



Multi-criteria group decision making through full multiplicative form under hesitant Fermatean fuzzy environment

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Abstract

The main purpose of this research work is to develop a novel approach for multi-objective optimization within the framework of hesitant Fermatean fuzzy methodology incorporating both, the ratio analysis and the complete multiplicative form techniques, to effectively tackle multi-criteria group decision-making (MCGDM) challenges arising from uncertain and ambiguous data about weight values of the criteria set. We propose a novel distance measure while formulating the framework to quantify the distinction among hesitation Fermatean fuzziness after integrating Hausdorff metric and Hamming distance. Subsequently, the developed distance measure is used to determine the unspecified weights of criteria using the weight entropy approach. The same distance measure is utilized in the reference point technique to attain the inclination implications of potential choices or alternatives. While the global economy has been developed vigorously, a big amount of complex and uncertain data get involved in the evaluations and judgments of evaluators in MCGDM. Decision-makers may assign a set of preference values because of their diverse knowledge backgrounds and dissimilarity of benchmarks. Thus, it is very challenging for evaluators to impart their evaluations through Fermatean fuzzy elements. To overcome the existing limitations and drawbacks of classical MULTIMOORA technique have been overcome through an integrated strategy, specifically, by amalgamating hesitant Fermatean fuzzy-MULTIMOORA with quantitative strategic planning matrix. Moreover, a MCGDM algorithm has also been presented in the current research work. The practical applicability and viability of the suggested approach is demonstrated through its implementation in a real life scenario involving the ranking of strategies for a tile assembling organization. The suggested technique's applicability and effectiveness, as well as the conclusions drawn using hesitant Fermatean fuzzy-MULTIMOORA, are distinguished from those derived using other existing method. These subsisting methodologies are examined not only from classical and fuzzy surroundings, but also in conjunction with certain other decision making approaches, including modified Pythagorean fuzzy-MOORA, considered as a special case of the created hesitant Fermatean fuzzy-MULTIMOORA framework.

Keywords Hesitant Fermatean fuzzy sets · Quantitative strategic planning matrix · MCGDM · Implementations · Discussions and comparative analysis · Sensitivity analysis

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1 Introduction

Multi-criteria group decision making (MCGDM) is contemplated as an approach of formulating decisions by which a finite collection of choices or options can be evaluated by a set of experts based on a collection of criteria. In the aforementioned procedure, first of all an evaluation matrix (or a response matrix) is formulated through the assessment experts (or decision makers (DMs')) in order to describe the preference estimations with respect to the possible choices that persuade a considered collection of specifications. It has been normally perceived that the obtained evaluated matrices comprise the ambiguous and vague data due to implicit uncertainty of DMs. To cope with such uncertainties, the proposal of fuzzy sets (FSs) was originated by Zadeh (1965) that was considered as an effective framework to solve DM (decision-making) issues Biswas and De (2018), Debnath and Biswas (2018) and Debnath et al. (2018). Further applications of FSs in MCGDM can be checked in Chen and Wang (2010), Chen et al. (2009), Chen and Jian (2017), Chen et al. (2019), Chen and Wang (1995) and Horng et al. (2005). Most of the times, it has been noted that FSs lack some knowledge to handle ambiguousness associated with DMs' evaluations in DM environments. As a consequence of these situations, FSs were extended to "Intuitionistic fuzzy sets" (IFSs) by Atanassov (1986) through associating membership degree (MD) along with non-membership degree (NMD) for the components of universal set, providing the constraint that the sum of these degrees must be less or equal to 1. There are various MCGDM techniques that are developed under IF environment Debnath et al. (2018), Kumar and Biswas (2019) and Sarkar and Biswas (2020) in the existing literature. After that, "Pythagorean fuzzy sets" (PFSs) were formulated by Yager (2013) and Yager (2014) which are considered as an efficient extension of IFSs. These sets were also characterized by MD and NMD providing the characteristic that the square sum must be less than 1. The formulation of PFSs clearly describes that these sets contain all the constraints of FSs and IFSs in order to handle uncertain data and information in DM applications. In this regard, various MCGDM frameworks, including Biswas and Sarkar (2018), Biswas and Sarkar (2019a), Chen (2017), Peng and Yang (2015), Rani et al. (2020) and Xue et al. (2018) have been developed utilizing PF numbers (PFNs) in accordance to deal with complicated problems involving multi criteria or objectives. Then, the PF "Superiority and Inferiority Ranking" (PF-SIR) technique was proposed by Peng and Yang (2015). After that, the "Fermatean fuzzy sets" (FFSs), as a novel extension of FSs, were presented by Senapati and Yager (2019a), the proposed new theory reduces the requisite of MD and

NMD. Note that, FFSs may accommodate the wider range of uncertain and vague information as compared to IFSs and PFSs. Considering the FF "linguistic term set" (LTS), some new "aggregation operators" (AOs) and a generalized formula to measure the similarity were developed to handle vague DM issues Liu et al. (2015). Even though, the FFS theory has been victoriously related to all types of extensive ranking applications, it still possesses certain drawbacks and restrictions. With the growing evolution of worldwide environment, a huge number of complex and uncertain knowledge impedes with the alternatives and evaluations of DMs' in MCGDM problems. It can be noted that DMs' can assign a set of index preference rankings because of their own background knowledge and the variety of indicators, instead of a single FFN. As a conclusion, it becomes a great challenge for DMs' to impart their evaluations between various potential FF components. To solve such kind of issues, the "hesitant fuzzy sets" (HFSs) Torra (2010) were utilized as these sets allow distinct MDs for a single criterion and greatly provide the hesitant evaluation activity of the DMs'. Motivating through this concept, the proposed article provides a discussion on a novel set, hesitant Fermatean fuzzy set (HFFS) Lai et al. (2022a, 2022b), combining the HFSs with FFs to bring the excellent characteristics of both theories. That is why, it is more useful for DMs' to utilize HFFSs in order to describe the uncertain data and knowledge in MCGDM problems.

There are various existing MCGDM techniques which were proposed by several researchers differ in the fundamental principles of accumulation. In case of ranking the alternatives, various scholars have developed a lot of effective ranking techniques, including TOPSIS Lai et al. (1994), VIKOR Opricovic and Tzeng (2007), DEMATEL Shieh et al. (2010), etc. On one hand, the above mentioned MCGDM techniques are completely based on the benefit value theory, thus these are complicated to contest the vagueness factor of DMs' in DM procedure. Furthermore, the decision experts are supposed to be fully rational, that leads to uncertain ranking orders. That is the reason that it is indiscreet for decision experts to only utilize an isolated kind of uncertainty to handle the MCGDM issues. In accordance to apprehend the effect of several vague components, it is obligatory to merge the characteristics of certain vague concepts to propose some more effective hybrid MCGDM techniques.

To this end, a mostly utilized MCGDM procedure, named as, "multi-objective optimization by ratio analysis" (MOORA) was proposed by Brauers and Zavadskas (2006). This technique contains two frameworks, namely, "ratio analysis approach and reference point approach" by which the alternatives are evaluated. Moreover, this technique was generalized by appending full multiplicative

form by Brauers and Zavadskas (2010) and developed a new method, namely, “MOORA plus full MULTIplicative form (MULTIMOORA) method”. In this method, a vector normalization technique was proposed to evaluate the choices. Therefore, MULTIMOORA Souzangarzadeh et al. (2020) has become a merged procedure along with three techniques (ratio analysis, reference point and full multiplicative form). First of all, MULTIMOORA under FSs was defined by Balezentis et al. (2012) in which the authors applied the dominance theory Brauers and Zavadskas (2011), Brauers and Zavadskas (2012) to achieve the final ranking order of alternatives from three distinct rankings included in MULTIMOORA. The MULTIMOORA technique has been extensively applied in various areas of DM through various fuzzy extensions, including Asante et al. (2020), Chen et al. (2020), Dai et al. (2020), Kutlu (2020) and Luo et al. (2019). After that, the MOORA method was generalized through PFSs by Perez-Dominguez et al. (2018) to solve a supplier selection application. In recent times, MULTIMOORA under PF theory was discussed by Li et al. (2020) and Liang et al. (2020), where the final ranking of alternatives has been obtained through dominance theory. Taking into consideration the effectiveness of linguistic terms adopted from artificial or real languages as compared to numerical values to represent evaluation degrees, the linguistic MULTIMOORA was proposed by Xian et al. (2020). Also, Akram et al. (2022a, b) proposed an integrated MULTIMOORA method with 2-tuple linguistic FFs and studied its application in urban quality of life selection problem. The authors also provided a variety of linguistic FF Hamy mean operators, including the linguistic FF Hamy mean operator, the linguistic FF dual Hamy mean operator, the linguistic FF weighted Hamy mean operator, and the linguistic FF weighted dual Hamy mean operator Akram et al. (2023a, b, c, d, e, f, g), Shahzadi et al. (2022) and Luqman and Shahzadi (2023b) among others. Gou et al. (2017) presented the double hierarchy HFLT set (DHHFLTS) and established the novel MULTIMOORA technique utilizing the proposed sets. A novel hybrid MCGDM technique under q -rung picture fuzzy information was developed by Rong and Yu (2024). Habib and Akram (2024) optimized the travelling salesman problem through tabu search algorithm along with PF information. Further work related to MCGDM can be seen in Akram et al. (2024a, 2024b), Pathak et al. (2024), Liu et al. (2024), Yang et al. (2022), Akram and Zahid (2023), etc. Further recent studies on MCGDM methods under fuzzy and hesitant information can be followed from Akram et al. (2021, 2022a; b), Luqman et al. (2021) and Luqman and Shahzadi (2023a). Despite that, there are some boundaries of existing MULTIMOORA process which are discussed as follows:

1. When the final ranking order of alternatives is obtained through dominance theory in MULTIMOORA, there are three orders of ranking which are utilized, however the corresponding values of score functions are not taken into consideration, and hence the problem of annular judging may appear.
2. Moreover, the enormous implementations of dominance theory lead to the multiple comparisons that may cause to same ranking. Further drawbacks of dominance theory may be observed in various applications, like transitiveness, total dominance, absolute dominance, and so on.
3. Another limitation of existing FF-MULTIMOORA technique is that no weight values of three approaches has been considered to compare the alternatives in order to achieve the final ranking order, which can produce arbitrary conclusions.
4. It can be noted clearly that distinct criteria weights can effect the final ranking order of alternatives to solve MCGDM issues. As a result, the level of certainty in the ranking of alternatives in MCGDM applications increases if weight values are allocated to each criterion in accordance with their relative relevance. Although, there is currently no approach for determining weight, specifically one based on the FF entropy measure, it is used to find unknown weight values for criteria using the MULTIMOORA methods that are now in use.

