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ORIGINAL PAPER

Multi-criteria decision-making based on bi-parametric exponential fuzzy information measures and weighted correlation coefficients

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Abstract
This paper proposes a new bi-parametric exponential fuzzy information measure. In addition to the validation of proposed fuzzy information measure, some of its major properties are also studied. Besides, the performance of proposed fuzzy information measure is demonstrated using two numerical examples. Further, based on the concept of TOPSIS (Technique for Order Preference by Similarity to Ideal Solutions) method, a new improved TOPSIS method based on weighted correlation coefficients has been introduced. Considering the importance of criteria weights in the solution of Multi-Criteria Decision-Making (MCDM) problems, two methods have been discussed for the evaluation of criteria weights. In first method, criteria weight evaluation from the partial information provided by experts is discussed. Second method proposes the criteria weight evaluation in case they are completely unknown or incompletely known. The proposed MCDM method is explained through a numerical example based on fault detection in an ill-functioning machine.

Keywords Fuzzy set · Fuzzy information measure · MCDM · TOPSIS

1 Introduction

The evolution of fuzzy set theory by Zadeh (1965) has drawn the attention of authors worldwide. Before this invention, probability was used to quantify the uncertainty. But to quantify the uncertainty using probability theory, the uncertainty should have been expressed as precise number. The vague terms for example 'very, more, slightly', etc. could not be quantified using probability theory. To quantify the uncertainty associated with such vague terms, the fuzzy set theory proposed by Zadeh (1965) proved to be an effective tool. Opposite to the classical set theory in which an element either belongs to the set or not, in fuzzy set, each element is assigned a number lying between 0 and 1 known as its membership degree. The practical relevance of fuzzy set in dealing with real world problems is now a proven fact (Chen and Chen (2014), Chen et al. (2009), Chen and Wang (2010), Chen et al. (2012a)). Fuzzy entropy is an important term related with fuzzy set. Fuzzy entropy measures the degree of fuzziness arising due to ambiguity that an element belongs to the set or not. Zadeh (1968) for the first time attempted to measure the fuzzy entropy as weighted Shannon entropy. But this proposal failed to serve the purpose. Another successful attempt to quantify the degree of fuzziness arising due to the ambiguity of being or not being the element of a set was made by Luca and Termini (1972). Also, they introduced a new fuzzy entropy based on Shannon entropy. In addition to this, Luca and Termini (1972) axiomatized the fuzzy entropy. With this axiomatization of fuzzy entropy, several authors proposed fuzzy entropies from their viewpoints and applied them in distinct fields like pattern recognition, image processing, etc. (Higashi and Klir (1982), Joshi and Kumar (2017a), Kaufman (1980), Yager (1979)).

Decision-making is another important field where the concept of fuzzy entropy is widely applied. Decision-making problems involve a set of alternatives satisfying a certain set of criteria. The main aim is to select an alternative which optimally satisfies the whole criteria. Such type of problems are also named as Multi-Criteria Decision-Making (MCDM) problems. Several methods have been proposed by researchers in the literature to solve MCDM problems (Arya and Kumar (1989) Chen et al.

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(2012b), Chen and Chang (2016), Zeng et al. (2019), Wang and Chen (2017), Chen and Chiou (2015), Chen et al. (2012c)). Criteria weights play an eminent role in the solution of a MCDM problem. Therefore, evaluation of criteria weights needs utmost attention. Estimating the importance of criteria weights in the solution od MCDM problems, Chen and Li (2010) bifurcated them into two parts: subjective weights and objective weights. Subjective weights are provided by the experts according to their preferences (Chu et al. (1979), Hwang and Lin (1987), Saaty (1980)). On the other hand, objective criteria weights are obtained by solving mathematical models without considering the experts' viewpoint. Entropy method, principle element analysis (Choo and Wedley (1985), Fan (1996)) etc. are the examples of objective criteria weights. Among the objective methods, the entropy method is one of the most trusted approaches of determining criteria weights and has gained much popularity with researchers. In present communication, we will be using the entropy method for determining the criteria weights. Several MCDM methods based on this approach have been proposed so far.

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is one of the well-known techniques used for solving MCDM problems (Joshi and Kumar 2017b, 2018b, c, d, e, c, Lalotra and Singh 2018, Singh et al. 2019, Lalotra and Singh 2020). In conventional TOPSIS method, an alternative which is nearest to the best solution and farthest from the worst solution is supposed to be the best alternative. The decision is made on the basis of relative closeness coefficients which are obtained by dividing the distance of an alternative from the worst solution by sum of the distances of the same alternative from the best and the worst solutions. To compute the distance between alternatives and the best as well as worst solutions, different distance measures are used. But the distance measures used to calculate the distances of an alternative from the best and the worst solutions just represent the distance between two numbers and do not give the sufficient information about their correlations. For instance, consider an example of a machine that is not functioning properly. Intuitively, we know that there is a strong relationship between symptoms and faulty parts of the machine. Now, in such a case, if we correlate the symptoms and the corresponding faulty part of the machine with the help of fuzzy sets then distance measure used will merely compute the distance between two numbers but not their correlation. This may cause the loss of necessary information and we may get erroneous results. To avoid such situation, in this paper, we propose a TOPSIS method based on weighted correlation coefficients introduced by Gerstenkorn and Manko (1991).

The parametric generalization of Shannon entropy (Shannon (1948)) proposed by Renyi (1961) caused the attention of authors from across the world. For the first time, the researchers observed the role of parameters in an information measure. The presence of parameters in an information measure makes it flexible from application point of view and widens its scope of applications. For example, if we face a problem based on some environmental issue, different parameters may be used to represent the different environmental factors such as humidity, pressure, and temperature. More the number of parameters an information measure has, more will it be flexible for applications in diverse fields. Therefore, depending upon the nature of problem, we need to develop more and more flexible information measures which not only cater the need of the hour but also are the better performers. This study is a sincere effort in this direction. In this communication, a new bi-parametric exponential fuzzy information measure is proposed. The proposed information measure is a two parametric extension of exponential entropy studied by Pal and Pal (1989). The prime aims of introducing this communication are as follows:

- to introduce a new bi-parametric exponential fuzzy information measure,
- to introduce a new MCDM method based on entropy weights based correlation coefficients, and
- to utilize the proposed information measure and proposed MCDM method in detecting the fault in a machine.

With this aim, the subject contents of this paper are managed as follows: the background literature of topic under consideration is covered in Sect. 1. The relevant definitions and concepts necessary related to topic are given in Sect. 2. A new fuzzy information measure along with its validation is proposed in Section 3. Besides discussing some major properties, the performance of proposed fuzzy information measure is also demonstrated through numerical examples in Sect. 3. A new MCDM method based on weighted correlation coefficients along with its justification is introduced in Sect. 4. The proposed method is explained using numerical examples in Sect.,5. Finally, the paper is concluded with conclusions in Sect. 6.

In next Section, we present some relevant concepts and definitions.

2 Preliminaries

Definition 2.1 (Fuzzy set (Zadeh (1965))). Let $\sqcup = (\ell_1, \ell_2, \ldots, \ell_n)$ be a finite universe of discourse. A fuzzy set (FS) $\odot$ in $\sqcup$ is defined as

$$\odot = \{\langle \ell_i, \mu_\odot(\ell_i)\rangle | \ell_i \in \sqcup\}, \tag{1}$$

where $\mu_\odot : \sqcup \to [0,1]$ is the membership function and $\mu_\odot(\ell_i)$ represents the membership degree of $\ell_i \in \sqcup$ to $\odot$.

Definition 2.2 (Set operations on FSs (Zadeh (1965))). For any two FSs $\odot_1$ and $\odot_2$, some basic set operations are given by

1. $\odot_1 \cup \odot_2 = \{\langle \ell_i, \max(\mu_{\odot_1}(\ell_i), \mu_{\odot_2}(\ell_i))\rangle | \ell_i \in \sqcup\}$;
2. $\odot_1 \cap \odot_2 = \{\langle \ell_i, \min(\mu_{\odot_1}(\ell_i), \mu_{\odot_2}(\ell_i))\rangle | \ell_i \in \sqcup\}$;
3. $\odot^c = \{\langle \ell_i, (1 - \mu_\odot(\ell_i))\rangle | \ell_i \in \sqcup\}$.

Definition 2.3 (Sharpened version (Luca and Termini (1972))). A FS $\odot_1$ is said to be a sharpened version of another fuzzy set $\odot_2$ if

1. $\mu_{\odot_1}(\ell_i)$ is less than or equal to $\mu_{\odot_2}(\ell_i)$ if $\mu_{\odot_2}(\ell_i) \le \frac{1}{2}$.
2. $\mu_{\odot_1}(\ell_i)$ is greater than or equal to $\mu_{\odot_2}(\ell_i)$ if $\mu_{\odot_2}(\ell_i) \ge \frac{1}{2}$.

