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ORIGINAL PAPER

Multi-attribute decision-making based on the advanced possibility degree measure of intuitionistic fuzzy numbers

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Abstract
In this paper, we propose a novel multi-attribute decision-making (MADM) approach for the intuitionistic fuzzy numbers (IFNs) environment. For this, we propose the advanced possibility degree measure (APDM) to rank the intuitionistic fuzzy numbers (IFNs). We also explore the properties of the proposed APDM of IFNs. The proposed APDM of IFNs can overcome the drawbacks of the existing possibility degree measure (PDM) of IFNs. Moreover, we propose a novel multi-attribute decision-making approach based on the proposed APDM of IFNs environment. We also explore the drawbacks of the existing MADM approach in the environment of IFNs, which has the drawback that it cannot distinguish the ranking order (RO) of the alternatives in some circumstances. The proposed MADM approach can overcome the drawbacks of the existing MADM approach. The proposed MADM approach offers us a very useful way to deal with MADM problems in the context of IFNs.

Keywords Advanced possibility degree · Aggregating operator · IFNs · Ranking order · MADM

1 Introduction

Multi-attribute decision-making (MADM) issues are common occurrence in today's society. The major problem for the decision-maker (DMk) in MADM problems is to select the appropriate environment for delivering performance ratings for alternatives towards the attributes. To deal with such types of problems of DMk, fuzzy set (FS) (Zadeh 1965) and its extension intuitionistic fuzzy set (IFS) (Atanassov 1986) are the most powerful environment. In the last two decades, various researchers (Arya and Kumar 2021; Rahman et al. 2021; Akram and Khan 2021; Rahman et al. 2020; Akram and Shahzadi 2021; Ashraf et al. 2021; Kumar and Gupta 2022; Gupta and Kumar 2022; Feng et al. 2022; Ma and Xu 2020; Liu and Wang 2020; Zhang 2020; Seikh and Mandal 2021; Joshi and Kumar 2022; Mishra et al. 2022; Yang et al. 2021; Dutta and Saikia 2021; Ganie 2022; Abdullah et al. 2022; Senapati et al. 2022; Hussain et al. 2022; Zhan and Sun 2020; Wang et al. 2021; Joshi 2018; Rani et al. 2019; Mishra et al. 2019; Garg and Kaur 2020; Chen and Chang 2016; Chen et al. 2016; Suresh et al. 2021; Ye et al. 2022; Ejegwa et al. 2022; Cheng et al. 2022; Kadian and Kumar 2021; Dutta and Doley 2021; Khan et al. 2019; Biswas and Deb 2021; Mishra et al. 2019; Verma 2022) have been developed different-different MADM methods based on the FSs and IFSs environment. Abdullah et al. (2022) defined the MADM approach based on the intuitionistic cubic fuzzy numbers. Akram and Shahzadi (2021) defined the hybrid decision-making method for the q-rung orthopair fuzzy environment. Feng et al. (2020) developed the decision-making approach based on the PROMETHEE method for intuitionistic fuzzy soft sets environment. Ma and Xu (2020) introduced a MADM approach based on fuzzy logical algebras for computing generalized linguistic term sets. Wang and Liu (2012) proposed the intuitionistic fuzzy Einstein weighted averaging (IFEWA) aggregation operator (AO) for IFNs. The geometric averaging AOs and MADM method for IFNs were defined by Chen and Chang (2016). Chen et al. (2016) developed the MADM approach based on the TOPSIS method under the IFNs environment. Feng et al. (2020) introduced a MADM technique based on Minkowski-weighted scoring functions of IFNs. Based on

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the set pair analysis (SPA) theory, Garg and Kumar (2018) established a MADM approach for IFNs environments. Zou et al. (2020) defined the improved intuitionistic fuzzy weighted geometric (IIFWG) AO and MADM approach for the IFNs environment. Garg and Kumar (2020) proposed the MADM approach for IFNs, which was based on the power geometric AOs of SPA theory. Kumar and Chen (2021) developed the improved intuitionistic fuzzy Einstein weighted averaging (IIFEWA) AO and MADM approach for IFNs environment. Ke et al. (2018) introduced the MADM approach for the IFNs based on the distance metric. Joshi (2018) defined the MADM approach for moderator IFNs. Kumar and Garg (2018) introduced a MADM approach using SPA theory and TOPSIS methodology in the context of IFNs. Zeng et al. (2019) proposed a MADM approach based on IFN's score function and a modified VIKOR method. Wei and Tang (2010) have introduced the possibility degree measure (PDM) for IFNs with application in MADM. Garg and Kumar (2019) found the limitations of PDM given in Wei and Tang (2010) and also defined the improved PDM for the MADM that can overcome the drawbacks of existing PDM Wei and Tang (2010).

The PDM between any two objects represents the possibility that one object is more likely than the other, and can be used to compare the objects. We observed in this study that the Garg and Kumar's (2019) PDM of the IFNs gives the incorrect ranking order (RO) in some circumstances, as illustrated in Examples 1 and 2 of Sect. 2. To overcome the drawbacks of Garg and Kumar's (2019) PDM of IFNs, we need to develop a new PDM of IFNs. Apart from this, however, the Garg and Kumar's MADM approach (Garg and Kumar 2019), based on existing PDM, has the drawback that it cannot distinguish the ranking order (RO) of the alternatives in some circumstances. Therefore, a new MADM approach under the IFNs environment must develop to overcome the drawbacks of Garg and Kumar's MADM approach (Garg and Kumar 2019).

In this article, we propose the advanced possibility degree measure (APDM) to rank the IFNs. The proposed APDM of IFNs can overcome drawbacks of the Garg and Kumar's (Garg and Kumar 2019) PDM of IFNs. We also provide proofs of the validity and some desirable properties of the proposed APDM of IFNs. Afterwards, based on the proposed APDM of IFNs, we propose a novel MADM approach in the IFNs environment. The proposed MADM approach can overcome the drawbacks of Garg and Kumar's MADM approach (Garg and Kumar 2019), which has the drawbacks that it cannot distinguish the ranking order (RO) of the alternatives in some circumstances. It gives us a very convenient way of dealing with MADM issues in IFNs environments.

The rest of this paper is organised as follows. The preliminaries related to this paper as well as the drawbacks of the Garg and Kumar's (2019) PDM of IFNs are described in Sect. 2. The proposed APDM of IFNs is shown in Sect. 3. The drawbacks of the Garg and Kumar's MADM approach (Garg and Kumar 2019) are discussed in Sect. 4. We provide a novel MADM approach based on the proposed APDM of IFNs environment in Sect. 5 to overcome the drawbacks of the Garg and Kumar's MADM approach (Garg and Kumar 2019). Finally, Sect. 6 brings the article to a conclusion.

2 Preliminaries

Definition 1 (Atanassov 1986) In universal set X, an IFS I_F is represented by

$$I_F = \left\{ \langle x, \eta(x), \upsilon(x) \rangle \mid x \in X \right\},$$

where $\eta(x)$ and $\upsilon(x)$ represent the membership grade (MG) and non-membership grade (NMG) of the element x to I_F, respectively, $x \in X$, $0 \le \eta(x) \le 1$, $0 \le \upsilon(x) \le 1$ and $0 \le \eta(x) + \upsilon(x) \le 1$. $\pi(x) = 1 - \eta(x) - \upsilon(x)$ is called the hesitance degree of x to I_F, where $0 \le \pi(x) \le 1, x \in X$.

