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ORIGINAL PAPER

Improved composite relation for pythagorean fuzzy sets and its application to medical diagnosis

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Abstract

Uncertainty is an important factor in any decision-making process. Different mathematical frameworks have been introduced to cope the ambiguity of decision-making. The concept of Pythagorean fuzzy sets (PFSs) is one of the latest mathematical frameworks that deals with uncertainty. Pythagorean fuzzy sets generalize intuitionistic fuzzy sets with a wider scope of applications, and hence, the motivation for investigating into its applicability in tackling uncertainty imbedded in medical diagnosis. This paper studies the approach of max–min–max composite relation for Pythagorean fuzzy sets, improves upon the approach, and applies its to medical diagnosis problem. The validity of the improved composite relation for Pythagorean fuzzy sets is carried out in comparison to the max–min–max composite relation for Pythagorean fuzzy sets using numerical experiments. The improved composite relation for Pythagorean fuzzy sets yields a better relation with a greater relational value when compared to the aforementioned composite relation and, hence, its choice to solving medical diagnosis problem. To this end, an application of the improved composite relation for Pythagorean fuzzy sets is explored in medical diagnosis using hypothetical medical database. This improved composite relation could be used as a sustainable approach in applying Pythagorean fuzzy sets to multi-criteria decision-making (MCDM) problems, multi-attribute decision-making (MADM) problems, pattern recognition problems, among others.

Keywords Fuzzy set · Intuitionistic fuzzy set · Medical diagnosis · Pythagorean fuzzy relation · Pythagorean fuzzy set

1 Introduction

The theory of fuzzy sets proposed by Zadeh (1965) has several applications because of its ability to handle the imprecision imbedded in real-life situations. A fuzzy set is characterized by a membership function μ which takes value from a crisp set to a unit interval $I = [0, 1]$. Many researchers have worked on the theory of fuzzy sets and its applications (see Chen et al. 2001; Chen and Tanuwijaya 2011; Chen and Chang 2011; Chen et al. 2012; Wang and Chen 2008). The inevitable presence of uncertainty and imprecision in the real world necessitated researchers to develop some mathematical frameworks to cope imprecision more accurately than fuzzy sets. The concept of intuitionistic fuzzy sets (IFSs) proposed in Atanassov (1983, 1986) is one of the generalizations of fuzzy sets with a better applicability.

After the introduction of IFSs theory, many studies that bother on its applications have been carried out, especially in areas like medical diagnosis, electoral system, career determination, appointment procedures, pattern recognition, learning techniques, among others (see Davvaz and Sadrabadi 2016; Chen and Chang 2015; Chen et al. 2016; De et al. 2001; Ejegwa et al. 2014a, b, c; Ejegwa 2015; Ejegwa and Modom 2015; Ejegwa et al. 2016; Hatzimichailidis et al. 2012; Liu and Chen 2017; Szmidt and Kacprzyk 2001, 2004).

Although the notion of IFSs is very resourceful, there are cases where the sum of the membership and nonmembership degrees is greater than one unlike the situation captured in IFSs; where the sum of the membership and nonmembership degrees is less than or equal to one only. This limitation in IFSs led to the introduction of intuitionistic fuzzy sets of second type (IFSST) by Atanassov (1989). The concept was later called Pythagorean fuzzy sets (PFSs) by Yager (2013a). PFS is a new tool to deal with vagueness involved

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in decision-making. As a generalized set, PFS has close relationship with IFS. The concept of PFSs can be used to characterize uncertain information more sufficiently and accurately when compare to IFSs. The theory of PFSs has been extensively studied in the literature since inception (see Beliakov and James 2014; Dick et al. 2016; Gou et al. 2016; He et al. 2016; Peng and Yang 2015; Peng and Selvachandran 2017; Yager 2013b, 2014; Ejegwa 2018).

Pythagorean fuzzy set has attracted great attentions of many researchers and subsequently, the concept has been applied to many application areas such as decision-making, aggregation operators, information measures, among others. Perez-Dominguez et al. (2018) presented a multi-objective optimization on the basis of ratio analysis (MOORA) under PFS setting and applied it to multi-criteria decision-making (MCDM) problems. Liang and Xu (2017) proposed the idea of PFSs in hesitant environment and its multi-criteria decision-making (MCDM) by employing the technique for order preference by similarity to ideal solution (TOPSIS) using energy project selection model. Rahman et al. (2017) worked on some geometric aggregation operators on interval-valued PFSs (IVPFSs) and applied the same to group decision-making problem. Mohagheghi et al. (2017) offered a novel last aggregation group decision-making process for the weight of decision makers using PFSs.