Definition 2.4 (Fuzzy entropy (Luca and Termini (1972))). For any FS $\odot$, a measure $H(\odot)$ is said to be fuzzy information measure if it satisfies the following properties:

Property 1. The value of $H(\odot)$ is zero if and only if $\odot$ ia a crisp set. This property is called Sharpness.

Property 2. The value of $H(\odot)$ is maximum if and only if $\odot$ is most fuzzy set. This property is called Maximality.

Property 3. The value of $H(\odot)$ is greater than equal to the value of $H(\tilde{\odot})$ if and only if $\tilde{\odot}$ is the sharpened version of $\odot$. This property is known as Resolution.

Property 4. The value of $H(\odot)$ is equal to the value of $H(\odot^c)$, where $\odot^c$ denotes the complement of $\odot$. This property is known as Symmetry.

Definition 2.5 (Correlation coefficients (Gerstenkorn and Manko (1991))) For any two FSs $\odot_1$ and $\odot_2$, the correlation coefficient $\sqsubset(\odot_1, \odot_2)$ is given by

$$\sqsubset(\odot_1, \odot_2) = \sum_{i=1}^{n} \frac{\Lambda(\odot_1, \odot_2)}{\sqrt{\delta(\odot_1) \cdot \delta(\odot_2)}}, \tag{2}$$

where $\Lambda(\odot_1, \odot_2) = \sum_{i=1}^{n} (\mu_{\odot_1}(\ell_i)\mu_{\odot_2}(\ell_i))$, $\delta(\odot_1) = \sum_{i=1}^{n} (\mu_{\odot_1}(\ell_i))^2$ and $\delta(\odot_2) = \sum_{i=1}^{n} (\mu_{\odot_2}(\ell_i))^2$.

The correlation coefficients $\sqsubset(\odot_1, \odot_2)$ satisfies the following properties:

1. $0 \le \sqsubset(\odot_1, \odot_2) \le 1$.
2. $\sqsubset(\odot_1, \odot_2) = \sqsubset(\odot_2, \odot_1)$.
3. $\sqsubset(\odot_1, \odot_2) = 1$ if $\odot_1 = \odot_2$.

3 A new bi-parametric exponential fuzzy information measure

3.1 Background

Let $\triangle_n = \{\hbar = (\hbar_1, \hbar_2, \ldots, \hbar_n); \hbar_i \ge 0, \sum_{i=1}^{n} \hbar_i = 1\}, n \ge 2$ be a set of complete probability distributions. For some $\hbar \in \triangle_n$, Shannon entropy (Shannon (1948)) is defined by

$${}_{Sh}H(\hbar) = -\sum_{i=1}^{n} \hbar_i \log(\hbar_i). \tag{3}$$

Afterwards, Renyi (1961) introduced a parametric extension of Shannon's concept given by

$${}_{Re}H^{\varsigma}(\hbar) = \begin{cases} \dfrac{1}{1-\varsigma} \log\left(\sum_{i=1}^{n} \hbar_i^{\varsigma}\right), \varsigma > 0 (\ne 1); \\ -\sum_{i=1}^{n} \hbar_i \log(\hbar_i), \varsigma = 1. \end{cases} \tag{4}$$

This development caused the several authors to propose parametric extensions of Shannon entropy (Boekee and Lubbe (1980), Havdra and Charavat (1967), Joshi and Kumar (2016, 2018g), Tsallis (1988)). Pointing out the limitations of Shannon (1948), Pal and Pal (1989) introduced a new entropy known as exponential entropy given by

$${}_{PP}H(\hbar) = \sum_{i=1}^{n} \hbar_i (e^{1-\hbar_i} - 1); \hbar \in \triangle_n. \tag{5}$$

The introduction of FS by Zadeh (1965) altered the way of quantifying the fuzziness. Moreover, the pioneering study of fuzzy entropy by Luca and Termini (1972) inspired the authors worldwide to come up with several fuzzy information measures. Extending the Shannon's concept, Luca and Termini (1972) introduced a fuzzy entropy. Following the way by Luca and Termini (1972), Pal and Pal (1989) proposed a new fuzzy entropy defined by

$$\begin{aligned} {}^{PP}H(\odot) = &\frac{1}{n(\sqrt{e}-1)} \sum_{i=1}^{n} \Big(\mu_\odot(\ell_i) e^{1-\mu_\odot(\ell_i)} \\ &+ (1-\mu_\odot(\ell_i)) e^{\mu_\odot(\ell_i)} - 1\Big). \end{aligned} \tag{6}$$

The concept of Pal and Pal (1989) was further generalized by Gupta et al. (2014) by introducing a parameter as follows:

$$ {}^{G}H_{\varsigma}(\odot) = \frac{1}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 1)} \sum_{i=1}^{n} \Big(\mu_{\odot}(\ell_i)^{\varsigma} e^{1-\mu_{\odot}(\ell_i)^{\varsigma}} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} - 1 \Big); 0<\varsigma<1. \tag{7} $$

From the above discussion, the importance of parameters in an information measure can be easily estimated. In fact, the problems involving several parameters may be conveniently represented by a multi-parametric information measure. For example, an environmental problem may involve the parameters such as temperature, pressure, and humidity. Motivated by this, we now propose a new bi-parametric exponential fuzzy information measure.

3.2 Definition

For some FS $\odot$, define

$$ {}^{\varrho}_{\varsigma}H(\odot) = \begin{cases} \frac{1}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}})} \sum_{i=1}^{n} \Big[\Big(\mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} \Big) \\ -\Big(\mu_{\odot}(\ell_i)^{\varrho} e^{(1-\mu_{\odot}(\ell_i)^{\varrho})} + (1-\mu_{\odot}(\ell_i))^{\varrho} e^{1-(1-\mu_{\odot}(\ell_i))^{\varrho}} \Big) \Big], \\ \qquad \text{either } 0<\varsigma<1, 1<\varrho<\infty \text{ or } 0<\varrho<1, 1<\varsigma<\infty; \\ \frac{1}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2)} \sum_{i=1}^{n} \Big(\mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} - 2 \Big), \varsigma = 0 \text{ or } \varrho = 0; \\ \frac{1}{n(\sqrt{e}-2)} \sum_{i=1}^{n} \Big(\mu_{\odot}(\ell_i) e^{1-\mu_{\odot}(\ell_i)} + (1-\mu_{\odot}(\ell_i)) e^{\mu_{\odot}(\ell_i)} - 2 \Big); \varsigma = 1, \varrho = 0 \text{ or } \varsigma = 0, \varrho = 1. \end{cases} \tag{8} $$

In (8), second and third cases, respectively, denote fuzzy entropies analogous to that of studied by Gupta et al. (2014) and Pal and Pal (1989).

The proposed information measure (8) being symmetric in nature, we will study only one case, that is, $0<\varsigma<1, 1<\varrho<\infty$.

In the next section, we validate the existence of (8) by satisfying the axioms in Definition (2.4).

3.3 Validation of proposed measure (8)

Theorem 3.1 *The proposed information measure (8) is a valid fuzzy information measure.*

Proof **Property 1.** First, we suppose that $\odot$ is a crisp set, that is, $\mu_{\odot}(\ell_i) = 1$ or 0 for all $\ell_i \in \sqcup$. Therefore, (8) gives

$$ {}^{\varrho}_{\varsigma}H(\odot) = 0 \text{for all} 0<\varsigma<1, 1<\varrho<\infty. \tag{9} $$

Conversely, suppose that ${}^{\varrho}_{\varsigma}H(\odot) = 0$. This implies

$$ \frac{1}{(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}})} \sum_{i=1}^{n} \Big[\Big(\mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} \Big) - \Big(\mu_{\odot}(\ell_i)^{\varrho} e^{(1-\mu_{\odot}(\ell_i)^{\varrho})} + (1-\mu_{\odot}(\ell_i))^{\varrho} e^{1-(1-\mu_{\odot}(\ell_i))^{\varrho}} \Big) \Big] = 0. \tag{10} $$

Since $0<\varsigma<1, 1<\varrho<\infty$, therefore, we have

$$ 2^{1-\varsigma}e^{1-2^{-\varsigma}} > 2^{1-\varrho}e^{1-2^{-\varrho}}. \tag{11} $$