Usually, Garg and Kumar (2019) called the pair $\langle \eta, \upsilon \rangle$ an intuitionistic fuzzy number (IFN) in the IFS $I_F = \left\{ \langle x, \eta(x), \upsilon(x) \rangle \mid x \in X \right\}$.

Definition 2 (Atanassov 1986) For comparing two IFNs $\chi_1 = \langle \eta_1, \upsilon_1 \rangle$ and $\chi_2 = \langle \eta_2, \upsilon_2 \rangle$ the operational rules are given as:

(i) $\chi_1 \succeq \chi_2 \Leftrightarrow \eta_1 \ge \eta_2$ and $\upsilon_1 \le \upsilon_2$;
(ii) $\chi_1 = \chi_2 \Leftrightarrow \eta_1 = \eta_2$ and $\upsilon_1 = \upsilon_2$.

Definition 3 (Garg 2016) For aggregating the IFNs $\chi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\chi_2 = \langle \eta_2, \upsilon_2 \rangle$, $\ldots$, $\chi_m = \langle \eta_m, \upsilon_m \rangle$, the intuitionistic fuzzy Einstein weighted geometric interactive averaging (IFEWGIA) aggregating operator (AO) is defined as follows:

$$\begin{aligned} &\mathrm{IFEWGIA}(\chi_1, \chi_2, \ldots, \chi_m) \\ &= \left\langle \frac{2\left\{ \prod_{t=1}^{m} (1-\upsilon_t)^{w_t} - \prod_{t=1}^{m} (1-\eta_t-\upsilon_t)^{w_t} \right\}}{\prod_{t=1}^{m} (1+\upsilon_t)^{w_t} + \prod_{t=1}^{m} (1-\upsilon_t)^{w_t}}, \right. \\ &\left. \frac{\prod_{t=1}^{m} (1+\upsilon_t)^{w_t} - \prod_{t=1}^{m} (1-\upsilon_t)^{w_t}}{\prod_{t=1}^{m} (1+\upsilon_t)^{w_t} + \prod_{t=1}^{m} (1-\upsilon_t)^{w_t}} \right\rangle, \end{aligned} \tag{1}$$

where the IFN χ_t has weight w_t with conditions $w_t \ge 0$ and

$\sum_{t=1}^{m} w_t = 1$.

Definition 4 (Kumar and Chen 2021) For aggregating the IFNs $\chi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\chi_2 = \langle \eta_2, \upsilon_2 \rangle$, ..., $\chi_m = \langle \eta_m, \upsilon_m \rangle$, the improved intuitionistic fuzzy Einstein weighted averaging (IIFEWA) AO is defined as follows:

$$\mathrm{IIFEWA}(\chi_1, \chi_2, \ldots, \chi_n) = \left\langle \frac{\prod_{t=1}^{m}(1-\eta_t)^{w_t} - \left[1 - \frac{1}{\varepsilon}\left(1 - \prod_{t=1}^{m}(1-\varepsilon\eta_t)^{w_t}\right)\right]}{\prod_{t=1}^{m}(1-\eta_t)^{w_t} + \left[1 - \frac{1}{\varepsilon}\left(1 - \prod_{t=1}^{m}(1-\varepsilon\eta_t)^{w_t}\right)\right]}, \frac{2(1 - \frac{1}{\varepsilon}(1 - \prod_{t=1}^{m}(1-\varepsilon(1-\theta_t))^{w_t}))}{\prod_{t=1}^{m}(2-\theta_t)^{w_t} + (1 - \frac{1}{\varepsilon}(1 - \prod_{t=1}^{m}(1-\varepsilon(1-\theta_t))^{w_t}))} \right\rangle, \quad (2)$$

where the IFN χ_t has weight w_t with conditions $w_t \geq 0$ and $\sum_{t=1}^{m} w_t = 1$.

The following is a brief overview of the existing possibile degree measure (PDM) (Garg and Kumar 2019) as well as the ranking principle based on the PDM given in (Garg and Kumar 2019).

Definition 5 (Garg and Kumar 2019) For two IFNs $\chi_1 = \langle \eta_1, \upsilon_1 \rangle$ and $\chi_2 = \langle \eta_2, \upsilon_2 \rangle$ the existing PDM $\rho'(\chi_1 \succeq \chi_2)$, of $\chi_1 \succeq \chi_2$ is defined as follows :

(i) If either $\pi_1 \neq 0$ or $\pi_2 \neq 0$ then

$$\rho'(\chi_1 \succeq \chi_2) = \min\left(\max\left(\frac{1 + \eta_1 - 2\eta_2 - \upsilon_2}{\pi_1 + \pi_2}, 0\right), 1\right) \quad (3)$$

(ii) If both $\pi_1 = \pi_2 = 0$ then

$$\rho'(\chi_1 \succeq \chi_2) = \begin{cases} 1 & : \eta_1 > \eta_2 \\ 0 & : \eta_1 < \eta_2 \\ 0.5 & : \eta_1 = \eta_2 \end{cases}$$

we obtain the possibility degree matrix (PDMx) $M' = [\rho'_{ti}]_{n\times n} = [\rho'(\chi_t \succeq \chi_i)]_{n\times n}$ for ordering n IFNs χ_1, χ_2, ..., χ_n by applying the Eq. (3) as follows:

$$M' = \begin{pmatrix} \rho'_{11} & \rho'_{12} & \cdots & \rho'_{1n} \\ \rho'_{21} & \rho'_{22} & \cdots & \rho'_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho'_{n1} & \rho'_{n2} & \cdots & \rho'_{nn} \end{pmatrix} \quad (4)$$

After that, calculate the ranking value (RV) φ'_t for IFNs χ_t as

$$\varphi'_t = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n} \rho'_{tj} + \frac{n}{2} - 1\right). \quad (5)$$

Hence, arrange the RVs φ'_t, $t = 1, 2, \ldots, n$, in descending order and choose the best IFNs χ_t.

Example 1 Let two IFNs $\chi_1 = \langle 0.3, 0.2 \rangle$ and $\chi_2 = \langle 0.3, 0.3 \rangle$. For comparing χ_1 and χ_2, we use the existing PDM ρ' as given in the Eq. (3) and obtain

$$\begin{aligned} \rho'(\chi_1 \succeq \chi_2) &= \min\left(\max\left(\frac{1 + 0.3 - 2\times 0.3 - 0.3}{0.5 + 0.4}, 0\right), 1\right) \\ &= \min\left(\max\left(\frac{0.4}{0.9}, 0\right), 1\right) \\ &= \min(\max(0.4444, 0), 1) \\ &= 0.4444, \end{aligned}$$

and

$$\begin{aligned} \rho'(\chi_2 \succeq \chi_1) &= \min\left(\max\left(\frac{1 + 0.3 - 2\times 0.3 - 0.2}{0.5 + 0.4}, 0\right), 1\right) \\ &= \min\left(\max\left(\frac{0.5}{0.9}, 0\right), 1\right) \\ &= \min(\max(0.5556, 0), 1) \\ &= 0.5556. \end{aligned}$$

Hence, we get the PDMx M' using Eq. (4) as

$$M' = \begin{pmatrix} 0.5 & 0.4444 \\ 0.5556 & 0.5 \end{pmatrix}.$$

Thus, by using Eq. (5), we obtain the RVs $\varphi'_1 = \frac{1}{2(2-1)}(0.5 + 0.4444 - \frac{2}{2} - 1) = 0.4722$ and $\varphi'_2 = \frac{1}{2(2-1)}(0.5556 + 0.5 - \frac{2}{2} - 1) = 0.5278$ of the IFNs χ_1 and χ_2 respectively. Since $\varphi'_2 > \varphi'_1$, therefore $\chi_2 \succ \chi_1$.