Rahman et al. (2018b) proposed some approaches to multi-attribute group decision-making based on induced interval-valued Pythagorean fuzzy Einstein aggregation operator. Garg (2018a) discussed a decision-making problem under Pythagorean fuzzy environment by proposing some generalized aggregation operators. Garg (2018b) proposed an improved score function for solving multi-criteria decision-making (MCDM) problem with partially known weight information, such that the preferences related to the criteria are taken in the form of interval-valued Pythagorean fuzzy sets. Garg (2018d, e) developed a new decision-making model with probabilistic information, using the concept of immediate probabilities to aggregate the information under the Pythagorean fuzzy set environment, and defined two new exponential operational laws about IVPFS and their corresponding aggregation operators with application to multi-criteria decision-making (MCDM). See Gao and Wei (2018); Rahman et al. (2018a); Rahman and Abdullah (2018); Khan et al. (2018a, b); Garg (2016, 2017, 2018c); Du et al. (2017); Hadi-Venchen and Mirjaberi (2014); Yager and Abbasov (2013); Yager (2016) for more applications of PFSs and IVPFSs, respectively.

In this paper, we propose an improved version of max–min–max composite relation for Pythagorean fuzzy sets with application to medical diagnosis. A juxtapositional analysis of the improved composite relation for Pythagorean fuzzy sets and the max–min–max composite relation for Pythagorean fuzzy sets is carried out using some numerical experiments. It follows that the improved version provides a reliable Pythagorean fuzzy relation when compared to max–min–max composite relation for Pythagorean fuzzy sets. Finally, an application of the improved composite relation for Pythagorean fuzzy sets is carried out in medical diagnosis because of its reliable output when compared to max–min–max composite relation. The paper is organized by presenting some mathematical preliminaries of fuzzy sets, IFSs and PFSs, in Sect. 2. In Sect. 3, we present max–min–max composite relation for PFSs, its improved version, and their numerical verifications. An application of the improved composite relation for PFSs to medical diagnosis is explored in Sect. 4. Finally, Sect. 5 concludes the paper and provides direction for future studies.

2 Some basic notions of Pythagorean fuzzy sets

In this section, we recall some mathematical preliminaries of fuzzy sets, IFSs and PFSs.

Definition 1 (*See* Zadeh 1965) Let X be a nonempty set. A fuzzy set A of X is characterized by a membership function:

$\mu_A : X \to [0, 1]$.

That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X \end{cases}$$

Alternatively, a fuzzy set A of X is an object having the form:

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\} \text{ or } A = \left\{\left\langle \frac{\mu_A(x)}{x} \right\rangle \mid x \in X\right\},$$

where the function

$\mu_A(x) : X \to [0, 1]$

defines the degree of membership of the element $x \in X$.

Definition 2 (*See* Atanassov 1983, 1986) Let a nonempty set X be fixed. An IFS A of X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

or

$$A = \left\{\left\langle \frac{\mu_A(x), \nu_A(x)}{x}\right\rangle \mid x \in X\right\},$$

where the functions

$$\mu_A(x) : X \to [0,1] \text{ and } \nu_A(x) : X \to [0,1]$$

define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

For each A in X:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is the intuitionistic fuzzy set index or hesitation margin of $x \in X$. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$, to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses the lack of knowledge of whether $x \in X$ or $x \notin X$. Thus:

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1.$$

Example 3 Let $X = \{x, y, z\}$ be a fixed universe of discourse and

$$A = \left\{\left\langle \frac{0.70, 0.10}{x}\right\rangle, \left\langle \frac{0.85, 0.05}{y}\right\rangle, \left\langle \frac{0.50, 0.20}{z}\right\rangle\right\}$$

be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are

$$\pi_A(x) = 0.20, \ \pi_A(y) = 0.10 \quad \text{and} \quad \pi_A(z) = 0.30.$$

Definition 4 (*See* Yager 2013a, b) Let X be a universal set. Then, a Pythagorean fuzzy set A which is a set of ordered pairs over X, is defined by the following:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

or

$$A = \left\{\left\langle \frac{\mu_A(x), \nu_A(x)}{x}\right\rangle \mid x \in X\right\},$$

where the functions

$$\mu_A(x) : X \to [0,1] \quad \text{and} \quad \nu_A(x) : X \to [0,1]$$

define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$:

$$0 \le (\mu_A(x))^2 + (\nu_A(x))^2 \le 1.$$

Supposing that $(\mu_A(x))^2 + (\nu_A(x))^2 \le 1$; then, there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\nu_A(x))^2 = 1$.