Moreover, $0 \le \mu_{\odot}(\ell_i) \le 1$ implies

$$ \mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} \ge \mu_{\odot}(\ell_i)^{\varrho} e^{(1-\mu_{\odot}(\ell_i)^{\varrho})} + (1-\mu_{\odot}(\ell_i))^{\varrho} e^{1-(1-\mu_{\odot}(\ell_i))^{\varrho}}. \tag{12} $$

Therefore, from (11) and (12), we have

$$ 0 < \frac{A_1 - B_1}{2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}}} \le 1, \tag{13} $$

where

$$ A_1 = \mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}}; $$
$$ B_1 = \mu_{\odot}(\ell_i)^{\varrho} e^{(1-\mu_{\odot}(\ell_i)^{\varrho})} + (1-\mu_{\odot}(\ell_i))^{\varrho} e^{1-(1-\mu_{\odot}(\ell_i))^{\varrho}}. $$

This shows that ${}^{\varrho}_{\varsigma}H(\odot) = 0$ implies

$$ \Big(\mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} + (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} \Big) = \Big(\mu_{\odot}(\ell_i)^{\varrho} e^{(1-\mu_{\odot}(\ell_i)^{\varrho})} + (1-\mu_{\odot}(\ell_i))^{\varrho} e^{1-(1-\mu_{\odot}(\ell_i))^{\varrho}} \Big). \tag{14} $$

Since $\varsigma, \varrho > 0$ and $\varsigma \neq \varrho$, then (14) will hold only if $\mu_{\odot}(\ell_i) = 1$ or 0. This proves Property 1.

Property 2. Consider a function ψ given by

$$ \psi(x, \varsigma, \varrho) = \frac{\big(x^{\varsigma}e^{1-x^{\varsigma}} + (1-x)^{\varsigma}e^{1-(1-x)^{\varsigma}}\big) - \big(x^{\varrho}e^{1-x^{\varrho}} + (1-x)^{\varrho}e^{1-(1-x)^{\varrho}}\big)}{2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}}}, \tag{15} $$

where $0<\varsigma<1, 1<\varrho<\infty$ and $0 \le x \le 1$.

Differentiating, (15) with respect to x, we get

$$\frac{\partial\psi}{\partial x}=\frac{A_2-B_2}{2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}}},\tag{16}$$

where

$$A_2=\varrho x^{\varrho-1}(1-x^{\varrho})e^{1-x^{\varrho}}-\varrho(1-x)^{\varrho-1}(1-(1-x)^{\varrho})e^{(1-(1-x)^{\varrho})},$$
$$B_2=\varsigma x^{\varsigma-1}(1-x^{\varsigma})e^{1-x^{\varsigma}}-\varsigma(1-x)^{\varsigma-1}(1-(1-x)^{\varsigma})e^{(1-(1-x)^{\varsigma})}.$$

To determine the stationary points, we express (16) as

$$\frac{\partial\psi}{\partial x}=\frac{\widehat{A}+\widehat{B}+\widehat{C}+\widehat{D}}{2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}}},\tag{17}$$

where

$$\begin{aligned}\widehat{A}&=\varrho x^{\varrho-1}(1-x^{\varrho})e^{1-x^{\varrho}};\\ \widehat{B}&=-\varrho(1-x)^{\varrho-1}(1-(1-x)^{\varrho})e^{(1-(1-x)^{\varrho})};\\ \widehat{C}&=-\varsigma x^{\varsigma-1}(1-x^{\varsigma})e^{1-x^{\varsigma}};\\ \widehat{D}&=\varsigma(1-x)^{\varsigma-1}(1-(1-x)^{\varsigma})e^{(1-(1-x)^{\varsigma})}.\end{aligned}\tag{18}$$

On replacing x with $1-x$ in $\widehat{A}$ and $\widehat{C}$, we find that $\widehat{A}+\widehat{B}=0=\widehat{C}+\widehat{D}$. Therefore, $\frac{\partial\psi}{\partial x}=0$ at $x=1-x$. This implies that $x=\frac{1}{2}$ is the stationary point.

To prove that ψ has maximum value $x=\frac{1}{2}$, we prove that ψ is concave at $x=\frac{1}{2}$. For this, differentiating (15) twice with respect to x and computing its value at $x=\frac{1}{2}$, we get

$$\left(\frac{\partial^2\psi}{\partial x^2}\right)_{x=\frac{1}{2}}=\frac{A_3-B_3}{2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}}},\tag{19}$$

where

$$\begin{aligned}A_3&=e^{1-.5^{\varrho}}\big(4\varrho^2 2^{2-2\varrho}-2\varrho^2 2^{2-3\varrho}-2\varrho(\varrho-1)2^{2-\varrho}\\&\quad+2\varrho(\varrho-1)2^{2-2\varrho}\big),\\ B_3&=e^{1-.5^{\varsigma}}\big(4\varsigma^2 2^{2-2\varsigma}-2\varsigma^2 2^{2-3\varsigma}-2\varsigma(\varsigma-1)2^{2-\varsigma}\\&\quad+2\varsigma(\varsigma-1)2^{2-2\varsigma}\big).\end{aligned}$$

Now, (19) is negative for all $0<\varsigma<1, 1<\varrho<\infty$ which shows that the function ψ is concave with maximum value at the point $x=\frac{1}{2}$. This proves that ${}^{\varrho}_{\varsigma}H(\odot)$ has maximum value when $\odot$ is the most fuzzy set.

Property 3. In Property 2, it is proved that the measure ${}^{\varrho}_{\varsigma}H(\odot)$ is a concave function with point of inflexion at $x=\frac{1}{2}$. From this, it may be easily concluded that the measure ${}^{\varrho}_{\varsigma}H(\odot)$ is an increasing function in the interval [0, 0.5) and decreasing function in the interval (0.5, 1]. This proves Property 3.

Property 4. This property follows directly from the definition.

Thus, the existence of proposed measure ${}^{\varrho}_{\varsigma}H(\odot)$ is established.

In next section, we discuss some major properties of proposed measure (8).

3.4 Properties of measure (8)

Theorem 3.2 *For any two fuzzy sets* $\odot_1$ *and* $\odot_2$, ${}^{\varrho}_{\varsigma}H(\odot_1\cup\odot_2)+{}^{\varrho}_{\varsigma}H(\odot_1\cap\odot_2)={}^{\varrho}_{\varsigma}H(\odot_1)+{}^{\varrho}_{\varsigma}H(\odot_2)$.

Corollary *For any fuzzy set* $\odot$ *and its complement* $\odot^c$, ${}^{\varrho}_{\varsigma}H(\odot\cup\odot^c)+{}^{\varrho}_{\varsigma}H(\odot\cap\odot^c)={}^{\varrho}_{\varsigma}H(\odot)+{}^{\varrho}_{\varsigma}H(\odot^c)$.

Theorem 3.3 *The maximum value of* ${}^{\varrho}_{\varsigma}H(\odot)$ *takes place when* $\odot$ *is most fuzzy set and minimum value takes place when* $\odot$ *is least fuzzy set. Moreover, both of the values are independent of* ς *and* ϱ.

Proof The proofs of the abovementioned theorems as well as corollary are given in Appendix 'A'. □

3.5 A demonstration of performance

The adjectives like 'slightly', 'more or less' are used to modify the linguistic variables. Fuzzy sets proposed by Zadeh (1965) have been proved to be an effective tool to characterize the linguistic variables. Therefore, the adjectives such as very and slightly can be supposed as operations on fuzzy sets. Using these operations on fuzzy sets, in this section, we compare the performance of proposed measure (8) with some other existing fuzzy information measures in the literature. For a fuzzy set $\odot$, the modifier of $\odot$ is defined as

$$\odot^n=\{\langle\ell_i,\mu_{\odot}(\ell_i)^n\rangle|\ell_i\in\sqcup\}.\tag{20}$$

Hwang and Yang (2008) and Hung and Yang (2008) define the concentration of a fuzzy set $\odot$ as $CON(\odot)=\odot^2$ and dilation of a fuzzy set $\odot$ as $DIL(\odot)=\odot^{\frac{1}{2}}$. The two terms, that is, concentration and dilation are frequently used to modify the fuzzy sets. Thus, we use these terms as mathematical operators to characterize the linguistic variables as follows:

$$\begin{aligned}&\text{More or Less}(\odot)=DIL(\odot)=\odot^{\frac{1}{2}},\text{Very}A=CON(\odot)=\odot^2;\\&\text{Quite Very}(\odot)=\odot^3,\text{Very Very}(\odot)=\odot^4.\end{aligned}\tag{21}$$

Many authors (De et al. (2001), Hung and Yang (2006), Hwang and Yang (2008), Joshi and Kumar (2018a), Xia and Xu (2012)) have utilized these operators as a criteria of performance of a fuzzy set as follows:

$$H(\odot^{\frac{1}{2}})>H(\odot)>H(\odot^2)>H(\odot^3)>H(\odot^4),\tag{22}$$

where $H(\cdot)$ denotes the fuzzy entropy of a fuzzy set $\odot$. Therefore, we use (22) as a criteria to compare the performance of proposed fuzzy information measure with other measures. Before comparison, we characterize the fuzzy sets $\odot^{\frac{1}{2}}, \odot^2, \odot^3, \odot^4$ in terms of linguistic variables as follows:

$$\begin{aligned} &\text{Characterize } \odot^{\frac{1}{2}} \; as \text{ ``More or Less Large''} \\ &\text{Characterize } \odot^{2} \; as \text{ ``Very Large''} \\ &\text{Characterize } \odot^{3} \; as \text{ ``Quite Very Large''} \\ &\text{Characterize } \odot^{4} \; as \text{ ``Very Very Large''}. \end{aligned} \tag{23}$$

For comparison sake, we take two examples adapted from Joshi and Kumar (2018a).