Furthermore, we found that $\eta_1 = \eta_2 = 0.3$ and $\upsilon_1 = 0.2 < 0.3 = \upsilon_2$ which implies that the MGs of χ_1 and χ_2 are same, and NMG of χ_1 is less than the NMG of χ_2. As a result, based on the Definition 2, we get $\chi_1 \succ \chi_2$. Thus, the existing PDM ρ' (Garg and Kumar 2019) fails to identify the correct ranking order (RO) of IFNs χ_1 and χ_2.

Example 2 Let two IFNs $\chi_1 = \langle 0.4, 0.5 \rangle$ and $\chi_2 = \langle 0.4, 0.3 \rangle$. For comparing χ_1 and χ_2, we use the existing PDM ρ' as given in the Eq. (3) and obtain

$$\begin{aligned} \rho'(\chi_1 \succeq \chi_2) &= \min\left(\max\left(\frac{1 + 0.4 - 2\times 0.4 - 0.3}{0.1 + 0.3}, 0\right), 1\right) \\ &= \min\left(\max\left(\frac{0.3}{0.4}, 0\right), 1\right) \\ &= \min(\max(0.7500, 0), 1) \\ &= 0.7500. \end{aligned}$$

and

$$\begin{aligned}\rho'(\chi_2 \succeq \chi_1) &= \min\left(\max\left(\frac{1+0.4-2\times 0.4-0.5}{0.1+0.3},0\right),1\right)\\ &= \min\left(\max\left(\frac{0.1}{0.4},0\right),1\right)\\ &= \min(\max(0.2500,0),1)\\ &= 0.2500.\end{aligned}$$

Hence, we get the PDMx M' using Eq. (4) as

$$M' = \begin{pmatrix} 0.5000 & 0.7500 \\ 0.2500 & 0.5000 \end{pmatrix}.$$

Thus, by using Eq. (5), we obtain the RVs $\varphi'_1 = \frac{1}{2(2-1)}(0.5+0.7500-\frac{2}{2}-1) = 0.6250$ and $\varphi'_2 = \frac{1}{2(2-1)}(0.2500+0.5-\frac{2}{2}-1) = 0.3750$ of the IFNs χ_1 and χ_2 respectively. Since $\varphi'_1 > \varphi'_2$, therefore $\chi_1 \succ \chi_2$.

Furthermore, we found that $\eta_1 = \eta_2 = 0.4$ and $\upsilon_1 = 0.5 > 0.3 = \upsilon_2$ which implies that the MGs of χ_1 and χ_2 are identical, and NMG of χ_2 is less than the NMG of χ_1. As a result, according to the Definition 2, $\chi_2 \succ \chi_1$. Thus, the existing PDM Garg and Kumar (2019) fails to identify ranking order (RO) of IFNs χ_1 and χ_2.

From the results of Examples 1 and 2, it is clear that Garg and Kumar's PDM (Garg and Kumar 2019) has the shortcomings that it provide the incorrect RO of the IFNs. To overcome the shortcomings of Garg and Kumar's PDM (Garg and Kumar 2019), we must develop a new PDM of IFNs.

3 Advanced possibility degree measure

In this section, we present an advanced possibility degree measure (APDM) for the ranking of IFNs.

Definition 6 For any two IFNs $\chi_1 = \langle \eta_1, \upsilon_1 \rangle$ and $\chi_2 = \langle \eta_2, \upsilon_2 \rangle$, the proposed APDM $\rho(\chi_1 \succeq \chi_2)$ of $\chi_1 \succeq \chi_2$ is given as follows :

(i) If either $\pi_1 \neq 0$ or $\pi_2 \neq 0$ then

$$\rho(\chi_1 \succeq \chi_2) = 1 - \min\left(\max\left(\frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2},0\right),1\right) \tag{6}$$

(ii) If both $\pi_1 = \pi_2 = 0$ then

$$\rho(\chi_1 \succeq \chi_2) = \begin{cases} 1 & : \eta_1 > \eta_2 \\ 0 & : \eta_1 < \eta_2 \\ 0.5 & : \eta_1 = \eta_2 \end{cases} \tag{7}$$

Theorem 1 *Consider χ_1 and χ_2 be any two IFNs, then*

(a) $0 \leq \rho(\chi_1 \succeq \chi_2) \leq 1$;
(b) $\rho(\chi_1 \succeq \chi_2) = 0.5$ if $\chi_1 = \chi_2$;
(c) $\rho(\chi_1 \succeq \chi_2) + \rho(\chi_2 \succeq \chi_1) = 1$.

Proof

(a) We do this by assuming

$$\kappa = \frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2}.$$

Now, there are the following three situations:

(1) If $\kappa \geq 1$ then

$$\begin{aligned}\rho(\chi_1 \succeq \chi_2) &= 1-\min(\max(\kappa,0),1)\\ &= 1-\min(\kappa,1) = 1-1 = 0.\end{aligned}$$

(2) If $0 < \kappa < 1$ then

$$\begin{aligned}\rho(\chi_1 \succeq \chi_2) &= 1-\min(\max(\kappa,0),1)\\ &= 1-\min(\kappa,1) = 1-\kappa.\end{aligned}$$

(3) If $\kappa \leq 0$ then

$$\begin{aligned}\rho(\chi_1 \succeq \chi_2) &= 1-\min(\max(\kappa,0),1)\\ &= 1-\min(0,1) = 1.\end{aligned}$$

As a result of the above three cases, we can conclude that $0 \leq \rho(\chi_1 \succeq \chi_2) \leq 1$.

(b) Let $\chi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\chi_2 = \langle \eta_2, \upsilon_2 \rangle$ be two IFNs. If $\chi_1 = \chi_2$, which implies that $\eta_1 = \eta_2$ and $\upsilon_1 = \upsilon_2$. Then, Eq. (6) becomes

$$\begin{aligned}\rho(\chi_1 \succeq \chi_2) &= 1-\min\left(\max\left(\frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2},0\right),1\right)\\ &= 1-\min\left(\max\left(\frac{1-\eta_1+\upsilon_1-2\upsilon_1}{\pi_1+\pi_1},0\right),1\right)\\ &= 1-\min\left(\max\left(\frac{\pi_1}{2\pi_1},0\right),1\right)\\ &= 1-\min(\max(0.5,0),1)\\ &= 1-0.5\\ &= 0.5.\end{aligned}$$

(c) Let

$$\kappa = \frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2},$$
$$\epsilon = \frac{1-\eta_2+\upsilon_2-2\upsilon_1}{\pi_1+\pi_2},$$

and we have

$$
\begin{aligned}
\kappa+\epsilon &= \frac{1-\eta_1+\upsilon_1-2\upsilon_2+1-\eta_2+\upsilon_2-2\upsilon_1}{\pi_1+\pi_2} \\
&= \frac{1-\eta_1-\upsilon_1+1-\eta_2-\upsilon_2}{\pi_1+\pi_2} \\
&= \frac{\pi_1+\pi_2}{\pi_1+\pi_2}=1.
\end{aligned}
$$

