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ORIGINAL PAPER

Medical diagnostic process based on modified composite relation on pythagorean fuzzy multi-sets

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Abstract

Pythagorean fuzzy multiset (PFMS) is a generalized Pythagorean fuzzy set (PFS) with a higher degree of accuracy. It is characterized by the capacity to handle imprecisions because of its inbuilt ability to allow repetitions of the orthodox parameters of PFSs. Max–min–max composite relation on PFMSs has been studied and proven to be resourceful. However, max–min–max approach used maximum and minimum values of the parameters of PFMS only without considering the average values. This paper proposes a modified version of the max–min–max composite relation on PFMSs to enhance reliable output by incorporating the average values of the PFMSs' parameters. Some numerical examples are given to juxtapose the correctness of the max–min–max composite relation on PFMSs with that of the modified version to ascertain reliability/superiority of the modified version. To demonstrate the applicability of the proposed composite relation on PFMSs, an illustration of medical diagnosis is considered assuming there are some patients whose symptoms are represented in Pythagorean fuzzy multi-values. To determine the diagnosis of the patients, the max–min–max composite relation and its modified version are deployed to find the correlation between each of the patients with some suspected diseases.

Keywords Composite relation · Intuitionistic fuzzy set · Intuitionistic fuzzy multiset · Pythagorean fuzzy set · Pythagorean fuzzy multiset · Medical diagnosis

1 Introduction

Medical diagnosis or diagnosis is the process of deciding which illness or disease describes a patient's signs and symptoms. The information necessary for diagnosis is usually collected from a history and frequently, physical examination of the patient seeking medical attention. Over and over again, one or more diagnosis processes, like medical tests, are also conducted during the procedure. Diagnosis is time and again thought-provoking, because many signs and symptoms are uncertain. For example, headache by itself, is a sign of numerous diseases and thus does not show the physician what the patient is suffering from Ejegwa and Onyeke (2020). Consequently, differential diagnosis, in which some possible explanations are juxtaposed, must be performed, which could be best done by Pythagorean fuzzy approach. This involves the correlation of many pieces of information followed by the recognition of patterns via composite relation. In fact, the process of medical diagnosis is more challenging when a patient is showing symptoms of some closely related diseases; this also posed a problem to therapeutic process.

Uncertainties are a huge barrier to reckon with in medical diagnostic processes because of fuzziness. The invention of fuzzy sets technology by Zadeh (1965) brought an amazing sight of relief to medical decision-makers, because of the ability of fuzzy model to curb the embedded uncertainties in medical diagnostic processes. Some medical decision-making problems could not be properly resolved with fuzzy approach because fuzzy set

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only considered membership grade whereas many medical diagnostic processes have the component of both membership grade and non-membership grade with the possibility of hesitation.

Subsequently, Atanassov (1983, 1986) introduced intuitionistic fuzzy sets (IFSs). This construct captured the nonmembership degree (NMD) ν together with MD μ of fuzzy set with a possibility of hesitation margin (HM) π such that $\mu + \nu \leq 1$ and $\mu + \nu + \pi = 1$. The concept of intuitionistic fuzzy sets and its generalizations have been applied to many decision-making problems (see Atanassov 1999; Chen et al. 2016; Chen and Chang 2016; Chen et al. 2016a; Cheng et al. 2016b; Liu et al. 2020; Zeng et al. 2019; De et al. 2001; Garg and Kumar 2018; Szmidt and Kacprzyk 2001; Jana et al. 2019a, 2020a, b; Jana and Pal 2019a, b). However, in some problems like medical diagnosis, the decision on the medical status of a patient may not be taken just once and so discarding the degrees of membership and non-membership in each of the consultations may lead to a compromised diagnosis. By considering the said degrees in each of the consultations, Shinoj and Sunil (2012) proposed intuitionistic fuzzy multiset (IFMS) which has the same features such as intuitionistic fuzzy set but allowing repetitions of membership grade, non-membership grade and hesitation margin. Some fundamentals of intuitionistic fuzzy multisets have been studied (Ejegwa 2016; Ejegwa and Awolola 2013), and the methods of transforming intuitionistic fuzzy multisets to intuitionistic fuzzy sets and fuzzy sets were explicated (Ejegwa 2015). Myriad of applications of IFMSs have been discussed (Das et al. 2013; Rajarajeswari and Uma 2013, 2014; Shinoj and Sunil 2013; Ulucay et al. 2019).

Assuming a decision-maker has a MD $\mu = 0.7$ and a NMD $\nu = 0.4$ for a particular problem, then the framework of intuitionistic fuzzy sets could not be used since $\mu + \nu \geq 1$. As a result, Yager (2013) proposed Pythagorean fuzzy set which has MD μ, NMD ν and HM π with the properties that $\mu + \nu \geq 1$ and $\pi = \sqrt{1 - (\mu^2 + \nu^2)}$. Pythagorean fuzzy set is a generalized intuitionistic fuzzy set with a more degree of accuracy. Many studies have applied Pythagorean fuzzy sets to several applicative areas (Ejegwa 2019a, b, c, d, e, 2020a, b; Khan et al. 2019; Rahman and Abdullah 2019; Ejegwa and Awolola 2019; Garg 2018a, b; Zhang 2016; Jana et al. 2019b, c). By allowing the repetitions of MD μ, NMD ν and HM π, the idea of Pythagorean fuzzy multisets (PFMSs) was introduced by Ejegwa (2020c) and applied in course placement using a max–min–max composite relation on PFMSs. Max–min–max approach use maximum and minimum values of the parameters of PFMS only without considering the average values of the parameters. To this end, we are motivated to modify max–min–max composite relation on PFMSs by incorporating the average values for accuracy sake and address its applicability in medical diagnostic processes. The objectives of this paper are to

(i) show the matrix representation of PFMSs and the transformation of PFMSs to Pythagorean fuzzy sets,
(ii) explicate the max–min–max composite relation on PFMSs and modify it for better output,
(iii) numerically verify the superiority of the proposed composite relation on PFMSs over the existing one,
(iv) apply the proposed composite relation on PFMSs in medical diagnosis to curb embedded fuzziness.