Example 1 Consider a fuzzy set $\odot$ defined on a universe of discourse $X = (a, b, c, d, e)$ given by

$$\odot = \{(a, .1), (b, .3), (c, .4), (d, .9), (e, 1)\}. \tag{24}$$

Characterizing the linguistic variables, we tag $\odot$ as "Large". Using this characterization, following fuzzy sets may be generated:

$$\begin{aligned} \odot^{\frac{1}{2}} &= \{(a, .316), (b, .548), (c, .632), (d, .949), (e, 1)\}; \\ \odot^{2} &= \{(a, .010), (b, .090), (c, .160), (d, .810), (e, 1)\}; \\ \odot^{3} &= \{(a, .001), (b, .027), (c, .064), (d, .0729), (e, 1)\}; \\ \odot^{4} &= \{(a, 0), (b, .008), (c, .026), (d, .656), (e, 1)\}. \end{aligned} \tag{25}$$

To compare the performance of proposed measure, we consider the following well-known fuzzy information measures proposed by various authors given by

1. Fuzzy information measure proposed by Hwang and Yang (2008),

$$\begin{aligned} H_{hy}(\odot) =& \frac{\sqrt{e}}{\sqrt{e}-1} \sum_{i=1}^{n} \Big[\Big(1 - e^{-\mu_{\odot^c}(\ell_i)}\Big) I_{[\mu_{\odot^c}(\ell_i) \geq \frac{1}{2}]} \\ &+ \Big(1 - e^{-\mu_{\odot}(\ell_i)}\Big) I_{[\mu_{D^c}(\ell_i) < \frac{1}{2}]} \Big]. \end{aligned} \tag{26}$$

2. Fuzzy information measure introduced by Pal and Pal (1989),

$$\begin{aligned} {}^{PP}H(\odot) =& \frac{1}{n} \sum_{i=1}^{n} \Big[\mu_{\odot}(\ell_i) e^{1-\mu_{\odot}(\ell_i)} \\ &+ (1-\mu_{\odot}(\ell_i)) e^{\mu_{\odot}(\ell_i)} - 1 \Big]. \end{aligned} \tag{27}$$

3. Fuzzy information measure proposed by Li and Liu (2008),

$$H_{ll}(\odot) = \sum_{i=1}^{n} S(Cr(\xi_{\odot} = x_i)). \tag{28}$$

4. Fuzzy information measure introduced by Yager (1979),

$$H_{yager}(\odot) = 1 - \frac{d_p(\odot, \odot^c)}{n^{\frac{1}{p}}}. \tag{29}$$

5. Fuzzy information studied by Gupta et al. (2014),

$$\begin{aligned} {}^{G}H_{\varsigma}(\odot) &= \frac{1}{n(2^{1-\varsigma} e^{1-2^{-\varsigma}} - 1)} \sum_{i=1}^{n} \Big(\mu_{\odot}(\ell_i)^{\varsigma} e^{(1-\mu_{\odot}(\ell_i)^{\varsigma})} \\ &+ (1-\mu_{\odot}(\ell_i))^{\varsigma} e^{1-(1-\mu_{\odot}(\ell_i))^{\varsigma}} - 1 \Big); \; 0 < \varsigma < 1. \end{aligned} \tag{30}$$

6. Fuzzy information measure studied by Kosko (1986),

$$H_{KO}(\odot) = \frac{d_p(\odot, \odot_{near})}{d_p(\odot, \odot_{far})}. \tag{31}$$

The computed values of fuzzy information measures (26) to (31) and proposed information measure (8) for the fuzzy sets defined in (25) are displayed in Table 1.

The observations obtained from Table 1 are summarized as follows:

$$\begin{aligned} & H_{hy}(\odot^{\frac{1}{2}}) > H_{hy}(\odot) > H_{hy}(\odot^2) > H_{hy}(\odot^3) > H_{hy}(\odot^4); \\ & {}^{PP}H(\odot^{\frac{1}{2}}) > {}^{PP}H(\odot) > {}^{PP}H(\odot^2) > {}^{PP}H(\odot^3) > {}^{PP}H(\odot^4); \\ & H_{KO}(\odot^{\frac{1}{2}}) < H_{KO}(\odot) > H_{KO}(\odot^2) > H_{KO}(\odot^3) > H_{KO}(\odot^4); \\ & H_{ll}(\odot^{\frac{1}{2}}) > H_{ll}(\odot) > H_{ll}(\odot^2) > H_{ll}(\odot^3) > H_{ll}(\odot^4); \\ & H_{yager}(\odot^{\frac{1}{2}}) > H_{yager}(\odot) > H_{yager}(\odot^2) > H_{yager}(\odot^3) \\ & \quad > H_{yager}(\odot^4); \\ & {}^{G}H_{\varsigma}(\odot^{\frac{1}{2}}) > {}^{G}H_{\varsigma}(\odot) > {}^{G}H_{\varsigma}(\odot^2) > {}^{G}H_{\varsigma}(\odot^3) > {}^{G}H_{\varsigma}(\odot^4); \\ & {}^{\varrho}_{\varsigma}H(\odot^{\frac{1}{2}}) > {}^{\varrho}_{\varsigma}H(\odot) > {}^{\varrho}_{\varsigma}H(\odot^2) > {}^{\varrho}_{\varsigma}H(\odot^3) > {}^{\varrho}_{\varsigma}H(\odot^4). \end{aligned} \tag{32}$$

From (32), it may be observed that $H_{hy}, {}^{PP}H, H_{ll}, H_{yager}, {}^{G}H_{\varsigma}$ and ${}^{\varrho}_{\varsigma}H$ follow the pattern (22) whereas H_{KO} do not. This shows that the performance of measure H_{KO} is not better as compared to other measures.

For further comparison, we consider another Example.

Example 2 On a universe of discourse $X = \{\widehat{a}, \widehat{b}, \widehat{c}, \widehat{d}, \widehat{e}\}$, consider another fuzzy set defined by

$$\widetilde{\odot} = \{(\widehat{a}, .2), (\widehat{b}, .3), (\widehat{c}, .4), (\widehat{d}, .7), (\widehat{e}, 1)\}. \tag{33}$$

Again, using (20), we generate the following fuzzy sets.