(a) If $\kappa \leq 0$ and $\epsilon \geq 1$ then

$$
\begin{aligned}
&\rho(\chi_1 \succeq \chi_2)+p(\chi_2 \succeq \chi_1) \\
&\quad = 1-\min(\max(\kappa,0),1)+1 \\
&\quad - \min(\max(\epsilon,0),1) \\
&= 2-\min(0,1)-\min(\epsilon,1)=1.
\end{aligned}
$$

(b) If $0<\kappa,\epsilon<1$ then

$$
\begin{aligned}
&\rho(\chi_1 \succeq \chi_2)+p(\chi_2 \succeq \chi_1) \\
&\quad = 1-\min(\max(\kappa,0),1)+1 \\
&\quad - \min(\max(\epsilon,0),1) \\
&= 2-\min(\kappa,1)-\min(\epsilon,1) \\
&= 2-\kappa-\epsilon=1.
\end{aligned}
$$

(c) If $\kappa \geq 1$ and $\epsilon \leq 0$ then

$$
\begin{aligned}
&\rho(\chi_1 \succeq \chi_2)+p(\chi_2 \succeq \chi_1) \\
&\quad = 1-\min(\max(\kappa,0),1) \\
&\quad + 1-\min(\max(\epsilon,0),1) \\
&= 2-\min(\kappa,1)-\min(0,1)=1.
\end{aligned}
$$

□

Theorem 2 *For any two IFNs $\chi_1=\langle\eta_1,\upsilon_1\rangle$ and $\chi_2=\langle\eta_2,\upsilon_2\rangle$, the proposed APDM $p(\chi_1 \succeq \chi_2)$ satisfies the following characteristics*:

(i) $\rho(\chi_1 \succeq \chi_2)=0$ if $\upsilon_1-\upsilon_2 \geq \pi_2/2$;
(ii) $\rho(\chi_1 \succeq \chi_2)=1$ if $\upsilon_2-\upsilon_1 \geq \pi_1/2$.

Proof For two IFNs $\chi_1=\langle\eta_1,\upsilon_1\rangle$ and $\chi_2=\langle\eta_2,\upsilon_2\rangle$, we have

(i) Let $\upsilon_1-\upsilon_2 \geq \pi_2/2$, then we have

$$
\begin{aligned}
\frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2} &= \frac{1-\eta_1-\upsilon_1+2\upsilon_1-2\upsilon_2}{\pi_1+\pi_2} \\
&= \frac{\pi_1+2\upsilon_1-2\upsilon_2}{\pi_1+\pi_2} \\
&\geq \frac{\pi_1+\pi_2}{\pi_1+\pi_2} \\
&= 1
\end{aligned}
$$

Therefore, $1-\min\left(\max\left(\frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2},0\right),1\right)=0.$
Hence $\rho(\chi_1 \succeq \chi_2)=0$.

(ii) Let $\upsilon_2-\upsilon_1 \geq \pi_1/2$, then we have

$$
\begin{aligned}
\frac{1-\eta_1+\upsilon_1-2\upsilon_2}{\pi_1+\pi_2} &= \frac{1-\eta_1-\upsilon_1+2\upsilon_1-2\upsilon_2}{\pi_1+\pi_2} \\
&= \frac{\pi_1-2(\upsilon_2-\upsilon_1)}{\pi_1+\pi_2} \\
&\leq \frac{\pi_1-\pi_1}{\pi_1+\pi_2} \\
&= 0
\end{aligned}
$$

Therefore, $1-\min\left(\max\left(\frac{1-\eta_1+\upsilon_1-\upsilon_2}{\pi_1+\pi_2},0\right),1\right)=1.$
Hence $\rho(\chi_1 \succeq \chi_2)=1$.

□

However, we develop the possibility degree matrix (PDMx) $M=[\rho_{tj}]_{n\times n}=[\rho(\chi_t \succeq \chi_i)]_{n\times n}$ where $t,i=1,2,\ldots,n$, to rank n IFNs χ_1, χ_2, …, χ_n, by applying the Eq. (6) as follows:

$$
M=\begin{pmatrix}
\rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\
\rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{n1} & \rho_{n2} & \cdots & \rho_{nn}
\end{pmatrix} \tag{8}
$$

Now, for IFNs χ_t, we calculate the ranking value (RV) φ_t as

$$
\varphi_t=\frac{1}{n(n-1)}\left(\sum_{j=1}^{n}\rho_{ti}+\frac{n}{2}-1\right). \tag{9}
$$

As a result, sort the RVs φ_t, $t=1,2,\ldots,n$, in descending order and select the best IFNs χ_t.

Example 3 Consider the same IFNs $\chi_1=\langle 0.3,0.2\rangle$ and $\chi_2=\langle 0.3,0.3\rangle$ as given in Example 1 for ranking using the proposed APDM. For this, we use Eq. (6) to obtain the APDMs $\rho(\chi_1 \succeq \chi_2)$ and $\rho(\chi_2 \succeq \chi_1)$ of $\chi_1 \succeq \chi_2$ and $\chi_2 \succeq \chi_1$, respectively, and shown as follows:

$$
\begin{aligned}
\rho(\chi_1 \succeq \chi_2) &= 1-\min\left(\max\left(\frac{1-0.3+0.2-2\times 0.3}{0.5+0.4},0\right),1\right) \\
&= 1-\min\left(\max\left(\frac{0.3}{0.9},0\right),1\right) \\
&= 1-\min(\max(0.3333,0),1) \\
&= 1-0.3333=0.6667,
\end{aligned}
$$

and

$$\rho(\chi_1 \succeq \chi_2) = 1 - \min\left(\max\left(\frac{1-0.3+0.3-2\times 0.2}{0.5+0.4}, 0\right), 1\right)$$
$$= 1 - \min\left(\max\left(\frac{0.6}{0.9}, 0\right), 1\right)$$
$$= 1 - \min(\max(0.6667, 0), 1)$$
$$= 1 - 0.6667 = 0.3333.$$

Thus, by utilizing the Eq. (8), we calculate the PDMx as

$$M = \begin{bmatrix} 0.5000 & 0.6667 \\ 0.3333 & 0.5000 \end{bmatrix}$$

The ranking values φ_1 and φ_2 of the IFNs χ_1 and χ_2 are calculated using Eq. (9), respectively, and obtain $\varphi_1 = \frac{1}{2(2-1)}(0.5 + 0.6667 - \frac{2}{2} - 1) = 0.5833$, $\varphi_2 = \frac{1}{2(2-1)}(0.3333 + 0.5 - \frac{2}{2} - 1) = 0.4167$. Since $\varphi_1 > \varphi_2$, therefore $\chi_1 \succ \chi_2$. As a result, the proposed APDM ρ can overcome the drawbacks of the existing PDM ρ' (Garg and Kumar 2019) as described in Sect. 2.