The rest of the paper are outlined as follow; Sect. 2 presents the basic notions of PFMSs, Sect. 3 discusses composite relation on PFMSs, its modified version and numerical verifications to ascertain the advantage of the proposed composite relation on PFMSs, Sect. 4 shows the application of the proposed composite relation in solving the problem of medical diagnosis where symptoms are represented in Pythagorean fuzzy setting with multi-values, and Sect. 5 concludes the findings in the paper.

2 Basic notions of Pythagorean fuzzy multisets

Definition 2.1 (Atanassov 1983) An intuitionistic fuzzy set A of X (where X is a non-empty set) is an object having the form

$$A = \left\{ \left\langle \frac{\mu_A(x), \nu_A(x)}{x} \right\rangle \mid x \in X \right\}, \tag{1}$$

where the functions $\mu_A(x),\ \nu_A(x) : X \to [0,1]$ define MD and NMD of the element $x \in X$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For any intuitionistic fuzzy set A of X, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of A.

Definition 2.2 (Shinoj and Sunil 2012) An intuitionistic fuzzy multiset A of X (where X is a non-empty set) is of the form

$$A = \left\{ \left\langle \frac{CM_A(x)}{x}, \frac{CN_A(x)}{x} \right\rangle | x \in X \right\} \tag{2}$$

where

$$CM_A(x) = \mu_A^1(x), \ldots, \mu_A^m(x)$$

and

$CN_A(x) = \nu_A^1(x), \ldots, \nu_A^m(x),$

or simply $CM_A(x) = \mu_A^j(x)$ and $CN_A(x) = \nu_A^j(x)$ for $j = 1, \ldots, m$ are the count MD and count NMD defined by the functions $CM_A(x),\ CN_A(x) : X \rightarrow N^{[0,1]}$ such that $0 \leq CM_A(x) + CN_A(x) \leq 1$, $N = \mathbb{N} \cup \{0\}$.

For each intuitionistic fuzzy multiset A of X, $CH_A(x) = 1 - CM_A(x) - CN_A(x)$ is the intuitionistic fuzzy multisets index or count HM of A, where $CH_A(x) = \pi_A^1(x), \ldots, \pi_A^m(x)$.

Definition 2.3 (Yager 2013) A Pythagorean fuzzy sets $\mathtt{A}$ of X (where X is a non-empty set) is the set of ordered pairs defined by

$$\mathtt{A} = \left\{ \left\langle \frac{\mu_{\mathtt{A}}(x), \nu_{\mathtt{A}}(x)}{x} \right\rangle \mid x \in X \right\}, \tag{3}$$

where the functions $\mu_{\mathtt{A}}(x),\ \nu_{\mathtt{A}}(x) : X \rightarrow [0,1]$ define the MD and NMD of the element $x \in X$ to A such that $0 \leq (\mu_{\mathtt{A}}(x))^2 + (\nu_{\mathtt{A}}(x))^2 \leq 1$. Assuming $(\mu_{\mathtt{A}}(x))^2 + (\nu_{\mathtt{A}}(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to $\mathtt{A}$ defined by $\pi_{\mathtt{A}}(x) = \sqrt{1 - [(\mu_{\mathtt{A}}(x))^2 + (\nu_{\mathtt{A}}(x))^2]}$ and $\pi_{\mathtt{A}}(x) \in [0,1]$.

Definition 2.4 (Ejegwa 2020c) A Pythagorean fuzzy multiset A of X (where X is a non-empty set) is characterized by

$$\mathsf{A} = \left\{ \left\langle \frac{CM_{\mathsf{A}}(x)}{x}, \frac{CN_{\mathsf{A}}(x)}{x} \right\rangle | x \in X \right\} \tag{4}$$

alternatively,

$$\mathsf{A} = \{\langle x, CM_{\mathsf{A}}(x), CN_{\mathsf{A}}(x)\rangle | x \in X\} \tag{5}$$

where

$$CM_{\mathsf{A}}(x) = \mu_{\mathsf{A}}^1(x), \ldots, \mu_{\mathsf{A}}^m(x)$$

and

$$CN_{\mathsf{A}}(x) = \nu_{\mathsf{A}}^1(x), \ldots, \nu_{\mathsf{A}}^m(x),$$

or simply $CM_{\mathsf{A}}(x) = \mu_{\mathsf{A}}^j(x)$ and $CN_{\mathsf{A}}(x) = \nu_{\mathsf{A}}^j(x)$ for $j = 1, \ldots, m$ are the count MD and count NMD defined by the functions $CM_{\mathsf{A}}(x),\ CN_{\mathsf{A}}(x) : X \rightarrow N^{[0,1]}$ such that $0 \leq [CM_{\mathsf{A}}(x)]^2 + [CN_{\mathsf{A}}(x)]^2 \leq 1$, $N = \mathbb{N} \cup \{0\}$.

For any Pythagorean fuzzy multiset A of X,

$$CH_{\mathsf{A}}(x) = \sqrt{1 - [CM_{\mathsf{A}}(x)]^2 - [CN_{\mathsf{A}}(x)]^2} \tag{6}$$

is the count HM of A, where

$$CH_{\mathsf{A}}(x) = \pi_{\mathsf{A}}^1(x), \ldots, \pi_{\mathsf{A}}^m(x).$$

The count HM $CH_{\mathsf{A}}(x)$ is the degree of non-determinacy of x in A and $CH_{\mathsf{A}}(x) \in [0,1]$. The count HM is the function that expresses lack of knowledge of whether $x \in \mathsf{A}$ or $x \notin \mathsf{A}$.

Throughout this paper $\mathrm{PFMS}(X)$ denotes the set of all PFMS of X.

Definition 2.5 (Ejegwa 2020c) Suppose $\mathsf{A} \in \mathrm{PFMS}(X)$. Then, the level/ground set of A is $\mathsf{A}_* = \{x \in X | CM_{\mathsf{A}}(x) > 0,\ CN_{\mathsf{A}}(x) < 1\}$. It follows that, A_* is a subset of X.

Definition 2.6 (Ejegwa 2020c) Let $\mathsf{A}, \mathsf{B} \in \mathrm{PFMS}(X)$. Then A and B are said to be equal if and only if $CM_{\mathsf{A}}(x) = CM_{\mathsf{B}}(x)$ and $CN_{\mathsf{A}}(x) = CN_{\mathsf{B}}(x)\ \forall x \in X$.