Taking into account the above described limitations and shortcomings of current methods, the proposed work aims to conquer the beyond described drawbacks and develops HFF-MULTIMOORA framework as an extend-ed and effective DM method to handle the MCGDM applications considering the fully known weights of criteria, where the values of evaluations are described by linguistic terms of a FF weighting scale. The novelty and contribution of the developed framework can be summed up as:

1. A novel extended distance measure is developed which is build on the amalgamation of Hamming distance and Hausdorff metric in order to appraise the distinctness among HFFs, and to compute the distance measures between the reference point and the possible choices in MULTIMOORA method.
2. The proposed work introduces the computation of unspecified weight values of criteria utilizing HFF entropy weight framework which is based on novel distance measures to solve MCGDM problems.
3. The limitations and drawbacks have been overcome through developing MULTIMOORA under HFF environment in order to deal with crisp, HF, HIF or HPF knowledge to solve HFF MCGDM issues.

4. In spite of utilizing the dominance theory, that is less suitable in case of very large number of criteria, an aggregated framework is developed to determine the final ranking order of alternatives through MULTIMOORA. This method can consider distinct weight values for each of the three techniques of the developed HFF-MULTIMOORA.
5. The developed HFF-MULTIMOORA approach is afterwards used as a successful MCGDM model in the quantitative strategic planning matrix (QSPM) procedure to build strategy, the first step of strategic management. In this way, a tool for HFF QSPM with HFF-MULTIMOORA is also suggested.

Briefly, we conclude that the evaluation of information values through HFFSs permits a more comprehensive analysis of the assessment process. Thus, the generalization of MULTIMOORA method along with HFFSs has made an academic contribution to applications that are relevant to social and economical evaluations.

Following is a description of the remainder of the article: A brief review of some fundamental ideas that are relevant to FFSs and the traditional MULTIMOORA technique is provided in Sect. 2. Certain HFF distance measures are discussed in Sect. 3. The HFF-MULTIMOORA's methodological development is presented in Sect. 4. The approach for HFF QSPM using the HFF-MULTIMOORA method is provided in Sect. 5. The same section consists the summary of proposed framework. The HFF-MULTIMOORA techniques is combined with HFF QSPM in the same Section. An example is provided in Sect. 6. In Sect. 7, comparative analysis are carried out. Conclusions are shown in Sect. 8.

2 Basic concepts

Definition 1 Senapati and Yager (2019a) A *Fermatean fuzzy set* (FFS) in the universal set \mathcal{X} is described as an object Q given by $Q = \{(x, T_Q(x), F_Q(x)) | x \in \mathcal{X}\}$, where the mappings $T_Q : \mathcal{X} \rightarrow [0, 1]$ and $F_Q : \mathcal{X} \rightarrow [0, 1]$ represent the MD and NMD of every component with $0 \leq T_Q^3(x) + F_Q^3(x) \leq 1$.

Moreover, $\pi_Q(x) = \sqrt[3]{1 - T_Q^3(x) - F_Q^3(x)}$ is defined as the FF index or hesitancy degree of x in Q .

Inspired by the notion of HPFSs Liang and Xu (2017) and Lia et al. (2022) extended the concept of FFSs to HFFSs.

2.1 Hesitant Fermatean fuzzy sets

Definition 2 Lai et al. (2022a, b) Suppose \mathcal{X} be a finite collection of components. An *hesitant Fermatean fuzzy set* (HFFS) \mathcal{H}_F on \mathcal{X} is described as

$$H_F = \{(u, t_{H_F}(u), f_{H_F}(u)) | u \in \mathcal{X}\},$$

$t_{H_F}(u)$ and $f_{H_F}(u)$ are regarded as two separate classes that correspond to certain values in the range of $[0, 1]$. Note that, $t_{H_F}(u)$ represents the possible Fermatean MDs and $f_{H_F}(u)$ describes the possible Fermatean NMDs of the component u to the set H_F , fulfilling the constraint

$$0 \leq g, h \leq 1, 0 \leq (g^+)^3 + (h^+)^3 \leq 1,$$

for every $g \in t_{H_F}(u), h \in f_{H_F}(u)$, and with $g^+ = \max_{g \in t_{H_F}(u)} \{g\}$ and $h^+ = \max_{h \in f_{H_F}(u)} \{h\}$.

Note that, $t_{H_F}(u)$ and $f_{H_F}(u)$ are known as *hesitant Fermatean fuzzy elements* (HFFEs). Their corresponding cardinalities are represented as $|t_{H_F}(u)|, |f_{H_F}(u)|$. Two distinct kinds of uncertainties may be resolved through HFFSs and a much wider and flexible framework is proposed to correspond values to each component in the universal set.

Definition 3 Let $H_E = (t_{H_F}, f_{H_F})$ be a HFFE. The maximum value and minimum value of each distinct element are formulated as

$$g^+ = \max_{g \in t_{H_F}} \{g\}, g^- = \min_{g \in t_{H_F}} \{g\}, \quad (1)$$

$$h^+ = \max_{h \in f_{H_F}} \{h\}, h^- = \min_{h \in f_{H_F}} \{h\}, \quad (2)$$

g^+, g^- are the greatest value and lowest value of component t_{H_F} and h^+, h^- are the greatest value and lowest value of component f_{H_F} .

Some operations on HFFEs are defined as follows:

Definition 4 Let $H_E = (t_{H_F}, f_{H_F}), H_{E_1} = (t_{H_F}^1, f_{H_F}^1), H_{E_2} = (t_{H_F}^2, f_{H_F}^2)$ be HPFEs. Then, the definitions of sum, product, and multiplication by a scalar on these elements are as follows:

- $H_{E_1} \oplus H_{E_2} = \bigcup_{g_1 \in t_{H_F}^1, h_1 \in f_{H_F}^1, g_2 \in t_{H_F}^2, h_2 \in f_{H_F}^2} \{\{\sqrt{(g_1)^3 + (g_2)^3 - (g_1)^3(g_2)^3}, \{h_1 h_2\}\},$
- $H_{E_1} \otimes H_{E_2} = \bigcup_{g_1 \in t_{H_F}^1, h_1 \in f_{H_F}^1, g_2 \in t_{H_F}^2, h_2 \in f_{H_F}^2} \{\{g_1 g_2\}, \{\sqrt{(h_1)^3 + (h_2)^3 - (h_1)^3(h_2)^3}\},$
- $\alpha H_E = \bigcup_{g \in t_{H_F}, h \in f_{H_F}} \{\{\sqrt{1 - (1 - g^3)^\alpha}, \{h^\alpha\}\},$
where $\alpha \in \mathbb{R}$ and $\alpha \geq 0$.

Definition 5 Let $H_E = (t_{H_F}, f_{H_F})$ be an HFFE. The *indeterminacy degree* of H_E is defined as,

$$I_{H_E} = \sqrt[3]{1 - \left(\frac{1}{|t_{H_F}|} \sum_{g \in t_{H_F}} g^3 + \frac{1}{|f_{H_F}|} \sum_{h \in t_{H_F}} h^3 \right)}. \tag{3}$$

The following numbers make up the score and accuracy functions in HFFNs:

Definition 6 Let $H_E = (t_{H_F}, f_{H_F})$ be a HFFE. Then the score function of H_E is given as

$$s(H_E) = \frac{1}{2} \left[\frac{1}{|t_{H_F}|} \sum_{g \in t_{H_F}} g^3 + \frac{1}{|f_{H_F}|} \sum_{h \in t_{H_F}} h^3 \right. \tag{4}$$

$$\left. + \frac{\frac{1}{|t_{H_F}|} \sum_{g \in t_{H_F}} g^3 + \frac{1}{|f_{H_F}|} \sum_{h \in t_{H_F}} h^3}{1 + I_{H_E}^3} \right]. \tag{5}$$

3 Hesitant Fermatean fuzzy distance measures

For the purpose of comparing two elements, the HFFEs are normalized. This normalization is accomplished as follows:

Definition 7 Let $H_E = (t_{H_F}, f_{H_F})$ be an HFFE, then its further degrees of truth and falsehood are determined by $\bar{g} = \gamma g^+ + (1 - \gamma)g^-$ and $\bar{h} = \gamma h^+ + (1 - \gamma)h^-$, respectively, where the greatest value and lowest value of elements of t_{H_F} are represented by g^+, g^- . The greatest value and lowest value of components of f_{H_F} are h^+ and h^- . The decision expert's parameter, γ , ($0 \leq \gamma \leq 1$), here corresponds to risk predilections. There exist three specific instances of the decision expert's consideration, to wit:

1. The decision expert, who is viewed as favorable, can add the maximum degree of truth (g^+) and maximum degree of falsity (h^+) when $\gamma = 1$.
2. When $\gamma = 0.5$, the assessment expert, who is viewed as unbiased, can add the MD of 0.5 ($g^+ + g^-$) and NMD of 0.5 ($h^+ + h^-$).
3. When $\gamma = 0$, the evaluator, who is viewed as negative, can add the minimum MD g^- and minimum NMD h^- .

