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ORIGINAL PAPER

Group decision based on trapezoidal neutrosophic Dombi fuzzy hybrid operator

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Abstract

The purpose of this paper is to introduce the concepts of Dombi t-norm and Dombi t-conorm to aggregate trapezoidal neutrosophic fuzzy information. First, we have proposed some new operational laws of trapezoidal neutrosophic fuzzy number based on Dombi t-norm and t-conorm. Furthermore, based on these operational laws, we have introduced trapezoidal neutrosophic fuzzy Dombi weighted arithmetic averaging operator, trapezoidal neutrosophic fuzzy Dombi order weighted arithmetic averaging operator, and trapezoidal neutrosophic fuzzy Dombi hybrid arithmetic averaging operator. Moreover, some suitable properties of these operators are also discussed. Then, utilizing these proposed operators, we have presented an algorithm to solve multiattribute decision-making (MADM) problems under trapezoidal neutrosophic fuzzy environment. Finally, we have utilized a numerical example to compare the flexibility of the proposed method with the other existing methods.

Keywords TrNDF aggregation operators · MADM · Numerical application

1 Introduction

In MCDM problem, a grade alternatives subsequently numerical charge of a determinate customary of reliant principles. Necessary alternative container be selected by if partiality data in footings of careful arithmetic value or interval. The conception of fuzzy sets was presented by Zadeh (1975) and it remained general to intuitionistic fuzzy sets by Atanassov (1986). Chen et al. (2012) introduced the relevance degree between the forecasted variation of the main factor and the forecasted variation of each elementary secondary factor. Chen et al. (2012b) defined the guarantees to produce normal interval type-2 Gaussian fuzzy sets. Chen et al. (2016a) defined the new similarity measure between intuitionistic fuzzy sets. Chen et al. (2016b) defined the positive similarity degrees and the negative similarity degrees between the evaluating IFVs. Chen and Jian (2017) defined the PSO-based optimal-intervals partition algorithm to get the optimal partition of the intervals in the universe of discourse (UOD) of the main factor TAIEX and to get the optimal partition of the intervals in the UOD of the secondary factor SF. Chen et al. (2015) introduced the probability of the "down-trend," the probability of the "equal-trend", and the probability of the "up-trend" of the two-factors second-order fuzzy-trend logical relationships in each two-factor second-order fuzzy-trend logical relationship group. Shen et al. (2020) established to guarantee the σ-error mean-square stability for the considered systems. The IFS has established additional and supplementary attention meanwhile its entrance, because the data about characteristic morals are usually undefined or fuzzy unpaid to the cumulative difficulty of the educational situation and the imprecision of essential individual wildlife of human intelligent. Then, it is necessary to deliberate the data of experts near the strictures as intuitionistic fuzzy data. These intuitionistic fuzzy strictures are considered by different intuitionistic fuzzy numbers as triangular, trapezoidal.

The theories and practices have created different and a lasting investigation experiment of the condition of creation with indecision in real-world difficulties. The different types of complications of fuzzy decision-making along their extensions if a varied variety of apparatuses that

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are gifted to contract with indecisiveness. Fuzzy decision-making methods have progress slowly general in decision-making for personnel variety. Many DM techniques have been upcoming to solve the problem. According to Li et al. (2017), forward opinions are then confirmed with data inoculated from trusted experts. In adding with deference to Li et al. (2017), such feelings are further adapted by pretending their development due to common impact. It is also confirmed that, under convinced molds, the evolution of feelings due to influence joins to a final cooperative opinion. If averages are not met, standard collection methods are used to select the best alternative. TOPSIS, VIKOR, and GRA (You et al. 2015; Hashemi et al. 2015; Wang et al. 2009) developed the three widely adopted group decision-making methods.

Cubic set showed by Jun et al. (2011). The CSs are the philosophies of FSs and IFSs, Here are two illustrations, one is charity for the level of involvement, and other is used for the level of non- involvement. The involvement capacity is grasp as interval, while non-involvement is overall viewed as the characteristic fuzzy set.

Yu built up THFS and its accommodation to assessment in 2013. Jun et al. (2011) considered CSs which prohibit an IVFs (1975) with the FS (1965). Dong et al. (2013) proposed semantic computational model dependent on 2-tuples and interims, which they creators called an interim variant of the 2-tuple fuzzy etymological portrayal demonstrate. Dong et al. (2015) created accord issue in the reluctant phonetic collective choice making issue. Chen and Chen (2015) created three sorts of combination approaches: the backhanded methodology, the improvement based methodology, and the immediate methodology for cooperative choice making in a study. Dong et al. (2016b) proposed an agreement achieving model in the perplexing and energetic MAGDM issue. Dong et al. (2016a, b) created association between etymological chain of command and the scale for the 2-tuple semantic model and its utilization to manage reluctant lopsided phonetic data. Liu et al. (2017a, b) proposed the inclination connection with self-assurance by taking various self-assurance levels into thought, and we consider it the inclination connection with fearlessness. Dong et al. (2017) proposed an overseeing agreement dependent on initiative in assessment elements. Dong et al. (2018) proposed a progression of blended 0–1 direct programming models to demonstrate the way toward structuring a vital quality weight vector. Cong et al. (2017) created customize singular semantics by methods for an interim numerical scale and the 2-tuple etymological model. Zhang et al. (2018) researched the 2-rank MAGDM issue under the multi-granular phonetic setting, and proposed a 2-rank agreement achieving structure with the base changes.