Definition 2.7 (Ejegwa 2020c) Suppose $\mathsf{A}, \mathsf{B} \in \mathrm{PFMS}(X)$, then $\mathsf{A} \subseteq \mathsf{B}$ if $CM_{\mathsf{A}}(x) \leq CM_{\mathsf{B}}(x)$ and $CN_{\mathsf{A}}(x) \geq CN_{\mathsf{B}}(x)$ $\forall x \in X$. Also $\mathsf{A} \subset \mathsf{B}$ if $\mathsf{A} \subseteq \mathsf{B}$ and $\mathsf{A} \neq \mathsf{B}$.

Definition 2.8 (Ejegwa 2020c) Let $\mathsf{A}, \mathsf{B} \in \mathrm{PFMS}(X)$. Then the following operations hold.

(i) $$\overline{\mathsf{A}} = \left\{ \left\langle \frac{CN_{\mathsf{A}}(x)}{x}, \frac{CM_{\mathsf{A}}(x)}{x} \right\rangle \mid x \in X \right\}$$

(ii) $$\mathsf{A} \cup \mathsf{B} = \left\{ \left\langle \frac{\max(CM_{\mathsf{A}}(x), CM_{\mathsf{B}}(x))}{x}, \frac{\min(CN_{\mathsf{A}}(x), CN_{\mathsf{B}}(x))}{x} \right\rangle \mid x \in X \right\}$$

(iii) $$\mathsf{A} \cup \mathsf{B} = \left\{ \left\langle \frac{\min(CM_{\mathsf{A}}(x), CM_{\mathsf{B}}(x))}{x}, \frac{\max(CN_{\mathsf{A}}(x), CN_{\mathsf{B}}(x))}{x} \right\rangle \mid x \in X \right\}$$

Definition 2.9 Let $X = \{x_i\}$ for $i = 1, \ldots, n$. Then, the PFMS A of X is a Pythagorean fuzzy set $\mathtt{A}$ of X by the computations:

$$\frac{1}{n}\ CM_{\mathsf{A}}(x_i) = \mu_{\mathtt{A}}(x_i),\ \frac{1}{n}\ CN_{\mathsf{A}}(x_i) = \nu_{\mathtt{A}}(x_i). \tag{7}$$

Clearly, every Pythagorean fuzzy set is a PFMS but the converse is not true (in particular, if $i = 1$).

Example 2.10 If A is an PFMS of $X = \{x, y\}$ such that

$$\mathsf{A} = \left\{ \left\langle \frac{(0.7, 0.5, 0.4), (0.3, 0.5, 0.6)}{x} \right\rangle, \left\langle \frac{(0.8, 0.6, 0.4), (0.4, 0.5, 0.5)}{y} \right\rangle \right\}.$$

To enhance computation, an PFMS A becomes a Pythagorean fuzzy set

$$\mathtt{A} = \left\{ \left\langle \frac{0.5333, 0.4667}{x} \right\rangle, \left\langle \frac{0.6, 0.4667}{y} \right\rangle \right\},$$

and the $CH_{\mathsf{A}}(x)$ and $CH_{\mathsf{A}}(y)$ can be computed using (6).

Definition 2.11 Let $X = \{x_i\}$ for $i = 1, \ldots, n$. If

$$\mathsf{A} = \left\{ \left\langle \frac{(\mu^1_{\mathsf{A}}(x_1), ..., \mu^m_{\mathsf{A}}(x_1)), (\nu^1_{\mathsf{A}}(x_1), ..., \nu^m_{\mathsf{A}}(x_1))}{x_1} \right\rangle, \ldots, \left\langle \frac{(\mu^1_{\mathsf{A}}(x_n), ..., \mu^m_{\mathsf{A}}(x_n)), (\nu^1_{\mathsf{A}}(x_n), \ldots, \nu^m_{\mathsf{A}}(x_n))}{x_n} \right\rangle \right\}$$

is a PFMS of X. Then, A can be represented in matrix form as

$$\mathsf{A} = \underset{x_1,}{\begin{pmatrix} \mu^1_{\mathsf{A}}(x_1) & \ldots & \mu^m_{\mathsf{A}}(x_1) \\ \nu^1_{\mathsf{A}}(x_1) & \ldots & \nu^m_{\mathsf{A}}(x_1) \end{pmatrix}} \ldots \underset{x_n}{\begin{pmatrix} \mu^1_{\mathsf{A}}(x_n) & \ldots & \mu^m_{\mathsf{A}}(x_n) \\ \nu^1_{\mathsf{A}}(x_n) & \ldots & \nu^m_{\mathsf{A}}(x_n) \end{pmatrix}}$$

Using Example 2.10, we have

$$\mathsf{A} = \underset{x}{\begin{pmatrix} 0.7 & 0.5 & 0.4 \\ 0.3 & 0.5 & 0.6 \end{pmatrix}}, \underset{y}{\begin{pmatrix} 0.8 & 0.6 & 0.4 \\ 0.4 & 0.5 & 0.5 \end{pmatrix}}$$

3 Composite relation on Pythagorean fuzzy multisets

In this section, we recall the composite relation on PFMSs (Ejegwa 2020c). Suppose X and Y are non-empty sets. Then, a Pythagorean fuzzy multi-relation (PFMR) R from X to Y is a PFMS of $X \times Y$ characterised by CM_{R} and CN_{R}, denoted by $\mathsf{R}(X \to Y)$.

