^+)^3 = 0.7300, (K_1^-)^1 = 0.6570, (K_1^-)^2 = 0.9368, (K_1^-)^3 = 0.9217, (K_2^-)^1 = 0.9393, (K_2^-)^2 = 0.9560, (K_2^-)^3 = 0.6098, (K_3^-)^1 = 0.9920, (K_3^-)^2 = 0.9937$ and $(K_3^-)^3 = 0.9349$.

Step 4: By utilizing Eq. (15) and (16), we get the PWCC (K_k^+) and NWCC (K_k^-) for each alternative ζ_k, where $k = 1,2,3$, $K_1^+ = 0.8885, K_2^+ = 0.8909, K_3^+ = 0.7800, K_1^- = 0.8483, K_2^- = 0.8471, K_3^- = 0.9756$.

Step 5: By utilizing Eq. (17), we get the relative closeness of coefficient ϕ_1, ϕ_2, and ϕ_3 of the alternative ζ_1, ζ_2, and ζ_3, respectively, where $\phi_1 = 0.5116, \phi_2 = 0.5126$, and $\phi_3 = 0.4443$.

Step 6: Because $\phi_2 \succ \phi_1 \succ \phi_3$ where $\phi_1 = 0.5116, \phi_2 = 0.5126$, and $\phi_3 = 0.4443$, the PO of the alternatives ζ_1, ζ_2, and ζ_3 is "$\zeta_2 \succ \zeta_1 \succ \zeta_3$". Therefore, ζ_2 is the best alternative among the alternatives ζ_1, ζ_2, and ζ_3.

Table 4 provides a comparison of the POs of the alternatives ζ_1, ζ_2 and ζ_3 obtained by the different MAGDM methods for *Example* 4. From Table 4, it is clear that the Liu et al.'s MAGDM approach (Liu et al. 2022) gets the PO "$\zeta_1 = \zeta_2 \succ \zeta_3$" of the alternatives ζ_1, ζ_2 and ζ_3, where it has the shortcomings that it cannot distinguish the PO of alternatives ζ_1 and ζ_2 in this case. Furthermore, it is also clear that the proposed MAGDM approach and the Liu and Liu's MAGDM approach (Liu and Liu 2019a) obtain the same PO "$\zeta_2 \succ \zeta_1 \succ \zeta_3$" of ζ_1, ζ_2 and ζ_3. Hence, the proposed MAGDM approach can overcome the shortcomings of Liu et al.'s MAGDM approach (Liu et al. 2022) in this case.

Example 5 Let ζ_1, ζ_2, ζ_3, and ζ_4 be four alternatives and C_1, C_2, C_3, and C_4 be four attributes where $w_1 = 0.2$, $w_2 = 0.3$, $w_3 = 0.2$, and $w_4 = 0.3$ are the weights of the C_1, C_2, C_3, and C_4, respectively. The weight of DMExs ξ_1, ξ_2, and ξ_3 is $\varsigma_1 = 0.25$, $\varsigma_2 = 0.35$, and $\varsigma_3 = 0.4$, respectively. The DMExs ξ_1, ξ_2 and ξ_3 evaluate the attribute C_i of the alternative ζ_k using a Lq-ROFN $\tilde{\varrho}_{ki}^j = \langle s_{\tilde{\theta}_{ki}^j}, s_{\tilde{\vartheta}_{ki}^j} \rangle$, where $\tilde{\varrho}_{ki}^j \in \Omega_{[0,8]}$, $k = 1,2,3,4$; $i = 1,2,3,4$ and $j = 1,2,3$, to

Table 3 PIA $(\zeta^+)^j$ and NIA $(\zeta^-)^j$ for each DMEx ξ_j for Example 4, where $j = 1, 2, 3$.