Recently, Wang et al. (2009) introduced a single-valued neutrosophic set, which is a subclass of a neutrosophic set presented by Smarandache (1998), as a generalization of the classic set, fuzzy set, and intuitionistic fuzzy set. The single-valued neutrosophic set can independently express truth-membership degree, indeterminacy membership degree, and falsity-membership degree and deal with incomplete, indeterminate and inconsistent information.

Fahmi et al. (2017a, b, c 2018a, b) established the different ideas. Amin et al. (2018) defined the GTCHFWG operator, GTCLHFOWA operator, GTCLHFOWG operator, GTCLHFHA operator, and GTCLHFHG operator. Fahmi et al. (2018c, d, e, f) defined the operators, approximately aggregation operators, TrCFEWA operator, TrCFEOWA operator, TrCFEHWA operator CFEWA operator, CFEOWA operator, and CFEHWA operator.

Liu and Jin (2012) introduced the intuitionistic uncertain linguistic weighted geometric average (IULWGA) operator and an intuitionistic uncertain linguistic ordered weighted geometric (IULOWG) operator. Liu (2013) introduced the interval-valued intuitionistic fuzzy Hamacher weighted averaging operator, interval-valued intuitionistic fuzzy Hamacher-ordered weighted averaging operator, interval-valued intuitionistic fuzzy Hamacher hybrid weighted averaging operator, interval-valued intuitionistic fuzzy Hamacher geometric weighted averaging operator, interval-valued intuitionistic fuzzy Hamacher geometric-ordered weighted averaging operator, and interval-valued intuitionistic fuzzy Hamacher geometric hybrid weighted averaging operator. Liu et al. (2017a, b) presented the interaction PBM (IFIPBM) operator for intuitionistic fuzzy numbers (IFNs), the weighted interaction PBM (IFWIPBM) operator for IFNs, the interaction PGBM (IFIPGBM) operator for IFNs, and the weighted interaction PGBM (IFWIPGBM) operator for IFNs. Liu et al. (2018) presented the new MAGDM method with the I2LI based on the proposed I2LGA operator. Liu et al. (2018) introduced the q-rung orthopair fuzzy operational rules using ATT. Liu et al. (2018) introduced the the q-rung orthopair fuzzy power MSM (q-ROFPMSM) operator and the q-rung orthopair fuzzy power weighed MSM (q-ROFPWMSM) operator of q-ROFNs. Liu et al. (2018) presented the O'Hagan's maximum entropy to obtain the quantity of each priority level in the generalized prioritized measure. Liu et al. (2018) introduced the q-rung orthopair fuzzy weighted averaging operator and the q-rung orthopair fuzzy weighted geometric operator. Liu et al. (2020) introduced the generalize the PMSM operator to propose the intuitionistic fuzzy PMSM (IFPMSM) operator for IFNs and its weighted form (IFWPMSM) for IFNs.

Liu et al. (2019) defined the weighted values of the alternatives can be obtained and the alternatives. Rahman and Ali (2019) defined the Pythagorean fuzzy Einstein-ordered weighted geometric aggregation operator weighs only the ordered positions of the Pythagorean fuzzy arguments instead of weighing the Pythagorean fuzzy arguments themselves. Rahman et al. (2020) defined the interval-valued Pythagorean fuzzy Einstein-weighted geometric operator, interval-valued Pythagorean fuzzy Einstein-ordered weighted geometric operator, and interval-valued Pythagorean fuzzy Einstein hybrid geometric. Singh and Huang (2019) defined the 4WIVDS method and validated with various benchmark datasets.

Motivation

With the view of circumventing these trapezoidal neutrosophic fuzzy number, we gain better motivation and set our proposal for a better decision-making process:

(1) The trapezoidal neutrosophic fuzzy (1) motivated our proposal towards solving the decision-making problem under the roof of linguistic preferences. The motivation for choosing linguistic preferences as the source for the rating is twofold: (a) from the DMs'point of view, attribute weighting rating is often easy and well distinguishable, and hence, DMs prefer this as a convenient source for rating alternatives. (b) From the methods point of view, linguistic ratings are used for the better representation of vagueness and imprecision which are properly handled using the trapezoidal neutrosophic fuzzy number.

(2) To better handle trapezoidal neutrosophic Dombi fuzzy number-based ranking is employed which is a computationally powerful and attractive tool for handling vagueness, and hence, this motivated us to set our proposal to convert linguistic terms to trapezoidal neutrosophic Dombi fuzzy number values for proper and sensible ranking.