3.1 Max–min–max composite relation on Pythagorean fuzzy multisets

Definition 3.1 Suppose $\mathsf{A} \in \mathrm{PFMS}(X)$. Then, the max–min–max composition of $\mathsf{R}(X \to Y)$ with A is a PFMS B of Y defined by $\mathsf{B} = \mathsf{R} \circ \mathsf{A}$ where

$$\left.\begin{array}{l} CM_{\mathsf{B}}(y) = \max_x(\min[CM_{\mathsf{A}}(x), CM_{\mathsf{R}}(x,y)]) \\ CN_{\mathsf{B}}(y) = \min_x(\max[CN_{\mathsf{A}}(x), CN_{\mathsf{R}}(x,y)]) \end{array}\right\}, \tag{8}$$

$\forall x \in X$ and $y \in Y$

Definition 3.2 If $\mathsf{Q}(X \to Y)$ and $\mathsf{R}(Y \to Z)$ are PFMRs. Then, the max–min–max composition $\mathsf{R} \circ \mathsf{Q}$ is a PFMR from X to Z where

$$\left.\begin{array}{l} CM_{\mathsf{R}\circ\mathsf{Q}}(x,z) = \max_y(\min[CM_{\mathsf{Q}}(x,y), CM_{\mathsf{R}}(y,z)]) \\ CN_{\mathsf{R}\circ\mathsf{Q}}(x,z) = \min_y(\max[CN_{\mathsf{Q}}(x,y), CN_{\mathsf{R}}(y,z)]) \end{array}\right\}, \tag{9}$$

$\forall (x, z) \in X \times Z$ and $\forall y \in Y$.

Remark 3.3 From Definitions 3.1 and 3.2, the max–min–max composite relation B or $\mathsf{R} \circ \mathsf{Q}$ can be computed by

$$\beta = CM_{\mathsf{B}}(y) - CN_{\mathsf{B}}(y)CH_{\mathsf{B}}(y) \tag{10}$$

$\forall y \in Y$. Alternatively,

$$\beta = CM_{\mathsf{R}\circ\mathsf{Q}}(x,z) - CN_{\mathsf{R}\circ\mathsf{Q}}(x,z)CH_{\mathsf{R}\circ\mathsf{Q}}(x,z) \tag{11}$$

$\forall (x, z) \in X \times Z$.

3.2 Modified composite relation on Pythagorean fuzzy multisets

Now, we propose the modified composite relation on PFMSs and demonstrate its advantage over the max–min–max composite relation using some numerical examples.

Definition 3.4 Suppose $\mathsf{A} \in \mathrm{PFMS}(X)$. Then, the modified composite relation of $\mathsf{R}(X \to Y)$ with A is a PFMS B^* of Y defined by $\mathsf{B}^* = \mathsf{R} \circ \mathsf{A}$ where

$$\left.\begin{array}{l} CM_{\mathsf{B}^*}(y) = \max_x\left[\dfrac{CM_{\mathsf{A}}(x) + CM_{\mathsf{R}}(x,y)}{2}\right] \\ CN_{\mathsf{B}^*}(y) = \min_x\left[\dfrac{CN_{\mathsf{A}}(x) + CN_{\mathsf{R}}(x,y)}{2}\right] \end{array}\right\}, \tag{12}$$

$\forall x \in X$ and $y \in Y$.

Definition 3.5 If $\mathsf{Q}(X \to Y)$ and $\mathsf{R}(Y \to Z)$ be two PFMRs. Then, the modified composite relation $(\mathsf{R} \circ \mathsf{Q})^*$ is a PFMR from X to Z where

$$\left.\begin{array}{l} CM_{(\mathsf{R}\circ\mathsf{Q})^*}(x,z) = \max_y\left[\dfrac{CM_{\mathsf{Q}}(x,y) + CM_{\mathsf{R}}(y,z)}{2}\right] \\ CN_{(\mathsf{R}\circ\mathsf{Q})^*}(x,z) = \min_y\left[\dfrac{CN_{\mathsf{Q}}(x,y) + CN_{\mathsf{R}}(y,z)}{2}\right] \end{array}\right\}, \tag{13}$$

$\forall (x, z) \in X \times Z$ and $\forall y \in Y$.

Remark 3.6 From Definitions 3.4 and 3.5, the modified composite relation B^* or $(\mathsf{R} \circ \mathsf{Q})^*$ can be computed by

$$\beta^* = CM_{\mathsf{B}^*}(y) - CN_{\mathsf{B}^*}(y)CH_{\mathsf{B}^*}(y) \tag{14}$$

$\forall y \in Y$. Alternatively,

$$\beta^* = CM_{(\mathsf{R}\circ\mathsf{Q})^*}(x,z) - CN_{(\mathsf{R}\circ\mathsf{Q})^*}(x,z) \times CH_{(\mathsf{R}\circ\mathsf{Q})^*}(x,z), \tag{15}$$

$\forall (x, z) \in X \times Z$.

Proposition 3.7 *Suppose R and Q are two PFMRs on $X \times Y$ and $Y \times Z$, respectively. Then*

(i) $(\mathsf{R}^{-1})^{-1} = \mathsf{R}$,
(ii) $(\mathsf{Q} \circ \mathsf{R})^{-1} = \mathsf{R}^{-1} \circ \mathsf{Q}^{-1}$.

3.3 Numerical examples

This subsection shows the performance index of the modified PFMR in comparison to the performance index of max–min–max PFMR using following examples.

Example 3.8 Suppose $\mathsf{E}, \mathsf{F} \in \mathrm{PFMS}(X)$ for $X = \{x_i\}$ for $i = 1, 2, 3$. If

$$\mathsf{E} = \underset{x_1}{\begin{pmatrix} 0.6 & 0.5 & 0.7 \\ 0.2 & 0.3 & 0.1 \end{pmatrix}}, \underset{x_2}{\begin{pmatrix} 0.4 & 0.7 & 0.1 \\ 0.6 & 0.4 & 0.8 \end{pmatrix}}, \underset{x_3}{\begin{pmatrix} 0.5 & 0.4 & 0.6 \\ 0.3 & 0.5 & 0.1 \end{pmatrix}}$$

$$\mathsf{F} = \underset{x_1}{\begin{pmatrix} 0.6 & 0.8 & 1.0 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}}, \underset{x_2}{\begin{pmatrix} 0.7 & 0.6 & 0.8 \\ 0.3 & 0.4 & 0.2 \end{pmatrix}}, \underset{x_3}{\begin{pmatrix} 0.6 & 0.5 & 0.7 \\ 0.1 & 0.2 & 0.0 \end{pmatrix}}$$

Using Definitions 3.1 and 3.2, respectively we have

$$\min[CM_{\mathsf{R}}(e_i, x_j), CM_{\mathsf{S}}(x_j, f_k)] = 0.6, 0.4, 0.5$$

implying that

$$CM_{\mathsf{B}}(e_i, f_k) = \max_{x_j \in X}[0.6, 0.4, 0.5] = 0.6.$$

Applying this to E and F, we see that the minimum of the membership values of the elements in E and F, respectively are 0.6, 0.4 and 0.5.