Now, many definitions of distance measures between normalized HFFEs are possible. To this purpose, let $H_{E_1} = (t_{H_F}^1, f_{H_F}^1)$ and $H_{E_2} = (t_{H_F}^2, f_{H_F}^2)$ be two HFFEs. Next, three methods for determining their distance from one another are produced by the definitions that follow:

Definition 8 The HFF Hamming distance among H_{E_1} and H_{E_2} is given as

$$d_h(H_{E_1}, H_{E_2}) = \frac{1}{3} \left(\frac{1}{|t_{H_F}^1|} \sum_{l=1}^{|t_{H_F}^1|} |(g_1^{\varepsilon(l)})^3 - (g_2^{\varepsilon(l)})^3| \right. \tag{6}$$

$$\left. + \frac{1}{|f_{H_F}^1|} \sum_{l=1}^{|f_{H_F}^1|} |(h_1^{\varepsilon(l)})^3 - (h_2^{\varepsilon(l)})^3| \right. \tag{7}$$

$$\left. + |(\pi_1)^2 - (\pi_2)^3| \right),$$

where $|t_{H_F}^1| = |t_{H_F}^2| = |t_{H_F}|$ and $|f_{H_F}^1| = |f_{H_F}^2| = |f_{H_F}|$. $g_1^{\varepsilon(l)}, g_2^{\varepsilon(l)}, h_1^{\varepsilon(l)}, h_2^{\varepsilon(l)}$ are the l th greatest values of MDs and NMDs of H_{E_1} and H_{E_2} , respectively. Here, π_1 and π_2 are the IDs (indeterminacy degrees) of H_{E_1} and H_{E_2} , respectively, which are computed through the formulas

$$\pi_1 = \sqrt[3]{1 - \left(\frac{1}{|t_{H_F}^1|} \sum_{g \in t_{H_F}^1} g^3 + \frac{1}{|f_{H_F}^1|} \sum_{h \in f_{H_F}^1} h^3 \right)}, \tag{7}$$

$$\pi_2 = \sqrt[3]{1 - \left(\frac{1}{|t_{H_F}^2|} \sum_{g \in t_{H_F}^2} g^3 + \frac{1}{|f_{H_F}^2|} \sum_{h \in f_{H_F}^2} h^3 \right)}. \tag{8}$$

Definition 9 The HFF Euclidean distance among H_{E_1} and H_{E_2} is given as

$$d_e(H_{E_1}, H_{E_2}) = \left(\frac{1}{2} \left(\frac{1}{|t_{H_F}^1|} \sum_{l=1}^{|t_{H_F}^1|} ((g_1^{\varepsilon(l)})^3 - (g_2^{\varepsilon(l)})^3)^3 \right. \tag{9}$$

$$\left. + \frac{1}{|f_{H_F}^1|} \sum_{l=1}^{|f_{H_F}^1|} ((h_1^{\varepsilon(l)})^3 - (h_2^{\varepsilon(l)})^3)^2 \right. \tag{10}$$

$$\left. + ((\pi_1)^3 - (\pi_2)^3)^3 \right)^{1/3}.$$

Definition 10 The HFF generalized distance between H_{E_1} and H_{E_2} is given as

$$d_G(H_{E_1}, H_{E_2}) = \left(\frac{1}{2} \left(\frac{1}{|t_{H_F}^1|} \sum_{l=1}^{|t_{H_F}^1|} |(g_1^{\varepsilon(l)})^3 - (g_2^{\varepsilon(l)})^3|^n \right. \tag{10}$$

$$\left. + \frac{1}{|f_{H_F}^1|} \sum_{l=1}^{|f_{H_F}^1|} |(h_1^{\varepsilon(l)})^3 - (h_2^{\varepsilon(l)})^3|^n \right. \tag{11}$$

$$\left. + |(\pi_1)^3 - (\pi_2)^3|^n \right)^{1/n},$$

where the constant $n > 0$ refers. Changes to n 's values, relations between $d_h(H_{E_1}, H_{E_2}), d_e(H_{E_1}, H_{E_2})$ and $d_G(H_{E_1}, H_{E_2})^n$ can be deduced.

In this subsection, different types of aggregation operators for HFFSs are defined. Suppose that, w_j be the weight vector of H_E such that $w_j > 0, \sum_{j=1}^n w_j = 1$. Then, AOs are defined as follows.

Definition 11 An HFF weighted averaging operator (HFFWAO) is defined as a mapping HFFWAO: $\Delta^n \rightarrow \Delta$ and is given as

$$\text{HFFWAO}(H_{E_1}, H_{E_2}, H_{E_3}, \dots, H_{E_n}) = \bigoplus_{j=1}^n w_j H_{E_j}. \quad (11)$$

Definition 12 An HFF ordered weighted averaging operator (HFFOWAO) is defined as a mapping HFFOWAO: $\Delta^n \rightarrow \Delta$ and is given as

$$\text{HFFOWAO}(H_{E_1}, H_{E_2}, H_{E_3}, \dots, H_{E_n}) = \bigoplus_{j=1}^n w_j H_{E_{\alpha(j)}}, \quad (12)$$

where $(\alpha(1), \alpha(2), \alpha(3), \dots, \alpha(n))$ is taken as the permutation of $(1, 2, 3, \dots, n)$, such that $H_{E_{\alpha(j-1)}} > H_{E_{\alpha(j)}}$, for all $j = 1, 2, 3, \dots, n$.

Definition 13 An HFF weighted geometric operator (HFFWGO) is defined as a mapping HFFWGO: $\Delta^n \rightarrow \Delta$ and is given as

$$\text{HFFWGO}(H_{E_1}, H_{E_2}, H_{E_3}, \dots, H_{E_n}) = \bigotimes_{j=1}^n H_{E_{\alpha(j)}}^{w_j}. \quad (13)$$

Definition 14 An HFF ordered weighted geometric operator (HFFOWGO) is defined as a mapping HFFOWGO: $\Delta^n \rightarrow \Delta$ and is given as

$$\text{HFFOWGO}(H_{E_1}, H_{E_2}, H_{E_3}, \dots, H_{E_n}) = \bigotimes_{j=1}^n H_{E_{\alpha(j)}}^{w_j}, \quad (14)$$

where $(\alpha(1), \alpha(2), \alpha(3), \dots, \alpha(n))$ is taken as the permutation of $(1, 2, 3, \dots, n)$, such that $H_{E_{\alpha(j-1)}} \geq H_{E_{\alpha(j)}}$, for all $j = 1, 2, 3, \dots, n$.

4 Methodological development of hesitant Fermatean fuzzy MULTIMOORA method

We first explain a method to formulate strategy in strategic management. Then, a novel generalized distance measure is formulated to evaluate the difference between two HFFEs. Finally, generalization of MULTIMOORA technique is developed using HFFEs to cope with vagueness and hesitance in MCGDM problems.

4.1 Quantitative strategic planning matrix

A systematic procedure to generate major decisions is called the strategic management. This process maximize the chances of the organizations to achieve their business

objectives. The effective decisions can be constructed through this process by consolidating the qualitative and quantitative information within various areas of uncertainty. There are three distinct stages of this procedure:

1. Strategy formulation,
2. Strategy implementation,
3. Strategy evaluation.

The strategy management is the most powerful and leading stage of strategy management. In this stage, strategies are formed, evaluated, and nominated in a framework.

Both an internal and external factor evaluation matrix (IFEM) are necessary for an effective process of strategy formulation. Strengths, Weaknesses, Opportunities, and Threat (SWOT) Analysis is the next step. In order to evaluate the strategies, the quantitative strategic planning matrix (QSPM) technique is used. EFEM and IFEM are taken into account as input in the QSPM process, and the SWOT analysis is used to produce workable solutions. The QSPM technique is used in the last stage of the strategy formulation process to rank the strategies by computing their total appealing scores. To choose the most desirable strategy, the QSPM procedure needs a decision-making technique.

4.2 Hesitant Fermatean fuzzy distance measure

In this subsection, a novel generalized dm (distance measure), which is constructed on the basis of the notion of weighted Hamming distance and Hausdorff metric, has been formulated in order to compute the dissimilarity among two distinct HFFSs as follows:

Let $G = \{G_1, G_2, G_3, \dots, G_n\}$ and $H = \{H_1, H_2, H_3, \dots, H_n\}$ be two families of HFFSs and $\delta \geq 1 (\in \mathbb{R})$. The *generalized weighted distance* is given as:

$$\begin{aligned} d_h(G, H) = & \frac{1}{3n} \left(\sum_{j=1}^n \lambda_j \left[\frac{1}{|t_{H_j}|} \sum_{l=1}^{|t_{H_j}|} |(g_{G_j}^{(l)})^3 - (g_{H_j}^{(l)})^3|^\delta \right. \right. \\ & + \frac{1}{|f_{H_j}|} \sum_{l=1}^{|f_{H_j}|} |(h_{G_j}^{(l)})^3 - (h_{H_j}^{(l)})^3|^\delta \\ & + |(\pi_{G_j})^3 - (\pi_{H_j})^3|^\delta \\ & + \left. \left\{ \max \left(\frac{1}{|t_{H_j}|} \sum_{l=1}^{|t_{H_j}|} |(g_{G_j}^{(l)})^3 - (g_{H_j}^{(l)})^3|^\delta, \right. \right. \right. \\ & \left. \left. \frac{1}{|f_{H_j}|} \sum_{l=1}^{|f_{H_j}|} |(h_{G_j}^{(l)})^3 - (h_{H_j}^{(l)})^3|^\delta, \right. \right. \\ & \left. \left. |(\pi_{G_j})^3 - (\pi_{H_j})^3|^\delta \right\} \right] \right), \end{aligned} \quad (15)$$

where $|t_{G_j}| = |t_{H_j}|$ and $|f_{G_j}| = |f_{H_j}|$ and λ_j is the weight that is assigned to G and H such that $\sum_{j=1}^n \lambda_j = 1$ and $g_{H_{F_j}}^{(l)}, h_{H_{F_j}}^{(l)}$ are the l th greatest values of MDs and NMDs of HFFEs H_{F_j} , respectively.

Note that, the dm that is defined in Eq. 15 fulfills the given characteristics:

1. $0 \leq d_h(G, H) \leq 1$,
2. $d_h(G, H) = 0$ if and only if $G = H$,
3. $d_h(G, H) = d_h(H, G)$.