(3) Since group decision-making and aggregation are an interesting process in MCDM (as per trapezoidal neutrosophic fuzzy Dombi number (3)), we gain motivation and consider these two aspects in our proposal as well. We propose a new aggregation operator called the trapezoidal neutrosophic Dombi fuzzy number for directly aggregating preferences without making any changes to the setup, with a view of preventing loss of information and maintaining a rational decision process. We further use trapezoidal neutrosophic Dombi fuzzy hybrid weighted geometric operator for aggregating trapezoidal neutrosophic Dombi fuzzy numbers.

Thus, it is very indispensable to introduce a new extension of FSs to address this matter. The goal of this paper is to present the notion of trapezoidal neutrosophic fuzzy set, which outspreads the fuzzy set to trapezoidal neutrosophic fuzzy environments and permits the connection of an element to be a set of several possible trapezoidal neutrosophic fuzzy numbers. Thus, TrNFN is a very valuable implement to compact with the positions in which the experts hesitate between several possible TrNFNs to evaluate the degree to which an alternative satisfies an attribute. In the current example, the degree to which the alternative placates the attribute can be represented by the TrNFN. Moreover, in many MAGDM problems, as that the estimations of the attribute values are TrNFNs, it therefore is very required to give some aggregation techniques to cumulative the TrNF information. Though, we are aware that the present aggregation techniques have difficulty with GDM problems with TrNF data. Therefore, we in the current paper propose a series of aggregation operators for aggregating the trapezoidal neutrosophic fuzzy information and inspect some chattels of these operators. Then, based on these aggregation operators, we develop an approach to MAGDM with TrNF data. Moreover, we use a mathematical case to show the presentation of the developed approach.

This paper are systematized as follows. In Sect. 2, we contribute some fundamental thought, properties of cubic set, TrNDFN, and OLs. In Sect. 3, we exhibit a series of aggregation operators for TrNDF data and among these total operators. Section 4 introduces a methodology to GDM with TrNF data, the application of the developed approach in GDM problems is shown by an illustrative example, and we discuss in comparison analysis. Finally, we give the conclusions in Sect. 5.

2 Preliminaries

Definition 1 (Zadeh 1965) Let H be a fixed set, a FS F in H is defined as

$$F = \{(\hbar, \Gamma_F(\hbar)|\hbar \in H\}, \tag{1}$$

where Γ_F is a mapping from H to the closed interval [0, 1], and for each $\hbar \in$H, $\Gamma_F(\hbar)$ is called the degree of membership of $\hbar$ in H.

Definition 2 (Zadeh 1975) Let H be a fixed set. An interval-valued fuzzy set I in H is defined as

$$I = \left\{\langle \hbar, \mathbb{R}_I^-(\hbar), \mathbb{R}_I^+(\hbar)\rangle | \hbar \in H\right\}, \tag{2}$$

where $\mathbb{R}_I^-, \mathbb{R}_I^+ : H \to [0,1]$. The $\mathbb{R}_I^-(\hbar)$ is lower membership and $\mathbb{R}_I^+(\hbar)$ is upper membership, such that $0 \le \mathbb{R}_I^-(\hbar) \le \mathbb{R}_I^+(\hbar) \le 1$.

Definition 3 (Atanassov 1986) An IFS Đ in H is given by

$$Đ = \{\langle \hbar, \mathbb{R}_{Đ}(\hbar), \Omega_{Đ}(\hbar)\rangle | \hbar \in H\}, \tag{3}$$

where $\mathbb{R}_{Đ} : H \to [0,1]$ and $\Omega_{Đ} : H \to [0,1]$, with the condition $0 \le \mathbb{R}_{Đ}(\hbar) + \Omega_{Đ}(\hbar) \le 1$. The numbers $\mathbb{R}_{Đ}(\hbar)$ and

$\Omega_{Đ}(\hbar)$ represent, respectively, the membership degree and non-membership degree of the element $\hbar$ to the set Đ.

2.1 Trapezoidal neutrosophic Dombi fuzzy aggregation operator

Definition 4 Let $h_1 = \left\{ \begin{array}{c} [r_1, s_1, t_1, u_1], \\ \langle F_1, G_1, I_1 \rangle \end{array} \right\}$ and $h_2 = \left\{ \begin{array}{c} [r_2, s_2, t_2, u_2], \\ \langle F_2, G_2, I_2 \rangle \end{array} \right\}$ be two trapezoidal neutrosophic fuzzy numbers (TrNFNs) and λ is a real number. Then, the operational laws are defined as follows:

$$h_1 + h_2 = \left\{ \begin{array}{c} \left[1 - \dfrac{1}{1 + \left\{ \left[\dfrac{r_1}{1-r_1} \right]^{\rho} + \left[\dfrac{r_2}{1-r_2} \right] \right\}^{\frac{1}{\rho}}}, \right. \\ 1 - \dfrac{1}{1 + \left\{ \left[\dfrac{s_1}{1-s_1} \right]^{\rho} + \left[\dfrac{s_2}{1-s_2} \right] \right\}^{\frac{1}{\rho}}}, \\ 1 - \dfrac{1}{1 + \left\{ \left[\dfrac{t_1}{1-t_1} \right]^{\rho} + \left[\dfrac{t_2}{1-t_2} \right] \right\}^{\frac{1}{\rho}}}, \\ \left. 1 - \dfrac{1}{1 + \left\{ \left[\dfrac{u_1}{1-u_1} \right]^{\rho} + \left[\dfrac{u_2}{1-u_2} \right] \right\}^{\frac{1}{\rho}}} \right], \\ \left\langle 1 - \dfrac{1}{1 + \left\{ \left[\dfrac{F_1}{1-F_1} \right]^{\rho} + \left[\dfrac{F_2}{1-F_2} \right] \right\}^{\frac{1}{\rho}}}, \right. \\ \dfrac{1}{1 + \left\{ \left[\dfrac{1-G_1}{G_1} \right]^{\rho} + \left[\dfrac{1-G_2}{G_2} \right] \right\}^{\frac{1}{\rho}}}, \\ \left. \dfrac{1}{1 + \left\{ \left[\dfrac{1-I_1}{I_1} \right]^{\rho} + \left[\dfrac{1-I_2}{I_2} \right] \right\}^{\frac{1}{\rho}}} \right\rangle \end{array} \right\}, \tag{4}$$

$$h_1 \otimes h_2 = \left\{ \begin{array}{c} \left[\dfrac{1}{1 + \left\{ \left[\dfrac{1-r_1}{r_1} \right]^{\rho} + \left[\dfrac{1-r_2}{r_2} \right] \right\}^{\frac{1}{\rho}}}, \right. \\ \dfrac{1}{1 + \left\{ \left[\dfrac{1-s_1}{s_1} \right]^{\rho} + \left[\dfrac{1-s_2}{s_2} \right] \right\}^{\frac{1}{\rho}}}, \\ \dfrac{1}{1 + \left\{ \left[\dfrac{1-t_1}{t_1} \right]^{\rho} + \left[\dfrac{1-t_2}{t_2} \right] \right\}^{\frac{1}{\rho}}}, \\ \left. \dfrac{1}{1 + \left\{ \left[\dfrac{1-u_1}{u_1} \right]^{\rho} + \left[\dfrac{1-u_2}{u_2} \right] \right\}^{\frac{1}{\rho}}} \right], \\ \left\langle \dfrac{1}{1 + \left\{ \left[\dfrac{1-F_1}{F_1} \right]^{\rho} + \left[\dfrac{1-F_2}{F_2} \right] \right\}^{\frac{1}{\rho}}}, \right. \\ 1 - \dfrac{1}{1 + \left\{ \left[\dfrac{G_1}{1-G_1} \right]^{\rho} + \left[\dfrac{G_2}{1-G_2} \right] \right\}^{\frac{1}{\rho}}} \\ \left. 1 - \dfrac{1}{1 + \left\{ \left[\dfrac{I_1}{1-I_1} \right]^{\rho} + \left[\dfrac{I_2}{1-I_2} \right] \right\}^{\frac{1}{\rho}}} \right\rangle \end{array} \right\}, \tag{5}$$

$$\lambda h_1 = \left\{ \begin{array}{c} \left[1 - \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{r_1}{1-r_1} \right]^{\rho} \right\}}, 1 - \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{s_1}{1-s_1} \right]^{\rho} \right\}}, \right. \\ \left. 1 - \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{t_1}{1-t_1} \right]^{\rho} \right\}}, 1 - \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{u_1}{1-u_1} \right]^{\rho} \right\}} \right], \\ \left\langle 1 - \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{F_1}{1-F_1} \right]^{\rho} \right\}}, \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{1-G_1}{G_1} \right]^{\rho} \right\}}, \right. \\ \left. \dfrac{1}{1 + \left\{ \lambda \left[\dfrac{1-I_1}{I_1} \right]^{\rho} \right\}} \right\rangle \end{array} \right\}, \tag{6}$$

$$h_1^{\lambda}=\left\{\begin{array}{c}\left[\dfrac{1}{1+\left\{\lambda\left[\dfrac{1-r_1}{r_1}\right]^{\rho}\right\}},\dfrac{1}{1+\left\{\lambda\left[\dfrac{1-s_1}{s_1}\right]^{\rho}\right\}},\right.\\ \left.\dfrac{1}{1+\left\{\lambda\left[\dfrac{1-t_1}{t_1}\right]^{\rho}\right\}},\dfrac{1}{1+\left\{\lambda\left[\dfrac{1-u_1}{u_1}\right]^{\rho}\right\}}\right],\\ \left\langle\dfrac{1}{1+\left\{\lambda\left[\dfrac{1-F_1}{F_1}\right]^{\rho}\right\}},1-\dfrac{1}{1+\left\{\lambda\left[\dfrac{G_1}{1-G_1}\right]^{\rho}\right\}}\right.\\ \left.1-\dfrac{1}{1+\left\{\lambda\left[\dfrac{I_1}{1-I_1}\right]^{\rho}\right\}}\right\rangle\end{array}\right\}. \tag{7}$$

Definition 5 Let $h_i=\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}$ be the trapezoidal neutrosophic fuzzy numbers (TrNFNs) and score function S. Then, score function are defined as follows:

$$S=\frac{\{[r_i+s_i+t_i+u_i],\langle F_i+1-G_i+1-I_i\rangle\}}{3}. \tag{8}$$

Definition 6 Let $h_i=\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}$ be the trapezoidal neutrosophic fuzzy numbers (TrNFNs) and accuracy function Z. Then, accuracy function are defined as follows:

$$Z=\frac{\{[r_i+s_i+t_i+u_i],\langle F_i+1+G_i+1+I_i\rangle\}}{3}. \tag{9}$$

3 Aggregation trapezoidal neutrosophic Dombi fuzzy information

In this section, the operators for aggregating the trapezoidal neutrosophic Dombi fuzzy information and investigating some properties of these operators are presented.