Also,

$$\max[CN_{\mathsf{R}}(e_i, x_j), CN_{\mathsf{S}}(x_j, f_k)] = 0.2, 0.6, 0.3$$

implying that

$$CN_{\mathsf{B}}(e_i, f_k) = \min_{x_j \in X}[0.2, 0.6, 0.3] = 0.2.$$

It follows that the maximum of the non-membership values of the elements in E and F, respectively are 0.2, 0.6 and 0.3.

Thus

$$\beta = 0.6 - (0.2 \times 0.7746) = 0.4451.$$

Now, by applying the modified composite relation on E and F using Definitions 3.4 and 3.5, we have

$$\frac{CM_{\mathsf{R}}(e_i, x_j) + CM_{\mathsf{S}}(x_j, f_k)}{2} = 0.7, 0.55, 0.55$$

that is,

$$CM_{\mathsf{B}^*}(e_i, f_k) = \max_{x_j \in X}[0.7, 0.55, 0.55] = 0.7.$$

Similarly,

$$\frac{CN_{\mathsf{R}}(e_i, x_j) + CN_{\mathsf{S}}(x_j, f_k)}{2} = 0.15, 0.45, 0.2$$

that is,

$$CN_{\mathsf{B}^*}(e_i, f_k) = \min_{x_j \in X}[0.15, 0.45, 0.2] = 0.15.$$

Thus

$$\beta^* = 0.7 - (0.15 \times 0.6982) = 0.5953.$$

From the computations, the modified composite relation gives a better relation between E and F when compare to max–min–max composite relation.

Example 3.9 Suppose $\mathsf{G}, \mathsf{H} \in \mathrm{PFMS}(X)$ for $X = \{x_i\}$ for $i = 1, \ldots, 5$.

$$\mathsf{G} = \underset{x_1}{\begin{pmatrix} 0.6 & 0.8 & 1.0 \\ 0.4 & 0.6 & 0.2 \end{pmatrix}}, \underset{x_2}{\begin{pmatrix} 0.5 & 0.6 & 0.4 \\ 0.7 & 0.4 & 1.0 \end{pmatrix}}, \underset{x_3}{\begin{pmatrix} 0.8 & 0.7 & 0.9 \\ 0.4 & 0.6 & 0.2 \end{pmatrix}}, \underset{x_5}{\begin{pmatrix} 0.7 & 0.5 & 0.9 \\ 0.2 & 0.1 & 0.3 \end{pmatrix}}$$

$$\mathsf{H} = \underset{x_1}{\begin{pmatrix} 0.7 & 0.6 & 0.8 \\ 0.3 & 0.2 & 0.4 \end{pmatrix}}, \underset{x_3}{\begin{pmatrix} 0.4 & 0.6 & 0.2 \\ 0.7 & 0.5 & 0.9 \end{pmatrix}}, \underset{x_4}{\begin{pmatrix} 0.9 & 0.8 & 1.0 \\ 0.2 & 0.4 & 0.0 \end{pmatrix}}$$

Certainly, $\mathsf{G}_* \neq \mathsf{H}_*$.

From Definitions 3.1 and 3.2, respectively, we have

$$\min[CM_{\mathsf{R}}(g_i, x_j), CM_{\mathsf{S}}(x_j, h_k)] = 0.7, 0.0, 0.4, 0.0, 0.0$$

and

$$CM_{\mathsf{B}}(g_i, h_k) = \max_{x_j \in X}[0.7, 0.5, 0.4, 0.9, 0.7] = 0.7.$$

Similarly,

$$\max[CN_{\mathsf{R}}(g_i, x_j), CN_{\mathsf{S}}(x_j, h_k)] = 0.4, 1.0, 0.7, 1.0, 1.0$$

and

$$CN_{\mathsf{B}}(g_i, h_k) = \min_{x_j \in X}[0.4, 1.0, 0.7, 1.0, 1.0] = 0.4.$$

Thus

$$\beta = 0.7 - (0.4 \times 0.5916) = 0.4634.$$

Also, computing β^* using Definitions 3.4 and 3.5, we get

$$\frac{CM_{\mathsf{R}}(g_i, x_j) + CM_{\mathsf{S}}(x_j, h_k)}{2} = 0.75, 0.25, 0.6, 0.45, 0.35$$

and

$$CM_{\mathsf{B}}(g_i, h_k) = \max_{x_j \in X}[0.75, 0.25, 0.6, 0.45, 0.35] = 0.75.$$

Again,

$$\frac{CN_R(g_i,x_j)+CN_{\mathsf{S}}(x_j,h_k)}{2}=0.35,0.85,0.55,0.6,0.6$$

and

$$CN_{\mathsf{B}}(g_i,h_k)=\min_{x_j\in X}[0.35,0.85,0.55,0.6,0.6]=0.35.$$

Then

$$\beta^*=0.75-(0.35\times 0.5612)=0.5536.$$

In this example also, the modified composite relation yields a better relation between G and H. Table 1 provides a quick comparison between the modified composite relation β^* and the max–min–max composite relation β on PFMSs.

4 Diagnostic processes using composite relations in Pythagorean fuzzy environment

Medical diagnosis/testing is a delicate exercise because failure to make the right decision may lead to the death of the patient. In this section, we present a scenario of mathematical approach of medical diagnosis to ascertain the medical conditions of some patients via a novel composite relation on PFMSs, where the symptoms or clinical manifestations of the diseases are represented in PFMSs framework using a hypothetical approach.

Assuming there are four patients represented by the set P_j for $j=1,2,3,4$, who are billed for medical diagnosis due to the manifestation of some symptoms. After critical analysis on the samples collected from C_j, the following major symptoms are observed;

$$S=\{s_1,s_2,s_3,s_4,s_5\},$$

where s_1 = fever, s_2 = cough, s_3 = shortness of breath or breathing difficulties, s_4 = sore throat, and s_5 = headache.

Suppose $\mathsf{D}=\{\mathsf{D}_1,\mathsf{D}_2,\mathsf{D}_3,\mathsf{D}_4,\mathsf{D}_5\}$ are set of diseases with relatively common symptoms, where D_1 = influenza, D_2 = viral fever, D_3 = hay fever, D_4 = pneumonia, and D_5 = common cold which P_1, P_2, P_3, and P_4 are likely to be infected with.

