Granular Computing (2023) 8:1013–1025
https://doi.org/10.1007/s41066-023-00367-0

ORIGINAL PAPER

Multi-attribute decision making based on the q-rung orthopair fuzzy Yager power weighted geometric aggregation operator of q-rung orthopair fuzzy values

Chirag Dhankhar[1] · **Kamal Kumar**[1]

Received: 14 January 2023 / Accepted: 24 January 2023 / Published online: 15 February 2023

Abstract

The power geometric (PG) operator has the significant advantage of reducing the effects of the incorrect information given by the biased experts. Therefore, in this paper, we propose the q-rung orthopair fuzzy Yager power weighted geometric (q-ROFYPWG) aggregation operator (AO) based on the PG operator and Yager's norm for aggregating the q-rung orthopair fuzzy values (q-ROFVs). The q-ROFYPWG AO proposed in this article can conquer the shortcomings of the q-rung orthopair fuzzy weighted geometric (q-ROFWG) AO and the q-rung orthopair fuzzy Einstein weighted geometric (q-ROFEWG) AO of q-ROFVs. We also present some characteristics of the proposed q-ROFYPWG AO. Moreover, by utilizing the proposed q-ROFYPWG AO of q-ROFVs, we develop a new multi-attribute decision making (MADM) approach for q-ROFVs environment. The proposed MADM approach can conquer the drawbacks of existing MADM approaches, where they cannot distinguish the ranking orders (ROs) of alternatives in some situations. It offers a highly effective approach for dealing with MADM issues in the context of q-ROFVs.

Keywords Power operator · Decision-making · Yager's norm · MADM · q-ROFVs

1 Introduction

Multi-attribute decision-making (MADM) is a common and important activity in our regular lifestyle. However, the most important step for solving the MADM problems is to choose an adequate environment to do performance assessments of the alternatives. Recently, "fuzzy sets" (Zadeh 1965) and its extensions, "intuitionistic fuzzy set" (IFS) (Atanassov 1986) and "Pythagorean fuzzy set" (PFS) (Yager 2013), have been widely used by the researchers for the alternative assessment. In these environments, various MADM approaches (Chen et al. 2016; Kumar and Chen 2021b, a; Dhankhar and Kumar 2022; Garg and Kumar 2020, 2019; Jiang et al. 2018; Zou et al. 2020; Abdullah et al. 2022; Gupta and Kumar 2022; Joshi and Kumar 2022; Seikh and Mandal 2021a; Garg and Kaur 2020; Ahmad and Sabir 2022; Saad and Rafiq 2022; Chabaane and Kheffache 2022; Ganie 2022; Ashraf et al. 2021; Suresh et al. 2021; Dutta and Doley 2021; Biswas and Deb 2021; Dutta and Saikia 2021; Mishra et al. 2022; Meng and Chen 2021; Pant and Kumar 2022; Ali and Ansari 2022; Ejegwa et al. 2022; Zeb et al. 2022; Chaurasiya and Jain 2022) have been developed by the researchers. For example, Garg and Kumar (2019) defined the improved possibility degree measure (PDM) for intuitionistic fuzzy numbers (IFNs) and a MADM approach based on it. Dhankhar and Kumar (2022) proposed the advanced PDM of IFNs and a MADM approach for the IFNs environment. The power aggregation operator (AO) has the significant advantage of reducing the effects incorrect information given by the biased experts. The power AO allows aggregated values to support each other throughout the aggregation process. Therefore, Garg and Kumar (2020) defined the power AOs for aggregating the connection numbers and a MADM approach for the IFNs environment. Jiang et al. (2018) defined the MADM approach based power AO and entropy measures for the IFNs. Biswas and Deb (2021)

✉ Kamal Kumar
kamalkumarrajput92@gmail.com

[1] Department of Mathematics, Amity University Haryana, Gurugram, Haryana 122413, India

presented the Schweizer and Sklar power AOs and MADM for PFSs environment.

Recently, Yager (2017) presented the idea of q-rung orthopair fuzzy set (q-ROFS), where a q-rung orthopair fuzzy value (q-ROFV) $\chi = \langle \zeta, \theta \rangle$ with membership grade (MG) ζ and non-MG (NMG) θ fulfils the constraints $0 \leq \zeta \leq 1$, $0 \leq \theta \leq 1$, $0 \leq \zeta^q + \theta^q \leq 1$ and $q \geq 1$. The q-ROFS is called IFS and PFS when $q = 1$ and $q = 2$, respectively. The q-ROFS provides experts with additional flexibility when evaluating alternatives. In last five years, various MADM approaches (Akram and Shahzadi 2021; Farid and Riaz 2021; Akram et al. 2021; Wei et al. 2018; Yang et al. 2021; Liu and Wang 2018; Liu et al. 2018; Khan et al. 2021; Kumar and Gupta 2023; Feng et al. 2022a; Seikh and Mandal 2021b; Feng et al. 2022b; Verma 2022) have been developed by the researchers in context of q-ROFVs. Akram and Shahzadi (2021) defined the AO based on the Yager'norm for aggregating the q-ROFVs for MADM under the q-ROFVs environment. Farid and Riaz (2021) defined the Einstein interactive geometric AOs for MADM of q-ROFVs. Akram et al. (2021) proposed the q-rung orthopair fuzzy Einstein weighted geometric (q-ROFEWG) AO to develop a MADM approach in the environment of q-ROFVs. Wei et al. (2018) presented the Heronian mean AO to develop a MADM approach in context of q-ROFVs. Yang et al. (2021) defined the interaction Maclaurin symmetric mean AO for MADM approach under the q-ROFVs environment. Liu and Wang (2018) proposed the q-rung orthopair fuzzy weighted geometric (q-ROFWG) AO for MADM in q-ROFVs context. Liu et al. (2018) presented the MADM approach based entropy measure for the q-ROFVs environment.

In this paper, we find that the q-ROFWG AO (Liu and Wang 2018) and the q-ROFEWG AO (Akram et al. 2021) both have the shortcomings that the MG and the NMG of their aggregated q-ROFVs are indeterminate in some instances. Moreover, the q-ROFWG AO (Liu and Wang 2018) and the q-ROFEWG AO (Akram et al. 2021) have the drawbacks that if there is only one q-ROFV with MG 0 among n aggregated q-ROFVs then the MG of the aggregated q-ROFV becomes 0; if there is only one q-ROFV with NMG 1 among n aggregated q-ROFVs then the NMG of the aggregated q-ROFV becomes 1. Therefore, we need a new AO of q-ROFVs to overcome the shortcomings of the q-ROFWG AO (Liu and Wang 2018) and the q-ROFEWG AO (Akram et al. 2021) of q-ROFVs. Moreover, we also find that Akram et al.'s MADM approach (Akram et al. 2021) based on q-ROFEWG AO and Liu and Wang's MADM approach (Liu and Wang 2018) based on q-ROFWG AO have the shortcomings that they cannot distinguish the ranking ordering (RO) of alternatives in some cases. In order to overcome the shortcomings of the Akram et al.'s MADM approach (Akram et al. 2021) and the Liu and Wang's MADM approach (Liu and Wang 2018), we need to develop a new MADM approach for the q-ROFVs environment.

In this article, we propose the q-rung orthopair fuzzy Yager power weighted geometric (q-ROFYPWG) AO for aggregating the q-ROFVs by combining the features of PG AO and Yager's norm. We also present the various properties of the proposed q-ROFYPWG AO. The proposed q-ROFYPWG AO can conquer the shortcomings of the q-ROFWG AO (Liu and Wang 2018) and the q-ROFEWG AO (Akram et al. 2021) of q-ROFVs. Based on the proposed q-ROFYPWG AO, we propose a new MADM approach for q-ROFVs environment. The MADM approach proposed in this paper can conquer the shortcomings of the Akram et al.'s MADM approach (Akram et al. 2021) and the Liu and Wang's MADM approach (Liu and Wang 2018), where they cannot distinguish the ranking order (RO) of alternatives in some cases. The proposed MADM approach provides us with a very effective method for dealing with MADM challenges in the q-ROFVs environment.

The remaining part of this article is organised as follows: The preliminary materials related to this paper are given in Sect. 2. In Sect. 3, we explore the drawbacks of the q-ROFWG AO (Liu and Wang 2018) and q-ROFEWG AO (Akram et al. 2021) of q-ROFVs. In Sect. 4, we propose the q-ROFYPWG AO of q-ROFVs to overcome the drawbacks of the q-ROFWG AO (Liu and Wang 2018) and q-ROFEWG AO (Akram et al. 2021) of q-ROFVs. Section 5 presents the drawbacks of the Akram et al.'s MADM approach (Akram et al. 2021). In Sect. 6, we develop a new MADM approach based on the proposed q-ROFYPWG AO of q-ROFVs to conquer the drawbacks of the MADM approach given in (Liu and Wang 2018) and (Akram et al. 2021). Finally, Sect. 7 discusses the conclusion of the paper.

2 Preliminaries

In the following, we present some preliminary materials related to this paper.

Definition 1 (Yager 2017) A q-ROFS Q in universal set U is defined as

$$Q = \left\{ \langle u, \zeta_Q(u), \theta_Q(u) \rangle \mid u \in U \right\} \quad (1)$$

where $\zeta(u)$ and $\theta(u)$ represent the membership grade (MG) and the non-MG (NMG) of element u of q-ROFS Q, respectively, $0 \leq \zeta_Q(u), \theta_Q(u) \leq 1$ and $0 \leq \zeta_Q^q(u) + \theta_Q^q(u) \leq 1$. The hesitance degree of u is given by $\pi_Q = (1 - \zeta_Q^q(u) - \theta_Q^q(u))^{1/q}$ where $q \geq 1$.