DMEx	PIA and NIA	C_1	C_2	C_3
ξ_1	$(\zeta^+)^1$	$\langle s_4, s_3\rangle$	$\langle s_8, s_0\rangle$	$\langle s_4, s_3\rangle$
	$(\zeta^-)^1$	$\langle s_1, s_4\rangle$	$\langle s_2, s_5\rangle$	$\langle s_1, s_5\rangle$
ξ_2	$(\zeta^+)^2$	$\langle s_5, s_0\rangle$	$\langle s_5, s_1\rangle$	$\langle s_4, s_0\rangle$
	$(\zeta^-)^2$	$\langle s_4, s_4\rangle$	$\langle s_3, s_3\rangle$	$\langle s_2, s_1\rangle$
ξ_3	$(\zeta^+)^3$	$\langle s_4, s_1\rangle$	$\langle s_8, s_0\rangle$	$\langle s_5, s_1\rangle$
	$(\zeta^-)^3$	$\langle s_2, s_5\rangle$	$\langle s_1, s_6\rangle$	$\langle s_3, s_5\rangle$

Table 4 A comparison of the POs of the alternatives obtained by different MAGDM approaches for Example 4

MAGDM methods	ROs
Liu and Liu's MAGDM methods (Liu and Liu 2019a)	$\zeta_2 \succ \zeta_1 \succ \zeta_3$
Liu et al.'s MAGDM approach (Liu et al. 2022)	$\zeta_1 = \zeta_2 \succ \zeta_3$
Proposed MAGDM method	$\zeta_2 \succ \zeta_1 \succ \zeta_3$

construct the DMxs $\tilde{R}^1 = (\tilde{\varrho}^1_{ki})_{4\times 4}$, $\tilde{R}^2 = (\tilde{\varrho}^2_{ki})_{4\times 4}$ and $\tilde{R}^3 = (\tilde{\varrho}^3_{ki})_{4\times 4}$, respectively, shown as follows:

$$\widetilde{R}^1 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle s_8, s_0\rangle & \langle s_5, s_1\rangle & \langle s_2, s_3\rangle & \langle s_2, s_3\rangle \\ \langle s_1, s_2\rangle & \langle s_4, s_2\rangle & \langle s_6, s_1\rangle & \langle s_8, s_0\rangle \\ \langle s_8, s_0\rangle & \langle s_8, s_0\rangle & \langle s_5, s_2\rangle & \langle s_4, s_4\rangle \\ \langle s_5, s_2\rangle & \langle s_4, s_3\rangle & \langle s_5, s_2\rangle & \langle s_3, s_2\rangle \end{pmatrix} \end{array},$$

$$\widetilde{R}^2 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle s_8, s_0\rangle & \langle s_5, s_2\rangle & \langle s_4, s_4\rangle & \langle s_8, s_0\rangle \\ \langle s_3, s_5\rangle & \langle s_8, s_0\rangle & \langle s_4, s_2\rangle & \langle s_4, s_3\rangle \\ \langle s_5, s_3\rangle & \langle s_4, s_4\rangle & \langle s_4, s_1\rangle & \langle s_4, s_1\rangle \\ \langle s_4, s_3\rangle & \langle s_5, s_3\rangle & \langle s_6, s_2\rangle & \langle s_4, s_3\rangle \end{pmatrix} \end{array},$$

$$\widetilde{R}^3 = \begin{array}{c} \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle s_5, s_1\rangle & \langle s_4, s_2\rangle & \langle s_3, s_5\rangle & \langle s_8, s_0\rangle \\ \langle s_4, s_3\rangle & \langle s_8, s_0\rangle & \langle s_4, s_4\rangle & \langle s_8, s_0\rangle \\ \langle s_5, s_2\rangle & \langle s_4, s_2\rangle & \langle s_4, s_3\rangle & \langle s_4, s_2\rangle \\ \langle s_4, s_1\rangle & \langle s_6, s_2\rangle & \langle s_5, s_3\rangle & \langle s_3, s_3\rangle \end{pmatrix} \end{array}.$$

In the following, we use the proposed MAGDM method to solve this MAGDM problem, shown as follows:

Step 1: Since all the attributes C_1, C_2, C_3 and C_4 are of benefit type, using Eq. (10), we get NDMxs $R^1 = (\tilde{\varrho}^1_{ki})_{4\times 4} = (\varrho^1_{ki})_{4\times 4} = (\langle s_{\theta^1_{ki}}, s_{\vartheta^1_{ki}}\rangle)_{4\times 4}$, $R^2 = (\tilde{\varrho}^2_{ki})_{4\times 4} = (\varrho^2_{ki})_{4\times 4} = (\langle s_{\theta^2_{ki}}, s_{\vartheta^2_{ki}}\rangle)_{4\times 4}$ and $R^3 = (\tilde{\varrho}^3_{ki})_{4\times 4} = (\varrho^3_{ki})_{4\times 4} = (\langle s_{\theta^3_{ki}}, s_{\vartheta^3_{ki}}\rangle)_{4\times 4}$.