Table 1 Computed values of information measures (26)–(31) and (8)

	$H_{hy}(\odot)$	${}^{PP}H(\odot)$	$H_{KO}(\odot)$	$H_{ll}(\odot)$	$H_{yager}(D)$	${}^{G}H_{\varsigma=.8}(\odot)$	${}^{\varrho(=7)}_{\varsigma(=.5)}H(\odot)$
$\odot^{\frac{1}{2}}$	.505	.599	.220	.810	.397	.630	.630
$\odot$	.397	.510	.311	.723	.360	.564	.566
$\odot^2$	.212	.311	.099	.378	.167	.378	.368
$\odot^3$	.167	.232	.078	.870	.145	.281	.287
$\odot^4$	.165	.209	.082	.692	.151	.238	.253

Table 2 Computed values of information measures (26)–(31) and (8)

	$H_{hy}(\tilde{\odot})$	${}^{PP}H(\tilde{\odot})$	${}^{G}H_{\varsigma=8}(\tilde{\odot})$	${}^{\varrho(=25)}_{\varsigma(=.8)}H(\tilde{\odot})$
$\tilde{\odot}^{\frac{1}{2}}$	.653	.694	.630	.761
$\tilde{\odot}$	.616	.661	.563	.754
$\tilde{\odot}^2$	.577	.409	.378	.549
$\tilde{\odot}^3$	.393	.259	.281	.333
$\tilde{\odot}^4$	.298	.177	.238	.238

$$\begin{aligned}\tilde{\odot}^{\frac{1}{2}} &= \{(\widehat{a}, .447), (\widehat{b}, .548), (\widehat{c}, .632), (\widehat{d}, .837), (\widehat{e}, 1)\};\\ \tilde{\odot}^{2} &= \{(\widehat{a}, .040), (\widehat{b}, .090), (\widehat{c}, .160), (\widehat{d}, .490), (\widehat{e}, 1)\};\\ \tilde{\odot}^{3} &= \{(\widehat{a}, .008), (\widehat{b}, .027), (\widehat{c}, .064), (\widehat{d}, .0343), (\widehat{e}, 1)\};\\ \tilde{\odot}^{4} &= \{(\widehat{a}, .002), (\widehat{b}, .008), (\widehat{c}, .026), (\widehat{d}, .240), (\widehat{e}, 1)\}.\end{aligned} \tag{34}$$

The computed values of $H_{hy}, {}^{PP}H, {}^{G}H_{\varsigma}$ and ${}^{\varrho}_{\varsigma}H$ are shown in Table 2.

A Discussion on performance: Thus, all the four measures, that is, $H_{hy}, {}^{PP}H, {}^{G}H_{\varsigma}$ and ${}^{\varrho}_{\varsigma}H$, considered in Table 2 follow the sequence given in (22). This conclusion generates a natural query in mind that which information measure is better than others. On analyzing the eight fuzzy information measures considered so far, we observe that all the information measures except the proposed one are either rigid or contains one parameter. Therefore, the measures (26), (27), (28), (29) and (31) are not suitable for the applications which involve parameters. Also, the measure (30) is one parametric which restricts its scope of application to the problems involving one parameter only. On the other hand, the proposed measure (8) is a bi-parametric information measure which enhances its scope of applications. This establishes the fact that the proposed measure (8) is effective and its performance is considerably good.

4 Application of proposed measure in decision-making

A multi-criteria decision-making (MCDM) is the process of selecting the most desirable alternative satisfying the established criteria. As discussed earlier, the criteria weights play a decisive role in the selection of the most appropriate alternative. The improper evaluation of criteria weights may lead to the selection of an inferior alternative which ultimately may appear in the form of loss. Therefore, the evaluation of criteria weights needs utmost attention while solving a MCDM problem. In the present study, we adopt two approaches to evaluate criteria weights: First approach includes the case when we have partial information about criteria weights and second approach is based on when the criteria weights are incompletely known or completely unknown.

4.1 A new TOPSIS method based on weighted correlation coefficients

4.1.1 Justification of proposed method

Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS) is one of the widely used techniques to solve MCDM problems. In TOPSIS method, two extreme solutions, that is, the best possible solution and the worst possible solution are determined and an alternative closest to the best solution and farthest from the worst solution is chosen as the most desirable alternative. To check the degree of closeness with extreme solutions, different distance measures are used. Now, it has been proved that results of TOPSIS method vary with the change of distance measure used (Joshi and Kumar (2018f)). Therefore, instead of using distance measure based TOPSIS method, a new improved TOPSIS method based on correlation coefficients is being proposed in this study. Further, Ye (2010) argued that the techniques used in fuzzy decision-making either use score function or accuracy functions that do not give sufficient information about alternatives. Therefore, these methods are not reliable and a substitute is necessary for decision-making. With this viewpoint, Ye

(2010) proposed an alternative method for fuzzy decision-making. But in his proposed technique, Ye (2010) considered the correlation between alternatives and the best solution only thus neglecting the correlation between alternatives and the worst solution.

In the next Subsection, we propose a MCDM method by considering the correlation of alternatives with the best and the worst possible solutions.

4.1.2 The proposed method

The procedural steps of proposed MCDM method are as follows:

Step 1. Preparation of fuzzy decision matrix

Represent the given MCDM problem in matrix form with rows representing alternatives $\Upsilon_i (i = 1, 2, \ldots, m)$ and columns denoting criteria $\Psi_j (j = 1, 2, \ldots, n)$ as follows:

$$\widetilde{M}_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} \Psi_1 & \Psi_2 & \ldots & \Psi_n \end{array} \\ \begin{array}{c} \Upsilon_1 \\ \Upsilon_2 \\ \vdots \\ \Upsilon_m \end{array} & \begin{pmatrix} \widehat{\mu}_{11} & \widehat{\mu}_{12} & \cdots & \widehat{\mu}_{1n} \\ \widehat{\mu}_{21} & \widehat{\mu}_{22} & \cdots & \widehat{\mu}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{\mu}_{m1} & \widehat{\mu}_{m2} & \cdots & \widehat{\mu}_{mn} \end{pmatrix} \end{array}; \tag{35}$$

where $\widehat{\mu}_{ij} = \mu(\Upsilon_i, \Psi_j)$ denotes the degree of satisfaction of the alternative Υ_i to the criteria Ψ_j. To determine the degree of satisfaction of Υ_i's corresponding to criteria Ψ_j's, we apply the following method:

$$\mu_{ij} = \frac{n_y(i,j)}{N} \tag{36}$$

where $n_y(i,j)$ denotes the number of experts who support Υ_i corresponding to criteria Ψ_j and N represents the total number of experts.

Step 2. Normalization of decision matrix

To treat all the criteria at a par, normalize the matrix obtained in Step 1 as follows:

$$\mu_{ij} = \begin{cases} \widehat{\mu}_{ij} & \text{for Benefit Criteria;} \\ (1 - \widehat{\mu}_{ij}) & \text{for Cost Criteria.} \end{cases} \tag{37}$$

Let the matrix so obtained be represented by M

$$M_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} \Psi_1 & \Psi_2 & \ldots & \Psi_n \end{array} \\ \begin{array}{c} \Upsilon_1 \\ \Upsilon_2 \\ \vdots \\ \Upsilon_m \end{array} & \begin{pmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{pmatrix} \end{array}; \tag{38}$$

. **Step 3. Determination of criteria weights**

As discussed earlier, the criteria weights play an important role in the solution of a MCDM problem. This implies that evaluation of criteria weights needs due attention. With this point in mind, we bifurcate the process of determination of criteria weights into two parts as follows:

Approach I: If the Information Available about criteria Weights is Partial

The advice of experts involved in decision-making process matters very much. But all the experts may not be well-versed with all the aspects of a problem. Therefore, it is not possible to have such reliable advice every time. This may also be due to lack of time, limited expertise about problem domain etc. that experts do not express themselves in the form of precise numbers. In fact, they prefer to express themselves in the form of intervals. We compile this available partial information about criteria weights in the form of a set denoted by Σ. To determine the criteria weights, we use the principle of minimum entropy suggested by Wang and Wang (2012) as follows:

The total entropy of an alternative Υ_i across all the criteria Ψ_j is given by

$$\begin{aligned} {}^{\varrho}_{\varsigma}H(\Upsilon_i) &= \sum_{j=1}^{n} {}^{\varrho}_{\varsigma}H(\mu_{ij}) \\ &= \sum_{j=1}^{n} \Bigg(\frac{1}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}})} \\ &\quad \Big[\Big(\mu_{ij}^{\varsigma} e^{(1-\mu_{ij}^{\varsigma})} + (1-\mu_{ij})^{\varsigma} e^{1-(1-\mu_{ij})^{\varsigma}} \Big) \\ &\quad - \Big(\mu_{ij}^{\varrho} e^{(1-\mu_{ij}^{\varrho})} + (1-\mu_{ij})^{\varrho} e^{1-(1-\mu_{ij})^{\varrho}} \Big) \Big] \Bigg); \\ &= \frac{1}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}})} \\ &\quad \sum_{j=1}^{n} \Big[\Big(\mu_{ij}^{\varsigma} e^{(1-\mu_{ij}^{\varsigma})} + (1-\mu_{ij})^{\varsigma} e^{1-(1-\mu_{ij})^{\varsigma}} \Big) \\ &\quad - \Big(\mu_{ij}^{\varrho} e^{(1-\mu_{ij}^{\varrho})} + (1-\mu_{ij})^{\varrho} e^{1-(1-\mu_{ij})^{\varrho}} \Big) \Big]. \end{aligned} \tag{39}$$

Since, each alternative is a fair competition, then, the weight coefficients corresponding to same criteria should also be same. Therefore, to determine the optimal criteria weights, we construct the following programming model:

$$\begin{aligned} \min(I) &= \sum_{i=1}^{m} \Big(w_j \Big({}^{\varrho}_{\varsigma}H(\Upsilon_i) \Big) \Big) \\ &= \frac{1}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}})} \\ &\quad \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(w_j \Big[\Big(\mu_{ij}^{\varsigma} e^{(1-\mu_{ij}^{\varsigma})} + (1-\mu_{ij})^{\varsigma} e^{1-(1-\mu_{ij})^{\varsigma}} \Big) \\ &\quad - \Big(\mu_{ij}^{\varrho} e^{(1-\mu_{ij}^{\varrho})} + (1-\mu_{ij})^{\varrho} e^{1-(1-\mu_{ij})^{\varrho}} \Big) \Big] \Big) \end{aligned} \tag{40}$$

subject to the condition $\sum_{j=1}^{n} w_j = 1, w_j \in \Sigma$. On solving (40), we get the criteria weight vector in the form $\arg\min E = (w_1, w_2, \ldots, w_n)'$ as an optimal solution, where $'$ denotes the transpose.