Frequently, Yager (2017) called the pair $\langle \zeta, \theta \rangle$ a q-ROFV in the q-ROFS $Q = \left\{ \langle u, \zeta_Q(u), \theta_Q(u) \rangle \mid u \in U \right\}$.

Definition 2 (Liu et al. 2019) The Euclidean distance $d_E(\chi_1, \chi_2)$ between two q-ROFVs $\chi_1 = \langle \zeta_1, \theta_1 \rangle$ and $\chi_2 = \langle \zeta_2, \theta_2 \rangle$ is defined as follows:

$$d_E(\chi_1, \chi_2) = \left(\frac{1}{2} (|\zeta_1^q - \zeta_2^q|^2 + |\theta_1^q - \theta_2^q|^2) \right)^{1/2} \tag{2}$$

Definition 3 (Yager 2017) Let $\chi = \langle \zeta, \theta \rangle$ be any q-ROFV. The score value $S(\chi)$ of χ is defined as

$$S(\chi) = \zeta^q - \theta^q, \tag{3}$$

where $S(\chi) \in [-1, 1]$.

Definition 4 (Yager 2017) Let $\chi = \langle \zeta, \theta \rangle$ be any q-ROFV. The accuracy value $A_c(\chi)$ of χ is defined as

$$A_c(\chi) = \zeta^q + \theta^q, \tag{4}$$

where $A_c(\chi) \in [0, 1]$.

Definition 5 (Yager 2017) Let χ_1 and χ_2 be two q-ROFVs. Then

(i) If $S(\chi_1) > S(\chi_2)$, then $\chi_1 \succ \chi_2$.
(ii) If $S(\chi_1) < S(\chi_2)$, then $\chi_1 \prec \chi_2$.
(iii) If $S(\chi_1) = S(\chi_2)$, then

  (a) If $A_c(\chi_1) > A_c(\chi_2)$, then $\chi_1 \succ \chi_2$.
  (b) If $A_c(\chi_1) < A_c(\chi_2)$, then $\chi_1 \prec \chi_2$.
  (c) If $A_c(\chi_1) = A_c(\chi_2)$, then $\chi_1 = \chi_2$.

Definition 6 (Xu and Yager 2010) For aggregating the real numbers $\chi_1, \chi_2, \ldots, \chi_s$, the PG AO is defined as follows:

$$\mathrm{PG}(\chi_1, \chi_2, \ldots, \chi_s) = \prod_{t=1}^{s} \chi_t^{\frac{(1+T(\chi_t))}{\sum_{t=1}^{s}(1+T(\chi_t))}}, \tag{5}$$

where $T(\chi_t) = \sum_{\substack{l=1 \\ l \neq t}}^{n} Sup(\chi_t, \chi_l)$, and $Sup(\chi_t, \chi_l)$ represents the support degree to which χ_l supports χ_t and meets the following criteria:

(i) $Sup(\chi_t, \chi_l) \in [0, 1]$;
(ii) $Sup(\chi_t, \chi_l) = Sup(\chi_l, \chi_t)$;
(iii) $Sup(\chi_t, \chi_l) \geq Sup(\chi_m, \chi_n)$ if $|\chi_t - \chi_l| \leq |\chi_m - \chi_n|$.

Definition 7 (Akram and Shahzadi 2021) Let $\chi_1 = \langle \zeta_1, \theta_1 \rangle$ and $\chi_2 = \langle \zeta_2, \theta_2 \rangle$ be two q-ROFVs, $\lambda > 0$ and $\delta > 0$. Then operation laws of q-ROFVs based on the Yager t-conorm and t-norm are defined as follows:

(a) $\chi_1 \oplus \chi_2 = \left\langle \sqrt[q]{\min\{1, ((\zeta_1^q)^\lambda + (\zeta_2^q)^\lambda)^{1/\lambda}\}}, \sqrt[q]{1 - \min\{1, ((1-\theta_1^q)^\lambda + (1-\theta_2^q)^\lambda)^{1/\lambda}\}} \right\rangle$;

(b) $\chi_1 \otimes \chi_2 = \left\langle \sqrt[q]{1 - \min\{1, ((1-\zeta_1^q)^\lambda + (1-\zeta_2^q)^\lambda)^{1/\lambda}\}}, \sqrt[q]{\min\{1, ((\theta_1^q)^\lambda + (\theta_2^q)^\lambda)^{1/\lambda}\}} \right\rangle$;

(c) $\delta\chi_1 = \left\langle \sqrt[q]{\min\{1, (\delta(\zeta_1^q)^\lambda)^{1/\lambda}\}}, \sqrt[q]{1 - \min\{1, (\delta(1-\theta_1^q)^\lambda)^{1/\lambda}\}} \right\rangle$;

(d) $\chi_1^\delta = \left\langle \sqrt[q]{1 - \min\{1, (\delta(1-\zeta_1^q)^\lambda)^{1/\lambda}\}}, \sqrt[q]{\min\{1, (\delta(\theta_1^q)^\lambda)^{1/\lambda}\}} \right\rangle$.

3 Analyzing the shortcomings of the existing aggregation operators of q-ROFVs

In this section, we explore the shortcomings of the q-ROFWG AO (Liu and Wang 2018) and q-ROFEWG AO (Akram et al. 2021) of q-ROFVs.

Let $\chi_1 = \langle \zeta_1, \theta_1 \rangle$, $\chi_2 = \langle \zeta_2, \theta_2 \rangle$,..., $\chi_s = \langle \zeta_s, \theta_s \rangle$ be q-ROFVs. The q-ROFWG AO (Liu and Wang 2018) and q-ROFEWG AO (Akram et al. 2021) of q-ROFVs are reviewed as below:

(1) q-ROFWG AO (Liu and Wang 2018):

$$\text{q-ROFWG}(\chi_1, \chi_2, \ldots, \chi_s) = \left\langle \prod_{t=1}^{s} \zeta_t^{w_t}, \left(1 - \prod_{t=1}^{s} (1-\theta_t^q)^{w_t}\right)^{1/q} \right\rangle \tag{6}$$

where $q \geq 1$, $w_t \in [0, 1]$, $t = 1, 2, \ldots, s$ and $\sum_{t=1}^{s} w_t = 1$.

(2) q-ROFEWG AO (Akram et al. 2021):

$$q\text{-ROFEWG}(\chi_1, \chi_2, \ldots, \chi_s) = \left\langle \frac{\sqrt[q]{2} \prod_{t=1}^{s} \zeta_t^{w_t}}{\sqrt[q]{\prod_{t=1}^{s}(2-\zeta_t^q)^{w_t} + \prod_{t=1}^{s}(\zeta_t^q)^{w_t}}}, \sqrt[q]{\frac{\prod_{t=1}^{s}(1+\theta_t^q)^{w_t} - \prod_{t=1}^{s}(1-\theta_t^q)^{w_t}}{\prod_{t=1}^{s}(1+\theta_t^q)^{w_t} + \prod_{t=1}^{s}(1-\theta_t^q)^{w_t}}} \right\rangle \tag{7}$$

where $q \geq 1$, $w_t \in [0, 1]$, $t = 1, 2, \ldots, s$ and $\sum_{t=1}^{s} w_t = 1$.

Example 1 Let $\chi_1 = \langle 0.2, 0.3\rangle$ and $\chi_2 = \langle 0, 1\rangle$ be two q-ROFVs with weights $w_1 = 0.3$ and $w_2 = 0.7$, respectively. By using the q-ROFWG AO (Liu and Wang 2018) given in Eq. (6) to aggregate the χ_1 and χ_2, it gets $q-ROFWG(\chi_1, \chi_2) = \langle 0, 1\rangle$. It can be seen that the MG of the q-ROFV χ_1 is 0.2, which is not 0 because $0.2 > 0$, but the MG of the aggregated result $q-ROFWG(\chi_1, \chi_2) = \langle 0, 1\rangle$ is 0, which is not satisfactory. Moreover, it can be seen that the NMG of the q-ROFV χ_1 is 0.3, which is not 1 because $0.3 < 1$, but the NMG of aggregated result $q-ROFWG(\chi_1, \chi_2) = \langle 0, 1\rangle$ is 1, which is not satisfactory. This aggregated value is unreasonable as the q-ROFV χ_1 loses its effect in the presence of the q-ROFV χ_2, which is practically infeasible.

Example 2 Let $\chi_1 = \langle 0.4, 0.6\rangle$ and $\chi_2 = \langle 0, 1\rangle$ be two q-ROFVs with weights $w_1 = 0.4$ and $w_2 = 0.6$, respectively. By using the q-ROFEWG AO (Akram et al. 2021) given in Eq. (7) to aggregate the χ_1 and χ_2, it gets $q-ROFEWG(\chi_1, \chi_2) = \langle 0, 1\rangle$. It can be seen that the MG of the q-ROFV χ_1 is 0.2, which is not 0 because $0.4 > 0$, but the MG of aggregated result $q-ROFEWG(\chi_1, \chi_2) = \langle 0, 1\rangle$ is 0, which is not satisfactory. Moreover, it can be seen that the NMG of the q-ROFV χ_1 is 0.6, which is not 1 because $0.6 < 1$, but the NMG of aggregated result $q-ROFEWG(\chi_1, \chi_2) = \langle 0, 1\rangle$ is 1, which is not satisfactory. This aggregated value is unreasonable as the q-ROFV χ_1 loses its effect in the presence of the q-ROFV χ_2, which is practically infeasible.