We denote the set of all *PFSs* over X by $PFS(X)$.

Example 5 Let $A \in PFS(X)$. Suppose that $\mu_A(x) = 0.70$ and $\nu_A(x) = 0.50$ for $X = \{x\}$. Clearly, $0.70 + 0.50 \nleq 1$, but $0.70^2 + 0.50^2 \le 1$. Thus, $\pi_A(x) = 0.5099$, and hence, $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$.

Table 1 explains the difference between Pythagorean fuzzy sets and intuitionistic fuzzy sets (Ejegwa 2018).

Definition 6 (*See* Yager 2013a, b, 2014) Let $A, B \in PFS(X)$. Then, we have the following:

(i) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$.
(ii) $A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in X\}$.
(iii) $A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in X\}$.
(iv) $A \oplus B = \{\langle x, \sqrt{(\mu_A(x))^2 + (\mu_B(x))^2 - (\mu_A(x))^2(\mu_B(x))^2}, \nu_A(x)\nu_B(x)\rangle | x \in X\}$.
(v) $A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \sqrt{(\nu_A(x))^2 + (\nu_B(x))^2 - (\nu_A(x))^2(\nu_B(x))^2}\rangle | x \in X\}$.

Table 1 Pythagorean fuzzy sets and Intuitionistic fuzzy sets

Intuitionistic fuzzy sets	Pythagorean fuzzy sets
$\mu + \nu \le 1$	$\mu + \nu \le 1$ or $\mu + \nu \ge 1$
$0 \le \mu + \nu \le 1$	$0 \le \mu^2 + \nu^2 \le 1$
$\pi = 1 - (\mu + \nu)$	$\pi = \sqrt{1 - [\mu^2 + \nu^2]}$
$\mu + \nu + \pi = 1$	$\mu^2 + \nu^2 + \pi^2 = 1$

Remark 7 (Ejegwa 2018) Let $A, B, C \in PFS(X)$. By Definition 6, the following properties hold:

$(A^c)^c = A$

$A \cap A = A$

$A \cup A = A$

$A \oplus A \neq A$

$A \otimes A \neq A$

$A \cap B = B \cap A$

$A \cup B = B \cup A$

$A \oplus B = B \oplus A$

$A \otimes B = B \otimes A$

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \oplus (B \oplus C) = (A \oplus B) \oplus C$

$A \otimes (B \otimes C) = (A \otimes B) \otimes C$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$

$A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$

$A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$

$A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$

$(A \cap B)^c = A^c \cup B^c$

$(A \cup B)^c = A^c \cap B^c$

$(A \oplus B)^c = A^c \otimes B^c$

$(A \otimes B)^c = A^c \oplus B^c$.

3 Pythagorean fuzzy relations

The author has proposed the notion of max–min–max composite relation for PFSs in the other communication. Albeit, in this section, we introduce an improved composite relation for PFSs.

Definition 8 Let X and Y be two nonempty sets. A Pythagorean fuzzy relation (PFR) R from X to Y is a PFS of $X \times Y$ characterized by the membership function μ_R and nonmembership function ν_R. A PF relation or PFR from X to Y is denoted by $R(X \to Y)$.

Definition 9 Let $A \in PFS(X)$. Then, the max–min–max composite relation of

$R(X \to Y)$

with A is a PFS B of Y denoted by $B = R \circ A$, such that its membership and nonmembership functions are defined by the following:

$$\mu_B(y) = \bigvee_x \{\min[\mu_A(x), \mu_R(x, y)]\}$$

and

$$\nu_B(y) = \bigwedge_x \{\max[\nu_A(x), \nu_R(x, y)]\}$$

$\forall x \in X$ and $y \in Y$, where $\bigvee =$ maximum; $\bigwedge =$ minimum.

Definition 10 Let $Q(X \to Y)$ and $R(Y \to Z)$ be two PFRs. Then, the max–min–max composite relation $R \circ Q$ is a PFR from X to Z, such that its membership and nonmembership functions are defined by the following:

$$\mu_{R \circ Q}(x, z) = \bigvee_y \{\min[\mu_Q(x, y), \mu_R(y, z)]\}$$

and

$$\nu_{R \circ Q}(x, z) = \bigwedge_y \{\max[\nu_Q(x, y), \nu_R(y, z)]\}$$

$\forall (x, z) \in X \times Z$ and $\forall y \in Y$.