4.3 Entropy measure of HFFSs

Entropy plays an efficient role in information technology while estimating the information which is contained in a specific note. In theory of information, entropy may be considered as an appraise of vagueness that is imposed within a description. Furthermore, dissimilarities among distinct sets of data can be explored using entropy. As there exists a certain amount of data regarding the collection of attributes in a decision matrix, criteria weights can be evaluated through entropy measure in state of affairs of MCGDM. While the information about a decision matrix (DM) is given and well known, the weights can be evaluated through the entropy method. The entropy measure of HFFSs, H_F , is developed to quantify the uncertainty of H_F in its space of preach. Using the generalized distance measure as given in Eq. 15, an HFF entropy measure of $H_F = \{H_{P_1}, H_{P_2}, H_{P_3}, \dots, H_{P_n}\}$ is given as follows:

$$\epsilon(H_F) = \frac{1}{n} \sum_{j=1}^n \left[\left\{ 1 - \frac{1}{|t_{H_{F_j}}|} \sum_{l=1}^{|t_{H_{F_j}}|} |(g_{H_{F_j}}^{(l)})^3 - (h_{H_{F_j}}^{(l)})^3| \right\} \right. \\ \left. \frac{3 - \sum_{l=1}^{|t_{H_{F_j}}|} |(g_{H_{F_j}}^{(l)})^3 - (h_{H_{F_j}}^{(l)})^3|}{3} \right], \tag{16}$$

where $|t_{H_{F_j}}| = |f_{H_{F_j}}|$ and $g_{H_{F_j}}^{(l)}, h_{H_{F_j}}^{(l)}$ are the l th maximum truth degrees and falsity degrees for HFFEs H_{F_j} , respectively.

5 Hesitant Fermatean fuzzy MULTIMOORA technique for MCGDM

In the following portion, the extended idea of HFFSs is combined with MULTIMOORA technique in order to formulate an innovative framework, named as hesitant Fermatean fuzzy MULTIMOORA (HFF MULTIMOORA) technique, to cope with a MCGDM incident through HFF

environment. The evaluation of k decision-makers (DMs) is expressed regarding the m alternatives, $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_m$, that satisfy the set of n criteria, $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \dots, \mathcal{C}_n$. The reciprocation matrices specified by the DMs which are evaluated through LTs (linguistic terms) are given in Table 1 as follows:, where “ $r_{ij}^{(k)}$ ”, $k = 1, 2, 3, \dots, \kappa$, $i = 1, 2, 3, \dots, m$, $j = 1, 2, 3, \dots, n$ expresses the terms of linguistic variables of a six level HFF weighting ranking as given in Table 3. Let $w = [\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \dots, \omega^{(k)}]^T$ be the weight vector of k DMs. The MCGDM application is resolved considering the subsequent procedure.

5.1 Construction of response matrices

The HFF response matrices are constructed using the six level HFF weighting scale as given in Table 3 as follows:

$$H^{(k)} = \begin{pmatrix} h_{11}^{(k)} & h_{12}^{(k)} & h_{13}^{(k)} & \dots & h_{1n}^{(k)} \\ h_{21}^{(k)} & h_{22}^{(k)} & h_{23}^{(k)} & \dots & h_{2n}^{(k)} \\ h_{31}^{(k)} & h_{32}^{(k)} & h_{33}^{(k)} & \dots & h_{3n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{m1}^{(k)} & h_{m2}^{(k)} & h_{m3}^{(k)} & \dots & h_{mn}^{(k)} \end{pmatrix},$$

where $h_{ij}^{(k)} = (t_{h_{ij}}^{(k)}, f_{h_{ij}}^{(k)})$ are HFFEs and $k = 1, 2, 3, \dots, \kappa$, $i = 1, 2, 3, \dots, m$, $j = 1, 2, 3, \dots, n$.

5.2 Normalization of response matrices

The response matrices are normalized as follows:

$$N^{(k)} = \begin{pmatrix} n_{11}^{(k)} & n_{12}^{(k)} & n_{13}^{(k)} & \dots & n_{1n}^{(k)} \\ n_{21}^{(k)} & n_{22}^{(k)} & n_{23}^{(k)} & \dots & n_{2n}^{(k)} \\ n_{31}^{(k)} & n_{32}^{(k)} & n_{33}^{(k)} & \dots & n_{3n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{m1}^{(k)} & n_{m2}^{(k)} & n_{m3}^{(k)} & \dots & n_{mn}^{(k)} \end{pmatrix},$$

where

Table 1 The response matrices R^k

$\mathcal{A}_m/\mathcal{C}_n$	\mathcal{C}_1	\mathcal{C}_2	\mathcal{C}_3	\dots	\mathcal{C}_n
\mathcal{A}_1	$r_{11}^{(k)}$	$r_{12}^{(k)}$	$r_{13}^{(k)}$	\dots	$r_{1n}^{(k)}$
\mathcal{A}_2	$r_{21}^{(k)}$	$r_{22}^{(k)}$	$r_{23}^{(k)}$	\dots	$r_{2n}^{(k)}$
\mathcal{A}_3	$r_{31}^{(k)}$	$r_{32}^{(k)}$	$r_{33}^{(k)}$	\dots	$r_{3n}^{(k)}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
\mathcal{A}_m	$r_{m1}^{(k)}$	$r_{m2}^{(k)}$	$r_{m3}^{(k)}$	\dots	$r_{mn}^{(k)}$

Table 2 The HFF weighting scale

Linguistic term	HFF scale
Very poor (vp)	{{0, 0, 0}, {1, 1, 1}}
Poor (p)	{{0.25, 0.20, 0.15}, {0.85, 0.80, 0.75}}
Fair (f)	{{0.45, 0.40, 0.35}, {0.65, 0.60, 0.55}}
Good (g)	{{0.65, 0.60, 0.55}, {0.45, 0.40, 0.35}}
Very good (vg)	{{0.85, 0.80, 0.75}, {0.25, 0.20, 0.15}}
Excellent (e)	{{1, 1, 1}, {0, 0, 0}}

$$n_{ij}^{(k)} = \begin{cases} h_{ij}^{(k)}, & C_j \in \mathcal{C}^b \\ (h_{ij}^{(k)})^c, & C_j \in \mathcal{C}^c. \end{cases} \tag{17}$$

Note that, $(h_{ij}^{(k)})^c$ is the compliment of $h_{ij}^{(k)}$, that is, $(h_{ij}^{(k)})^c = (f_{h_{ij}^{(k)}}^{(k)}, t_{h_{ij}^{(k)}}^{(k)})$, for $k = 1, 2, 3, \dots, \kappa$, $i = 1, 2, 3, \dots, m, j = 1, 2, 3, \dots, n''$.

5.3 Aggregation of DMs' evaluations

The evaluation of DMs' has been aggregated through HFFOWA operator (or any operator can be used as given in Eqs. 11–14) as follows:

$$n_{ij}^\star = \bigoplus_{k=1}^\kappa w^{(k)} n_{ij}^{(k)} = \bigcup_{\substack{g_{ij}^{(k)} \in t_{n_{ij}^{(k)}}^{(k)}, \\ h_{ij}^{(k)} \in f_{n_{ij}^{(k)}}^{(k)}}} \left\{ \left\{ \sqrt[3]{1 - \prod_{k=1}^\kappa (1 - (g_{ij}^{(k)})^3)^{\omega^{(k)}}} \right\}, \left\{ \prod_{k=1}^\kappa ((h_{ij}^{(k)})^3)^{\omega^{(k)}} \right\} \right\}, \tag{18}$$

Table 3 The tile company's QSPM

SWOT	Criteria	Weight	Attractive value of strategies						
			\mathcal{X}_1	\mathcal{X}_2	\mathcal{X}_3	\mathcal{X}_4	\mathcal{X}_5	\mathcal{X}_6	
Strength	C_1	Consistent and high product quality	0.05	3	4	2	4	2	1
	C_2	Reputation for innovative design	0.07	4	2	3	1	5	1
	C_3	Experience in exporting to other countries	0.12	2	4	4	3	1	1
Weakness	C_4	Insufficient cash flow	0.08	1	2	4	4	2	1
	C_5	Problems with on-time delivery	0.07	4	4	2	2	4	4
	C_6	Relatively high costs of labour	0.15	4	3	2	2	2	1
Opportunity	C_7	Demand increasing by 10%	0.09	2	4	17	2	4	2
	C_8	Luxurious products	0.11	4	4	1	1	1	4
	C_9	Easier access to markets in accession	0.03	4	1	3	4	4	4
Threat	C_{10}	Increased competition of producers	0.12	3	4	4	1	1	3
	C_{11}	From countries with lower costs of labour	0.07	4	4	2	4	1	1
	C_{12}	Recession in many countries markets	0.04	3	4	3	3	1	5

where $n_{ij}^{(k)}$ attains the k th highest score value among $n_{ij}^{(k)}$, $k = 1, 2, 3, \dots, \kappa''$, $w^{(k)}$ is the value of weight of k th DM.

5.4 Deduction of criteria weights

If the weight values of criteria are not known, HFF entropy measure method, as developed in Eq. 16, is utilized to obtain the criteria weights given as follows:

$$\xi_j = \frac{1 - \epsilon_{M_j}}{n - \sum_{j=1}^n \epsilon_{M_j}}, \tag{19}$$

where $\epsilon_{M_j} = \epsilon\{n_{1j}^\star, n_{2j}^\star, n_{3j}^\star, \dots, n_{mj}^\star\}$, $j = 1, 2, 3, \dots, n$, and $\epsilon(\cdot)$ represents the HFF entropy measures as given in Eq. 16.

5.5 Formulation of response matrix considering weights

The HFF weighted response matrix is constructed as

$$W^\omega = (N_{ij}^\omega)_{m \times n}, \text{ where } N_{ij}^\omega = \xi_j n_{ij}^\star = \bigcup_{g_{ij} \in t_{n_{ij}^\star}, h_{ij} \in f_{n_{ij}^\star}} \left\{ \left\{ \sqrt{1 - (1 - g_{ij}^2)^{\xi_j}} \right\}, \left\{ (h_{ij}^2)^{\xi_j} \right\} \right\}. \tag{20}$$

Note that, in case of known attractive scores then HFF weighted response matrix is computed as:

$$N_{ij}^\omega = A_{S_j}^w n_{ij}^\star, \tag{21}$$

where $A_{S_{ij}}^w$ is the attractive weighted value of i th substitute with respect to j th criteria.