Example 3 Let $\chi_1 = \langle 0.5, 0.3\rangle$ and $\chi_2 = \langle 0, 1\rangle$ be two q-ROFVs with weights $w_1 = 1$ and $w_2 = 0$, respectively. By using the q-ROFWG AO (Liu and Wang 2018) given in Eq. (6) to aggregate the χ_1 and χ_2, it can be seen that the terms $(\zeta_t)^{w_t}$ and $\left(1-\theta_t^q\right)^{w_t}$ given in Eq. (6) of the q-ROFWG AO (Liu and Wang 2018) are $(\zeta_2)^{w_2} = 0^0$ and $\left(1-\zeta_2^q\right)^{w_2} = 0^0$, respectively, where 0^0 and 0^0 are indeterminated values. Hence, the q-ROFWG AO (Liu and Wang 2018) given in Eq. (6) has the above shortcoming in this case.

Example 4 Let $\chi_1 = \langle 0, 1\rangle$ and $\chi_2 = \langle 0.6, 0.2\rangle$ be two q-ROFVs with weights $w_1 = 0$ and $w_2 = 1$, respectively. By using the q-ROFEWG AO (Akram et al. 2021) given in Eq. (7) to aggregate the χ_1 and χ_2, it can be seen that the terms $(\zeta_t)^{w_t}$ and $\left(1-\theta_t^q\right)^{w_t}$ given in Eq. (7) of the q-ROFEWG AO (Akram et al. 2021) are $(\zeta_1)^{w_1} = 0^0$ and $\left(1-\zeta_1^q\right)^{w_1} = 0^0$, respectively, where 0^0 and 0^0 are indeterminated values. Hence, the q-ROFWG AO (Akram et al. 2021) given in Eq. (7) has the above shortcoming in this case.

4 The proposed *q*-rung orthopair fuzzy Yager power weighted geometric aggregation operator

In this section, based on the Yager's operations of q-ROFVs given in Definition 7 and the PG AO given in Eq. (5), we propose the q-rung orthopair fuzzy Yager power weighted geometric (q-ROFYPWG) AO to conquer the shortcomings of the q-ROFWG AO (Liu and Wang 2018) and the q-ROFEWG AO (Akram et al. 2021) of the q-ROFVs.

Definition 8 Let $\chi_1 = \langle \zeta_1, \theta_1\rangle$, $\chi_2 = \langle \zeta_2, \theta_2\rangle$, ..., $\chi_s = \langle \zeta_s, \theta_s\rangle$ be q-ROFVs. For aggregating the $\chi_1 = \langle \zeta_1, \theta_1\rangle$, $\chi_2 = \langle \zeta_2, \theta_2\rangle$, ..., $\chi_s = \langle \zeta_s, \theta_s\rangle$, the proposed q-ROFYPWG AO is as follows:

$$
\begin{aligned}
&q\text{-ROFYPWG}(\chi_1, \chi_2, \ldots, \chi_s) \\
&= \bigotimes_{t=1}^{s} (\chi_t)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}}, \\
&= \left\langle \sqrt[q]{1-\min\left\{1, \left(\sum_{t=1}^{s} \frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}\left(1-\zeta_t^q\right)^{\lambda}\right)^{1/\lambda}\right\}}, \right. \\
&\left. \sqrt[q]{\min\left\{1, \left(\sum_{t=1}^{s} \frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}\left(\theta_t^q\right)^{\lambda}\right)^{1/\lambda}\right\}} \right\rangle,
\end{aligned}
\tag{8}
$$

where w_t represents the weight of χ_t, $w_t \geq 0$, $\sum_{t=1}^{s} w_t = 1$, $q \geq 1$, $\lambda > 0$, $T(\chi_t) = \sum_{l=1, l \neq t}^{s} Sup(\chi_t, \chi_l)$, and $Sup(\chi_t, \chi_l) = 1 - d_E(\chi_t, \chi_l) = 1 - \left(\frac{1}{2}\left(|\zeta_t^q - \zeta_l^q|^2 + |\theta_t^q - \theta_l^q|^2\right)\right)^{1/2}$ represents the support degree to which χ_l supports χ_t and meets the following criteria:

(i) $Sup(\chi_t, \chi_l) \in [0, 1]$;
(ii) $Sup(\chi_t, \chi_l) = Sup(\chi_l, \chi_t)$;
(iii) $Sup(\chi_t, \chi_l) \geq Sup(\chi_m, \chi_n)$ if $|\chi_t - \chi_l| \leq |\chi_m - \chi_n|$.

Example 5 Let $\chi_1 = \langle 0.4, 0.5\rangle$, $\chi_2 = \langle 0.3, 0.6\rangle$ and $\chi_3 = \langle 0.5, 0.2\rangle$ be three q-ROFVs with weights $w_1 = 0.4$, $w_2 = 0.3$ and $w_3 = 0.3$, respectively. First, we calculate the $Sup(\chi_t, \chi_l)$ between q-ROFVs χ_t and χ_l, where $q = 3$, $t, l = 1, 2, 3$, $t \neq l$,

$$\begin{aligned} Sup(\chi_1,\chi_2) =& 1-\left(\frac{1}{2}(|\zeta_1^q-\zeta_2^q|^2+|\theta_1^q-\theta_2^q|^2)\right)^{1/2}\\ =&1-\left(\frac{1}{2}(|0.4^3-0.3^3|^2+|0.5^3-0.6^3|^2)\right)^{1/2}\\ =&0.9305,\\ Sup(\chi_1,\chi_3) =& 1-\left(\frac{1}{2}(|\zeta_1^q-\zeta_3^q|^2+|\theta_1^q-\theta_3^q|^2)\right)^{1/2}\\ =&1-\left(\frac{1}{2}(|0.4^3-0.5^3|^2+|0.5^3-0.2^3|^2)\right)^{1/2}\\ =&0.9067,\\ Sup(\chi_2,\chi_3) =& 1-\left(\frac{1}{2}(|\zeta_2^q-\zeta_3^q|^2+|\theta_2^q-\theta_3^q|^2)\right)^{1/2}\\ =&1-\left(\frac{1}{2}(|0.3^3-0.5^3|^2+|0.6^3-0.2^3|^2)\right)^{1/2}\\ =&0.8374. \end{aligned}$$

In next step, we calculate the $T(\chi_1)$, $T(\chi_2)$ and $T(\chi_3)$ of the q-ROFVs χ_1, χ_2 and χ_3, respectively, as follows:

$$\begin{aligned} T(\chi_1) =& Sup(\chi_1,\chi_2)+Sup(\chi_1,\chi_3)\\ =&0.9305+0.9067=1.8372,\\ T(\chi_2) =& Sup(\chi_2,\chi_1)+Sup(\chi_2,\chi_3)\\ =&0.9305+0.8374=1.7680,\\ T(\chi_3) =& Sup(\chi_3,\chi_2)+Sup(\chi_3,\chi_1)\\ =&0.8374+0.9067=1.7441. \end{aligned}$$

Then, by using the Eq. (8), we aggregate the q-ROFVs χ_1, χ_2 and χ_3, where $q=3$, $\lambda=2$,

$$\begin{aligned} &q-ROFYPWG(\chi_1,\chi_2,\chi_3)\\ =&\left\langle \sqrt[3]{1-\min\left\{1,\left(\frac{0.4(1+1.8372)}{0.4(1+1.8372)+0.3(1+1.7680)+0.3(1+1.7441)}(1-0.4^3)^2 + \frac{0.3(1+1.7680)}{0.4(1+1.8372)+0.3(1+1.7680)+0.3(1+1.7441)}(1-0.3^3)^2 + \frac{0.3(1+1.7441)}{0.4(1+1.8372)+0.3(1+1.7680)+0.3(1+1.7441)}(1-0.5^3)^2\right)^{1/2}\right\}},\right.\\ &\left.\sqrt[3]{\min\left\{1,\left(\frac{0.4(1+1.8372)}{0.4(1+1.8372)+0.3(1+1.7680)+0.3(1+1.7441)}(0.5^3)^2 + \frac{0.3(1+1.7680)}{0.4(1+1.8372)+0.3(1+1.7680)+0.3(1+1.7441)}(0.6^3)^2 + \frac{0.3(1+1.7441)}{0.4(1+1.8372)+0.3(1+1.7680)+0.3(1+1.7441)}(0.2^3)^2\right)^{1/2}\right\}}\right\rangle\\ =&\langle 0.4125, 0.5222\rangle. \end{aligned}$$

In the following, we present a few characteristics of the proposed q-ROFYPWG AO given in Eq. (8).