Approach II: If the criteria Weights are Unknown

To obtain the criteria weights in this case, we employ the method suggested by Chen and Li (2010) as follows:

$$w_j = \frac{1 - {}^{\varrho}_{\varsigma}H(\mu_{ij})}{n - \sum_{j=1}^{n}\left({}^{\varrho}_{\varsigma}H(\mu_{ij})\right)}, \tag{41}$$

where

$$\begin{aligned} {}^{\varrho}_{\varsigma}H(\mu_{ij}) = & \frac{1}{(2^{1-\varsigma}e^{1-2^{-\varsigma}} - 2^{1-\varrho}e^{1-2^{-\varrho}})} \\ & \sum_{i=1}^{m}\left[\left(\mu_{ij}^{\varsigma}e^{(1-\mu_{ij}^{\varsigma})} + (1-\mu_{ij})^{\varsigma}e^{1-(1-\mu_{ij})^{\varsigma}}\right)\right. \\ & \left.-\left(\mu_{ij}^{\varrho}e^{(1-\mu_{ij}^{\varrho})} + (1-\mu_{ij})^{\varrho}e^{1-(1-\mu_{ij})^{\varrho}}\right)\right]. \end{aligned} \tag{42}$$

Step 4. Determination of extreme solutions

Determine the best solution $(\Upsilon^{\star})$ as well as worst solution $(\Upsilon_{\star})$ as follows:

$$\begin{aligned} \Upsilon^{\star} = \max(\mu_{ij}) \text{ and } \Upsilon_{\star} = \min(\mu_{ij}); \text{ for all} \\ i = 1, 2, \ldots m; j = 1, 2, \ldots, n. \end{aligned} \tag{43}$$

Step 5. Determination of weighted correlation coefficients

Determine the weighted correlation coefficients Υ_i's$(i = 1, 2, \ldots, m)$ with $\Upsilon^{\star}$ and $\Upsilon_{\star}$ using Definition (2.5) as follows:

$$\begin{aligned} \sqsubset(\Upsilon_i, \Upsilon^{\star}) &= \frac{w_j \Lambda(\Upsilon_i, \Upsilon^{\star})}{\sqrt{w_j(\delta(\Upsilon_i)\cdot\delta(\Upsilon^{\star}))}}; \\ \sqsubset(\Upsilon_i, \Upsilon_{\star}) &= \frac{w_j \Lambda(\Upsilon_i, \Upsilon_{\star})}{\sqrt{w_j(\delta(\Upsilon_i)\cdot\delta(\Upsilon_{\star}))}}. \end{aligned} \tag{44}$$

Step 6. Determination of relative closeness coefficients

Determine the relative correlation coefficients $\chi_i(i = 1, 2, \ldots, m)$ as follows:

$$\chi_i = \frac{\sqsubset(\Upsilon_i, \Upsilon_{\star})}{\sqsubset(\Upsilon_i, \Upsilon_{\star}) + \sqsubset(\Upsilon_i, \Upsilon^{\star})}. \tag{45}$$

Step 7. Ranking of alternatives

Arranging the Υ_i's according to the values of χ_i's$(i = 1, 2, \ldots, m)$ in descending order, we obtain the preferential sequence of alternatives.

In subsequent Section, we explain the proposed MCDM method through numerical examples.

5 Illustrated numerical examples

Vagueness in scientific studies is presenting a challenging dimensions. Fuzzy set theory proposed by Zadeh (1965) has proved to be effective tool to characterize such vagueness. To counter the menace of vagueness in scientific studies, there is a need to develop information measures and relevant methods which can measure the vagueness in the underlying characterizing fuzzy sets. The example considered in this Section highlights the need of such measures and methods. Consider an example of a factory in which a machine is not functioning properly. In large-scale units, such occurrences are frequent and needs immediate attention otherwise it may slow down the production rate which ultimately may appear in the form of loss to the company. To keep the machine going on smoothly, the immediate detection of fault in the machine is necessary. To locate the fault in the machine at the earliest can be one of the best examples of the applied intelligence. Since, symptoms the machine is showing and the part of the machine in which fault is located are strongly correlated, therefore, the proposed MCDM method can be applied successfully in this case.

5.1 For partially known criteria weights

Example To perform the task, we broadly divide the machine under observation into five parts say $\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5$. Let $(\Psi_1, \Psi_2, \Psi_3, \Psi_4)$ denote the set of symptoms bearing certain relationship with faulty sections that the machine is indicating on the basis of which the faulty section is to be fixed. Sometimes, different faults have common symptoms which give rise to fuzziness in determining the exact faulty part of the machine. Fuzzy sets proposed by Zadeh (1965) are helpful in dealing with such vagueness. Therefore, we use the proposed fuzzy information measure for the purpose. Using (36), let the

Table 3 Fuzzy decision matrix $(M)_{5\times4}$

	Ψ_1	Ψ_2	Ψ_3	Ψ_4	
Υ_1	.7	.4	.5	.6	(46)
Υ_2	.7	.8	.6	.3	
Υ_3	.6	.3	.4	.7	
Υ_4	.8	.1	.4	.5	
Υ_5	.6	.2	.1	.6	
${}^{\varrho}_{\varsigma}H(\Psi_j)$	.9601	.8742	.9221	.9821	

degree of relationship that a symptom bears with a particular part of the machine under observation be represented by the fuzzy decision matrix given by

Let the information available about criteria weights be denoted by the set Σ given by

$$\Sigma = \{0 \le w_1 \le .3, .1 \le w_2 \le .2, .2 \le w_3 \le .5, .1 \le w_4 \le .3\} \tag{47}$$

subject to the condition $\sum_{j=1}^{4} w_j = 1$.

Now, we apply the proposed MCDM method to select the faulty section of the machine. The computational procedure is as follows:

Step 1. The given MCDM problem is represented by the matrix (46).

Step 2. Since, we are dealing with the problem of detection of fault in the machine, therefore, there is no benefit or cost criteria. Therefore, the normalized fuzzy decision matrix is represented by the fuzzy decision matrix (46) itself.

Step 3. To determine the weight-age of symptoms, we proceed as follows:

1. Using proposed information measure (8), the computed values of information for each symptom $\Psi_j; (j = 1, 2, 3, 4)$, that is, ${}^{\varrho}_{\varsigma}H(\Psi_j)$ are shown in last row of Table 1.
2. Using (40), we construct the the following programming model:

$$\min(I) = .9601w_1 + .8742w_2 + .9221w_3 + .9821w_4$$
$$\text{subject to} \begin{cases} 0 \le w_1 \le .3; \\ .1 \le w_2 \le .2; \\ .2 \le w_3 \le .5; \\ .1 \le w_4 \le .3; \\ \sum_{i=1}^{4} w_i = 1. \end{cases} \tag{48}$$

3. Solving the linear programming model (48) using MATLAB, the criteria weights so obtained are given by

$$w_1 = .2, w_2 = .2, w_3 = .5, w_4 = .1. \tag{49}$$

   The syntax for MATLAB code is provided in Appendix 'B'.