Trapezoidal neutrosophic fuzzy Dombi weighted arithmetic averaging operator

Definition 7 The collections of TrNFNs are $h_i=\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}$ and $\ddot{\omega}=(\ddot{\omega}_1,\ddot{\omega}_2,\ldots,\ddot{\omega}_n)^T$ be the weight vector, with $\ddot{\omega}_j\in[0,1]$, $\sum_{j=1}^{n}\ddot{\omega}_j=1$. Hence, TrNDFWAA operator of dimension n is a mapping

$$\mathrm{TrNDFWAA}: h^n\rightarrow h$$

and defined by

$$\mathrm{TrNDFWAA}(h_1,h_2,\ldots,h_n)=\ddot{\omega}_1h_1+\ddot{\omega}_2h_2,\ldots,\ddot{\omega}_nh_n. \tag{10}$$

If $\ddot{\omega}=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})^T$. Hence, the TrNDFWAA operator is reduced to TrNDFWAA operator of dimension n. It can be defined as follows:

$$\mathrm{TrNDFWA}(h_1,h_2,\ldots,h_n)=\frac{1}{n}(h_1,h_2,\ldots,h_n).$$

Theorem 1 *The gathering of TrNFNs are* $h_i=\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}$ *and* $\ddot{\omega}=(\ddot{\omega}_1,\ddot{\omega}_2,\ldots,\ddot{\omega}_n)^T$ *be the weight vector, with* $\ddot{\omega}_j\in[0,1]$, $\sum_{j=1}^{n}\ddot{\omega}_j=1$. *The aggregated value of the trapezoidal neutrosophic fuzzy Dombi weighted averaging operator is also a TrNFN and*

$$\mathrm{TrNDFWAA}\,(h_1,h_2,\ldots,h_n)=$$

$$\left\{\begin{array}{c}\left[1-\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{r_i}{1-r_i}\right]^{\rho}\right\}},1-\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{s_i}{1-s_i}\right]^{\rho}\right\}},\right.\\ \left.1-\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{t_i}{1-t_i}\right]^{\rho}\right\}},1-\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{u_i}{1-u_i}\right]^{\rho}\right\}}\right],\\ \left\langle 1-\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{F_i}{1-F_i}\right]^{\rho}\right\}},\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{1-G_i}{G_i}\right]^{\rho}\right\}},\right.\\ \left.\dfrac{1}{1+\left\{\sum_{i=1}^{n}\lambda_i\left[\dfrac{1-I_i}{I_i}\right]^{\rho}\right\}}\right\rangle\end{array}\right\}.$$

Theorem 2 (*Idempotency*): *The gathering of TrNFNs are* $h_i=\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}$, *then TrNDFWAA* $(h_1,h_2,\ldots,h_n)=h$.

Theorem 3 *Boundedness*: *The trapezoidal neutrosophic fuzzy Dombi weighted arithmetic averaging operator of TrNFNs satisfies*

$$\min\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}\leq\mathrm{TrNDFWAA}\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}\leq\max\left\{\begin{array}{c}[r_i,s_i,t_i,u_i],\\ \langle F_i,G_i,I_i\rangle\end{array}\right\}.$$

Theorem 4 (*Commutativity*). *Let* $(\tilde{h_1}, \tilde{h_2}, \ldots, \tilde{h_n})$ *be a permutation of* $(h_1, h_2, \ldots, h_n)$, *and then, TrNDFWAA* $(\tilde{h_1}, \tilde{h_2}, \ldots, \tilde{h_n})$ =*TrNDFWAA* $(h_1, h_2, \ldots, h_n)$.

Theorem 5 *Monotonicity*: *The collections of TrNCFNs are* h_i. *If* $h_i \le \tilde{h_i}$ *for* $i = 1, 2, \ldots, n$:*then TrNDFWAA* $(h_1, h_2, \ldots, h_n)$*TrNDFWAA* $(\tilde{h_1}, \tilde{h_2}, \ldots, \tilde{h_n})$.