Property 1 (Idempotency) Let $\chi_1, \chi_2, \ldots, \chi_s$ be q-ROFVs with weights $w_1, w_2, \ldots, w_s$, respectively, where $w_t \geq 0$ and $\sum_{t=1}^{s} w_t = 1$. If $\chi_1 = \chi_2, \ldots = \chi_s = \chi$, then

$$q-ROFYPWG(\chi_1,\chi_2,\ldots,\chi_s)=\chi.$$

Proof The weights of q-ROFVs $\chi_1, \chi_2, \ldots, \chi_s$ are $w_1, w_2, \ldots, w_s$, respectively, where $w_t \geq 0$ and $\sum_{t=1}^{s} w_t = 1$. If $\chi_1 = \chi_2, \ldots = \chi_s = \chi$, then by using Eq. (8), we have

$$\begin{aligned} q\text{-ROFYPWG}(\chi_1,\chi_2,\ldots,\chi_s) =& \bigotimes_{t=1}^{s}(\chi_t)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}}\\ =& \bigotimes_{t=1}^{s}(\chi)^{\frac{w_t(1+T(\chi))}{\sum_{t=1}^{s} w_t(1+T(\chi))}}\\ =& \chi^{\sum_{t=1}^{s} w_t}\\ =& \chi. \end{aligned}$$

Property 2 Let $\chi_1, \chi_2, \ldots, \chi_s$ be q-RONs, let $\chi^- = \min\{\chi_1, \chi_2, \ldots, \chi_s\}$ and let $\chi^+ = \max\{\chi_1, \chi_2, \ldots, \chi_s\}$. Then,

$$\chi^- \leq \text{q-ROFYPWG}(\chi_1,\chi_2,\ldots,\chi_s) \leq \chi^+.$$

Proof Because $\chi^- = \min\{\chi_1, \chi_2, \ldots, \chi_n\}$ and $\chi^+ = \max\{\chi_1, \chi_2, \ldots, \chi_n\}$, by using Eq. (8), we obtain

$$q\text{-ROFYPWG}(\chi_1, \chi_2, \ldots, \chi_s) = \bigotimes_{t=1}^{s} (\chi_t)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}} \leq \bigotimes_{t=1}^{s} (\chi^+)^{\frac{w_t(1+T(\chi^+))}{\sum_{t=1}^{s} w_t(1+T(\chi^+))}} = (\chi^+)^{\sum_{t=1}^{s} w_t} = \chi^+,$$

$$q\text{-ROFYPWG}(\chi_1, \chi_2, \ldots, \chi_s) = \bigotimes_{t=1}^{s} (\chi_t)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}} \geq \bigotimes_{t=1}^{s} (\chi^-)^{\frac{w_t(1+T(\chi^-))}{\sum_{t=1}^{s} w_t(1+T(\chi^-))}} = (\chi^-)^{\sum_{t=1}^{s} w_t} = \chi^-.$$

Hence, we get $\chi^- \leq$ q-ROFYPWG$(\chi_1, \chi_2, \ldots, \chi_s) \leq \chi^+$.

Property 3 (Monotonicity) Let $\chi_1, \chi_2, \ldots, \chi_s, \chi_1^*, \chi_2^*, \ldots,$ and χ_s^* be q-ROFVs. If $\chi_t \leq \chi_t^*$, where $t = 1, 2, \ldots, n$, then

$$q - ROFYPWG(\chi_1, \chi_2, \ldots, \chi_n) \leq q - ROFYPWG(\chi_1^*, \chi_2^*, \ldots, \chi_n^*).$$

Proof By utilizing Eq. (8), we obtain

$$q - ROFYPWG(\chi_1, \chi_2, \ldots, \chi_s) = \bigotimes_{t=1}^{s} (\chi_t)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}},$$

$$q - ROFYPWG(\chi_1^*, \chi_2^*, \ldots, \chi_s^*) = \bigotimes_{t=1}^{s} (\chi_t^*)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}}$$

Because $\chi_t \leq \chi_t^*$, $\forall t = 1, 2, \ldots, n$, we obtain

$$\bigotimes_{t=1}^{s} (\chi_t)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}} \leq \bigotimes_{t=1}^{s} (\chi_t^*)^{\frac{w_t(1+T(\chi_t))}{\sum_{t=1}^{s} w_t(1+T(\chi_t))}}.$$

Hence, $q - ROFYPWG(\chi_1, \chi_2, \ldots, \chi_s) \leq q - ROFYPWG(\chi_1^*, \chi_2^*, \ldots, \chi_s^*)$.

In the following, we show how the proposed q-ROFYPWG AO can conquer the shortcomings of the q-ROFWG AO (Liu and Wang 2018) and q-ROFEWG AO (Akram et al. 2021) of the q-ROFVs shown in Sect. 3, as follows:

Example 6 Let us consider same q-ROFVs $\chi_1 = \langle 0.2, 0.3 \rangle$ and $\chi_2 = \langle 0, 1 \rangle$ with weights $w_1 = 0.3$ and $w_2 = 0.7$, respectively, given in Example 1. By utilizing the q-ROFYPWG AO given in Eq. (8), we obtain

$$q - ROFYPWG(\chi_1, \chi_2) = \langle 0.1338, 0.9423 \rangle.$$

Hence, the q-ROFYPWG AO given in Eq. (8) can conquer the shortcoming of q-ROFWG AO (Liu and Wang 2018), shown in Example 1.

Example 7 Let us consider same q-ROFVs $\chi_1 = \langle 0.4, 0.6 \rangle$ and $\chi_2 = \langle 0, 1 \rangle$ with weights $w_1 = 0.4$ and $w_2 = 0.6$, respectively, given in Example 2. By utilizing the q-ROFYPWG AO given in Eq. (8), we obtain

$$q - ROFYPWG(\chi_1, \chi_2) = \langle 0.2928, 0.9231 \rangle.$$

Hence, the q-ROFYPWG AO given in Eq. (8) can conquer the shortcoming of q-ROFEWG AO (Akram et al. 2021), shown in Example 2.

Example 8 Let us consider same q-ROFVs $\chi_1 = \langle 0.5, 0.3 \rangle$ and $\chi_2 = \langle 0, 1 \rangle$ with weights $w_1 = 1$ and $w_2 = 0$, respectively, given in Example 3. By utilizing the q-ROFYPWG AO given in Eq. (8), we obtain

$$q - ROFYPWG(\chi_1, \chi_2) = \langle 0.5, 0.3 \rangle.$$

Hence, the proposed q-ROFYPWG AO given in Eq. (8) can conquer the shortcoming of q-ROFWG AO (Liu and Wang 2018), shown in Example 3.

Example 9 Let us consider same q-ROFVs $\chi_1 = \langle 0, 1 \rangle$ and $\chi_2 = \langle 0.6, 0.2 \rangle$ with weights $w_1 = 0$ and $w_2 = 1$, respectively, given in Example 4. By utilizing the q-ROFYPWG AO given in Eq. (8), we obtain

$$q - ROFYPWG(\chi_1, \chi_2) = \langle 0.6, 0.2 \rangle.$$

Hence, the q-ROFYPWG AO given in Eq. (8) can conquer the shortcoming of q-ROFEWG AO (Akram et al. 2021), shown in Example 4.

5 Analyzing the shortcomings of the Akram et al.'s MADM approach

In this section, we explore the shortcomings of Akram et al.'s MADM approach (Akram et al. 2021). Let $O_1, O_2, \ldots, O_r$ be alternatives and $C_1, C_2, \ldots, C_s$ be attributes with weights $w_1, w_2, \ldots, w_s$ such that $w_t \in [0, 1]$ and $\sum_{t=1}^{s} w_t = 1$. The expert assess the alternative O_k with respect to the attribute C_t by utilizing the q-ROFVs $\widetilde{\chi}_{kt} = \langle \widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt} \rangle$ to construct the decision matrix (DMx) $\widetilde{D} = (\widetilde{\chi}_{kt})_{r\times s}$ as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ \vdots \\ O_r \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_s \end{array} \\ \begin{pmatrix} \widetilde{\chi}_{11} & \widetilde{\chi}_{12} & \ldots & \widetilde{\chi}_{1s} \\ \widetilde{\chi}_{21} & \widetilde{\chi}_{22} & \ldots & \widetilde{\chi}_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\chi}_{r1} & \widetilde{\chi}_{r2} & \ldots & \widetilde{\chi}_{rs} \end{pmatrix} \end{array}.$$

where $k = 1, 2, \ldots, r$ and $t = 1, 2, \ldots, s$. We review the Akram et al.'s MADM approach (Akram et al. 2021) as follows:

Step 1: By using the q-ROFEWG AO given in Eq. (7), aggregate the q-ROFVs χ_{k1}, $\chi_{k2},\ldots,\chi_{ks}$ appeared in k^{th} row of NDMx $D=(\chi_{kt})_{r\times s}$ to obtain the overall aggregated q-ROFV $\chi_k = \langle\zeta_k,\theta_k\rangle$ of alternative O_k, where

$$\begin{aligned}\chi_k =& q\text{-ROFEWG}(\chi_{k1},\chi_{k2},\ldots,\chi_{ks})\\ =&\left\langle \frac{\sqrt[q]{2}\prod_{t=1}^{s}{\zeta_{kt}}^{w_t}}{\sqrt[q]{\prod_{t=1}^{s}(2-\zeta_{kt}^q)^{w_t}+\prod_{t=1}^{s}(\zeta_{kt}^q)^{w_t}}},\right.\\ &\left.\sqrt{q\frac{\prod_{t=1}^{s}(1+\theta_{kt}^q)^{w_t}-\prod_{t=1}^{s}(1-\theta_{kt}^q)^{w_t}}{\prod_{t=1}^{s}(1+\theta_{kt}^q)^{w_t}+\prod_{t=1}^{s}(1-\theta_{kt}^q)^{w_t}}}\right\rangle,\end{aligned} \tag{9}$$

$k=1,2,\ldots,r$, $w_t\in[0,1]$ and $\sum_{t=1}^{s}w_t=1$.

Step 2: Compute the score value $S(\chi_k)$ of the overall q-ROFV $\chi_k=\langle\zeta_k,\theta_k\rangle$ of alternative O_k obtained in Step 1, shown as follows:

$$S(\chi_k)=(\zeta_k)^q-(\theta_k)^q, \tag{10}$$

and compute the accuracy value $A_c(\chi_k)$ of the overall q-ROFV $\chi_k=\langle\zeta_k,\theta_k\rangle$ obtained in Step 1 as follows:

$$A_c(\chi_k)=(\zeta_k)^q+(\theta_k)^q, \tag{11}$$

where $k=1,2,\ldots,r$.