Step 4. Using (43), the compute values of $\Upsilon^{\star}$ and $\Upsilon_{\star}$ are given by

$$\Upsilon^{\star} = (.8, .8, .6, .7) \text{ and } \Upsilon_{\star} = (.6, .1, .1, .3). \tag{50}$$

Step 5. The computed values of weighted correlation coefficients using (44) are given by

$$\begin{aligned} \sqsubset(\Upsilon_1, \Upsilon^{\star}) &= .9636,\ \sqsubset(\Upsilon_2, \Upsilon^{\star}) = .9536,\ \sqsubset(\Upsilon_3, \Upsilon^{\star}) \\ &= .9420,\ \sqsubset(\Upsilon_4, \Upsilon^{\star}) = .8482,\ \sqsubset(\Upsilon_5, \Upsilon^{\star}) \\ &= .7525; \\ \sqsubset(\Upsilon_1, \Upsilon_{\star}) &= .6742,\ \sqsubset(\Upsilon_2, \Upsilon_{\star}) = .7023,\ \sqsubset(\Upsilon_3, \Upsilon_{\star}) \\ &= .6725,\ \sqsubset(\Upsilon_4, \Upsilon_{\star}) \\ &= .5999,\ \sqsubset(\Upsilon_5, \Upsilon_{\star}) = .5785. \end{aligned} \tag{51}$$

Step 6. The computed values of relative correlation coefficients $\chi_i (i = 1, 2, 3, 4, 5)$ using (45) are given by

$$\chi_1 = .4117, \chi_2 = .4241, \chi_3 = .4166, \chi_4 = .4143, \chi_5 = .4346. \tag{52}$$

Step 7. Arranging the sections $\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5$ of the machine, according to the values of χ_is in descending order, the preferential sequence of faulty sections so obtained is given by

$$\Upsilon_5 \succ \Upsilon_2 \succ \Upsilon_3 \succ \Upsilon_4 \succ \Upsilon_1. \tag{53}$$

Thus, Υ_5 is the section of the machine in which fault is located.

A comparative analysis: To compare the performance of proposed method, the same Example was computed using the weighted correlation coefficients method proposed by Ye (2010) and the output so obtained is given by $\Upsilon_1 \succ \Upsilon_2 \succ \Upsilon_3 \succ \Upsilon_4 \succ \Upsilon_5$ with Υ_1 as the best alternative. If we compute the above example by using conventional fuzzy TOPSIS method and using the Hamming distance as the distance measure, the preferential sequence of alternatives so obtained is given by: $\Upsilon_2 \succ \Upsilon_1 \succ \Upsilon_3 \succ \Upsilon_4 \succ \Upsilon_5$ with Υ_2 as the best alternative. Thus, all the three methods used to determine the best alternative in same example are producing the different outputs. Now, a natural question that arise in mind is that which output is most reliable. This is an established fact that different algorithms are defined with different viewpoints (Joshi and Kumar (2017c)). Due to this difference in viewpoints, the weight-age given to different criteria in a problem may vary which ultimately may change the output. We start with Ye's method (Ye (2010)). Ye's method (Ye (2010)) is based on the correlation of alternatives with the best solution only whereas the correlation with the worst solution is not considered. Even in TOPSIS method, which is one of the widely used method, the best alternative is decided on the basis of relative closeness coefficients in which distances of the alternative from both solutions, that is, the best solution and the worst solution are considered. This is due to this reason that the most preferred alternative obtained using Ye's method may not be always be the best one. Moreover, as mentioned earlier in Sect. 1, that is, 'Introduction', a distance measure based TOPSIS method do not consider the correlation between two objects due to which a certain amount of useful information may be lost. Therefore, the output of a such a TOPSIS method may not be reliable. Thus, in MCDM problems where the criteria bears certain correlation with alternatives, the proposed MCDM method may be the best choice. This establishes the efficacy of proposed MCDM method.

5.2 If criteria weights are unknown

Now, we compute the above Example for the case when criteria weights are unknown to us. The computational steps are as follows:

Step 1. The computed values of symptoms weights using (41) are given by

$$w_1 = .1526; w_2 = .4811; w_3 = .2979; w_4 = .0685. \tag{54}$$

Step 2. The computed values of $\Upsilon^{\star}$ and $\Upsilon_{\star}$ are same as given in (50).

Step 3. The computed values of weighted correlation coefficients using (44) are given by

$$\begin{aligned} &\sqsubset(\Upsilon_1, \Upsilon^{\star}) = .9695,\ \sqsubset(\Upsilon_2, \Upsilon^{\star}) = .9621,\\ &\sqsubset(\Upsilon_3, \Upsilon^{\star}) = .9458,\ \sqsubset(\Upsilon_4, \Upsilon^{\star}) = .7673,\ \sqsubset(\Upsilon_5, \Upsilon^{\star}) = .7995;\\ &\sqsubset(\Upsilon_1, \Upsilon_{\star}) = .6315,\ \sqsubset(\Upsilon_2, \Upsilon_{\star}) = .7120,\ \sqsubset(\Upsilon_3, \Upsilon_{\star}) = .6213,\\ &\sqsubset(\Upsilon_4, \Upsilon_{\star}) = .5196,\ \sqsubset(\Upsilon_5, \Upsilon_{\star}) = .5358. \end{aligned} \tag{55}$$

Step 4. The calculated values of χ_i's$(i = 1, 2, \ldots, m)$ are given by

$$\chi_1 = .3944, \chi_2 = .4253, \chi_3 = .3965, \chi_4 = .4038, \chi_5 = .4013. \tag{56}$$

Step 5. Applying Step 7 of proposed method, the preferential sequence so obtained is given by

$$\Upsilon_2 \succ \Upsilon_4 \succ \Upsilon_5 \succ \Upsilon_3 \succ \Upsilon_1. \tag{57}$$

Thus, this is the Υ_2 section in which fault is located.

Moreover, the proposed decision-making model may also be applied in other real-world applications, for example, to select the most appropriate supplier for a company, to select the most suitable site for project installation, for purchasing a suitable flat meeting almost all the requirements of a common man, to invest a sum of money to have good returns and any other problem involving conflicting criteria and fuzziness in decision-making.

6 Conclusions

In this paper, we have successfully introduced a new fuzzy information measure as an extension of the fuzzy information measure studied by Gupta et al. (2014). A new modified TOPSIS method based on weighted correlation coefficients is proposed. Estimating the importance of criteria weights, two methods of determining the criteria weights are discussed: for partially known criteria weights and for unknown criteria weights. Further, the proposed information measure and proposed MCDM method have been utilized in locating the fault in a machine.

Limitations and future scope: One of the major limitations of the proposed work is that it is based on the complete probability distribution, that is, $(\hbar_1, \hbar_2, \ldots, \hbar_n)$ such that $\sum_{i=1}^{n} \hbar_i = 1$. In practice, this may not always the case, that is, sum of probabilities may exceed one or lesser than one, for example D-numbers studied by Deng (2012). Another limitation of this study is that the proposed decision-making method is based on fuzzy information only whereas in practice there exists several generalizations of fuzzy sets, for example, intuitionistic fuzzy sets studied by Atanassov (1986), hesitant fuzzy set introduced by Torra and Narukawa (2009), neutrosophic set Smarandache (2006) etc. Therefore, there is need to develop more general decision-making methods. The work is under consideration and will be reported somewhere else.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

Appendix A

Proof of Theorem (3.2) To prove the theorem, we bifurcate the universe of discourse $X = (\ell_1, \ell_2, \ldots, \ell_n)$ as follows:

$$\begin{aligned} X_1 = &\{\ell_i \in X |\ \odot_1(\ell_i) \subseteq \odot_2(\ell_i)\}\\ &\text{and}\ X_2 = \{\ell_i \in X |\ \odot_1(\ell_i) \supseteq \odot_2(\ell_i)\}. \end{aligned} \tag{58}$$

This implies that for all $\ell_i \in X_1$, $\mu_{\odot_1}(\ell_i) \le \mu_{\odot_2}(\ell_i)$ and for all $\ell_i \in X_2$, $\mu_{\odot_2}(\ell_i) \le \mu_{\odot_1}(\ell_i)$, where $\mu_{\odot_1}(\ell_i)$ and $\mu_{\odot_2}(\ell_i)$ denote the membership degrees of $\odot_1$ and $\odot_2$, respectively. This gives

$$\text{For all}\,\ell_i \in X_1;\,\mu_{\odot_1\cup\odot_2}(\ell_i)=\max(\mu_{\odot_1}(\ell_i),\mu_{\odot_2}(\ell_i))=\mu_{\odot_2}(\ell_i);\ \text{and}\,\mu_{\odot_1\cap\odot_2}(\ell_i)=\min(\mu_{\odot_1}(\ell_i),\mu_{\odot_2}(\ell_i))=\mu_{\odot_1}(\ell_i). \tag{59}$$