Example 4 Consider the same IFNs $\chi_1 = \langle 0.4, 0.5\rangle$ and $\chi_2 = \langle 0.4, 0.3\rangle$ as given in Example 2 for ranking using the proposed APDM. For this, we use Eq. (6) to obtain the APDMs $\rho(\chi_1 \succeq \chi_2)$ and $\rho(\chi_2 \succeq \chi_1)$ of $\chi_1 \succeq \chi_2$ and $\chi_2 \succeq \chi_1$, respectively, and shown as follows:

$$\rho(\chi_1 \succeq \chi_2) = 1 - \min\left(\max\left(\frac{1 - 0.4 + 0.5 - 2\times 0.3}{0.1 + 0.3}, 0\right), 1\right)$$
$$= 1 - \min\left(\max\left(\frac{0.5}{0.4}, 0\right), 1\right)$$
$$= 1 - \min(\max(1.2500, 0), 1)$$
$$= 1 - 1 = 0,$$

and

$$\rho(\chi_1 \succeq \chi_2) = 1 - \min\left(\max\left(\frac{1-0.4+0.3-2\times 0.5}{0.1+0.3}, 0\right), 1\right)$$
$$= 1 - \min\left(\max\left(\frac{-0.1}{0.4}, 0\right), 1\right)$$
$$= 1 - \min(\max(0, 0), 1)$$
$$= 1 - 0 = 1.$$

Thus, by utilizing Eq. (8), we calculate the PDMx as

$$M = \begin{bmatrix} 0.5000 & 0.0 \\ 1.0 & 0.5000 \end{bmatrix}$$

The ranking values φ_1 and φ_2 of the IFNs χ_1 and χ_2 are calculated using Eq. (9), respectively, and get $\varphi_1 = \frac{1}{2(2-1)}(0.5 + 0 - \frac{2}{2} - 1) = 0.2500$, $\varphi_2 = \frac{1}{2(2-1)}(1.0 + 0.5 - \frac{2}{2} - 1) = 0.7500$. Since $\varphi_2 > \varphi_1$, therefore, $\chi_2 \succ \chi_1$. As a result, the proposed APDM ρ can overcome the drawbacks of the existing PDM ρ' (Garg and Kumar 2019) as described in Sect. 2.

4 Analyzing the limitations of the Garg and Kumar's MADM approach (Garg and Kumar 2019)

In this section, we analyze the drawbacks of the Garg and Kumar's MADM approach (Garg and Kumar 2019). Let the alternatives $O_1, O_2, \ldots, O_m$ and the attributes $C_1, C_2, \ldots, C_n$ with weights $w_1, w_2, \ldots, w_n$ such that $w_t > 0$ and $\sum_{t=1}^{n} w_t = 1$. DMk assess the alternative O_k under the attributes C_t by utilizing the IFNs $\widetilde{\chi}_{kt} = \langle \widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt}\rangle$, $k = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, n$.

The operating steps of the MADM approach (Garg and Kumar 2019) are discussed as follows:

Step 1: Construct the decision matrix (DMx) $\widetilde{D} = (\widetilde{\chi}_{kt})_{m\times n}$ using the DMk's assessment as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ \vdots \\ O_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \widetilde{\chi}_{11} & \widetilde{\chi}_{12} & \cdots & \widetilde{\chi}_{1n} \\ \widetilde{\chi}_{21} & \widetilde{\chi}_{22} & \cdots & \widetilde{\chi}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\chi}_{m1} & \widetilde{\chi}_{m2} & \cdots & \widetilde{\chi}_{mn} \end{pmatrix} \end{array}$$

Step 2: Transform the DMx $\widetilde{D} = (\widetilde{\chi}_{kt})_{m\times n} = (\langle \widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt}\rangle)_{m\times n}$ into the normalized DMx (NDMx) $D = (\chi_{kt})_{m\times n} = \langle \eta_{kt}, \upsilon_{kt}\rangle$ as follows :

$$\chi_{kt} = \begin{cases} \langle \widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt}\rangle & : \text{if } C_t \text{ is a benefit type attribute} \\ \langle \widetilde{\upsilon}_{kt}, \widetilde{\eta}_{kt}\rangle & : \text{if } C_t \text{ is a cost type attribute} \end{cases}$$

Step 3: Obtain the aggregated IFN χ_k of alternative O_k by combining the IFNs $\chi_{k1}, \chi_{k2}, \ldots, \chi_{kn}$ appeared in the k^{th} row of the NDMx $D = (\chi_{kt})_{m\times n}$ using the IFEWGIA AO and shown as follows:

$$\begin{aligned}\chi_k &= \langle \eta_k, \upsilon_k \rangle \\ &= \text{IFEWGIA}(\chi_{k1}, \chi_{k2}, \ldots, \chi_{kn}) \\ &= \left\langle \frac{2\left\{\prod_{t=1}^{n}(1-\upsilon_{kt})^{w_t} - (1-\eta_{kt}-\upsilon_{kt})^{w_t}\right\}}{\prod_{t=1}^{n}(1+\upsilon_{kt})^{w_t} + \prod_{t=1}^{n}(1-\upsilon_{kt})^{w_t}}, \frac{\prod_{t=1}^{n}(1+\upsilon_{kt})^{w_t} - \prod_{t=1}^{n}(1-\upsilon_{kt})^{w_t}}{\prod_{t=1}^{n}(1+\upsilon_{kt})^{w_t} + \prod_{t=1}^{n}(1-\upsilon_{kt})^{w_t}} \right\rangle,\end{aligned} \tag{10}$$

where w_t is the weight of C_t, $w_t \geq 0$ and $\sum_{t=1}^{n} w_t = 1$.

Step 4: Based on the Definition 5, MADM approach (Garg and Kumar 2019) obtains the PDMx $M' = [\rho'_{ki}]_{m \times m}$ as shown below:

$$M' = \begin{pmatrix} \rho'_{11} & \rho'_{12} & \cdots & \rho'_{1n} \\ \rho'_{21} & \rho'_{22} & \cdots & \rho'_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho'_{n1} & \rho'_{n2} & \cdots & \rho'_{nn} \end{pmatrix} \tag{11}$$

where $\rho'_{ki} = \rho'(\chi_k \succeq \chi_i)$ and

(i) when either $\pi_k \neq 0$ or $\pi_i \neq 0$ then

$$\rho'(\chi_k \succeq \chi_i) = \min\left(\max\left(\frac{1+\eta_k - 2\eta_i - \upsilon_i}{\pi_k + \pi_i}, 0\right), 1\right)$$

(ii) when both $\pi_k = \pi_i = 0$ then

$$\rho'(\chi_k \succeq \chi_i) = \begin{cases} 1 & : \eta_k > \eta_i \\ 0 & : \eta_k < \eta_i \\ 0.5 & : \eta_k = \eta_i \end{cases}$$

Step 5: Compute the ranking value (RV) φ'_k of alternative $O_k, k = 1, 2, \ldots, m$, as follows :

$$\varphi'_k = \frac{1}{m(m-1)}\left(\sum_{j=1}^{m} \rho'_{kj} + \frac{m}{2} - 1\right), \tag{12}$$

sort the RVs φ'_1, φ'_2,..., φ'_m of alternatives O_1, O_2,...,O_m in decreasing order and get the RO of the alternatives O_1, O_2,...,O_m.

Example 5 Let O_1, O_2 and O_3 be three alternatives and let C_1, C_2 and C_3 be three beneficiary type attributes with weights $w_1 = 0.3$, $w_2 = 0.4$ and $w_3 = 0.3$. The DMk assess the alternatives O_1, O_2, and O_3 under the attribute C_1, C_2 and C_3 using an IFN $\widetilde{\chi}_{kt}$ to obtain the DMx $\widetilde{D} = (\chi_{kt})_{3\times3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times3}$.