The Pythagorean fuzzy multi-relation $\mathsf{N}(\mathsf{S}\to\mathsf{D})$ is hypothetically given in Table 2 based on the medical knowledge of the enlisted diseases. The Pythagorean fuzzy multi-relation $\mathsf{M}(\mathsf{P}\to\mathsf{S})$ is hypothetically given in

Table 1 Comparison between β and β^*

PFMRs	Example 3.8	Example 3.9
β	0.4451	0.4634
β^*	0.5953	0.5536

Table 3 based on the medical analysis on $\mathsf{P}_j=\{\mathsf{P}_1,\mathsf{P}_2,\mathsf{P}_3,\mathsf{P}_4\}$. After applying Eq. (7) on Tables 2 and 3 (for the ease of computations), we obtain the values of the membership and non-membership grades of B and B^* in Tables 4 and 6. After computing the degree of hesitation count via Eq. (6), we calculate β and β^* as presented in Tables 5 and 7.

The first row represents the count membership degrees while the second row represents the count non-membership degrees, respectively.

The Pythagorean fuzzy multi-values in Table 3 were taken at three different times to curb any chance of error; this is unlike in the instance of Pythagorean fuzzy values. The first column represents the first result in both membership and non-membership grades, the second column and the third column likewise.

Applying max–min–max composite relation, we obtain Tables 4 and 5, respectively.

The following diagnoses are obtained from Table 5; P_1 is diagnosed with influenza and viral fever with some symptoms of pneumonia and common cold in that order, P_2 is diagnosed with influenza (but should be treated for viral fever and pneumonia), P_3 is diagnosed of influenza and viral fever (but should be treated for pneumonia), P_4 is diagnosed with pneumonia and common cold (but should be treated for influenza). It is noticed that P_1 and P_3 are diagnosed of the same ailments (but P_1 has a common cold in addition.)

Using the modified approach, we obtain Tables 6 and 7, respectively.

From Table 7, we infer that P_1 is tested positive for viral fever in a severe situation with very prominent symptoms of influenza, pneumonia, common cold and hay fever in that order. P_2 is tested positive for viral fever in a mild situation with less viral load. P_3 is tested positive for viral fever in a severe situation with very prominent symptoms of influenza, pneumonia, hay fever and common cold in that order. P_4 is tested positive for both viral fever and common cold in a mild situation with some symptoms of influenza and pneumonia in that order.

It is observed that P_1 and P_3 situations are very severe with the same viral load, follow by P_4 and P_2 in that order. In fact, P_2 has a very less viral load in comparison to the other cases. All the suspected cases test positive for viral fever but with a different viral load to enhance treatment and attention.

Synthesizing the information in Tables 5 and 7, we observe that the diagnoses given by max–min–max composite relation are different from the diagnostic results gotten from the modified approach. Notwithstanding, the diagnostic results of the modified approach are reliable because the average of the membership and non-

Table 2 N(S → D)

N	D_1	D_2	D_3	D_4	D_5
s_1	$\begin{pmatrix} 0.8 & 0.7 & 0.9 \\ 0.1 & 0.2 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 1.0 & 0.9 & 0.8 \\ 0.2 & 0.3 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.7 & 0.9 \\ 0.3 & 0.3 & 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.4 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.1 \end{pmatrix}$
s_2	$\begin{pmatrix} 0.9 & 0.8 & 0.7 \\ 0.3 & 0.2 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 1.0 & 0.9 & 0.8 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.5 & 0.7 \\ 0.3 & 0.1 & 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.7 & 0.6 & 0.5 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.5 & 0.4 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}$
s_3	$\begin{pmatrix} 0.6 & 0.5 & 0.4 \\ 0.3 & 0.2 & 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.9 & 0.7 & 0.8 \\ 0.2 & 0.1 & 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.3 & 0.1 & 0.2 \\ 0.8 & 0.6 & 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.7 & 0.5 \\ 0.2 & 0.3 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.0 & 0.0 & 0.0 \\ 0.7 & 0.8 & 0.9 \end{pmatrix}$
s_4	$\begin{pmatrix} 0.3 & 0.4 & 0.2 \\ 0.8 & 0.6 & 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.4 & 0.3 \\ 0.4 & 0.3 & 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.9 & 0.8 & 0.7 \\ 0.3 & 0.2 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.7 & 0.8 & 0.9 \\ 0.1 & 0.2 & 0.3 \end{pmatrix}$
s_5	$\begin{pmatrix} 0.8 & 0.7 & 0.9 \\ 0.2 & 0.3 & 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.5 & 0.7 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.2 & 0.3 & 0.4 \\ 0.7 & 0.6 & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.4 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.7 & 0.8 & 0.9 \end{pmatrix}$

membership grades of the symptoms were considered unlike the max–min–max approach where the minimum of the membership grades and the maximum of the non-membership grades of the symptoms were considered. Thus, it is meet to say that the results of the modified approach are more precise and reliable.

5 Conclusion

Medical diagnosis is an essential exercise because it determines recovery and so, a wrong diagnosis could lead to death or very critical cases. In this paper, we have addressed the mathematical approach of medical diagnosis using a modified composite relation on PFMSs, where symptoms were captured in Pythagorean fuzzy multi-values to prevent any chance of error-influence on the diagnostic processes. The nexus between Pythagorean fuzzy multi-values and Pythagorean fuzzy values was established with the aid of a proposed formula, and the matrix representation of PFMSs was introduced. Max–min–max composite relation on PFMSs has been studied (Ejegwa 2020c). However, max–min–max approach used maximum and minimum values of the parameters of PFMS only without considering the average values. To remedy this limitation, we have modified max–min–max composite relation on PFMSs to enhance reliable output by incorporating the average values of the PFMSs' parameters. The modified composite relation on PFMSs was verified to have high performance over the max–min–max composite relation. A hypothetical case of medical diagnosis on some selected patients was considered via the modified composite relation on PFMSs. Medical diagnosis on four patients was considered in the hypothetical case. An algorithmic approach embedded with the modified approach could be employed to address the medical diagnosis of more patients in future research.