Definition 11 Let $A \in PFS(X)$. Then, the improved composite relation of

$R(X \to Y)$

with A is a PFS $\mathbf{B}$ of Y denoted by $\mathbf{B} = R \circ A$, such that its membership and nonmembership functions are defined by the following:

$$\mu_{\mathbf{B}}(y) = \bigvee_x \left\{ \frac{\mu_A(x) + \mu_R(x, y)}{2} \right\}$$

and

$$\nu_{\mathbf{B}}(y) = \bigwedge_x \left\{ \frac{\nu_A(x) + \nu_R(x, y)}{2} \right\}$$

$\forall x \in X$ and $y \in Y$, where $\bigvee =$ maximum; $\bigwedge =$ minimum.

Definition 12 Let $Q(X \to Y)$ and $R(Y \to Z)$ be two PFRs. Then, the improved composite relation $\mathbf{R} \circ \mathbf{Q}$ is a PFR from X to Z, such that its membership and nonmembership functions are defined by the following:

$$\mu_{\mathbf{R} \circ \mathbf{Q}}(x, z) = \bigvee_y \left\{ \frac{\mu_Q(x, y) + \mu_R(y, z)}{2} \right\}$$

and

$$\nu_{\mathbf{R}\circ\mathbf{Q}}(x,z) = \bigwedge_y \left\{ \frac{\nu_Q(x,y) + \nu_R(y,z)}{2} \right\}$$

$\forall (x,z) \in X \times Z$ and $\forall y \in Y$.

Remark 13 From Definitions 11 and 12, the improved composite relation $\mathbf{B}$ or $\mathbf{R}\circ\mathbf{Q}$ is calculated by the following:

$\mathbf{B} = \mu_{\mathbf{B}}(y) - \nu_{\mathbf{B}}(y)\pi_{\mathbf{B}}(y)$

$\forall y \in Y$ or

$\mathbf{R}\circ\mathbf{Q} = \mu_{R\circ Q}(x,z) - \nu_{R\circ Q}(x,z)\pi_{R\circ Q}(x,z)$

$\forall (x,z) \in X \times Z$.

Proposition 14 *If R and S are two PFRs on $X \times Y$ and $Y \times Z$,* respectively. Then:

(i) $(R^{-1})^{-1} = R$.
(ii) $(S\circ R)^{-1} = R^{-1}\circ S^{-1}$.

3.1 Numerical examples

Before applying this relation to medical diagnosis, we discuss the procedures of the approach step-wisely. First, we use max–min–max composite relation and, then, the improved composite relation. A reliability analysis is conducted to ascertain which of the composite relation provides the best relation by comparing the relational values.

Example 15 Let $E, F \in PFS(X)$ for $X = \{x_1, x_2, x_3\}$. Suppose

$$E = \left\{ \left\langle \frac{0.6, 0.2}{x_1} \right\rangle, \left\langle \frac{0.4, 0.6}{x_2} \right\rangle, \left\langle \frac{0.5, 0.3}{x_3} \right\rangle \right\}$$

and

$$F = \left\{ \left\langle \frac{0.8, 0.1}{x_1} \right\rangle, \left\langle \frac{0.7, 0.3}{x_2} \right\rangle, \left\langle \frac{0.6, 0.1}{x_3} \right\rangle \right\}.$$

We find the composite relation B using Definitions 9 and 10, respectively:

$\min[\mu_R(e_i, x_j), \mu_S(x_j, f_k)] = 0.6, 0.4, 0.5,$

implying that

$$\mu_B(e_i, f_k) = \bigvee_{x_j \in X} \{0.6, 0.4, 0.5\} = 0.6.$$

Clearly, $\min[\mu_R(e_i, x_j), \mu_S(x_j, f_k)]$ is gotten by synthesizing Definitions 9 and 10. Applying this to E and F as given in the above example, we observe that the minimum value of the membership values of the elements (i.e., x_1, x_2, x_3) in E and F, respectively, is 0.6, 0.4, and 0.5.

Again:

$\max[\nu_R(e_i, x_j), \nu_S(x_j, f_k)] = 0.2, 0.6, 0.3,$

implying that

$$\nu_B(e_i, f_k) = \bigwedge_{x_j \in X} \{0.2, 0.6, 0.3\} = 0.2.$$

By explanation, $\max[\nu_R(e_i, x_j), \nu_S(x_j, f_k)]$ is gotten by synthesizing Definitions 9 and 10. Applying this to E and F as given in the above example, we observe that the maximum value of the nonmembership values of the elements (i.e., x_1, x_2, x_3) in E and F, respectively, is 0.2, 0.6, and 0.3.