We use the Garg and Kumar's MADM approach (Garg and Kumar 2019) to solve this MADM problem as follows:

Step 1: DMk's assessments of the alternative O_1, O_2 and O_3 with respect to attributes C_1, C_2 and C_3 in the form of the DMx $\widetilde{D} = (\chi_{kt})_{3\times3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times3}$ and shown as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.6 \rangle & \langle 0, 1 \rangle & \langle 0.7, 0.2 \rangle \\ \langle 0, 1 \rangle & \langle 0.6, 0.3 \rangle & \langle 0.4, 0.2 \rangle \\ \langle 0.5, 0.4 \rangle & \langle 0, 1 \rangle & \langle 0.2, 0.8 \rangle \end{pmatrix} \end{array}$$

Step 2: Because all the attributes are of the benefit type, Garg and Kumar's MADM approach (Garg and Kumar 2019) obtains the NDMx $R = (\eta_{kt}, \upsilon_{kt})_{3\times3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times3}$.

Step 3: By utilizing Eq. (10), Garg and Kumar's MADM approach (Garg and Kumar 2019) obtains the aggregated IFNs $\chi_1 = \langle 0, 1 \rangle$, $\chi_2 = \langle 0, 1 \rangle$, and $\chi_3 = \langle 0, 1 \rangle$ of the alternatives O_1, O_2 and O_3 respectively.

Step 4: Using Eq. (11), Garg and Kumar's MADM approach (Garg and Kumar 2019) calculate the PDMx $M' = [\rho'_{ki}]_{3\times3}$ as follows:

$$M' = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$$

Step 5: Using Eq. (12), Garg and Kumar's MADM approach (Garg and Kumar 2019) gets the RV φ'_k of the alternative O_k, where $k = 1, 2, 3$, $\varphi'_1 = 0.3333$, $\varphi'_2 = 0.3333$ and $\varphi'_3 = 0.3333$.

Step 6: Since, $\varphi'_1 = \varphi'_2 = \varphi'_3$, therefore RO of alternatives O_1, O_2 and O_3 obtained by Garg and Kumar's MADM approach (Garg and Kumar 2019) is "$O_1 = O_2 = O_3$". As a result, Garg and Kumar's MADM approach (Garg and Kumar 2019) fails to distinguish the RO of O_1, O_2 and O_3 in this case.

Example 6 Let O_1, O_2 and O_3 be three alternatives and let C_1, C_2 and C_3 be three beneficiary type attributes with weights $w_1 = 0.3$, $w_2 = 0.3$ and $w_3 = 0.4$. The DMk assesses the alternative O_1, O_2 and O_3 under the attribute C_1, C_2 and C_3 using an IFN $\widetilde{\chi}_{kt}$ to obtain the DMx $\widetilde{D} = (\chi_{kt})_{3\times3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times3}$.

We use the Garg and Kumar's MADM approach (Garg and Kumar 2019) to solve this MADM problem as follows:

Step 1: DMk's assessments of the alternative O_1, O_2 and O_3 with respect to attributes C_1, C_2 and C_3 in the form of the DMx $\widetilde{D} = (\chi_{kt})_{3\times3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times3}$ and shown as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.2, 0.5\rangle & \langle 0.5, 0.4\rangle & \langle 0, 1\rangle \\ \langle 0.4, 0.3\rangle & \langle 0.5, 0.2\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.4, 0.4\rangle & \langle 0.1, 0.3\rangle \end{pmatrix} \end{array}$$

Step 2: Because all the attributes are of the benefit type, Garg and Kumar's MADM approach (Garg and Kumar 2019) obtains the NDMx $D = (\eta_{kt}, \upsilon_{kt})_{3\times3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times3}$.

Step 3: By utilizing Eq. (10), Garg and Kumar's MADM approach (Garg and Kumar 2019) obtains the aggregated IFNs $\chi_1 = \langle 0, 1\rangle$, $\chi_2 = \langle 0, 1\rangle$, and $\chi_3 = \langle 0, 1\rangle$ of the alternatives O_1, O_2 and O_3 respectively.

Step 4: Using Eq. (11), Garg and Kumar's MADM approach (Garg and Kumar 2019) calculate the PDMx $M' = [\rho'_{ki}]_{3\times3}$ as follows:

$$M' = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$$

Step 5: Using Eq. (12), Garg and Kumar'sMADM approach (Garg and Kumar 2019) gets the RV φ'_k of the alternative O_k, where $k = 1, 2, 3$, $\varphi'_1 = 0.3333$, $\varphi'_2 = 0.3333$ and $\varphi'_3 = 0.3333$.

Step 6: Because $\varphi'_1 = \varphi'_2 = \varphi'_3$, the RO of alternatives O_1, O_2 and O_3 obtained by Garg and Kumar's MADM approach (Garg and Kumar 2019) is "$O_1 = O_2 = O''_3$. As a result, Garg and Kumar' MADM approach (Garg and Kumar 2019) fails to distinguish the RO of O_1, O_2 and O_3 in this case.

From the results of *Example* 5 and *Example* 6, it can be seen that Garg and Kumar's MADM approach (Garg and Kumar 2019) based on existing PDM has the drawback that it cannot distinguish the ROs of the alternatives in some situations. Therefore, we need to develop a new MADM approach under the IFNs environment to overcome the drawbacks of Garg and Kumar's MADM approach (Garg and Kumar 2019).

5 Proposed MADM approach based on the proposed APDM of IFNs

In this section, using the proposed APDM of IFNs, we develop a novel MADM method for IFNs.

Assume the alternatives $O_1, O_2, \ldots, O_m$ and the attributes $C_1, C_2, \ldots, C_n$ with weights $w_1, w_2, \ldots, w_n$ such that $w_t > 0$ and $\sum_{t=1}^{n} w_t = 1$. Decision maker (DMk) assess the alternative O_k towards the attributes $C_t (t = 1, 2, \ldots, n)$ using the IFNs $\widetilde{\chi}_{kt} = \langle \widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt}\rangle$, $k = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, n$

Step 1: Assemble the DMk assessment in the form of the decision matrix (DMx) $\widetilde{D} = (\widetilde{\chi}_{kt})_{m\times n}$ as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ \vdots \\ O_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \widetilde{\chi}_{11} & \widetilde{\chi}_{12} & \cdots & \widetilde{\chi}_{1n} \\ \widetilde{\chi}_{21} & \widetilde{\chi}_{22} & \cdots & \widetilde{\chi}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\chi}_{m1} & \widetilde{\chi}_{m2} & \cdots & \widetilde{\chi}_{mn} \end{pmatrix} \end{array}$$

Step 2: Transform the DMx $\widetilde{D} = (\widetilde{\chi}_{kt})_{m\times n} = (\langle \widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt}\rangle)_{m\times n}$ to the normalized DMx (NDMx) $D = (\chi_{kt})_{m\times n} = \langle \eta_{kt}, \upsilon_{kt}\rangle$ as follows :

$$\chi_{kt} = \begin{cases} \langle \widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt}\rangle & : \text{if } C_t \text{ is a benefit type attribute} \\ \langle \widetilde{\upsilon}_{kt}, \widetilde{\eta}_{kt}\rangle & : \text{if } C_t \text{ is a cost type attribute} \end{cases} \tag{13}$$