Trapezoidal neutrosophic fuzzy Dombi ordered weighted arithmetic averaging operator

Definition 8 The collections of TrNFNs are $h_i = \left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\}$ and $\ddot{\omega} = (\ddot{\omega}_1, \ddot{\omega}_2, \ldots, \ddot{\omega}_n)^T$ be the weight vector, with $\ddot{\omega}_j \in [0,1]$, $\sum_{j=1}^{n} \ddot{\omega}_j = 1$. Hence, TrNDFOWAA operator of dimension n is a mapping

$$\text{TrNDFOWAA} : h^n \to h$$

and defined by

$$\text{TrNDFOWAA}(h_1, h_2, \ldots, h_n) = \ddot{\omega}_1 h_1 + \ddot{\omega}_2 h_2, \ldots, \ddot{\omega}_n h_n. \tag{11}$$

If $\ddot{\omega} = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$. Hence the TrNDFOWAA operator is reduced to TrNDFWAA operator of dimension n. It can be defined as follows:

$$\text{TrNDFOWAA}(h_1, h_2, \ldots, h_n) = \frac{1}{n}(h_1, h_2, \ldots, h_n).$$

Theorem 6 *The gathering of TrNFNs are* $h_i = \left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\}$ *and* $\ddot{\omega} = (\ddot{\omega}_1, \ddot{\omega}_2, \ldots, \ddot{\omega}_n)^T$ *be the weight vector, with* $\ddot{\omega}_j \in [0,1]$, $\sum_{j=1}^{n} \ddot{\omega}_j = 1$. *The aggregated value of the TrNDFOWAA operator is also a TrNFN and*

$$TrNDFOWAA(h_1, h_2, \ldots, h_n) = \left\{ \begin{matrix} [1 - \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{r_i}{1 - r_i} \right]^{\rho} \right\}}, 1 - \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{s_i}{1 - s_i} \right]^{\rho} \right\}}, \\ 1 - \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{t_i}{1 - t_i} \right]^{\rho} \right\}}, 1 - \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{u_i}{1 - u_i} \right]^{\rho} \right\}}], \\ \left\langle 1 - \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{F_i}{1 - F_i} \right]^{\rho} \right\}}, \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{1 - G_i}{G_i} \right]^{\rho} \right\}}, \right. \\ \left. \dfrac{1}{1 + \left\{ \sum_{i=1}^{n} \lambda_i \left[\dfrac{1 - I_i}{I_i} \right]^{\rho} \right\}} \right\rangle \end{matrix} \right\}.$$

Theorem 7 (*Idempotency*): *The gathering of TrNFNs are* $h_i = \left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\}$, *then TrNDFOWAA* $(h_1, h_2, \ldots, h_n) = h$.

Theorem 8 *Boundedness*: *The trapezoidal neutrosophic fuzzy Dombi ordered weighted arithmetic averaging operator of TrNFNs satisfies*

$$min\left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\} \le TrNDFOWAA\left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\} \le \max\left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\}.$$

Theorem 9 (*Commutativity*). *Let* $(\tilde{h_1}, \tilde{h_2}, \ldots, \tilde{h_n})$ *be a permutation of* $(h_1, h_2, \ldots, h_n)$, *then TrNDFOWAA* $(\tilde{h_1}, \tilde{h_2}, \ldots, \tilde{h_n})$ =*TrNDFOWAA* $(h_1, h_2, \ldots, h_n)$.

Theorem 10 *Monotonicity*: *The collections of TrNCFNs are* h_i. *If* $h_i \le \tilde{h_i}$ *for* $i = 1, 2, \ldots, n$:*then TrNDFOWAA* $(h_1, h_2, \ldots, h_n)$*TrNDFOWAA* $(\tilde{h_1}, \tilde{h_2}, \ldots, \tilde{h_n})$.

Trapezoidal neutrosophic fuzzy Dombi hybrid weighted arithmetic averaging operator

Definition 9 The collections of TrNFNs are $h_i = \left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\}$ and $\ddot{\omega} = (\ddot{\omega}_1, \ddot{\omega}_2, \ldots, \ddot{\omega}_n)^T$ be the weight vector, with $\ddot{\omega}_j \in [0,1]$, $\sum_{j=1}^{n} \ddot{\omega}_j = 1$. Hence, TrNDFHWAA operator of dimension n is a mapping

$$\text{TrNDFHWAA} : h^n \to h$$

and defined by

$$\text{TrNDFHWAA}(h_1, h_2, \ldots, h_n) = \ddot{\omega}_1 h_1 + \ddot{\omega}_2 h_2, \ldots, \ddot{\omega}_n h_n. \tag{12}$$

If $\ddot{\omega} = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$. Hence, the TrNDFHWAA operator is reduced to TrNDFWAA operator of dimension n. It can be defined as follows:

$$\text{TrNDFHWAA}(h_1, h_2, \ldots, h_n) = \frac{1}{n}(h_1, h_2, \ldots, h_n).$$