5.6 HFF ratio analysis

Additive utility function is considered as a basis for ratio analysis. Here, HFFWAO, as given in Eq. 11, is utilized. The evaluation intimation or assessment values for every substitute is obtained by aggregating the values of criteria that are calculated in Eqs. 20–21. The aggregation of criteria values is done as:

$$\begin{aligned}
 U_i^{EI} &= \bigoplus_{j=1}^n N_{ij}^{\omega} = \bigoplus_{j=1}^n (g_{N_{ij}^{\omega}}, h_{N_{ij}^{\omega}}) \\
 &= \bigcup_{g_{ij} \in t_{N_{ij}^{\omega}}, h_{ij} \in f_{N_{ij}^{\omega}}} \left\{ \left\{ \sqrt[3]{1 - \prod_{j=1}^n (1 - g_{ij}^3)} \right\}, \right. \\
 &\quad \left. \left\{ \prod_{j=1}^n (h_{ij}^3) \right\} \right\}.
 \end{aligned}
 \tag{22}$$

The evaluation index of every alternative is normalized as:

$$N_i^{EI} = \frac{s(U_i^{EI})}{\max_i \{s(U_i^{EI})\}},
 \tag{23}$$

for $i = 1, 2, 3, \dots, m$. The larger value of N_i^{EI} corresponds to the higher rank.

5.7 HFF reference point approach

In the given step, the maximal objective recommendation point is identified by performing the HFF reference point technique and then the distances of every alternative are computed from that reference point. The greatest value of objective reference point is computed as follows:

$$R_{p_j} = \{N_{ij}^{\omega} : \max [s(N_{ij}^{\omega})]\}.
 \tag{24}$$

Then, the index of preference of every distinct choice is computed by determining the greatest value of the distances of reference point from the possible choice. Here, the distance measure as developed in Eq. 15 is utilized and the preference index is obtained as follows:

$$P_{ind}^i = \max_i \{d_h(N_{ij}^{\omega}, R_{p_j})\}, \quad i = 1, 2, 3, \dots, m.
 \tag{25}$$

The normalization of preference index of each alternative is performed as:

$$N_i^{P_{ind}} = \frac{\min_i \{P_{ind}^i\}}{P_{ind}^i}.
 \tag{26}$$

The maximum value of $N_i^{P_{ind}}$ corresponds to the higher rank.

5.8 HFF full multiplicative form

In this step, the HFF full multiplicative function is derived. To obtain the degree of utility, the HFFWGO as given in Eq. 13 has been utilized. The grade of usefulness of every distinct option is calculated as:

$$\begin{aligned}
 U_i &= \bigotimes_{j=1}^n N_{ij}^{\omega} = \bigcup_{g_{ij} \in t_{N_{ij}^{\omega}}, h_{ij} \in f_{N_{ij}^{\omega}}} \left\{ \left\{ \prod_{j=1}^n (g_{ij}) \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[3]{1 - \prod_{j=1}^n (1 - h_{ij}^3)} \right\} \right\}.
 \end{aligned}
 \tag{27}$$

The obtained grade of usefulness of every possible option has been regularized as:

$$U_i^{F_m} = \begin{cases} \frac{\max_i \{s(U_i)\}}{s(U_i)}, & \text{if } s(U_i) < 0, \\ \frac{s(U_i)}{\max_i \{s(U_i)\}}, & \text{otherwise.} \end{cases}
 \tag{28}$$

The higher rank is represented by the larger value of $U_i^{F_m}$.

5.9 Conclusive order of alternatives

The evaluated value S_i of every choice is obtained by aggregating N_i^{EI} , $N_i^{P_{ind}}$, and $U_i^{F_m}$ as:

$$S_i = \eta N_i^{EI} + \theta N_i^{P_{ind}} + (1 - \eta - \theta) U_i^{F_m},
 \tag{29}$$

where the degrees of importance are expressed by η , θ , and $(1 - \eta - \theta)$ that are allocated to HFF perusal corresponding to ratio, HFF perspective with respect to reference point, and HFF full multiplicative type, respectively. Note that, $\eta > 0$, $\theta > 0$, and $\eta + \theta \leq 1$. The final score having the largest value expresses the best alternative.

The implications of all steps that are involved in the developed technique are summarized in an as follows:

- Step 1. The HFF weighting scale listed in Table 1 should be used to construct the HFF response matrices.
- Step 2. Normalize the HFF response matrices based on the types of criteria using Eq. 17.
- Step 3. Aggregate the DMs' evaluation by applying the HFFOWA operator as defined in Eq. 18.
- Step 4. Calculate the unknown weights of criteria using the HFF entropy measures as developed in Eq. 19.
- Step 5. Use Eq. 20 or Eq. 21 to obtain the weighted HFF response matrix.
- Step 6. Use Eq. 23 to determine the evaluation indicator or assessment grade of every choice which is based on the HFF ratio analysis.

- Step 7. Utilizing Eq. 26, calculate the preference index for each alternative using the HFF reference point approach.
- Step 8. Eq. 28, which is based on the HFF complete multiplicative form, can be used to calculate the level of utility of each alternative.
- Step 9. Using Eq. 29, the evaluation degree, weightage index, as well as the degree of usefulness has been merged to create the resulting evaluation for each possible choice.
- Step 10. The ranking order of considered choices is determined by the descending values of resulting values of score functions.

QSPM is broadly utilized to rank feasible techniques (alternatives) as a crucial tool of strategy components. Because the QSPM is totally dependent on the data and knowledge that are allocated through the evaluators in addition to the DM techniques they have chosen, it requires careful judgement on the part of humans (experts) in order for it to be successful. In crisp QSPM, ratings under crisp conditions are used to evaluate final ratings based entirely on DM's assessment degrees. While general appealing grades are computed in fuzzy QSPM using fuzzy integers and their corresponding MDs. There is, therefore, no chance for decision experts to consider the non-preference value in the procedure of appraising scheme while considering the fuzzy QSPM. Hence, the utilization of fuzzy numbers or crisp numbers in decision-making methods can effect the accuracy of final results. That is because the finalized attractiveness ratings are possibly too strict to distinguish between different methodologies. An effective tool using FFNs in QSPM paired with an HFF-MULTIMOORA-based technique is proposed in the process of formulating strategies to address this problem. The following is a narration of the developed HFF QSPM algorithms:

- Step 1. To determine an organization's goal and vision. Basically, a vision statement is considered as the declaration about which an organization wishes to adopt or become.
- Step 2. To understand the relationship between the organization's internal and external components. Note that the external components are those which affect an organization economically, socially, politically, or technologically. Whereas, the internal factors are those which come within or under the control of a company, for example, organizational structure, human resources, corporate culture, etc.
- Step 3. SWOT analysis should be performed before developing the organization's strategies. The SWOT analysis is considered as a useful

technique that can help to evaluate that what a company does best now and to adopt a successful strategy for future.

- Step 4. To extract DM's thoughts on the linguistic value-based techniques. Because, certain DMs prefer social, narrative, or qualitative data, while the other prefer firm, specific, numerical or qualitative information.
- Step 5. To build an HFF evaluation matrix through altering the LTs in accordance with their corresponding FFNs using an HFF weighting scale.
- Step 6. To calculate the unknown values of the criterion using the HFF entropy weight approach. The entropy weight technique is commonly adopted to measure dispersion of values in DM procedures. The greater degree of dispersion leads to the greater differentiation and more knowledge can be derived.
- Step 7. To assess the ultimate preference degrees of the techniques through HFF-MULTIMOORA. The considered strategies are then ranked according to their preference degrees.
- Step 8. To determine the order in which the methods should be ranked in relation to ultimate attractive values and take the techniques into consideration.

6 Application example

In the following section, an explanatory case study (Sarkar and Biswas 2022) corresponding to grading a strategy in a Tile Company, is considered under HFF phenomenon and the solutions are obtained by the developed method. The following was listed as the company's mission and vision:

- Vision: The Tile Company wants to be the leader in a highly competitive field, so it bases its future growth on the satisfaction of its customers.
- Mission: The company always seeks to improve the quality of its engineering and manufacturing processes in order to appear as the leader in trading as well as governmental innards. The organization pays close attention to the needs of its clients in this regard so that the satisfaction of all stakeholders is maximized.

First of all, organization's basic goal and vision are determined. Basically, a vision statement is considered as the declaration about which an organization wishes to adopt or become. Then, the relationship between the organization's internal and external components is adopted. Note that, the external components are those which affect an organization economically, socially, politically, or technologically. Whereas, the internal factors are those

Table 4 Weighted attractive values of considered strategies

Criteria	Weight	Weighted attractive score of strategies					
		\mathcal{X}_1	\mathcal{X}_2	\mathcal{X}_3	\mathcal{X}_4	\mathcal{X}_5	\mathcal{X}_6
C_1	0.05	0.15	0.20	0.10	0.20	0.10	0.05
C_2	0.07	0.28	0.14	0.21	0.07	0.35	0.07
C_3	0.12	0.24	0.48	0.48	0.36	0.12	0.12
C_4	0.08	0.08	0.16	0.32	0.32	0.16	0.08
C_5	0.07	0.28	0.28	0.14	0.14	0.28	0.28
C_6	0.15	0.60	0.45	0.30	0.30	0.30	0.15
C_7	0.09	0.18	0.36	0.09	0.18	0.36	0.18
C_8	0.11	0.44	0.44	0.11	0.11	0.11	0.44
C_9	0.03	0.12	0.03	0.09	0.12	0.12	0.12
C_{10}	0.12	0.36	0.48	0.48	0.12	0.12	0.36
C_{11}	0.07	0.28	0.28	0.14	0.28	0.07	0.07
C_{12}	0.04	0.12	0.16	0.12	0.12	0.04	0.20
Total attractive scores		3.13	3.46	2.58	2.32	2.13	2.12

Table 5 The response matrix

Strategy	Criterion											
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
\mathcal{X}_1	G	VG	F	P	VG	VG	F	VG	VG	G	VG	G
\mathcal{X}_2	VG	F	VG	F	VG	G	VG	VG	P	VG	VG	VG
\mathcal{X}_3	P	G	VG	G	P	VG	VP	VP	G	VG	P	G
\mathcal{X}_4	VG	VP	G	VG	P	VG	P	VP	VP	G	VG	VP
\mathcal{X}_5	VP	G	G	P	VG	VP	G	G	VP	VG	G	G
\mathcal{X}_6	P	VP	G	P	G	G	P	VG	VG	G	P	VP

which come within or under the control of a company, for example, organizational structure, human resources, corporate culture, etc. After that, the SWOT analysis should be performed before developing the organization’s strategies. The SWOT analysis is considered as a useful technique that can help to evaluate that what a company does best now and to adopt a successful strategy for future. Now, extract the DM’s thoughts which are based on the linguistic term sets. Because, certain DMs prefer social, narrative, or qualitative data, while the other prefer firm, specific, numerical or qualitative information. The below mentioned strategies are determined by the DMs after concerning the internal and external characteristics of the company in order to acquire the mission and vision.