Step 3: By applying the Eq. (2), compute the overall IFN $\chi_k = \langle \eta_k, \upsilon_k\rangle$ of alternative O_k, $k = 1, 2, \ldots, m$, as follows:

$$\begin{aligned} \chi_k &= \langle \eta_k, \upsilon_k\rangle \\ &= \mathrm{IIFEWA}(\chi_{k1}, \chi_{k2}, \ldots, \chi_{kn}) \\ &= \left\langle \frac{\prod_{t=1}^{n}(1-\eta_{kt})^{w_t} - \left[1-\frac{1}{\varepsilon}\left(1-\prod_{t=1}^{n}(1-\varepsilon\eta_{kt})^{w_t}\right)\right]}{\prod_{t=1}^{n}(1-\eta_{kt})^{w_t} + \left[1-\frac{1}{\varepsilon}\left(1-\prod_{t=1}^{n}(1-\varepsilon\eta_{kt})^{w_t}\right)\right]}, \right. \\ &\quad \left. \frac{2\left(1-\frac{1}{\varepsilon}\left(1-\prod_{t=1}^{n}(1-\varepsilon(1-\theta_{kt}))^{w_t}\right)\right)}{\prod_{t=1}^{n}(2-\theta_{kt})^{w_t} + \left(1-\frac{1}{\varepsilon}\left(1-\prod_{t=1}^{n}(1-\varepsilon(1-\theta_{kt}))^{w_t}\right)\right)} \right\rangle \end{aligned} \tag{14}$$

Step 4: Based on the Definition 6, compute the PDMx $M = [\rho_{ki}]_{m\times m}$, $k, i = 1, 2, \ldots, m$ as:

$$M = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1i} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{k1} & \rho_{k2} & \cdots & \rho_{ki}, \end{pmatrix} \tag{15}$$

where $\rho_{ki} = \rho(\chi_k \succeq \chi_i)$ and

(i) If either $\pi_k \neq 0$ or $\pi_i \neq 0$ then

$$\rho(\chi_k \succeq \chi_i) = -\max\left(\min\left(\frac{1-\eta_k+\upsilon_k-2\upsilon_j}{\pi_k+\pi_i},1\right),0\right)$$

(ii) If both $\pi_k = \pi_i = 0$ then

$$\rho(\chi_k \succeq \chi_i) = \begin{cases} 1 & : \eta_k > \eta_i \\ 0 & : \eta_k < \eta_i \\ 0.5 & : \eta_k = \eta_i \end{cases}$$

Step 5: Using Eq. (9), compute the alternative's ranking value (RV) $\varphi_1, \varphi_2, \ldots, \varphi_m$ of alternatives $O_1, O_2, \ldots, O_m$ as follows :

$$\varphi_k = \frac{1}{m(m-1)}\left(\sum_{i=1}^{m} \rho_{ki} + \frac{m}{2} - 1\right). \tag{16}$$

Step 6: Arrange the RVs $\varphi_1, \varphi_2, \ldots, \varphi_m$ of alternatives $O_1, O_2, \ldots, O_m$ in decreasing order and get the RO of the alternatives $O_1, O_2, \ldots, O_m$.

Example 7 (Garg and Kumar 2019) The government wants to choose a contractor among the contractors, "PNC Infratech Ltd." (O_1)," Hindustan construction company" (O_2), "J.P. Construction" (O_3) and "Gammon India Ltd." (O_4) for any construction project. For this assignment, the government created five attributes: "project cost" (G_1), "completion time" (C_2), "technical capability" (C_3), "financial status"' (C_4) and "company background" (C_5) with the weights $w_1 = 0.3$, $w_2 = 0.25$, $w_3 = 0.1$, $w_4 = 0.15$ and $w_5 = 0.2$. The main goal of this MADM problem is to select the best firm for the task from among all of them.

To deal with this problem, we use the proposed approach as follows:

Step 1: The DMk obtains the DMx $\widetilde{R} = (\widetilde{\chi}_{kt})_{4\times5}$ by evaluating the alternatives towards the attributes using the IFNs and shown as follows:

$$\widetilde{D} = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{pmatrix} \langle 0.3,0.6\rangle & \langle 0.5,0.4\rangle & \langle 0.7,0.2\rangle & \langle 0.5,0.2\rangle & \langle 0.7,0.1\rangle \\ \langle 0.5,0.3\rangle & \langle 0.6,0.2\rangle & \langle 0.5,0.4\rangle & \langle 0.6,0.3\rangle & \langle 0.4,0.2\rangle \\ \langle 0.5,0.4\rangle & \langle 0.7,0.2\rangle & \langle 0.8,0.1\rangle & \langle 0.6,0.2\rangle & \langle 0.5,0.3\rangle \\ \langle 0.6,0.2\rangle & \langle 0.4,0.3\rangle & \langle 0.7,0.2\rangle & \langle 0.4,0.4\rangle & \langle 0.2,0.8\rangle \end{pmatrix} \end{array}$$

Step 2: Because the attributes C_1 and C_2 are of the cost type and the others are of the benefit type, the proposed MADM approach obtains the NDMx by using Eq. (13) and shown as follows:

$$D = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{pmatrix} \langle 0.6,0.3\rangle & \langle 0.4,0.5\rangle & \langle 0.7,0.2\rangle & \langle 0.5,0.2\rangle & \langle 0.7,0.1\rangle \\ \langle 0.3,0.5\rangle & \langle 0.2,0.6\rangle & \langle 0.5,0.4\rangle & \langle 0.6,0.3\rangle & \langle 0.4,0.2\rangle \\ \langle 0.4,0.5\rangle & \langle 0.2,0.7\rangle & \langle 0.8,0.1\rangle & \langle 0.6,0.2\rangle & \langle 0.5,0.3\rangle \\ \langle 0.2,0.6\rangle & \langle 0.3,0.4\rangle & \langle 0.7,0.2\rangle & \langle 0.4,0.4\rangle & \langle 0.2,0.8\rangle \end{pmatrix} \end{array}$$

Step 3: Using Eq. (14), the proposed MADM approach obtains the overall IFN χ_k of the alternative O_k, where $k = 1, 2, 3, 4$, $\varepsilon = 0.99$, $\chi_1 = \langle 0.5757, 0.2543\rangle$, $\chi_2 = \langle 0.3687, 0.4029\rangle$, $\chi_3 = \langle 0.4626, 0.3802\rangle$, and $\chi_4 = \langle 0.3173, 0.4966\rangle$.

Step 4: The proposed MADM approach obtains the PDMx $M = [\rho_{ki}]_{4\times4}$ using Eq. (15) and shown as follows:

$$M = \begin{pmatrix} 0.5000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 0.5000 & 0.2903 & 0.9012 \\ 0 & 0.7097 & 0.5000 & 1.0000 \\ 0 & 0.0988 & 0 & 0.5000 \end{pmatrix}$$

Step 5: Using Eq. (16), proposed MADM approach gets the RVs $\varphi_1 = 0.3750$, $\varphi_2 = 0.2243$, $\varphi_3 = 0.2675$ and $\varphi_4 = 0.1332$ of the alternative O_1, O_2, O_3 and O_4, respectively.

Step 6: Since, $\varphi_1 > \varphi_3 > \varphi_2 > \varphi_4$, therefore alternative's RO is $O_1 \succ O_3 \succ O_2 \succ O_4$. Hence, O_1 is the best alternative for this MADM problem.