Step 3: Find the ranking order (RO) of the alternatives $O_1, O_2,\ldots,O_r$ based on Definition 5 and select the best alternative.

Example 10 Let O_1, O_2 and O_3 be three alternatives and let C_1, C_2 and C_3 be three beneficiary type attributes with weights $w_1=0.3$, $w_2=0.4$ and $w_3=0.3$. The expert evaluates the alternatives O_1, O_2, and O_3 with respect to the attribute C_1, C_2 and C_3 by using an q-ROFV $\widetilde{\chi}_{kt}$ to obtain the DMx $\widetilde{D}=(\chi_{kt})_{3\times3}=\left(\widetilde{\zeta}_{kt},\widetilde{\theta}_{kt}\right)_{3\times3}$, shown as follows:

$$\widetilde{D}=\begin{array}{c} \\ O_1\\ O_2\\ O_3\end{array}\begin{array}{c}\begin{array}{ccc}C_1 & C_2 & C_3\end{array}\\ \begin{pmatrix}\langle0.2,0.3\rangle & \langle0.4,0.6\rangle & \langle0,1\rangle\\ \langle0.6,0.7\rangle & \langle0,1\rangle & \langle0.2,0.3\rangle\\ \langle0,1\rangle & \langle0.3,0.6\rangle & \langle0.1,0.4\rangle\end{pmatrix}\end{array}.$$

Step 1: By utilizing Eq. (9), Akram et al's MADM approach (Akram et al. 2021) obtains the overall q-ROFVs χ_1, χ_2 and χ_3 of the alternatives O_1, O_2 and O_3, respectively, where $q=3$, $\chi_1=\langle0,1\rangle$, $\chi_2=\langle0,1\rangle$, and $\chi_3=\langle0,1\rangle$.

Step 2: By utilizing Eq. (10), Akram et al's MADM approach (Akram et al. 2021) gets the score values $S(\chi_1)$, $S(\chi_2)$ and $S(\chi_3)$ of the overall q-ROFVs χ_1, χ_2 and χ_3 obtained in Step 1, respectively, where $q=3$, $S(\chi_1)=-1$, $S(\chi_2)=-1$ and $S(\chi_3)=-1$. Because $S(\chi_1)=S(\chi_2)=S(\chi_3)$, where $S(\chi_1)=-1$, $S(\chi_2)=-1$ and $S(\chi_3)=-1$, by using Eq. (11), Akram et al's MADM approach (Akram et al. 2021) gets the accuracy values $A_c(\chi_1)$, $A_c(\chi_2)$ and $A_c(\chi_3)$ of the overall q-ROFVs χ_1, χ_2 and χ_3, respectively, where $A_c(\chi_1)=1$, $A_c(\chi_2)=1$ and $A_c(\chi_3)=1$.

Step 3: Since $S(\chi_1)=S(\chi_2)=S(\chi_3)$, where $S(\chi_1)=-1$, $S(\chi_2)=-1$ and $S(\chi_3)=-1$, and because $A_c(\chi_1)=A_c(\chi_2)=A_c(\chi_3)$, where $A_c(\chi_1)=1$, $A_c(\chi_2)=1$ and $A_c(\chi_3)=1$, based on Definition 5, Akram et al's MADM approach (Akram et al. 2021) obtains the RO "$O_1=O_2=O_3$" of the alternatives O_1, O_2 and O_3.

Example 11 Let O_1, O_2 and O_3 be three alternatives and let C_1, C_2 and C_3 be three beneficiary type attributes with weights $w_1=0.28$, $w_2=0.35$ and $w_3=0.37$. The expert evaluates the alternatives O_1, O_2, and O_3 with respect to the attribute C_1, C_2 and C_3 by using an q-ROFV $\widetilde{\chi}_{kt}$ to obtain the DMx $\widetilde{D}=(\chi_{kt})_{3\times3}=\left(\widetilde{\zeta}_{kt},\widetilde{\theta}_{kt}\right)_{3\times3}$, shown as follows:

$$\widetilde{D}=\begin{array}{c} \\ O_1\\ O_2\\ O_3\end{array}\begin{array}{c}\begin{array}{ccc}C_1 & C_2 & C_3\end{array}\\ \begin{pmatrix}\langle0,1\rangle & \langle0.4,0.5\rangle & \langle0.8,0.9\rangle\\ \langle0.7,0.4\rangle & \langle0.2,0.3\rangle & \langle0,1\rangle\\ \langle0.2,0.3\rangle & \langle0,1\rangle & \langle0.6,0.7\rangle\end{pmatrix}\end{array}.$$

Step 1: By utilizing Eq. (9), Akram et al's MADM approach (Akram et al. 2021) obtains the overall q-ROFVs χ_1, χ_2 and χ_3 of the alternatives O_1, O_2 and O_3, respectively, where $q=3$, $\chi_1=\langle0,1\rangle$, $\chi_2=\langle0,1\rangle$, and $\chi_3=\langle0,1\rangle$.

Step 2: By utilizing Eq. (10), Akram et al's MADM approach (Akram et al. 2021) gets the score values $S(\chi_1)$, $S(\chi_2)$ and $S(\chi_3)$ of the overall q-ROFVs χ_1, χ_2 and χ_3 obtained in Step 1, respectively, where $q=3$, $S(\chi_1)=-1$, $S(\chi_2)=-1$ and $S(\chi_3)=-1$. Because $S(\chi_1)=S(\chi_2)=S(\chi_3)$, where $S(\chi_1)=-1$, $S(\chi_2)=-1$ and $S(\chi_3)=-1$, by using Eq. (11), Akram et al's MADM approach (Akram et al. 2021) gets the accuracy values $A_c(\chi_1)$, $A_c(\chi_2)$ and $A_c(\chi_3)$ of the overall q-ROFVs χ_1, χ_2 and χ_3, respectively, where $A_c(\chi_1)=1$, $A_c(\chi_2)=1$ and $A_c(\chi_3)=1$.

Step 3: Since $S(\chi_1) = S(\chi_2) = S(\chi_3)$, where $S(\chi_1) = -1$, $S(\chi_2) = -1$ and $S(\chi_3) = -1$, and because $A_c(\chi_1) = A_c(\chi_2) = A_c(\chi_3)$, where $A_c(\chi_1) = 1$, $A_c(\chi_2) = 1$ and $A_c(\chi_3) = 1$, based on Definition 5, Akram et al's MADM approach (Akram et al. 2021) obtains the RO "$O_1 = O_2 = O_3$" of the alternatives O_1, O_2 and O_3.

6 A new MADM approach based on the proposed q-ROFYWG AO of q-ROFVs

In this section, we develop a new MADM approach by using the proposed q-ROFYWG AO of q-ROFVs. Let $O_1, O_2, \ldots, O_r$ be alternatives and $C_1, C_2, \ldots, C_s$ be attributes with weights $w_1, w_2, \ldots, w_s$, respectively, where $w_t \in [0,1]$ and $\sum_{t=1}^{s} w_t = 1$. By using the q-ROFVs $\widetilde{\chi}_{kt} = \langle \widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt} \rangle$, the expert evaluates the alternative O_k with respect to the attribute C_t to construct the DMx $\widetilde{D} = (\widetilde{\chi}_{kt})_{r\times s}$, shown as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ \vdots \\ O_r \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_s \end{array} \\ \begin{pmatrix} \widetilde{\chi}_{11} & \widetilde{\chi}_{12} & \cdots & \widetilde{\chi}_{1s} \\ \widetilde{\chi}_{21} & \widetilde{\chi}_{22} & \cdots & \widetilde{\chi}_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\chi}_{r1} & \widetilde{\chi}_{r2} & \cdots & \widetilde{\chi}_{rs} \end{pmatrix} \end{array}.$$

where $k = 1, 2, \ldots, r$ and $t = 1, 2, \ldots, s$. The proposed MADM approach is presented as follows:

Step 1: Transform the DMx $\widetilde{D} = (\widetilde{\chi}_{kt})_{r\times s} = \left(\langle \widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt} \rangle\right)_{r\times s}$ into NDMx $D = (\chi_{kt})_{r\times s} = \langle \zeta_{kt}, \theta_{kt} \rangle$ as follows:

$$\chi_{kt} = \begin{cases} \langle \widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt} \rangle & : \text{if } C_t \text{ is a benefit type attribute} \\ \langle \widetilde{\theta}_{kt}, \widetilde{\zeta}_{kt} \rangle & : \text{if } C_t \text{ is a cost type attribute} \end{cases} \tag{12}$$

Step 2: Based on Eq. (2), calculate the support measure $Sup(\chi_{kt}, \chi_{kl})$ between q-ROFVs χ_{kt} and χ_{kl} as follows:

$$\begin{aligned} Sup(\chi_{kt}, \chi_{kl}) &= 1 - d_E(\chi_{kt}, \mu_{kl}) \\ &= 1 - \left(\frac{1}{2}(|\zeta_{kt}^q - \zeta_{kl}^q|^2 + |\theta_{kt}^q - \theta_{kl}^q|^2)\right)^{1/2} \end{aligned} \tag{13}$$

where $k = 1, 2, \ldots, r;\ t, l = 1, 2, \ldots, s;\ l \neq t$, $d(\chi_{kt}, \chi_{kl})$ is Hamming distance between q-ROFVs χ_{kt} and χ_{kl}.