- \mathcal{X}_1 : Upgrading the current production line.
- \mathcal{X}_2 : Adding an additional production line.
- \mathcal{X}_3 : Integrative progress.
- \mathcal{X}_4 : Integrating backward.
- \mathcal{X}_5 : Integrating horizontally.
- \mathcal{X}_6 : Diversification in concentration.

The characteristics of the criteria are obtained by applying SWOT analysis Sarkar and Biswas (2022), as

given in Table 3. The preference scores and weights are determined by the DMs using the traditional values listed in Table 3 which appeared in the QSPM of the under consideration tile company. These ratings are dependent on both intrinsic and extrinsic variables specific to the tile company.

In a traditional QSPM, the weighted attractive score of a strategy is determined by multiplying the weights of the criteria by the associated appealing scores, and is displayed in Table 4. A response matrix is created, as illustrated in Table 5 (Sarkar and Biswas 2022), and the expert evaluates the plans based on internal and external corporate aspects. To prioritize the strategies generated, the proposed HFF-MULTIMOORA is now used. The feed in values of the aforementioned assessment matrix are translated into the corresponding HFFNs using the HFF weighting scale, which is explained in Table 1, and are then shown in Table 6.

In this example, c_4 , c_5 and c_6 demonstrate minimizing (or cost) criteria. Thus, the Eq. 17 is utilized to normalize these criteria, and the resulting response matrix is given in Table 7.

Table 6 HFF response matrix

Strategy	c_1	c_2	c_3
\mathcal{X}_1	{0.65, 0.45, 0.25}, {0.85, 0.15, 0.52}	{0.55, 0.35, 0.15}, {0.75, 0.05, 0.42}	{0.45, 0.25, 0.05}, {0.65, 0.05, 0.32}
\mathcal{X}_2	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.45, 0.35, 0.25}, {0.65, 0.55, 0.45}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}
\mathcal{X}_3	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}
\mathcal{X}_4	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}
\mathcal{X}_5	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}
\mathcal{X}_6	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}
	c_4	c_5	c_6
\mathcal{X}_1	{0.55, 0.05, 0.22}, {0.35, 0.15, 0.05}	{0.45, 0.05, 0.12}, {0.35, 0.05, 0.05}	{0.35, 0.05, 0.02}, {0.25, 0.05, 0.05}
\mathcal{X}_2	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.45, 0.35, 0.25}, {0.65, 0.55, 0.45}
\mathcal{X}_3	{0.45, 0.35, 0.25}, {0.65, 0.55, 0.45}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}
\mathcal{X}_4	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}
\mathcal{X}_5	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{1.00, 1.00, 1.00}, {0.00, 0.00, 0.00}
\mathcal{X}_6	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.45, 0.35, 0.25}, {0.65, 0.55, 0.45}	{0.45, 0.35, 0.25}, {0.65, 0.55, 0.45}
	c_7	c_8	c_9
\mathcal{X}_1	{0.34, 0.04, 0.04}, {0.44, 0.04, 0.11}	{0.34, 0.04, 0.04}, {0.44, 0.04, 0.11}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}
\mathcal{X}_2	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}
\mathcal{X}_3	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}
\mathcal{X}_4	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_5	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_6	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}
	c_{10}	c_{11}	c_{12}
\mathcal{X}_1	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}
\mathcal{X}_2	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}
\mathcal{X}_3	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}
\mathcal{X}_4	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_5	{0.85, 0.75, 0.65}, {0.25, 0.15, 0.05}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}	{0.65, 0.55, 0.45}, {0.45, 0.35, 0.25}
\mathcal{X}_6	{0.25, 0.15, 0.05}, {0.85, 0.75, 0.65}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}

The example is solved using the proposed HFF-MULTIMOORA approach with the assumption that $\delta = 2$ in Eq. 15. In Table 8, the weights of the criteria determined by the suggested methodology are shown. The weightage values of HFF evaluation matrix have been produced by Eq. 20 using the estimated weights of the criterion as shown in the previous table, and is then shown in Table 9. The weighted HFF response matrix is then created using Eq. 21, taking into account the considered weighted values of attractive scores as shown in Table 4, in contrast if the expert provides the attractive score of the i -th alternative matching the j -th criteria. Table 10 presents the weighted HFF response matrix that was produced.

Applying Eqs. 23–25 to Table 9, evaluation indexes, fondness indication, and grades of usefulness of the possible choices are derived in accordance with Steps 6 through 8 of the proposed methodology. Now, using $\eta = \theta = 0.33$ in Eq. 29, the final values of attractive scores of

the corresponding schemes are calculated, as shown in Table 11.

As a result, it is determined that the strategies are ranked as follows: $\mathcal{X}_5 > \mathcal{X}_2 > \mathcal{X}_1 > \mathcal{X}_6 > \mathcal{X}_3 > \mathcal{X}_4$. The grading of the alternatives that corresponds to Table 10 can be derived in the same format. The computed total appealing ratings (final scores) of the options in this case are shown in the table below 12. As a result, the choices in this circumstance are ranked in the following order: $\mathcal{X}_5 > \mathcal{X}_2 > \mathcal{X}_1 > \mathcal{X}_3 > \mathcal{X}_6 > \mathcal{X}_4$.

6.1 Sensitivity analysis

After different weights have been assigned to each of the four SWOT analysis criteria components as shown in Table 13, the sensitivity analysis is carried out. The weights of the criterion would have an impact on the order in which the strategies are prioritized, according to the sensitivity analysis. To properly prioritise the techniques, it

Table 7 Normalized HFF response matrix

Strategy	c_1	c_2	c_3
\mathcal{X}_1	$(\{0.65, 0.45, 0.25\}, \{0.85, 0.15, 0.52\})$	$(\{0.55, 0.35, 0.15\}, \{0.75, 0.05, 0.42\})$	$(\{0.45, 0.25, 0.05\}, \{0.65, 0.05, 0.32\})$
\mathcal{X}_2	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.45, 0.35, 0.25\}, \{0.65, 0.55, 0.45\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_3	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_4	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
\mathcal{X}_5	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
\mathcal{X}_6	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
	c_4	c_5	c_6
\mathcal{X}_1	$(\{0.35, 0.15, 0.05\}, \{0.55, 0.05, 0.22\})$	$(\{0.35, 0.05, 0.05\}, \{0.45, 0.05, 0.12\})$	$(\{0.25, 0.05, 0.05\}, \{0.35, 0.05, 0.02\})$
\mathcal{X}_2	$(\{0.45, 0.35, 0.25\}, \{0.65, 0.55, 0.45\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
\mathcal{X}_3	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_4	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_5	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$
\mathcal{X}_6	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
	c_7	c_8	c_9
\mathcal{X}_1	$(\{0.34, 0.04, 0.04\}, \{0.44, 0.04, 0.11\})$	$(\{0.34, 0.04, 0.04\}, \{0.44, 0.04, 0.11\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_2	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$
\mathcal{X}_3	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
\mathcal{X}_4	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$
\mathcal{X}_5	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$
\mathcal{X}_6	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$
	c_{10}	c_{11}	c_{12}
\mathcal{X}_1	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
\mathcal{X}_2	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_3	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$
\mathcal{X}_4	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$
\mathcal{X}_5	$(\{0.85, 0.75, 0.65\}, \{0.25, 0.15, 0.05\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$	$(\{0.65, 0.55, 0.45\}, \{0.45, 0.35, 0.25\})$
\mathcal{X}_6	$(\{0.25, 0.15, 0.05\}, \{0.85, 0.75, 0.65\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$	$(\{0.00, 0.00, 0.00\}, \{1.00, 1.00, 1.00\})$

Table 8 The criteria weights

Method	Criterion											
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
HFF QSPM	0.09	0.08	0.07	0.08	0.09	0.09	0.08	0.09	0.09	0.07	0.09	0.08

is therefore essential to compute the criteria weights. The advantage of the suggested method is that it can compute the weight values of the criteria by evaluating them using an entropy framework and then computing the weight values through the DMs. This method enables us to evaluate the considered alternatives not only corresponding to distinct weights of the criteria but also according to different decision scenarios.

7 Comparative evaluation

The applicability and efficiency of the proposed HFF-MULTIMOORA framework has been demonstrated through a comparative analysis in certain distinct manners.