Step 2: By utilizing Eqs. (11) and (12), we obtain the PIAs $(\zeta^+)^1$, $(\zeta^+)^2$, amd $(\zeta^+)^3$ and the NIAs $(\zeta^-)^1$, $(\zeta^-)^2$, and $(\zeta^-)^3$ for the DMExs ξ_1, ξ_2, and ξ_3, respectively, as given in Table 5.

Step 3: By utilizing Eqs. (13) and (14), we obtain the weighted correlation coefficient $(K_k^+)^j$ between the alternative ζ_k and the PIA $(\zeta^+)^j$ and the weighted correlation coefficient $(K_k^-)^j$ between the alternatives ζ_k and the NIA $(\zeta^-)^j$, where $q = 2$, $j = 1, 2, 3$, $k = 1, 2, 3, 4$, $(K_k^+)^j = K_w^j(\zeta_k, (\zeta^+)^j)$, $(K_k^-)^j = K_w^j(\zeta_k, (\zeta^-)^j)$, $(K_1^+)^1 = 0.4568$, $(K_1^+)^2 = 0.7657$, $(K_1^+)^3 = 0.6953$, $(K_2^+)^1 = 0.5307$, $(K_2^+)^2 = 0.5445$, $(K_2^+)^3 = 0.9824$, $(K_3^+)^1 = 0.7401$, $(K_3^+)^2 = 0.3540$, $(K_3^+)^3 = 0.4186$, $(K_4^+)^1 = 0.3177$, $(K_4^+)^2 = 0.3752$, $(K_4^+)^3 = 0.5082$, $(K_1^-)^1 = 0.6600$, $(K_1^-)^2 = 0.4133$, $(K_1^-)^3 = 0.5843$, $(K_2^-)^1 = 0.5623$, $(K_2^-)^2 = 0.6008$, $(K_2^-)^3 = 0.3674$, $(K_3^-)^1 =$- , $(K_3^-)^3 = 0.9623$, $(K_4^-)^1 = 0.8402$, $(K_4^-)^2 = 0.8935$ and $(K_4^-)^3 = 0.9047$.

Step 4: By utilizing Eqs. (15) and (16), we get the PWCC (K_k^+) and NWCC (K_k^-) for each alternative ζ_k, where $k = 1, 2, 3, 4$, $K_1^+ = 0.6603$, $K_2^+ = 0.7162$, $K_3^+ = 0.4764$, $K_4^+ = 0.4140$, $K_1^- = 0.5434$, $K_2^- = 0.4978$, $K_3^- = 0.7889$, $K_4^- = 0.8846$.

Step 5: By utilizing Eq. (17), we get the relative closeness of coefficient ϕ_1, ϕ_2, ϕ_3, and ϕ_4 of the alternative ζ_1, ζ_2, ζ_3, and ζ_4, respectively, where $\phi_1 = 0.5486$, $\phi_2 = 0.5899$, $\phi_3 = 0.3765$, and $\phi_4 = 0.3188$.

Table 5 PIA $(\zeta^+)^j$ and NIA $(\zeta^-)^j$ for each DMEx ξ_j for Example 5, where $j = 1, 2, 3$.

DMEx	PIA and NIA	C_1	C_2	C_3	C_4
ξ_1	$(\zeta^+)^1$	$\langle s_8, s_0\rangle$	$\langle s_8, s_0\rangle$	$\langle s_6, s_1\rangle$	$\langle s_8, s_0\rangle$
	$(\zeta^-)^1$	$\langle s_1, s_2\rangle$	$\langle s_4, s_3\rangle$	$\langle s_2, s_3\rangle$	$\langle s_2, s_4\rangle$
ξ_2	$(\zeta^+)^2$	$\langle s_8, s_0\rangle$	$\langle s_8, s_0\rangle$	$\langle s_6, s_1\rangle$	$\langle s_8, s_0\rangle$
	$(\zeta^-)^2$	$\langle s_3, s_5\rangle$	$\langle s_4, s_4\rangle$	$\langle s_4, s_4\rangle$	$\langle s_4, s_3\rangle$
ξ_3	$(\zeta^+)^3$	$\langle s_5, s_1\rangle$	$\langle s_8, s_0\rangle$	$\langle s_5, s_3\rangle$	$\langle s_8, s_0\rangle$
	$(\zeta^-)^3$	$\langle s_4, s_3\rangle$	$\langle s_4, s_2\rangle$	$\langle s_3, s_5\rangle$	$\langle s_3, s_3\rangle$