Step 3: Calculate support T_{kt} and weight δ_{kt} for the q-ROFV χ_{kt} as follows:

$$T_{kt} = \sum_{\substack{l=1 \\ l \neq t}}^{s} Sup(\chi_{kt}, \chi_{kl}), \tag{14}$$

and

$$\delta_{kt} = \frac{w_t(1 + T(\chi_{kt}))}{\sum_{t=1}^{n} w_t(1 + T(\chi_{kt}))}, \tag{15}$$

where $\delta_{kt} \in [0,1]$ and $\sum_{t=1}^{s} \delta_{kt} = 1$.

Step 4: By applying the proposed q-ROFYWG AO given in Eq. (8), aggregate the q-ROFVs $\chi_{k1}, \chi_{k2}, \ldots, \chi_{ks}$ appeared in k^{th} row of NDMx $D = (\chi_{kt})_{m\times n}$ to obtain the overall aggregated q-ROFV $\chi_k = \langle \zeta_k, \theta_k \rangle$ of alternative O_k, shown as follows:

$$\begin{aligned} \chi_k &= \langle \eta_k, \upsilon_k \rangle \\ &= q\text{-ROFYWG}(\chi_{k1}, \chi_{k2}, \ldots, \chi_{ks}) \\ &= \left\langle \left(1 - \min\left\{1, \left(\sum_{t=1}^{s} \delta_{kt}(1 - \zeta_{kt}^q)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}\right)^{\frac{1}{q}}, \right. \\ &\quad \left. \left(\min\left\{1, \left(\sum_{t=1}^{s} \delta_{kt}(\theta_{kt}^q)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}\right)^{\frac{1}{q}} \right\rangle, \end{aligned} \tag{16}$$

where $q \geq 1$, $\lambda > 0$, $\delta_{kt} \geq 0$ and $\sum_{t=1}^{s} \delta_{kt} = 1$.

Step 5: Calculate the ranking value (RV) $\psi(\chi_k)$ of the overall q-ROFV $\chi_k = \langle \zeta_k, \theta_k \rangle$ of alternative O_k obtained in Step 4, shown as follows:

$$\psi(\chi_k) = \left((\zeta_k)^q + \frac{(\pi_k)^q}{2}\right)((\zeta_k)^q + (\theta_k)^q), \tag{17}$$

where $(\pi_k)^q = 1 - (\zeta_k)^q - (\theta_k)^q$ and $k = 1, 2, \ldots, r$.

Step 6: Arrange the RVs $\psi(\chi_1)$, $\psi(\chi_2)$,..., $\psi(\chi_r)$ of alternatives O_1, O_2,...,O_r in decreasing sequence and obtain the RO of the alternatives O_1, O_2,...,O_r. If the RVs $\psi(\chi_\alpha)$ and $\psi(\chi_\beta)$ of alternatives O_α and O_β, respectively, are equal, i.e., $\psi(\chi_\alpha) = \psi(\chi_\beta)$, then calculate the accuracy values $H(\chi_\alpha) = \zeta_\alpha^q + \theta_\alpha^q$ and $H(\chi_\beta) = \zeta_\beta^q + \theta_\beta^q$ of the overall q-ROFVs $\chi_\beta = \langle\zeta_\alpha, \theta_\alpha\rangle$ and $\chi_\beta = \langle\zeta_\alpha, \theta_\alpha\rangle$ respectively. The larger the accuracy value $H(\chi_k)$, the better the RO of alternative O_k, where $k = 1, 2, ..., r$. If $\psi(\chi_\alpha) = \psi(\chi_\beta)$ and $H(\chi_\alpha) = H(\chi_\beta)$, the alternatives O_α and O_β have the same RO.

Example 12 (Akram et al. 2021) The government wants to choose the ideal location for a thermal power station (TPS) plant to cover the requirements of electric power. Let O_1, O_2, O_3 and O_4 be the possible locations as alternatives to setup a TPS plant. Let the C_1 ("Availability of coal"), C_2 ("Availability of water"), C_3 ("Transportation facilities") be the three attributes for the judgement of a best TPS location. The attributes C_1, C_2 and C_3 have the weights $w_1 = 0.4$, $w_2 = 0.3$ and $w_3 = 0.3$, respectively. The expert evaluates the TPS locations $O_1, O_2,$, O_3 and O_4 with respect to the attribute C_1, C_2 and C_3 by using an q-ROFV $\widetilde{\chi}_{kt}$ to construct the DMx $\widetilde{D} = (\chi_{kt})_{4\times3} = \left(\widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt}\right)_{4\times3}$, shown as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.6, 0.4\rangle & \langle 0.7, 0.3\rangle & \langle 0.8, 0.2\rangle \\ \langle 0.4, 0.6\rangle & \langle 0.7, 0.2\rangle & \langle 0.5, 0.4\rangle \\ \langle 0.5, 0.4\rangle & \langle 0.8, 0.1\rangle & \langle 0.4, 0.5\rangle \\ \langle 0.4, 0.5\rangle & \langle 0.9, 0.1\rangle & \langle 0.3, 0.5\rangle \end{pmatrix} \end{array}$$

We utilize the MADM approach presented in this paper to solve this MADM problem as follows:

Step 1: Since all the attributes C_1, C_2 and C_3 are of benefit type, therefore we get the NDMx $D = (\zeta_{kt}, \theta_{kt})_{4\times3} = \left(\widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt}\right)_{4\times3}$.

Step 2: By utilizing Eq. (13), we calculate the support measure $Sup(\chi_{kt}, \chi_{kl})$ between q-ROFVs χ_{kt} and χ_{kl}, where $q = 3$, $k = 1, 2, 3, 4$; $t, l = 1, 2, 3$; $l \neq t$, $Sup(\chi_{11}, \chi_{12}) = 0.9065$, $Sup(\chi_{11}, \chi_{13}) = 0.7870$, $Sup(\chi_{12}, \chi_{13}) = 0.8797$, $Sup(\chi_{21}, \chi_{22}) = 0.7539$, $Sup(\chi_{21}, \chi_{23}) = 0.8842$, $Sup(\chi_{22}, \chi_{23}) = 0.8408$, $Sup(\chi_{31}, \chi_{32}) = 0.7227$, $Sup(\chi_{31}, \chi_{33}) = 0.9390$, $Sup(\chi_{32}, \chi_{33}) = 0.6713$, $Sup(\chi_{41}, \chi_{42}) = 0.5217$, $Sup(\chi_{41}, \chi_{43}) = 0.9738$ and $Sup(\chi_{32}, \chi_{33}) = 0.4959$.

Step 3: By utilizing Eq. (14), we calculate the support T_{kt} of the q-ROFVs χ_{kt}, where

$$T_{11} = 1.6934, T_{12} = 1.7862, T_{13} = 1.6667,$$
$$T_{21} = 1.6381, T_{22} = 1.5948, T_{23} = 1.7250,$$
$$T_{31} = 1.6617, T_{32} = 1.3941, T_{33} = 1.6103,$$
$$T_{41} = 1.4955, T_{42} = 1.0176, T_{43} = 1.4698,$$

and, based on Eq. (15), we calculate the weight δ_{kt} of the q-ROFV χ_{kt}, where

$$\delta_{11} = 0.3971, \delta_{12} = 0.3081, \delta_{13} = 0.2949,$$
$$\delta_{21} = 0.3980, \delta_{22} = 0.2936, \delta_{23} = 0.3084,$$
$$\delta_{31} = 0.4149, \delta_{32} = 0.2799, \delta_{33} = 0.3052,$$
$$\delta_{41} = 0.4258, \delta_{42} = 0.2582, \delta_{43} = 0.3160.$$

Step 4: By utilizing Eq. (16), we obtain the overall q-ROFV χ_k of the alternative O_k, where $\lambda = 2$, $q = 3$, $k = 1, 2, 3, 4$, $\chi_1 = \langle 0.6919, 0.3510\rangle$, $\chi_2 = \langle 0.5389, 0.5204\rangle$, $\chi_3 = \langle 0.5776, 0.4316\rangle$ and $\chi_4 = \langle 0.5525, 0.4757\rangle$.

Step 5: By utilizing Eq. (17), we obtain the RVs $\psi(\chi_1)$, $\psi(\chi_2)$, $\psi(\chi_3)$ and $\psi(\chi_4)$ of the overall q-ROFVs χ_1, χ_2, χ_3 and χ_4 obtained in Step 4, respectively, where $\psi(\chi_1) = 0.2412$, $\psi(\chi_2) = 0.1510$, $\psi(\chi_3) = 0.1519$ and $\psi(\chi_4) = 0.1466$.

Step 6: Because $\psi(\chi_1) > \psi(\chi_3) > \psi(\chi_2) > \psi(\chi_4)$, where $\psi(\chi_1) = 0.2412$, $\psi(\chi_2) = 0.1510$, $\psi(\chi_3) = 0.1519$ and $\psi(\chi_4) = 0.1466$, the RO of the alternatives O_1, O_2, O_3 and O_4 is "$O_1 \succ O_3 \succ O_2 \succ O_4$", where O_1 is the best alternative among the alternatives O_1, O_2, O_3 and O_4.

Table 1 presents the ROs of the alternatives obtained by using different MADM approaches for Example 12. It shows that Akram et al.'s MADM approach based on the q-

Table 1 The ROs of the alternatives achieved by different MADM approaches for Example 12

MADM approaches	ROs
Akram et al.'s MADM approach(Akram et al. 2021)	$O_1 \succ O_3 \succ O_2 \succ O_4$
Liu and Wang's MADM approach (Liu and Wang 2018)	$O_1 \succ O_3 \succ O_2 \succ O_4$
Proposed MADM approach	$O_1 \succ O_3 \succ O_2 \succ O_4$

Table 2 The ROs of the alternatives achieved by different MADM approaches for Example 13

MADM approaches	ROs
Akram et al.'s MADM approach (Akram et al. 2021)	$O_1 = O_2 = O_3$
Liu and Wang's MADM approach (Liu and Wang 2018)	$O_1 = O_2 = O_3$
Proposed MADM approach	$O_2 \succ O_1 \succ O_3$

ROFEWG AO (Akram et al. 2021), Liu and Wang's MADM approach based on the q-ROFWG AO (Liu and Wang 2018) and the proposed GDM approach achieve the same RO "$O_1 \succ O_3 \succ O_2 \succ O_4$" of the alternatives O_1, O_2, O_3 and O_4.