Theorem 11 *The gathering of TrNFNs are* $h_i = \left\{ \begin{matrix} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{matrix} \right\}$ *and* $\ddot{\omega} = (\ddot{\omega}_1, \ddot{\omega}_2, \ldots, \ddot{\omega}_n)^T$ *be the weight vector, with* $\ddot{\omega}_j \in [0,1]$, $\sum_{j=1}^{n} \ddot{\omega}_j = 1$. *The aggregated value of the TrNDFHWAA operator is also a TrNFN and*

$$\text{TrNDFHWAA}(h_1, h_2, \ldots, h_n) = \left\{ \begin{array}{c} \left[1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{r_i}{1 - r_i}\right]^{\rho}\right\}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{s_i}{1 - s_i}\right]^{\rho}\right\}}, \right. \\ \left. 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{t_i}{1 - t_i}\right]^{\rho}\right\}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{u_i}{1 - u_i}\right]^{\rho}\right\}}\right], \\ \langle 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{F_i}{1 - F_i}\right]^{\rho}\right\}}, \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{1 - G_i}{G_i}\right]^{\rho}\right\}}, \\ \frac{1}{1 + \left\{\sum_{i=1}^{n} \lambda_i \left[\frac{1 - I_i}{I_i}\right]^{\rho}\right\}} \rangle \end{array} \right\}.$$

Theorem 12 (*Idempotency*): *The gathering of TrNFNs are* $h_i = \left\{ \begin{array}{c} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{array} \right\}$, *then TrNDFHWAA* $(h_1, h_2, \ldots, h_n) = h$.

Theorem 13 *Boundedness*: *The trapezoidal neutrosophic fuzzy Dombi hybrid weighted arithmetic averaging operator of TrNFNs satisfies*

$$min\left\{ \begin{array}{c} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{array} \right\} \leq TrNDFHWAA\left\{ \begin{array}{c} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{array} \right\} \leq \max\left\{ \begin{array}{c} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{array} \right\}.$$

Theorem 14 (*Commutativity*). *Let* $(\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_n)$ *be a permutation of* $(h_1, h_2, \ldots, h_n)$, *then TrNDFHWAA* $(\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_n)$ =*TrNDFHWAA* $(h_1, h_2, \ldots, h_n)$.

Theorem 15 *Monotonicity*: *The collections of TrNFNs are* h_i. *If* $h_i \leq \tilde{h}_i$ *for* $i = 1, 2, \ldots, n$:*then TrNDFHWAA* $(h_1, h_2, \ldots, h_n)$*TrNDFHWAA* $(\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_n)$.

4 An approach to MADM with TrNFNs data

Let us suppose the discrete set is $h = \{h_1, h_2, \ldots, h_n\}$ and $G = \{g_1, g_2, \ldots, g_n\}$ be the attributes. Consider that value of alternatives h_i $(i = 1, 2, \ldots, n)$ on attributes g_j $(j = 1, 2, \ldots, m)$ given by decision-maker are TrNFNs: $A = \left\{ \begin{array}{c} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{array} \right\}$,

a MADM problem is expressed in the TrN fuzzy decision matrix $D = (A_{ij})_{m \times n} = \left\{ \begin{array}{c} [r_i, s_i, t_i, u_i], \\ \langle F_i, G_i, I_i \rangle \end{array} \right\}$.

Step 1: Calculate the TrNF decision matrix.
Step 2: Utilize the TrNDFWA operator to mixture all the values $\ddot{\beta}_{ij}$ $(j = 1, 2, \ldots, m)$ of the ith line and obtain the overall value.
Step 3: Calculate the score function
Step 4: Find the ranking.

4.1 Numerical application

In this subsection, we are profitable to display an demonstrative case of the innovative methodology in a basic leadership issue. We examine an organization that works in Australia and South America that needs to put some cash in another arcade. They deliberate three conceivable options $A_1 (i = 1; 2; 3)$

$A_1 =$ Antarctica advertise.
$A_2 =$ North American market.
$A_3 =$ Asia showcase.

To assess these choices, the speculator has united a gathering of three options.

In the wake of breaking down the data, this gathering thinks about the key factor is the monetary circumstance of the ecosphere budget for the following time frame.

They consider three fundamental conceivable conditions of nature that could occur later on.

There are three attributes $C_j (J = 1; 2; 3)$. Let $w = (0.4; 0.2; 0.4)$ are WV.

$C_1 =$ Financial circumstance.
$C_2 =$ Fixed financial circumstance.
$C_3 =$ Good financial circumstance.

The authorities of the rule appraise and their own opinions concerning the results attained with each alternative. The weights of specialists are given as $w = (0.4, 0.2, 0.4)$: As the situation is very indeterminate, the collections of specialists' requirements is to evaluate material using TrNFNs. The results given in the form of TrNFNs depending on the characteristic C_j and the alternative A_i are shown in Table 1. Such that

Step 1: Arrange the TrNF decision matrices.

Step 2: Calculate the TrNDFWA operator and $\omega = (0.2, 0.3, 0.5)$ (Table 2).

Step 3: Calculate the score value $S_1 = 0.2121, S_2 = 0.2927, S_3 = 0.4214$.

Step 4: Find the ranking $S_3 > S_2 > S_1$ and S_3 is the best.

4.2 Comparison analysis

So as to check the legitimacy and viability of the proposed methodology, a near report is led utilizing the techniques different methods, which are unique instances of TNFNs, to the equivalent illustrative model.

We define the comparison analysis with different MCDM methods as shown in Table 3.