Table 9 Weighted HFF response matrix based on computes criteria weights

Strategy	c_1	c_2	c_3
\mathcal{X}_1	{0.22, 0.21, 0.20}, {0.93, 0.83, 0.73}	{0.32, 0.42, 0.52}, {0.86, 0.76, 0.66}	{0.12, 0.22, 0.32}, {0.97, 0.87, 0.77}
\mathcal{X}_2	{0.33, 0.23, 0.43}, {0.88, 0.78, 0.68}	{0.14, 0.04, 0.24}, {0.97, 0.87, 0.77}	{0.29, 0.39, 0.49}, {0.91, 0.81, 0.71}
\mathcal{X}_3	{0.08, 0.18, 0.28}, {0.99, 0.89, 0.79}	{0.21, 0.31, 0.41}, {0.94, 0.84, 0.74}	{0.29, 0.39, 0.49}, {0.91, 0.81, 0.71}
\mathcal{X}_4	{0.33, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.19, 0.29, 0.39}, {0.95, 0.85, 0.75}
\mathcal{X}_5	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.21, 0.31, 0.41}, {0.94, 0.84, 0.74}	{0.19, 0.29, 0.39}, {0.95, 0.85, 0.75}
\mathcal{X}_6	{0.08, 0.07, 0.06}, {0.99, 0.89, 0.79}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.19, 0.29, 0.39}, {0.95, 0.85, 0.75}
	c_4	c_5	c_6
\mathcal{X}_1	{0.31, 0.41, 0.51}, {0.90, 0.80, 0.70}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}
\mathcal{X}_2	{0.21, 0.31, 0.41}, {0.94, 0.84, 0.74}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}
\mathcal{X}_3	{0.13, 0.23, 0.33}, {0.97, 0.87, 0.77}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}
\mathcal{X}_4	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}
\mathcal{X}_5	{0.31, 0.41, 0.51}, {0.90, 0.80, 0.70}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{1.00, 1.00, 1.00}, {0.00, 0.00, 0.00}
\mathcal{X}_6	{0.31, 0.41, 0.51}, {0.90, 0.80, 0.70}	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}
	c_7	c_8	c_9
\mathcal{X}_1	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}	{0.33, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.33, 0.43, 0.53}, {0.88, 0.78, 0.68}
\mathcal{X}_2	{0.32, 0.33, 0.34}, {0.89, 0.79, 0.69}	{0.34, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.08, 0.06, 0.05}, {0.98, 0.89, 0.79}
\mathcal{X}_3	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.22, 0.31, 0.41}, {0.93, 0.84, 0.74}
\mathcal{X}_4	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_5	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}	{0.22, 0.41, 0.51}, {0.93, 0.80, 0.70}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_6	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.34, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.34, 0.43, 0.53}, {0.88, 0.78, 0.68}
	c_{10}	c_{11}	c_{12}
\mathcal{X}_1	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}
\mathcal{X}_2	{0.30, 0.40, 0.50}, {0.90, 0.80, 0.70}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.31, 0.42, 0.52}, {0.89, 0.79, 0.69}
\mathcal{X}_3	{0.30, 0.40, 0.50}, {0.90, 0.80, 0.70}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}
\mathcal{X}_4	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_5	{0.30, 0.40, 0.50}, {0.90, 0.80, 0.70}	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}
\mathcal{X}_6	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}

7.1 Comparative attractiveness-based comparison

Table 14 compares the proposed method to the prevailed TOPSIS-based technique Nasab and Milani (2012), which computes the entire alluring scores of techniques in the surroundings of QSPM utilizing both fuzzy and non-fuzzy information. The prioritization of the chosen techniques may change depending on whether HFFNs are used and the suggested methodology is adopted, as shown in Table 14. Additionally, using the entropy weight framework to compute the unknown weight values of the criterion would aid in more accurately prioritizing methods in the context of the HFF QSPM. It should be noted that, depending on which final strategy the organization chooses, the size of

disparities in final scores indicates how desirable one plan is compared to the others.

7.2 Different weighting methodologies-based comparison

Second, three separate factors are used to compare the suggested methodology: unknown criteria weights as well as known criteria weights, and weighted appealing scores of methods that correspond to the criteria Table 10. Table 15 presents the analysis' findings. Table 15 reveals that in every instance, the approach \mathcal{X}_5 is chosen as the best one and shows that utilizing familiar or unfamiliar weight values of each criterion will allow the suggested method to more accurately identify the best strategy as compared to utilize weighted values of attractive scores

Table 10 Weighted HFF response matrix corresponding to weighted attractive value

Strategy	c_1	c_2	c_3
\mathcal{X}_1	{0.22, 0.21, 0.20}, {0.93, 0.83, 0.73}	{0.32, 0.42, 0.52}, {0.86, 0.76, 0.66}	{0.12, 0.22, 0.32}, {0.97, 0.87, 0.77}
\mathcal{X}_2	{0.33, 0.23, 0.43}, {0.88, 0.78, 0.68}	{0.14, 0.04, 0.24}, {0.97, 0.87, 0.77}	{0.29, 0.39, 0.49}, {0.91, 0.81, 0.71}
\mathcal{X}_3	{0.08, 0.18, 0.28}, {0.99, 0.89, 0.79}	{0.21, 0.31, 0.41}, {0.94, 0.84, 0.74}	{0.29, 0.39, 0.49}, {0.91, 0.81, 0.71}
\mathcal{X}_4	{0.33, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.19, 0.29, 0.39}, {0.95, 0.85, 0.75}
\mathcal{X}_5	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.21, 0.31, 0.41}, {0.94, 0.84, 0.74}	{0.19, 0.29, 0.39}, {0.95, 0.85, 0.75}
\mathcal{X}_6	{0.08, 0.07, 0.06}, {0.99, 0.89, 0.79}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.19, 0.29, 0.39}, {0.95, 0.85, 0.75}
	c_4	c_5	c_6
\mathcal{X}_1	{0.31, 0.41, 0.51}, {0.90, 0.80, 0.70}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}
\mathcal{X}_2	{0.21, 0.31, 0.41}, {0.94, 0.84, 0.74}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}
\mathcal{X}_3	{0.13, 0.23, 0.33}, {0.97, 0.87, 0.77}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}
\mathcal{X}_4	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}
\mathcal{X}_5	{0.31, 0.41, 0.51}, {0.90, 0.80, 0.70}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{1.00, 1.00, 1.00}, {0.00, 0.00, 0.00}
\mathcal{X}_6	{0.31, 0.41, 0.51}, {0.90, 0.80, 0.70}	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}
	c_7	c_8	c_9
\mathcal{X}_1	{0.14, 0.24, 0.34}, {0.96, 0.86, 0.76}	{0.33, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.33, 0.43, 0.53}, {0.88, 0.78, 0.68}
\mathcal{X}_2	{0.32, 0.33, 0.34}, {0.89, 0.79, 0.69}	{0.34, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.08, 0.06, 0.05}, {0.98, 0.89, 0.79}
\mathcal{X}_3	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.22, 0.31, 0.41}, {0.93, 0.84, 0.74}
\mathcal{X}_4	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_5	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}	{0.22, 0.41, 0.51}, {0.93, 0.80, 0.70}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_6	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.34, 0.43, 0.53}, {0.88, 0.78, 0.68}	{0.34, 0.43, 0.53}, {0.88, 0.78, 0.68}
	c_{10}	c_{11}	c_{12}
\mathcal{X}_1	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}
\mathcal{X}_2	{0.30, 0.40, 0.50}, {0.90, 0.80, 0.70}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.31, 0.42, 0.52}, {0.89, 0.79, 0.69}
\mathcal{X}_3	{0.30, 0.40, 0.50}, {0.90, 0.80, 0.70}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}
\mathcal{X}_4	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}	{0.32, 0.42, 0.52}, {0.89, 0.79, 0.69}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}
\mathcal{X}_5	{0.30, 0.40, 0.50}, {0.90, 0.80, 0.70}	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}	{0.21, 0.41, 0.51}, {0.94, 0.80, 0.70}
\mathcal{X}_6	{0.20, 0.30, 0.40}, {0.94, 0.84, 0.74}	{0.07, 0.06, 0.05}, {0.99, 0.89, 0.79}	{0.00, 0.00, 0.00}, {1.00, 1.00, 1.00}

Table 11 Final attractive values of alternatives

Strategy	\mathcal{X}_1	\mathcal{X}_2	\mathcal{X}_3	\mathcal{X}_4	\mathcal{X}_5	\mathcal{X}_6
Attractive score	0.5263	0.5449	0.4268	0.3495	1.000	0.4311

Table 12 Final attractive values of alternatives

Strategy	\mathcal{X}_1	\mathcal{X}_2	\mathcal{X}_3	\mathcal{X}_4	\mathcal{X}_5	\mathcal{X}_6
Final score	0.8099	0.8599	0.7848	0.6876	1.000	0.7347

given by the DMs. As a result, the suggested framework is fruitful in that manner that it can use all three weighting schemes depending on the context of the DM process.

7.3 Different techniques to obtain final ranking

Third, the rankings obtained using the proposed method are compared to those obtained using the dominance theory Brauers and Zavadskas (2011). In order to produce a single rank of strategies, the suggested HFF-MULTIMOORA uses three techniques, which are shown in Table 16, and dominance theory is applied to those ranks. With just one variation in the second and third positions, Table 16 demonstrates that the proposed aggregation technique and

Table 13 Weighting techniques applied in developed HFF-MULTIMOORA method

Criteria		Case 1	Case 2	Case 3	Case 4	Case 5
Strength		0.25	0.5	0.17	0.17	0.17
C_1	Consistent and high product quality	0.083	0.17	0.057	0.057	0.057
C_2	Reputation for innovative design	0.083	0.17	0.057	0.057	0.057
C_3	Experience in exporting to other countries	0.083	0.17	0.057	0.057	0.057
Weakness		0.25	0.17	0.5	0.17	0.17
C_4	Insufficient cash flow	0.083	0.057	0.17	0.085	0.057
C_5	Problems with on-time delivery	0.083	0.057	0.17	0.085	0.057
C_6	Relatively high costs of labour	0.083	0.057	0.17	0.000	0.057
Opportunity		0.25	0.17	0.17	0.5	0.17
C_7	Demand increasing by 10%	0.083	0.057	0.17	0.17	0.057
C_8	Luxurious products	0.083	0.057	0.17	0.17	0.057
C_9	Easier access to markets in accession	0.083	0.057	0.17	0.17	0.057
Threat		0.25	0.17	0.17	0.17	0.5
C_{10}	Increased competition of producers	0.083	0.057	0.057	0.057	0.17
C_{11}	From countries with lower costs of labour	0.083	0.057	0.057	0.057	0.17
C_{12}	Recession in many countries markets	0.083	0.057	0.057	0.057	0.17