Then:

$B = 0.6 - (0.2 \times 0.7746) = 0.4451.$

Again, finding $\mathbf{B}$ using Definitions 11 and 12 with application to E and F, we obtain the following:

$$\frac{\mu_R(e_i, x_j) + \mu_S(x_j, f_k)}{2} = 0.7, 0.55, 0.55,$$

implying that

$$\mu_B(e_i, f_k) = \bigvee_{x_j \in X} \{0.7, 0.55, 0.55\} = 0.7.$$

Again:

$$\frac{\nu_R(e_i, x_j) + \nu_S(x_j, f_k)}{2} = 0.15, 0.45, 0.2,$$

implying that

$$\nu_B(e_i, f_k) = \bigwedge_{x_j \in X} \{0.15, 0.45, 0.2\} = 0.15.$$

Then:

$\mathbf{B} = 0.7 - (0.15 \times 0.6982) = 0.5953.$

From the aforesaid, the improved composite relation yields a better relation with greater relational value when compared to max–min–max composite relation.

Now, we consider a situation where the elements of PFSs are not equal.

Example 16 Let $G, H \in PFS(X)$ for $X = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose that

$$G = \left\{ \left\langle \frac{0.8, 0.4}{x_1} \right\rangle, \left\langle \frac{0.5, 0.7}{x_2} \right\rangle, \left\langle \frac{0.8, 0.4}{x_3} \right\rangle, \left\langle \frac{0.7, 0.2}{x_5} \right\rangle \right\}$$

and

$$H = \left\{ \left\langle \frac{0.7, 0.3}{x_1} \right\rangle, \left\langle \frac{0.4, 0.7}{x_3} \right\rangle, \left\langle \frac{0.9, 0.2}{x_4} \right\rangle \right\}.$$

Before calculating the composite relations, we rewrite the PFSs; thus:

$$G = \left\{ \left\langle \frac{0.8, 0.4}{x_1} \right\rangle, \left\langle \frac{0.5, 0.7}{x_2} \right\rangle, \left\langle \frac{0.8, 0.4}{x_3} \right\rangle, \left\langle \frac{0.0, 1.0}{x_4} \right\rangle, \left\langle \frac{0.7, 0.2}{x_5} \right\rangle \right\}$$

and

$$H = \left\{ \left\langle \frac{0.7, 0.3}{x_1} \right\rangle, \left\langle \frac{0.0, 1.0}{x_2} \right\rangle, \left\langle \frac{0.4, 0.7}{x_3} \right\rangle, \left\langle \frac{0.9, 0.2}{x_4} \right\rangle, \left\langle \frac{0.0, 1.0}{x_5} \right\rangle \right\}.$$

Now, we find B using Definitions 9 and 10, respectively, as follows:

$$\min[\mu_R(g_i, x_j), \mu_S(x_j, h_k)] = 0.7, 0.0, 0.4, 0.0, 0.0,$$

implying that

$$\mu_B(g_i, h_k) = \bigvee_{x_j \in X} \{0.7, 0.5, 0.4, 0.9, 0.7\} = 0.7.$$

Again,

$$\max[\nu_R(g_i, x_j), \nu_S(x_j, h_k)] = 0.4, 1.0, 0.7, 1.0, 1.0,$$

implying that

$$\nu_B(g_i, h_k) = \bigwedge_{x_j \in X} \{0.4, 1.0, 0.7, 1.0, 1.0\} = 0.4.$$

Thus:

$$B = 0.7 - (0.4 \times 0.5916) = 0.4634.$$

In addition, finding $\mathbf{B}$ using Definitions 11 and 12, we obtain the following:

$$\frac{\mu_R(g_i, x_j) + \mu_S(x_j, h_k)}{2} = 0.75, 0.25, 0.6, 0.45, 0.35,$$

implying that

$$\mu_B(g_i, h_k) = \bigvee_{x_j \in X} \{0.75, 0.25, 0.6, 0.45, 0.35\} = 0.75.$$

Again:

$$\frac{\nu_R(g_i, x_j) + \nu_S(x_j, h_k)}{2} = 0.35, 0.85, 0.55, 0.6, 0.6,$$

implying that

$$\nu_B(g_i, h_k) = \bigwedge_{x_j \in X} \{0.35, 0.85, 0.55, 0.6, 0.6\} = 0.35.$$

Then:

$$\mathbf{B} = 0.75 - (0.35 \times 0.5612) = 0.5536.$$

In this case also, the improved composite relation gives a better relation compared to max–min–max composite relation.