Example 13 We utilize the MADM approach presented in this paper to solve the Example 10, shown as follows:

Step 1: Since all the attributes C_1, C_2 and C_3 are of benefit type, therefore we obtain the NDMx $D = (\zeta_{kt}, \theta_{kt})_{3\times3} = \left(\widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt}\right)_{3\times3}$.

Step 2: Based on Eq. (13), we calculate the support measure $Sup(\chi_{kt}, \chi_{kl}) = S_{kt,kl}$ between q-ROFVs χ_{kt} and χ_{kl}, where $q = 3$, $k = 1, 2, 3$; $t, l = 1, 2, 3$; $l \neq t$, $Sup(\chi_{11}, \chi_{12}) = 0.8606$, $Sup(\chi_{11}, \chi_{13}) = 0.3120$, $Sup(\chi_{12}, \chi_{13}) = 0.4438$, $Sup(\chi_{21}, \chi_{22}) = 0.5110$, $Sup(\chi_{21}, \chi_{23}) = 0.7325$, $Sup(\chi_{22}, \chi_{23}) = 0.3120$, $Sup(\chi_{31}, \chi_{32}) = 0.4443$, $Sup(\chi_{31}, \chi_{33}) = 0.3381$ and $Sup(\chi_{32}, \chi_{33}) = 0.8910$.

Step 3: By utilizing Eq. (14), we calculate the support T_{kt} of the q-ROFVs χ_{kt}, where

$$T_{11} = 1.1726, T_{12} = 1.3044, T_{13} = 0.7557,$$
$$T_{21} = 1.2435, T_{22} = 0.8229, T_{23} = 1.0445,$$
$$T_{31} = 0.7834, T_{32} = 1.3363, T_{33} = 1.2291,$$

and, by utilizing Eq. (15), we calculate the weight δ_{kt} of the q-ROFV χ_{kt}, where

$$\delta_{11} = 0.3103, \delta_{12} = 0.4389, \delta_{13} = 0.2508,$$
$$\delta_{21} = 0.3339, \delta_{22} = 0.3618, \delta_{23} = 0.3043,$$
$$\delta_{31} = 0.2502, \delta_{32} = 0.4370, \delta_{33} = 0.3127.$$

Step 4: By utilizing Eq. (16), we obtain the overall q-ROFV χ_k of the alternative O_k, where $\lambda = 2$, $q = 3$, $k = 1, 2, 3, 4$, $\chi_1 = \langle 0.3111, 0.8047\rangle$, $\chi_2 = \langle 0.4105, 0.8588\rangle$ and $\chi_3 = \langle 0.2291, 0.8049\rangle$.

Step 5: By utilizing the Eq. (17), we obtain the RVs $\psi(\chi_1)$, $\psi(\chi_2)$ and $\psi(\chi_3)$ of the overall q-ROFVs χ_1, χ_2 and χ_3 obtained in Step 4, respectively, where $\psi(\chi_1) = 0.1403$, $\psi(\chi_2) = 0.1531$ and $\psi(\chi_3) = 0.1309$.

Step 6: Because $\psi(\chi_2) > \psi(\chi_1) > \psi(\chi_3)$, where $\psi(\chi_1) = 0.1403$, $\psi(\chi_2) = 0.1531$ and $\psi(\chi_3) = 0.1309$, the RO of the alternatives O_1, O_2 and O_3 is "$O_2 \succ O_1 \succ O_3$", where O_2 is the best alternative among the alternatives O_1, O_2 and O_3.

Table 2 represents the ROs of the alternatives obtained by using different MADM approaches for Example 13. It shows that Akram et al.'s MADM approach based on the q-ROFEWG AO (Akram et al. 2021), Liu and Wang's MADM approach based on the q-ROFWG AO (Liu and Wang 2018) achieve the same RO "$O_1 = O_2 = O_3$" of the alternatives O_1, O_2 and O_3, where they have the drawbacks that they cannot distinguish the RO of the alternatives O_1, O_2 and O_3 in this situation; the proposed MADM approach gets the RO "$O_2 \succ O_1 \succ O_3$" of the alternatives O_1, O_2 and O_3. Hence, the proposed MADM approach can conquer the shortcomings of Akram et al.'s MADM approach based on the q-ROFEWG AO (Akram et al. 2021), Liu and Wang's MADM approach based on the q-ROFWG AO (Liu and Wang 2018) in this situation.

Example 14 We utilize the MADM approach presented in this paper to solve the Example 11, shown as follows:

Step 1: Since all the attributes C_1, C_2 and C_3 are of benefit type, therefore we obtain the NDMx $D = (\zeta_{kt}, \theta_{kt})_{3\times3} = \left(\widetilde{\zeta}_{kt}, \widetilde{\theta}_{kt}\right)_{3\times3}$.

Step 2: By utilizing Eq. (13), we calculate the support measure $Sup(\chi_{kt}, \chi_{kl}) = S_{kt,kl}$ between q-ROFVs χ_{kt} and χ_{kl}, where $q = 3$, $k = 1, 2, 3$; $t, l = 1, 2, 3$; $l \neq t$, $Sup(\chi_{11}, \chi_{12}) = 0.3796$, $Sup(\chi_{11}, \chi_{13}) = 0.5904$, $Sup(\chi_{12}, \chi_{13}) = 0.4682$, $Sup(\chi_{21}, \chi_{22}) = 0.7617$, $Sup(\chi_{21}, \chi_{23}) = 0.2951$, $Sup(\chi_{22}, \chi_{23}) = 0.3120$, $Sup(\chi_{31}, \chi_{32}) = 0.3120$, $Sup(\chi_{31}, \chi_{33}) = 0.7325$ and $Sup(\chi_{32}, \chi_{33}) = 0.5110$.

Table 3 The ROs of the alternatives achieved by different MADM approaches for Example 14

MADM approaches	ROs
Akram et al.'s MADM approach (Akram et al. 2021)	$O_1 = O_2 = O_3$
Liu and Wang's MADM approach (Liu and Wang 2018)	$O_1 = O_2 = O_3$
Proposed MADM approach	$O_1 \succ O_2 \succ O_3$

Step 3: By utilizing Eq. (14), we calculate the support T_{kt} of the q-ROFVs χ_{kt}, where

$$T_{11} = 0.9700, T_{12} = 0.8479, T_{13} = 1.0586,$$
$$T_{21} = 1.0568, T_{22} = 1.0736, T_{23} = 0.6071,$$
$$T_{31} = 1.0445, T_{32} = 0.8229, T_{33} = 1.2435,$$

and, by utilizing Eq. (15), we calculate the weight δ_{kt} of the q-ROFV χ_{kt}, where

$$\delta_{11} = 0.2814, \delta_{12} = 0.3300, \delta_{13} = 0.3886,$$
$$\delta_{21} = 0.3037, \delta_{22} = 0.3827, \delta_{23} = 0.3136,$$
$$\delta_{31} = 0.2805, \delta_{32} = 0.3127, \delta_{33} = 0.4068.$$

Step 4: By utilizing Eq. (16), we obtain the overall q-ROFV χ_k of the alternative O_k, where $\lambda = 2$, $q = 3, k = 1, 2, 3, 4, \chi_1 = \langle 0.3111, 0.8047 \rangle, \chi_2 = \langle 0.4105, 0.8588 \rangle$ and $\chi_3 = \langle 0.2291, 0.8049 \rangle$.

Step 5: By utilizing Eq. (17), we obtain the RVs $\psi(\chi_1)$, $\psi(\chi_2)$ and $\psi(\chi_3)$ of the overall q-ROFVs χ_1, χ_2 and χ_3 obtained in Step 4, respectively, where $\psi(\chi_1) = 0.2147$, $\psi(\chi_2) = 0.1735$ and $\psi(\chi_3) = 0.1656$.

Step 6: Because $\psi(\chi_1) > \psi(\chi_2) > \psi(\chi_3)$, where $\psi(\chi_1) = 0.2147$, $\psi(\chi_2) = 0.1735$ and $\psi(\chi_3) = 0.1656$, the RO of the alternatives O_1, O_2 and O_3 is "$O_1 \succ O_2 \succ O_3$" where O_1 is the best alternative among the alternatives O_1, O_2 and O_3.

Table 3 represents the ROs of the alternatives obtained by using different MADM approaches for Example 14. It shows that Akram et al.'s MADM approach MADM approach based on the q-ROFEWG AO (Akram et al. 2021), Liu and Wang's MADM approach based on the q-ROFWG AO (Liu and Wang 2018) achieve the same RO "$O_1 = O_2 = O_3$" of the alternatives O_1, O_2 and O_3, where they have the drawbacks that they cannot distinguish the RO of the alternatives O_1, O_2 and O_3 in this situation; the proposed MADM approach gets the RO "$O_1 \succ O_2 \succ O_3$" of the alternatives O_1, O_2 and O_3. Hence, the proposed MADM approach can conquer the shortcomings of Akram et al.'s MADM approach based on the q-ROFEWG AO (Akram et al. 2021), Liu and Wang's MADM approach based on the q-ROFWG AO (Liu and Wang 2018) in this situation.