Table 14 Combined QSPM-based strategy attractiveness scores

Method	\mathcal{X}_1	\mathcal{X}_2	\mathcal{X}_3	\mathcal{X}_4	\mathcal{X}_5	\mathcal{X}_6
Crisp QSPM with weighted sum (Nasab and Milani 2012)	3.13	3.46	2.58	2.32	2.13	2.12
Fuzzy QSPM with weighted sum (Nasab and Milani 2012)	0.424	0.393	0.442	0.425	0.389	0.396
Fuzzy QSPM with TOPSIS (Nasab and Milani 2012)	0.131	0.128	0.217	0.183	0.105	0.129
PF QSPM with PF-MULTIMOORA (Sarkar and Biswas 2022)	0.5326	0.4495	0.2684	0.4953	1.0000	0.3114
HFF QSPM with HFF-MULTIMOORA	0.5263	0.5449	0.4268	0.3495	1.0000	0.4311

Table 15 Under various weighting techniques, the total appealing scores and order of the strategies

Strategy	Using weighted attractive scores		Known criteria weight		Unknown criteria weight	
	Score value	Rank	Score value	Rank	Score value	Rank
\mathcal{X}_1	0.8099	3	0.4728	2	0.5263	3
\mathcal{X}_2	0.8599	2	0.5304	3	0.5449	2
\mathcal{X}_3	0.7848	4	0.4157	5	0.4268	5
\mathcal{X}_4	0.6876	6	0.3246	6	0.3495	6
\mathcal{X}_5	1.0000	1	1.0000	1	1.0000	1
\mathcal{X}_6	0.7347	5	0.4124	4	0.4311	4

dominance theory both recommend identical ranks. It is important to note in this context that using the dominance theory has various drawbacks, including absolute dominance, transitivity, general dominance, and more. It should be emphasised that, in a cyclic form, one choice may dominate the others or vice versa. Multiple comparisons and circular reasoning are the two main drawbacks it produces on a wide scale. In order to acquire final ranking,

the proposed aggregation method would get beyond the issues mentioned above. It should be emphasized that the HFF-MULTIMOORA approach is more effective in determining the relative attractiveness of options than dominance theory, which becomes inapplicable in this situation.

Table 16 Comparison of final results acquired through distinct techniques

Strategy	HFF-MULTIMOORA				
	Ratio analysis	Reference point approach	Full multiplicative form approach	Final ranking by dominance theory	Final ranking by proposed aggregation technique (Equation 14)
\mathcal{X}_1	2	4	2	2	2
\mathcal{X}_2	3	2	1	3	3
\mathcal{X}_3	5	5	3	5	5
\mathcal{X}_4	6	6	4	6	6
\mathcal{X}_5	1	1	5	1	1
\mathcal{X}_6	4	3	6	4	4

Table 17 Comparison of final results acquired through distinct techniques

	k_{m1}	Ranks	k_{m2}	Ranks	k_{m3}	Ranks
\mathcal{X}_1	0.1890	5	2.113	5	0.8412	5
\mathcal{X}_2	0.1860	6	2.000	6	0.8373	6
\mathcal{X}_3	0.2030	2	2.6285	2	0.9250	2
\mathcal{X}_4	0.2125	3	2.6065	3	0.8621	3
\mathcal{X}_5	0.2195	1	3.2856	1	1.000	1
\mathcal{X}_6	0.2021	4	2.6061	4	0.8511	4

7.4 Comparison with HFF CoCoSo MCGDM method

The HFF CoCoSo (combined compromise solution) method for MCGDM was introduced by Lai et al. (2022a, b). In this subsection, the case study of Tile company as adapted in current study is solved through the HFF CoCoSo method as given in Lai et al. (2022a, b). Then, we compare the obtained results of ranking with the final ranking of strategies as given through the developed methodology. The HFF response and normalized matrices are considered as given in Tables 6 and 7, respectively. The computation through HFF CoCoSo technique are given as follows: The arithmetic weighted sum (W_m) and geometrically weighted sum (G_m) corresponding to m alternatives are computed through Eqs. 30 and 31, respectively.

$$W_m = \sum_{n=1}^{12} \xi_n N_{mn}^\omega, m = 1, 2, 3, \dots, 6, \tag{30}$$

$$G_m = \sum_{n=1}^{12} (N_{mn}^\omega)^{\xi_n}, m = 1, 2, 3, \dots, 6. \tag{31}$$

Then, the subordinate preference value and subordinate rankings of each alternative are generated utilizing Eqs. 32–34, given as follows:

$$K_{m1} = \frac{s(W_m) + s(G_m)}{\sum_{m=1}^6 (s(W_m) + s(G_m))}, \tag{32}$$

$$K_{m2} = \frac{s(W_m)}{\min_m s(W_m)} + \frac{s(G_m)}{\min_m s(G_m)}, \tag{33}$$

$$K_{m3} = \frac{\phi s(W_m) + (1 - \phi)s(G_m)}{\phi \max_m s(W_m) + (1 - \phi) \max_m s(G_m)}, \tag{34}$$

where $m = 1, 2, 3, \dots, 6$. Thus, the obtained three appraisal scores and the rank of each alternative are shown in Table 17.

Note that, corresponding to the final ranking of each alternative under each subordinate aggregation operator, the final ranking of each alternative through the proposed ensemble ranking approach can be obtained as, i.e., $\mathcal{X}_5 \succ \mathcal{X}_3 \succ \mathcal{X}_4 \succ \mathcal{X}_6 \succ \mathcal{X}_1 \succ \mathcal{X}_2$. Therefore, we conclude that the best option is \mathcal{X}_5 . The comparative analysis shows that the proposed HFF-MULTIMOORA method performs better than the other methods at obtaining a more reasonable ranking because it enables DMs to represent their opinions in linguistic terms that are formulated by HFFNs to address vagueness and uncertainties occurring in DM applications. Additionally, the suggested framework uses the MD, NMD, and hesitant values of HFF evaluation degrees throughout the work, as opposed to the crisp value or MD of FS, which are utilized in previous techniques. Hence, adaptation of the suggested HFF-MULTIMOORA approach would enhance the evaluation and rankings of alternatives (strategies) and assure that the Tile Company would select \mathcal{X}_5 as the optimal plan.

8 Conclusions

This research creates the HFF-MULTIMOORA MC-GDM technique to determine the concluding order of ranking and evaluate the under consideration alternatives or choices. To get over the difficulties of using dominance theory to

establish the conclusive ranking order of choices, an accumulating strategy that is dependent on the utility grades of three MULTIMOORA strategies is applied. The proposed HFF QSPM framework is also used in the task of formulating strategies to address the shortcomings of the conventional QSPM, such as the DMs' assignment of ratings and attractive scores to strategies, their method for determining the total preference values of alternatives or strategies and their formulation of weight values of criteria. Thus, it can be concluded that the evaluation of information values through HFFs permits a more comprehensive analysis of the assessment process. Also, the generalization of MULTIMOORA method along with HFFs has made an academic contribution to applications that are relevant to social and economical evaluations. An example involving strategy prioritization at a tile firm is taken into account along with HFF data to demonstrate the suggested technique and demonstrate how it would improve the outcomes of the proposed HFF QSPM procedure. The HFF entropy weight model is used in this study's proposed approach to calculate the weights for the problem, which involves taking into account unknown criteria weights. The ranks are sensitive to the different weights of the criteria, the sensitivity study reveals. The results of considering different criterion weighting schemes have been represented by comparison studies. Further, comparison work is conducted based on the use of distinct HFFN values of accuracy and score functions to rate various methods. The suggested method is however constrained by the use of HFF aggregation operators in complete multiplicative methods and ratio analysis. Any HFFN decision value that has zero membership or non-membership degree will, in accordance, have either zero MD or zero NMD in the final aggregated value. Therefore, regardless of how significant a particular criterion is, it may be given precedence over all other criterion values in this case. The impact of the other factors would, therefore, be diminished during the assessment process, which is surprising.

The following is a list of benefits of the planned HFF-MULTIMOORA:

1. The DMs' find it simpler to use linguistic expressions to convey their judgement values.
2. To establish the subordinate ranks that serve as the foundation for the final rankings, three different methods are used.
3. Instead of selecting one of the three subordinate rankings using dominance theory, an aggregate technique is used to determine the final ranking.
4. The MCGDM technique is much reliable as compared to utilizing two distinct multi-objective optimization methods, simultaneously. Since, it incorporates three

distinct orders of ranking algorithms of the developed HFF-MULTIMOORA.

The suggested HFF-MULTIMOORA approach and a modified MOORA method are also shown to be simple and straightforward to use in comparison to other well-known MCGDM methods that are already in use. Consequently, the suggested HFF QSPM and HFF-MULTI-MOORA may be useful for strategy creation that requires both quantitative and qualitative information at once.

The efficiency of the suggested methodology in comparison to other HFF methods in various decision-making sectors may be investigated in future research projects. Additionally, a number of recently created FF aggregation operators, like FF interaction power Bonferroni mean aggregation operators, FF interactive Hamacher power aggregation operators, and others, may be added to the suggested MCGDM approach to enhance the outcomes in some way. Additionally, to more effectively capture the imprecision, the established HFF-MULTI-MOORA approach can be generalized in q -ROF, hesitant q -ROF and other extended spaces of uncertainties and ambiguity.

Author contributions Authors created and developed the project. They also conducted data analysis and report writing. Anam Luqman: concept, design, analysis, writing, or revision of the manuscript. Saba Siddique: concept, design, analysis, writing, or revision of the manuscript. Gulfam Shahzadi: concept, design, analysis, writing, or revision of the manuscript. Muhammad Akram: concept, design, analysis, writing, or revision of the manuscript.

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Declarations

Conflict of interest The authors say they have no conflict of interest.

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