Table 2 gives the comparative analysis of the improved composite relation $\mathbf{B}$ and max–min–max composite relation B for Pythagorean fuzzy sets. In what follows, the relational value of $\mathbf{B}$ is greater than that of B. This shows that $\mathbf{B}$ provides a better Pythagorean fuzzy relation when compared to B.

Table 2 Comparative analysis

PFR	Example 15	Example 16
B	0.4451	0.4634
$\mathbf{B}$	0.5953	0.5536

4 Improved composite relation for Pythagorean fuzzy sets in medical diagnosis

In this section, we present an application of Pythagorean fuzzy set theory to medical diagnosis using the proposed composite relation for PFSs. In a given pathology, suppose that S is a set of symptoms, D is a set of diseases, and P is a set of patients. We define *Pythagorean medical knowledge* as a Pythagorean fuzzy relation R from the set of symptoms S to the set of diseases D (i.e., on $S \times D$) which reveals the degree of association and the degree of nonassociation between symptoms and diseases.

Now, we discuss the notion of *Pythagorean fuzzy medical diagnosis* via the following methodology:

(i) determination of symptoms;
(ii) formulation of medical knowledge based on Pythagorean fuzzy relations;
(iii) determination of diagnosis on the basis of composition of Pythagorean fuzzy relations.

Let A be a PFS of the set S, and R be a PFR from S to D. Then, the improved composite relation $\mathbf{B}$ for PFS A with the IFR $R(S \to D)$ denoted by

$$\mathbf{B} = A \circ R$$

Table 3 $Q(P \rightarrow S)$

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
Lil	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.1, 0.6\rangle$
Jones	$\langle 0.0, 0.8\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.1, 0.7\rangle$	$\langle 0.1, 0.8\rangle$
Deby	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.0, 0.6\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.0, 0.5\rangle$
Inas	$\langle 0.6, 0.1\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.3, 0.4\rangle$

signifies the state of the patient in terms of diagnosis as a PFS $\mathbf{B}$ of D with the membership function given by the following:

$$\mu_{\mathbf{B}}(d) = \bigvee_{s \in S} \left\{ \frac{\mu_A(s) + \mu_R(s, d)}{2} \right\},$$

and the nonmembership function is given by the following:

$$\nu_{\mathbf{B}}(d) = \bigwedge_{s \in S} \left\{ \frac{\nu_A(s) + \nu_R(s, d)}{2} \right\}$$

$\forall d \in D$.

If the state of a given patient P is described in terms of a PFS A of S, then P is assumed to be assigned diagnosis in terms of PFS $\mathbf{B}$ of D, through a PFR R of *Pythagorean medical knowledge* from S to D which is assumed to be given by a doctor who is able to translate his own observation of the fuzziness involved in degrees of association and nonassociation, respectively, between symptoms and diagnosis.

Now, we extend this concept to a finite number of patients. Let there be n patients p_i for $i = 1, 2, \ldots, n$ in a given laboratory. Thus, $p_i \in P$ (or simply, $p \in P$). Let R be a PFR $(S \rightarrow D)$ and construct a PFR Q from the set of patients P to the set of symptoms S. Clearly, the composite relation $\mathbf{B}$ of PFRs R and $Q(\mathbf{B} = R \circ Q)$ designates the state of patients p in terms of the diagnosis as a PFR from P to D given by the membership function:

$$\mu_{\mathbf{B}}(p, d) = \bigvee_{s \in S} \left\{ \frac{\mu_Q(p, s) + \mu_R(s, d)}{2} \right\}$$

and the nonmembership function is given by the following:

$$\nu_{\mathbf{B}}(p, d) = \bigwedge_{s \in S} \left\{ \frac{\nu_Q(p, s) + \nu_R(s, d)}{2} \right\}$$

$\forall p \in P$ and $\forall d \in D$.

For a given R and Q, the relation $\mathbf{B} = R \circ Q$ can be computed. From the knowledge of Q and R, one may find $\mathbf{B}$ of the PFR for which

$$\mathbf{B} = \mu_{\mathbf{B}}(p, d) - \nu_{\mathbf{B}}(p, d)\pi_{\mathbf{B}}(p, d)$$

is the greatest.