Table 3 M(P → S)

M	s_1	s_2	s_3	s_4	s_5
P_1	$\begin{pmatrix} 0.8 & 0.7 & 0.9 \\ 0.1 & 0.2 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.5 & 0.7 \\ 0.1 & 0.2 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.2 & 0.3 & 0.1 \\ 0.8 & 0.7 & 0.9 \end{pmatrix}$	$\begin{pmatrix} 0.6 & 0.5 & 0.7 \\ 0.1 & 0.2 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.1 & 0.2 & 0.0 \\ 0.6 & 0.5 & 0.7 \end{pmatrix}$
P_2	$\begin{pmatrix} 0.8 & 0.9 & 0.7 \\ 0.0 & 0.0 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.4 & 0.3 & 0.5 \\ 0.4 & 0.4 & 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.6 & 0.7 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.2 & 0.1 & 0.0 \\ 0.7 & 0.5 & 0.9 \end{pmatrix}$	$\begin{pmatrix} 0.1 & 0.2 & 0.0 \\ 0.9 & 0.7 & 0.8 \end{pmatrix}$
P_3	$\begin{pmatrix} 0.7 & 0.8 & 0.9 \\ 0.2 & 0.0 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.9 & 0.7 & 0.8 \\ 0.1 & 0.2 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.0 & 0.0 & 0.0 \\ 0.7 & 0.5 & 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.5 & 0.7 & 0.9 \end{pmatrix}$	$\begin{pmatrix} 0.0 & 0.0 & 0.0 \\ 0.5 & 0.6 & 0.4 \end{pmatrix}$
P_4	$\begin{pmatrix} 0.5 & 0.6 & 0.7 \\ 0.2 & 0.1 & 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.4 & 0.6 & 0.5 \\ 0.4 & 0.3 & 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.3 & 0.2 & 0.4 \\ 0.4 & 0.4 & 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.7 & 0.6 & 0.8 \\ 0.3 & 0.2 & 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.4 & 0.3 & 0.2 \\ 0.5 & 0.4 & 0.3 \end{pmatrix}$

Table 4 $CM_{\mathrm{B}}(c,d)$ and $CN_{\mathrm{B}}(c,d)$

	D_1	D_2	D_3	D_4	D_5
P_1	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.6\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7333\\0.1\end{pmatrix}$	$\begin{pmatrix}0.6\\0.1\end{pmatrix}$
P_2	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8\\0.2\end{pmatrix}$	$\begin{pmatrix}0.5\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7333\\0.2\end{pmatrix}$	$\begin{pmatrix}0.4\\0.2\end{pmatrix}$
P_3	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.6\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7333\\0.1\end{pmatrix}$	$\begin{pmatrix}0.5\\0.1\end{pmatrix}$
P_4	$\begin{pmatrix}0.6\\0.1\end{pmatrix}$	$\begin{pmatrix}0.6\\0.2\end{pmatrix}$	$\begin{pmatrix}0.5\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7\\0.2\end{pmatrix}$

Table 5 Result outputs

β	D_1	D_2	D_3	D_4	D_5
P_1	0.7408	0.7408	0.4451	0.6661	0.5206
P_2	0.7408	0.6869	0.3315	0.6033	0.2211
P_3	0.7408	0.7408	0.4451	0.6661	0.4140
P_4	0.5206	0.4451	0.3315	0.5629	0.5629

Table 6 $CM_{\mathrm{B}^*}(c,d)$ and $CN_{\mathrm{B}^*}(c,d)$

	D_1	D_2	D_3	D_4	D_5
P_1	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.85\\0.1\end{pmatrix}$	$\begin{pmatrix}0.65\\0.15\end{pmatrix}$	$\begin{pmatrix}0.75\\0.1\end{pmatrix}$	$\begin{pmatrix}0.7\\0.1\end{pmatrix}$
P_2	$\begin{pmatrix}0.6\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7\\0.15\end{pmatrix}$	$\begin{pmatrix}0.5\\0.3\end{pmatrix}$	$\begin{pmatrix}0.6\\0.15\end{pmatrix}$	$\begin{pmatrix}0.45\\0.25\end{pmatrix}$
P_3	$\begin{pmatrix}0.8\\0.1\end{pmatrix}$	$\begin{pmatrix}0.85\\0.1\end{pmatrix}$	$\begin{pmatrix}0.7\\0.15\end{pmatrix}$	$\begin{pmatrix}0.75\\0.1\end{pmatrix}$	$\begin{pmatrix}0.65\\0.1\end{pmatrix}$
P_4	$\begin{pmatrix}0.7\\0.1\end{pmatrix}$	$\begin{pmatrix}0.75\\0.15\end{pmatrix}$	$\begin{pmatrix}0.55\\0.15\end{pmatrix}$	$\begin{pmatrix}0.75\\0.2\end{pmatrix}$	$\begin{pmatrix}0.75\\0.15\end{pmatrix}$

Table 7 Result outputs

β^*	D_1	D_2	D_3	D_4	D_5
P_1	0.7408	0.7983	0.5383	0.6846	0.6293
P_2	0.4451	0.5953	0.2563	0.4821	0.2357
P_3	0.7408	0.7983	0.5953	0.6793	0.5747
P_4	0.6293	0.6534	0.4268	0.6239	0.6534

Compliance with ethical standards

Conflict of interest The author declares that there is no conflict of interest toward the publication of this manuscript.