Similarly,

$$\text{For all}\,\ell_i \in X_2;\,\mu_{\odot_1\cup\odot_2}(\ell_i)=\max(\mu_{\odot}(\ell_i),\mu_{\odot_2}(\ell_i))=\mu_{\odot_1}(\ell_i);\ \text{and}\,\mu_{\odot_1\cap\odot_2}(\ell_i)=\min(\mu_{\odot_1}(\ell_i),\mu_{\odot_2}(\ell_i))=\mu_{\odot_2}(\ell_i). \tag{60}$$

Now, to prove theorem (3.2), consider

$$^{\varrho}_{\varsigma}H(\odot_1\cup\odot_2)+{}^{\varrho}_{\varsigma}H(\odot_1\cap\odot_2)=\frac{A_4+B_4}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}})}, \tag{61}$$

where

$$A_4=A_{41}-A_{42}, B_4=B_{41}-B_{42}$$

and

$$\begin{aligned}
A_{41}&=\sum_{i=1}^{n}\Big(\mu_{\odot_1\cup\odot_2}(\ell_i)^{\varsigma}e^{(1-\mu_{\odot_1\cup\odot_2}(\ell_i)^{\varsigma})}\\
&\quad+(1-\mu_{\odot_1\cup\odot_2}(\ell_i))^{\varsigma}e^{1-(1-\mu_{\odot_1\cup\odot_2}(\ell_i))^{\varsigma}}\Big);\\
A_{42}&=\sum_{i=1}^{n}\Big(\mu_{\odot_1\cup\odot_2}(\ell_i)^{\varrho}e^{(1-\mu_{\odot_1\cup\odot_2}(\ell_i)^{\varrho})}\\
&\quad+(1-\mu_{\odot_1\cup\odot_2}(\ell_i))^{\varrho}e^{1-(1-\mu_{\odot_1\cup\odot_2}(\ell_i))^{\varrho}}\Big);\\
B_{41}&=\sum_{i=1}^{n}\Big(\mu_{\odot_1\cap\odot_2}(\ell_i)^{\varsigma}e^{(1-\mu_{\odot_1\cap\odot_2}(\ell_i)^{\varsigma})}\\
&\quad+(1-\mu_{\odot_1\cap\odot_2}(\ell_i))^{\varsigma}e^{1-(1-\mu_{\odot_1\cap\odot_2}(\ell_i))^{\varsigma}}\Big);\\
B_{42}&=\sum_{i=1}^{n}\Big(\mu_{\odot_1\cap\odot_2}(\ell_i)^{\varrho}e^{(1-\mu_{\odot_1\cap\odot_2}(\ell_i)^{\varrho})}\\
&\quad+(1-\mu_{\odot_1\cap\odot_2}(\ell_i))^{\varrho}e^{1-(1-\mu_{\odot_1\cap\odot_2}(\ell_i))^{\varrho}}\Big).
\end{aligned}$$

Using (59) and (60), we get

$$^{\varrho}_{\varsigma}H(\odot_1\cup\odot_2)+{}^{\varrho}_{\varsigma}H(\odot_1\cap\odot_2)=\frac{\left[(A'_{41}+A''_{41})-(A'_{42}+A''_{42})\right]+\left[(B'_{41}+B''_{41})-(B'_{42}+B''_{42})\right]}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}})}, \tag{62}$$

where

$$\begin{aligned}
A'_{41}&=\sum_{X_1}\Big(\mu_{\odot_2}(\ell_i)^{\varsigma}e^{(1-\mu_{\odot_2}(\ell_i)^{\varsigma})}+(1-\mu_{\odot_2}(\ell_i))^{\varsigma}e^{1-(1-\mu_{\odot_2}(\ell_i))^{\varsigma}}\Big);\\
A''_{41}&=\sum_{X_2}\Big(\mu_{\odot_1}(\ell_i)^{\varsigma}e^{(1-\mu_{\odot_1}(\ell_i)^{\varsigma})}+(1-\mu_{\odot_1}(\ell_i))^{\varsigma}e^{1-(1-\mu_{\odot_1}(\ell_i))^{\varsigma}}\Big);\\
A'_{42}&=\sum_{X_1}\Big(\mu_{\odot_2}(\ell_i)^{\varrho}e^{(1-\mu_{\odot_2}(\ell_i)^{\varrho})}+(1-\mu_{\odot_2}(\ell_i))^{\varrho}e^{1-(1-\mu_{\odot_2}(\ell_i))^{\varrho}}\Big);\\
A''_{42}&=\sum_{X_2}\Big(\mu_{\odot_1}(\ell_i)^{\varrho}e^{(1-\mu_{\odot_1}(\ell_i)^{\varrho})}+(1-\mu_{\odot_1}(\ell_i))^{\varrho}e^{1-(1-\mu_{\odot_1}(\ell_i))^{\varrho}}\Big);\\
B'_{41}&=\sum_{X_1}\Big(\mu_{\odot_1}(\ell_i)^{\varsigma}e^{(1-\mu_{\odot_1}(\ell_i)^{\varsigma})}+(1-\mu_{\odot_1}(\ell_i))^{\varsigma}e^{1-(1-\mu_{\odot_1}(\ell_i))^{\varsigma}}\Big);\\
B''_{41}&=\sum_{X_2}\Big(\mu_{\odot_2}(\ell_i)^{\varsigma}e^{(1-\mu_{\odot_2}(\ell_i)^{\varsigma})}+(1-\mu_{\odot_2}(\ell_i))^{\varsigma}e^{1-(1-\mu_{\odot_2}(\ell_i))^{\varsigma}}\Big);\\
B'_{42}&=\sum_{X_1}\Big(\mu_{\odot_1}(\ell_i)^{\varrho}e^{(1-\mu_{\odot_1}(\ell_i)^{\varrho})}+(1-\mu_{\odot_1}(\ell_i))^{\varrho}e^{1-(1-\mu_{\odot_1}(\ell_i))^{\varrho}}\Big);\\
B''_{42}&=\sum_{X_2}\Big(\mu_{\odot_2}(\ell_i)^{\varrho}e^{(1-\mu_{\odot_2}(\ell_i)^{\varrho})}+(1-\mu_{\odot_2}(\ell_i))^{\varrho}e^{1-(1-\mu_{\odot_2}(\ell_i))^{\varrho}}\Big).
\end{aligned}$$

On computing (62), we get

$$^{\varrho}_{\varsigma}H(\odot_1\cup\odot_2)+{}^{\varrho}_{\varsigma}H(\odot_1\cap\odot_2)={}^{\varrho}_{\varsigma}H(\odot_1)+{}^{\varrho}_{\varsigma}H(\odot_2). \tag{63}$$

Corollary *Proof follows directly from the proof of theorem (3.2) by taking* $\odot_2=\odot_1^c$.

Proof of Theorem (3.3): First, we prove that $^{\varrho}_{\varsigma}H(\odot)$ is independent of ς when $\odot$ is most fuzzy set, that is, $\mu_{\odot}(\ell_i)=0.5$ for all $\ell_i\in X$. Therefore, substituting $\mu_{\odot}(\ell_i)=.5$ in (8), we get

$$^{\varrho}_{\varsigma}H(\odot)=\frac{n(2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}})}{n(2^{1-\varsigma}e^{1-2^{-\varsigma}}-2^{1-\varrho}e^{1-2^{-\varrho}})}=1, \tag{64}$$

which is free of ς and ϱ.

Similarly, if $\odot$ is least fuzzy set, that is, taking $\mu_{\odot}(\ell_i)=$ 1or0 in (8), we find that $^{\varrho}_{\varsigma}H(\odot)=0$ which is again free of ς and ϱ. This proves the theorem.

Appendix B

Consider an linear programming problem (LPP) of minimization type defined by

$$Z=\min_X f^TX; \tag{65}$$

$$\text{such that}\begin{cases}M\cdot X\le ub,\\ M_{eq}\cdot X=b_{eq},\\ lb\le X\le ub,\end{cases} \tag{66}$$

where f, X, ub, b_{eq}, lb are vectors and M and M_{eq} are matrices.

The syntax for MATLAB code for solving above LPP is given by

$$[X, fval] = linprog(f, M, b, M_{eq}, b_{eq}, lb, ub). \tag{67}$$

The Example discussed in Sect. 5.1 is solved as follows: $X = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$ represents the function to be optimized and $f = \begin{bmatrix} .9601 \\ .8742 \\ .9221 \\ .9821 \end{bmatrix}$ denotes the coefficient vector. M is matrix of constraints given by

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{68}$$

$ub = (.3, .2, .5, .3)$ is the upper bound of constraints and $lb = (0, .1, .2, .1)$ represents the lower bound of constraints. Further, $M_{eq} = (1, 1, 1, 1)$, $b_{eq} = 1$ and $fval$ is the optimum value of X.

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