Table 4 $R(S \rightarrow D)$

R	Viral fever	Malaria fever	Typhoid fever	Stomach problem	Chest problem
Temperature	$\langle 0.4, 0.0\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.3, 0.3\rangle$	$\langle 0.1, 0.7\rangle$	$\langle 0.1, 0.8\rangle$
Headache	$\langle 0.3, 0.5\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.2, 0.4\rangle$	$\langle 0.0, 0.8\rangle$
Stomach pain	$\langle 0.1, 0.7\rangle$	$\langle 0.0, 0.9\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.8, 0.0\rangle$	$\langle 0.2, 0.8\rangle$
Cough	$\langle 0.4, 0.3\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.2, 0.8\rangle$
Chest pain	$\langle 0.1, 0.7\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.8, 0.1\rangle$

Table 5 $\mu_{\mathbf{B}}(p, d)$ and $\nu_{\mathbf{B}}(p, d)$

$\mu_{\mathbf{B}}, \nu_{\mathbf{B}}$	Viral fever	Malaria fever	Typhoid fever	Stomach problem	Chest problem
Lil	$\langle 0.60, 0.05\rangle$	$\langle 0.75, 0.05\rangle$	$\langle 0.60, 0.10\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.45, 0.35\rangle$
Jones	$\langle 0.35, 0.40\rangle$	$\langle 0.40, 0.35\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.70, 0.05\rangle$	$\langle 0.45, 0.45\rangle$
Deby	$\langle 0.60, 0.05\rangle$	$\langle 0.75, 0.05\rangle$	$\langle 0.70, 0.10\rangle$	$\langle 0.50, 0.25\rangle$	$\langle 0.45, 0.30\rangle$
Inas	$\langle 0.55, 0.05\rangle$	$\langle 0.70, 0.05\rangle$	$\langle 0.55, 0.20\rangle$	$\langle 0.55, 0.20\rangle$	$\langle 0.55, 0.25\rangle$

Table 6 $\mathbf{B} = \mu_{\mathbf{B}} - \nu_{\mathbf{B}}\pi_{\mathbf{B}}$

$\mathbf{B}$	Viral fever	Malaria fever	Typhoid fever	Stomach problem	Chest problem
Lil	0.5601	0.7170	0.5206	0.2927	0.1624
Jones	0.0112	0.1035	0.2927	0.6644	0.1029
Deby	0.5601	0.7170	0.6293	0.2927	0.1977
Inas	0.5083	0.6644	0.3878	0.3878	0.3508

Obviously, R is a significant IFR translating the higher degrees of association and lower degrees of nonassociation of symptoms as well as degrees of hesitation to the diseases, an approach to *Pythagorean medical Knowledge*. From this approach, one may infer diagnosis from symptoms in the sense of a paired value: one being the degree of association and other the degree of nonassociation.

4.1 Application example

Suppose that four patients, viz, Lil, Jones, Deby, and Inas, visit a given laboratory for medical diagnosis. They are observed to have the following symptoms: temperature, headache, stomach pain, cough, and chest pain. That is, the set of patients P is as follows:

$$P = \{\text{Lil, Jones, Deby, and Inas}\},$$

and the set of symptoms S is as follows:

$$S = \{\text{temperature, headache, stomach pain, cough, and chest pain}\}.$$

The Pythagorean fuzzy relation $Q(P \to S)$ is given hypothetically, in Table 3.

Let the set of diseases D be

$$D = \{\text{viral fever, malaria, typhoid, stomach problem, and heart problem}\}.$$

The Pythagorean fuzzy relation $R(S \to D)$ is given hypothetically, in Table 4. The values of the membership and non-membership functions of the composite relation $\mathbf{B} = R \circ Q$ are given in Table 5. Note that, the data in Tables 3 and 4 are extracted from De et al. (2001). After finding the degree of hesitation in Pythagorean fuzzy sense ($\pi = \sqrt{1 - [\mu^2 + \nu^2]}$), we calculate $\mathbf{B}$, and is given in Table 6.

4.2 Decisions on the patients medical conditions

With the aid of Table 6, we present the decision-making. Decisions are made based on the greatest value of relation between patients and diseases. In doing this, we present two forms of decision-making approaches.