References

Atanassov KT (1983) Intuitionistic fuzzy sets. VII ITKR's Session, Sofia

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Set Syst 20:87–96

Atanassov KT (1999) Intuitionistic fuzzy sets: theory and applications. Physica-Verlag, Heidelberg

Chen SM, Chang CH (2016) Fuzzy multiattribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators. Inf Sci 352–353:133–149

Chen SM, Cheng SH, Chiou CH (2016) Fuzzy multiattribute group decision making based on intuitionistic fuzzy sets and evidential reasoning methodology. Inf Fusion 27:215–227

Chen SM, Cheng SH, Lan TC (2016a) Multicriteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values. Inf Sci 367–368:279–295

Cheng SH, Chen SM, Lan TC (2016b) A novel similarity measure between intuitionistic fuzzy sets based on the centroid points of transformed fuzzy numbers with applications to pattern recognition. Inf Sci 343–344:15–40

Das S, Kar MB, Kar S (2013) Group multi-criteria decision making using intuitionistic multi-fuzzy sets. J Uncertain Anal Appl. https://doi.org/10.1186/2195-5468-1-10

De SK, Biswas R, Roy AR (2001) An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Set Syst 117(2):209–213

Ejegwa PA (2015) Mathematical techniques to transform intuitionistic fuzzy multisets to fuzzy sets. J Inf Comput Sci 10(2):169–172

Ejegwa PA (2016) Some operations on intuitionistic fuzzy multisets. J Fuzzy Math 24(4):761–768

Ejegwa PA (2019a) Improved composite relation for Pythagorean fuzzy sets and its application to medical diagnosis. Granul Comput 5(2):277–286

Ejegwa PA (2019b) Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. Complex Intell Syst 5:165–175

Ejegwa PA (2019c) Modified Zhang and Xu's distance measure of Pythagorean fuzzy sets and its application to pattern recognition problems. Neural Comput Appl 32(14):10199–10208

Ejegwa PA (2019d) Personnel appointments: a Pythagorean fuzzy sets approach using similarity measure. J Inf Comput Sci 14(2):94–102

Ejegwa PA (2019e) Modal operators on Pythagorean fuzzy sets and some of their properties. J Fuzzy Math 27(4):939–956

Ejegwa PA (2020a) New similarity measures for Pythagorean fuzzy sets with applications. Int J Fuzzy Comput Model 3(1):75–94

Ejegwa PA (2020b) Generalized triparametric correlation coefficient for Pythagorean fuzzy sets with application to MCDM problems. Granul Comput. https://doi.org/10.1007/s41066-020-00215-5

Ejegwa PA (2020c) Pythagorean fuzzy multiset and its application to course placements. Open J Discrete Appl Math 3(1):55–74

Ejegwa PA, Awolola JA (2013) Some algebraic structures of intuitionistic fuzzy multisets. Int J Sci Technol 2(5):373–376

Ejegwa PA, Awolola JA (2019) Novel distance measures for Pythagorean fuzzy sets with applications to pattern recognition problems. Granul Comput. https://doi.org/10.1007/s41066-019-00176-4

Ejegwa PA, Onyeke IC (2020) Medical diagnostic analysis on some selected patients based on modified Thao et al.'s correlation coefficient of intuitionistic fuzzy sets via an algorithmic approach. J Fuzzy Exten Appl 1(2):130–141

Garg H (2018a) Some methods for strategic decision-making problems with immediate probabilities in Pythagorean fuzzy environment. Int J Intell Syst 33(4):687–712

Garg H (2018b) A new exponential operational laws and their aggregation operators of interval-valued Pythagorean fuzzy information. Int J Intell Syst 33(3):653–683

Garg H, Kumar K (2018) An advance study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making. Soft Comput 22(15):4959–4970

Jana C, Pal M (2019a) Assessment of enterprise performance based on picture fuzzy Hamacher aggregation operators. Symmetry 11(1):75. https://doi.org/10.3390/sym11010075

Jana C, Pal M (2019b) A robust single-Valued neutrosophic soft aggregation operators in multi-criteria decision-making. Symmetry 11(1):110. https://doi.org/10.3390/sym11010110

Jana C, Pal M, Wang JQ (2019) Bipolar fuzzy Dombi aggregation operators and its application in multiple attribute decision-making process. J Amb Intell Human Comput 10(9):3533–3549

Jana C, Senapati T, Pal M (2019a) Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision-making. Int J Intell Syst 34(9):2019–2038

Jana C, Senapati T, Pal M, Yager RR (2019b) Picture fuzzy Dombi aggregation operators: application to MADM process. Appl Soft Comput 74(1):99–109

Jana C, Pal M, Karaaslan F, Wang JQ (2020a) Trapezoidal neutrosophic aggregation operators and its application in multiple attribute decision-making process. Sci Iran 27(3):1655–1673

Jana C, Pal M, Wang JQ (2020b) Bipolar fuzzy Dombi prioritized aggregation operators in multiple attribute decision making. Soft Comput 24:3631–3646

Khan MSA, Abdullah S, Ali A, Amin F (2019) Pythagorean fuzzy prioritized aggregation operators and their application to multiattribute group decision making. Granul Comput 4:249–263

Liu P, Chen SM, Wang Y (2020) Multiattribute group decision making based on intuitionistic fuzzy partitioned Maclaurin symmetric mean operators. Inf Sci 512:830–854

Rahman K, Abdullah S (2019) Generalized interval-valued Pythagorean fuzzy aggregation operators and their application to group decision making. Granul Comput 4:15–25

Rajarajeswari P, Uma N (2013) Hausdroff similarity measures for intuitionistic fuzzy multisets and its application in medical diagnosis. Int J Math Arch 4(10):106–111

Rajarajeswari P, Uma N (2014) Normalized Hamming measure for intuitionistic fuzzy multisets and its application in medical diagnosis. Int J Math Trends Technol 5(3):219–225

Shinoj TK, Sunil JJ (2012) Intuitionistic fuzzy multisets and its application in medical diagnosis. Int J Math Comput Sci 6:34–38

Shinoj TK, Sunil JJ (2013) Accuracy in collaborative robotics: an intuitionistic fuzzy multiset approach. Glob J Sci Front Res Math Decis Sci 13:21–28

Szmidt E, Kacprzyk J (2001) Intuitionistic fuzzy sets in some medical applications. Note IFS 7(4):58–64

Ulucay V, Deli I, Sahin M (2019) Intuitionistic trapezoidal fuzzy multi-numbers and its application to multi-criteria decision-making problems. Complex Intell Syst 5:65–78

Yager RR (2013) Pythagorean membership grades in multicriteria decision making. Technical Report MII-3301 Machine Intelligence Institute, Iona College, New Rochelle, NY

Zadeh LA (1965) Fuzzy sets. Inf Control 8:38–353

Zeng S, Chen SM, Kuo LW (2019) Multiattribute decision making based on novel score function of intuitionistic fuzzy values and modified VIKOR method. Inf Sci 488:76–92

Zhang X (2016) A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making. Int J Intell Syst 31:593–661