Table 6 A comparison of the POs of the alternatives obtained by different MAGDM approaches for Example 5

MAGDM approaches	POs
Liu and Liu's MAGDM approach (Liu and Liu 2019a)	$\zeta_1 = \zeta_2 \succ \zeta_3 \succ \zeta_4$
Liu et al.'s MAGDM approach (Liu et al. 2022)	$\zeta_2 \succ \zeta_1 \succ \zeta_3 \succ \zeta_4$
The proposed MAGDM approach	$\zeta_2 \succ \zeta_1 \succ \zeta_3 \succ \zeta_4$

Step 6: Because $\phi_2 \succ \phi_1 \succ \phi_3 \succ \phi_4$ where $\phi_1 = 0.5486, \phi_2 = 0.5899, \phi_3 = 0.3765$, and $\phi_4 = 0.3188$, the ranking order of the alternatives $\zeta_1, \zeta_2, \zeta_3$, and ζ_4 is "$\zeta_2 \succ \zeta_1 \succ \zeta_3 \succ \zeta_4$". Therefore, ζ_2 is the best alternative among the alternatives $\zeta_1, \zeta_2, \zeta_3$, and ζ_4.

Table 6 provides a comparison of the POs of the alternatives $\zeta_1, \zeta_2, \zeta_3$, and ζ_4 obtained by the different MAGDM approaches for *Example*5. From Table 6, it is clear that the MAGDM approach by Liu and Liu (2019a) gets the PO "$\zeta_1 = \zeta_2 \succ \zeta_3 \succ \zeta_4$" of the alternatives $\zeta_1, \zeta_2, \zeta_3$ a,nd ζ_4, it has the shortcomings that it cannot distinguish the PO of alternatives ζ_1 and ζ_2 in this case. Furthermore, it is also clear that the proposed MAGDM approach and the Liu et al.'s MAGDM method (Liu et al. 2022) obtain the same PO "$\zeta_2 \succ \zeta_1 \succ \zeta_3 \succ \zeta_4$" of $\zeta_1, \zeta_2, \zeta_3$ and ζ_4. Hence, the proposed MAGDM approach can overcome the limitations of Liu and Liu's MAGDM approach (Liu and Liu 2019a) in this case.

5 Conclusion

In this paper, we have developed a multiattribute group decision making (MAGDM) approach based on the proposed weighted correlation coefficient of linguistic q-rung orthopair fuzzy sets (Lq-ROFSs) and the TOPSIS method under linguistic q-rung orthopair fuzzy numbers (Lq-ROFNs) environment. For this, first, we have proposed the correlation coefficient and weighted correlation coefficient of Lq-ROFSs, which measure the strength of the relationship between two Lq-ROFSs. We have also provided the various properties of the proposed correlation coefficient and weighted correlation coefficient of Lq-ROFSs. Furthermore, we have developed the MAGDM approach under the Lq-ROFNs environment, which is based on the TOPSIS method and the proposed weighted correlation coefficient of Lq-ROFSs. We have also solved the different MAGDM problems by utilizing the proposed MAGDM approach to illustrate the applicability and practicality of the proposed MAGDM approach. The results of *Example* 3, *Example* 4, and *Example* 5 show that the proposed MAGDM approach can overcome the shortcomings of the Liu and Liu's MAGDM approach (Liu and Liu 2019a) and Liu et al.'s MAGDM approach (Liu et al. 2022), where they cannot distinguish the preference orders of alternatives in some situations. The proposed MAGDM approach provides a valuable tools for tackling MAGDM problems in the context of Lq-ROFNs.

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Declarations

Conflicts of interest The authors declare that they have no conflicts of interest.

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