4.2.1 Existing methods and its advantages

This technique can act as a decision support method for the advantage decision-makers. The proposed method has the following benefits.

Uses expert's implicit knowledge, employments linguistic variables to extract tacit knowledge of experts, using the TrNDF numbers tactic to cover ambiguity and uncertainty and risk in the expert tacit knowledge and information. The skill is to use the classical in settings with a high degree of uncertainty and indecision and risk in the information. We define the existing methods to its advantages Table 4.

5 Conclusion

In this article, we have studied an MADM problem using trapezoidal neutrosophic fuzzy information. We have introduced arithmetic operations to expand a few trapezoidal neutrosophic fuzzy Dombi aggregation operators from the motivation of Dombi operations as trapezoidal neutrosophic Dombi fuzzy weighted averaging operator, trapezoidal neutrosophic Dombi fuzzy ordered weighted averaging operator, and trapezoidal neutrosophic Dombi fuzzy hybrid weighted averaging operators. The different features of those recommended operators are deliberated. Then, we have used those operators to expand a few strategies to remedy MADM issues. Ultimately, a realistic instance to implement the EPR system is provided to develop a plan and in accordance with expounding the utility and then effectiveness about the proposed method. In the future, the application of our proposed model can be applied in decision-making theory, risk evaluation, and other domains under ambiguous environments.

Table 1 Decision matrix

	C_1	C_2	C_3
A_1	$\left\{\begin{matrix}[0.2, 0.3,\\ 0.4, 0.5],\\ \langle 0.3, 0.5,\\ 0.7\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.1, 0.2,\\ 0.3, 0.4],\\ \langle 0.11, 0.13,\\ 0.15\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.11, 0.13,\\ 0.14, 0.15],\\ \langle 0.22, 0.24,\\ 0.26\rangle\end{matrix}\right\}$
A_2	$\left\{\begin{matrix}[0.1, 0.2,\\ 0.3, 0.4],\\ \langle 0.11, 0.13,\\ 0.15\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.11, 0.13,\\ 0.14, 0.15],\\ \langle 0.22, 0.24,\\ 0.26\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.2, 0.3,\\ 0.4, 0.5],\\ \langle 0.3, 0.5,\\ 0.7\rangle\end{matrix}\right\}$
A_3	$\left\{\begin{matrix}[0.11, 0.13,\\ 0.14, 0.15],\\ \langle 0.22, 0.24,\\ 0.26\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.2, 0.3,\\ 0.4, 0.5],\\ \langle 0.3, 0.5,\\ 0.7\rangle\end{matrix}\right\}$	$\left\{\begin{matrix}[0.1, 0.2,\\ 0.3, 0.4],\\ \langle 0.11, 0.13,\\ 0.15\rangle\end{matrix}\right\}$

Table 2 TrNDFWA operator

A_1	$\left\{\begin{matrix}[0.0883, 0.1421, 0.1791, 0.2269],\\ \langle 0.1429, 0.3152, 0.3586\rangle\end{matrix}\right\}$
A_2	$\left\{\begin{matrix}[0.1269, 0.1989, 0.2466, 0.3057],\\ \langle 0.2001, 0.2348, 0.2715\rangle\end{matrix}\right\}$
A_3	$\left\{\begin{matrix}[0.1951, 0.2927, 0.3531, 0.4233],\\ \langle 0.2943, 0.1555, 0.1827\rangle\end{matrix}\right\}$

Table 3 Comparison method

Method	Ranking
CT method [Fahmi 2017]	$S_3 > S_2 > S_1$
TCFV method [Fahmi 2018]	$S_3 > S_2 > S_1$
TCFNs [Fahmi 2018]	$S_3 > S_2 > S_1$
Geometry operator [Fahmi 2018]	$S_3 > S_2 > S_1$
CFEF operator [Fahmi 2018]	$S_3 > S_2 > S_1$
GDM [Liu et al. (2017a, b)]	$S_3 > S_2 > S_1$
FT method [Wang et al. (2009)]	$S_3 > S_2 > S_1$

Table 4 The advantages of existing technique

Method	Score values	Ranking
IVI data [Liu (2013)]	$\left\{\begin{matrix}S_1 = 0.0123,\\ S_2 = 0.1291,\\ S_3 = 0.1654\end{matrix}\right\}$	$S_3 > S_2 > S_1$
IFIBP [Liu et al. (2017a, b)]	$\left\{\begin{matrix}S_1 = 0.0879,\\ S_2 = 0.0982,\\ S_3 = 0.2367\end{matrix}\right\}$	$S_3 > S_2 > S_1$
q-ROFN [Liu et al. (2018)]	$\left\{\begin{matrix}S_1 = 0.0109,\\ S_2 = 0.0897,\\ S_3 = 0.2189\end{matrix}\right\}$	$S_3 > S_2 > S_1$
IFPMSM [Liu et al. (2020)]	$\left\{\begin{matrix}S_1 = 0.0005,\\ S_2 = 0.1109,\\ S_3 = 0.2137\end{matrix}\right\}$	$S_3 > S_2 > S_1$

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