7 Conclusion

In this paper, we have proposed the q-rung orthopair fuzzy Yager power weighted geometric (q-ROFYPWG) aggregation operator (AO) based on the PG operator and Yager's norm for aggregating the q-rung orthopair fuzzy values (q-ROFVs). The proposed q-ROFYPWG AO can reduce the effect of incorrect information given by the biased experts and also allows aggregated values to support each other throughout the aggregation process. The proposed q-ROFYPWG AO can conquer the shortcomings of the q-rung orthopair fuzzy weighted geometric (q-ROFWG) AO (Liu and Wang 2018) and the q-rung orthopair fuzzy Einstein weighted geometric (q-ROFEWG) AO (Akram et al. 2021) of q-ROFVs. Moreover, by utilizing the proposed q-ROFYPWG AO, we have proposed a new multi-attribute decision making (MADM) approach for q-ROFVs environment. The proposed MADM approach can conquer the drawbacks of the Akram et al.'s MADM approach MADM approach based on the q-ROFEWG AO (Akram et al. 2021), Liu and Wang's MADM approach based on the q-ROFWG AO (Liu and Wang 2018), where they cannot distinguish the ranking orders (ROs) of alternatives in some situations. In future, we can also define the group decision making methods based on the proposed q-ROFYPWG AO under the q-ROFVs environment.

Author Contributions Each author has equal contribution.

Funding Information Authors have no funding.

Data Availability The numerical data used to support the findings of this study are available from the corresponding author upon request.

Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

References

Abdullah S, Qiyas M, Khan MA et al (2022) Multi-criteria decision making based on intuitionistic cubic fuzzy numbers. Granul Comput 7(1):217–227

Ahmad U, Sabir M (2022) Multicriteria decision-making based on the degree and distance-based indices of fuzzy graphs. Granul Comput. https://doi.org/10.1007/s41066-022-00354-x

Akram M, Shahzadi G (2021) A hybrid decision-making model under q-rung orthopair fuzzy Yager aggregation operators. Granul Comput 6(4):763–777

Akram M, Shahzadi G, Peng X (2021) Extension of Einstein geometric operators to multi-attribute decision making under q-rung orthopair fuzzy information. Granul Comput 6(4):779–795

Ali G, Ansari MN (2022) Multiattribute decision-making under Fermatean fuzzy bipolar soft framework. Granul Comput 7(2):337–352

Ashraf S, Abdullah S (2021) Some novel aggregation operators for cubic picture fuzzy information: application in multi-attribute decision support problem. Granul Comput 6(3):603–618

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96

Biswas A, Deb N (2021) Pythagorean fuzzy Schweizer and Sklar power aggregation operators for solving multi-attribute decision-making problems. Granul Comput 6(4):991–1007

Chabaane D, Kheffache D (2022) Multi-criteria decision-making based on novel fuzzy generalized divergence and knowledge measures. Granul Comput. https://doi.org/10.1007/s41066-022-00352-z

Chaurasiya R, Jain D (2022) Pythagorean fuzzy entropy measure-based complex proportional assessment technique for solving multi-criteria healthcare waste treatment problem. Granul Comput 7(4):917–930

Chen SM, Cheng SH, Lan TC (2016) Multicriteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values. Inf Sci 367:279–295

Dhankhar C, Kumar K (2022) Multi-attribute decision-making based on the advanced possibility degree measure of intuitionistic fuzzy numbers. Granul Comput. https://doi.org/10.1007/s41066-022-00343-0

Dutta P, Doley D (2021) Fuzzy decision making for medical diagnosis using arithmetic of generalised parabolic fuzzy numbers. Granul Comput 6(2):377–388

Dutta P, Saikia B (2021) Arithmetic operations on normal semi elliptic intuitionistic fuzzy numbers and their application in decision-making. Granul Comput 6(1):163–179

Ejegwa PA, Adah V, Onyeke IC (2022) Some modified pythagorean fuzzy correlation measures with application in determining some selected decision-making problems. Granul Comput 7(2):381–391

Farid HMA, Riaz M (2021) Some generalized q-rung orthopair fuzzy Einstein interactive geometric aggregation operators with improved operational laws. Int J Intell Syst 36(12):7239–7273

Feng F, Zhang C, Akram M, Zhang J (2022) Multiple attribute decision making based on probabilistic generalized orthopair fuzzy sets. Granul Comput. https://doi.org/10.1007/s41066-022-00358-7

Feng F, Zheng Y, Sun B, Akram M (2022b) Novel score functions of generalized orthopair fuzzy membership grades with application to multiple attribute decision making. Granul Comput 7(1):95–111

Ganie AH (2022) Multicriteria decision-making based on distance measures and knowledge measures of fermatean fuzzy sets. Granul Comput 7(4):979–998

Garg H, Kaur G (2020) Novel distance measures for cubic intuitionistic fuzzy sets and their applications to pattern recognitions and medical diagnosis. Granul Comput 5(2):169–184

Garg H, Kumar K (2019) Improved possibility degree method for ranking intuitionistic fuzzy numbers and their application in multiattribute decision-making. Granul Comput 4(2):237–247

Garg H, Kumar K (2020) Power geometric aggregation operators based on connection number of set pair analysis under intuitionistic fuzzy environment. Arabian J Sci Eng 45(3):2049–2063

Gupta R, Kumar S (2022) Intuitionistic fuzzy scale-invariant entropy with correlation coefficients-based VIKOR approach for multi-criteria decision-making. Granul Comput 7(1):77–93

Jiang W, Wei B, Liu X, Li X, Zheng H (2018) Intuitionistic fuzzy power aggregation operator based on entropy and its application in decision making. Int J Intell Syst 33(1):49–67

Joshi R, Kumar S (2022) A novel VIKOR approach based on weighted correlation coefficients and picture fuzzy information for multicriteria decision making. Granul Comput 7(2):323–336

Khan MJ, Kumam P, Alreshidi NA, Kumam W (2021) Improved cosine and cotangent function-based similarity measures for q-rung orthopair fuzzy sets and TOSIS method. Complex Intell Syst 7(5):2679–2696

Kumar K, Chen SM (2021) Multiattribute decision making based on converted decision matrices, probability density functions, and interval-valued intuitionistic fuzzy values. Inf Sci 554:313–324

Kumar K, Chen SM (2021) Multiattribute decision making based on the improved intuitionistic fuzzy Einstein weighted averaging operator of intuitionistic fuzzy values. Inf Sci 568:369–383

Kumar M, Gupta S (2023) Multicriteria decision-making based on the confidence level Q-rung orthopair normal fuzzy aggregation operator. Granul Comput 8:77–96

Liu P, Wang P (2018) Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. Int J Intell Syst 33(2):259–280

Liu Z, Liu P, Liang X (2018) Multiple attribute decision-making method for dealing with heterogeneous relationship among attributes and unknown attribute weight information under q-rung orthopair fuzzy environment. Int J Intell Syst 33(9):1900–1928

Liu D, Chen X, Peng D (2019) Some cosine similarity measures and distance measures between q-rung orthopair fuzzy sets. Int J Intell Syst 34(7):1572–1587

Meng F, Chen SM (2021) A framework for group decision making with multiplicative trapezoidal fuzzy preference relations. Inf Sci 577:722–747

Mishra AR, Chen SM, Rani P (2022) Multiattribute decision making based on Fermatean hesitant fuzzy sets and modified VIKOR method. Inf Sci 607:1532–1549

Pant M, Kumar S (2022) Particle swarm optimization and intuitionistic fuzzy set-based novel method for fuzzy time series forecasting. Granul Comput 7(2):285–303

Saad M, Rafiq A (2022) Correlation coefficients for T-spherical fuzzy sets and their applications in pattern analysis and multi-attribute decision-making. Granul Comput. https://doi.org/10.1007/s41066-022-00355-w

Seikh MR, Mandal U (2021) Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making. Granul Comput 6(3):473–488

Seikh MR, Mandal U (2021) Q-rung orthopair Fuzzy frank aggregation operators and its application in multiple attribute decision-making with unknown attribute weights. Granul Comput 7(3):709–730

Suresh M, Arun Prakash K, Vengataasalam S (2021) Multi-criteria decision making based on ranking of neutrosophic trapezoidal fuzzy numbers. Granul Comput 6(4):943–952

Verma R (2022) Generalized similarity measures under linguistic q-rung orthopair fuzzy environment with application to multiple attribute decision-making. Granul Comput 7(2):253–275

Wei G, Gao H, Wei Y (2018) Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making. Int J Intell Syst 33(7):1426–1458

Xu Z, Yager RR (2010) Power-geometric operators and their use in group decision making. IEEE Trans Fuzzy Syst 18(1):94–105

Yager RR (2013) Pythagorean membership grades in multicriteria decision making. IEEE Trans on Fuzzy Syst 22(4):958–965
Yager RR (2017) Generalized orthopair fuzzy sets. IEEE Trans on Fuzzy Syst 25(5):1222–1230
Yang Z, Garg H, Li J, Srivastava G, Cao Z (2021) Investigation of multiple heterogeneous relationships using a q-rung orthopair fuzzy multi-criteria decision algorithm. Neural Comput Appl. 33(17):10771–10786
Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353
Zeb A, Khan A, Fayaz M, Izhar M (2022) Aggregation operators of Pythagorean fuzzy bi-polar soft sets with application in multiple attribute decision making. Granul Comput 7(4):931–950
Zou XY, Chen SM, Fan KY (2020) Multiple attribute decision making using improved intuitionistic fuzzy weighted geometric operators of intuitionistic fuzzy values. Inf Sci 535:242–253