For Example 7, we make a comparative analysis of the alternative's RO obtained by the proposed MADM approach with the alternative's RO obtained by the Garg and Kumar's MADM approach (Garg and Kumar 2019). The Garg and Kumar's MADM approach (Garg and Kumar 2019) gets the RO $O_1 \succ O_4 \succ O_3 \succ O_2$ of the alternatives O_1, O_2, O_3 and O_4. Whereas, the proposed MADM approach obtains the RO $O_1 \succ O_3 \succ O_2 \succ O_4$ of the alternatives O_1, O_2, O_3 and O_4. Hence, O_1 is the best alternative for both MADM approaches for this task.

Example 8 Consider the same data set as given in *Example* 5. To solve this MADM problem, we use the proposed MADM approach as follows:

Step 1: The DMx $\widetilde{D} = (\widetilde{\chi}_{kt})_{3\times 3}$ is same as given in Step 1 of *Example* 5.

Step 2: Since all the attributes are of benefit type, therefore the proposed MADM approach gets the NDMx $D = (\eta_{kt}, \upsilon_{kt})_{3\times 3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times 3}$.

Step 3: Using Eq. (14), the proposed MADM approach gets the aggregated IFN χ_k of the alternative O_k, where $k = 1, 2, 3$, $\varepsilon = 0.99$, $\chi_1 = \langle 0.3381, 0.5747 \rangle$, $\chi_2 = \langle 0.3832, 0.4124 \rangle$, and $\chi_3 = \langle 0.2216, 0.7381 \rangle$.

Step 4: The proposed MADM approach obtains the PDMx $M = [\rho_{ki}]_{3\times 3}$ using Eq. (15) and shown as follows:

$$M = \begin{pmatrix} 0.5 & 0 & 1.0 \\ 1.0 & 0.5 & 1.0 \\ 0 & 0 & 0.5 \end{pmatrix}$$

Step 5: Using Eq. (16), the proposed MADM approach gets the RVs $\varphi_1 = 0.3333$, $\varphi_2 = 0.5$, and $\varphi_3 = 0.1667$ of the alternatives O_1, O_2 and O_3.

Step 6: Since, $\varphi_2 > \varphi_1 > \varphi_3$, therefore alternative's RO is $O_2 \succ O_1 \succ O_3$.

For Example 8, we make a comparative analysis of the alternative's RO obtained by the proposed MADM approach with the alternative's RO obtained by the Garg and Kumar's MADM approach (Garg and Kumar 2019). The Garg and Kumar's MADM approach (Garg and Kumar 2019) based on the PDM obtains the RO "$O_1 = O_2 = O_3$" of the alternatives O_1, O_2, and O_3, which has the drawbacks that it cannot distinguish the RO of the alternatives O_1, O_2, and O_3 in this case. While, the proposed MADM approach obtains the RO "$O_2 \succ O_1 \succ O_3$" of the alternatives O_1, O_2, and O_3. Therefore, the proposed MADM approach based on the APDM can overcome the drawbacks of Garg and Kumar's MADM approach (Garg and Kumar 2019).

Example 9 Consider the same data set as given in Example 6. To solve this MADM problem, we use the proposed MADM approach as follows:

Step 1: The DMx $\widetilde{D} = (\widetilde{\chi}_{kt})_{3\times 3}$ is same as given in Step 1 of Example 6.

Step 2: Since all the attributes are of benefit type, therefore the proposed MADM approach gets the NDMx $D = (\eta_{kt}, \upsilon_{kt})_{3\times 3} = (\widetilde{\eta}_{kt}, \widetilde{\upsilon}_{kt})_{3\times 3}$.

Step 3: Using Eq. (14), the proposed MADM approach gets the aggregated IFN χ_k of the alternative O_k, where $k = 1, 2, 3$, $\varepsilon = 0.99$, $\chi_1 = \langle 0.2216, 0.6442 \rangle$, $\chi_2 = \langle 0.2836, 0.4723 \rangle$, and $\chi_3 = \langle 0.1655, 0.4968 \rangle$.

Step 4: The proposed MADM approach obtains the PDMx M using Eq. (15) and as follows:

$$M = \begin{pmatrix} 0.5 & 0 & 0.0908 \\ 1.0 & 0.5 & 0.6645 \\ 0.9092 & 0.3355 & 0.5 \end{pmatrix}$$

Step 5: Using Eq. (16), the proposed MADM approach gets the RVs $\varphi_1 = 0.1818$, $\varphi_2 = 0.4441$, and $\varphi_3 = 0.3741$ of the alternatives O_1, O_2 and O_3 respectively.

Step 6: Since, $\varphi_2 > \varphi_3 > \varphi_1$, therefore alternative's RO is $O_2 \succ O_3 \succ O_1$.

For Example 9, we make a comparative analysis of the alternative's RO obtained by the proposed MADM approach with the alternative's RO obtained by the Garg and Kumar's MADM approach (Garg and Kumar 2019). The Garg and Kumar's MADM approach (Garg and Kumar 2019) based on the PDM obtains the RO "$O_1 = O_2 = O_3$" of the alternatives O_1, O_2, and O_3, which has the drawbacks that it cannot distinguish the RO of the alternatives O_1, O_2, and O_3 in this case. While, the proposed MADM approach obtains the RO "$O_2 \succ O_1 \succ O_3$" of the alternatives O_1, O_2, and O_3. Therefore, the proposed MADM approach based on the APDM can overcome the drawbacks of Garg and Kumar's MADM approach (Garg and Kumar 2019).

6 Conclusion

In this study, we have proposed the a novel multi-attribute decision making (MADM) approach for the intuitionistic fuzzy numbers (IFNs) environment. For this, we have proposed an advanced possibility degree measure (APDM) for ranking of IFNs. The proposed APDM between two IFNs indicates the possibility of one IFN being larger than the other IFN. The proposed APDM of IFNs can overcome the drawbacks of Garg and Kumar's PDM (Garg and Kumar 2019) of IFNs, which gives the incorrect ranking order of the IFNs in some cases. Moreover, based on the proposed APDM of IFNs, we have introduced a novel MADM approach under the IFNs context. The proposed MADM approach can overcome the drawback the Garg and Kumar's MADM approach (Garg and Kumar 2019), which has drawback that it cannot distinguish the RO between the alternatives in some circumstances. The proposed MADM approach provides a very convenient way for solving the MADM problems in IFNs contexts. Abdullah et al. (2022) defined the MADM approach based on the intuitionistic cubic fuzzy numbers. Akram and Shahzadi (2021) defined the hybrid decision-making method for the q-rung orthopair fuzzy environment. Feng et al. (2020) develop the decision making approach based on the PROMETHEE method for intuitionistic fuzzy soft sets environment. Zhang (2020) defined the MADM approach based on the dual hesitant fuzzy environment. Ma and Xu (2020) introduced a MADM approach based on fuzzy logical algebras for computing generalized linguistic term sets. Future research can focus on developing new MADM algorithms based on (Abdullah et al. 2022; Akram and Shahzadi 2021; Feng et al. 2020; Zhang 2020; Ma and Xu 2020). In future, we can also extend the proposed MADM approach for solving the multi-attribute group decision-making problems under the IFNs environment.

Data availability The numerical data used to support the findings of this study are available from the corresponding author upon request.

Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

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