4.2.1 Horizontal decision

This approach is with respect to the patient against diseases. From the horizontal view of Table 6, we see that:

1. Lil is suffering from malaria fever (0.7170) with some elements of viral fever (0.5601) and typhoid fever (0.5206), respectively.
2. Jones is suffering from stomach problem (0.6644).
3. Deby is suffering from malaria fever (0.7170) with some elements of typhoid fever (0.6293) and viral fever (0.5601), respectively.
4. Inas is suffering from malaria fever (0.6644) with some element of viral fever (0.5083).

4.2.2 Vertical decision

This approach is taken from the vertical view of Table 6, thus:

1. Lil and Deby are suffering from malaria fever with equal severity (0.7170), follows by Inas (0.6644) with less severity compare to Lil and Deby's cases.
2. Deby is also suffering from typhoid fever (0.6293) and likewise, Lil (0.5206). Clearly, the severity of Deby's case is more acute than Lil's case.
3. Lil and Deby are also suffering from viral fever with equal severity (0.5601). Inas is also suffering from viral fever (0.5083) with less severity compare to the cases of Lil and Deby.
4. Jones is suffering from stomach problem with relational value 0.6644.

4.3 Some observations

1. From both horizontal and vertical decision approaches, we notice a similarity among malaria fever, typhoid fever, and viral fever, which conforms to the practice in human medicine.
2. None of the patients is suffering from chest problem, since the relational values are less than 0.5.
3. With the exception to Jones, none of the patients is suffering from stomach problem. It shows the lack of similarity between stomach problem and the set of diseases like malaria fever, typhoid fever, and viral fever.
4. For a better diagnosis, it is expedient to consider and synthesize both horizontal and vertical decision approaches together.

5 Conclusion

The concept of Pythagorean fuzzy sets is a novel mathematical framework in the fuzzy family with higher ability to tackle uncertainty imbedded in decision-making. Some applications of Pythagorean fuzzy sets have been discussed in the literature using different approaches (see Perez-Dominguez et al. 2018; Liang and Xu 2017; Rahman et al. 2017; Mohagheghi et al. 2017; Garg 2018a; Gao and Wei 2018; Khan et al. 2018b; Du et al. 2017; Hadi-Venchen and Mirjaberi 2014; Yager 2016). In this paper, the notion of max–min–max composite relation for Pythagorean fuzzy sets was studied, and the approach was improved and applied to medical diagnosis. A juxtapositional analysis of the improved composite relation for Pythagorean fuzzy sets and the max–min–max composite relation for Pythagorean fuzzy sets was carried out with the aid of numerical experiments. It was shown that the improved version provides a better Pythagorean fuzzy relation when compared to max–min–max composite relation and, hence, the need for its usage to solve medical diagnosis problem. Finally, an application of the improved composite relation for Pythagorean fuzzy sets was carried out in medical diagnosis case using medical database extracted from De et al. (2001), but in Pythagorean fuzzy context. The improved composite relation proposed in this paper could be used as a viable tool in applying Pythagorean fuzzy sets to multi-criteria decision-making (MCDM) problems, multi-attribute decision-making (MADM) problems, pattern recognition problems, etc. Albeit, it is suggestible to consider this approach from object-oriented perspective for quick output in further research.

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Compliance with ethical standards

Conflict of interest The author declares that there is no conflict of interest toward the publication of this manuscript.

References

Atanassov KT (1983) Intuitionistic fuzzy sets. VII ITKR's Session, Sofia

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Set Syst 20:87–96

Atanassov KT (1989) Geometrical interpretation of the elements of the intuitionistic fuzzy objects. Preprint IM-MFAIS-1-89, Sofia

Beliakov G, James S (2014) Averaging aggregation functions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs. In: Proc IEEE Int Conf Fuzzy Syst (FUZZ-IEEE), pp 298–305

Chen SM, Chang CH (2015) A novel similarity measure between atanassov's intuitionistic fuzzy sets based on transformation techniques with applications to pattern recognition. Inform Sci 291:96–114

Chen SM, Chang YC (2011) Weighted fuzzy rule interpolation based on ga-based weight-learning techniques. IEEE Trans Fuzzy Syst 19(4):729–744

Chen SM, Cheng SH, Chiou CH (2016) Fuzzy multiattribute group decision making based on intuitionistic fuzzy sets and evidential reasoning methodology. Inform Fusion 27:215–227

Chen SM, Lee SH, Lee CH (2001) A new method for generating fuzzy rules from numerical data for handling classification problems. Appl Artif Intell 15(7):645–664

Chen SM, Munif A, Chen GS, Liu HC, Kuo BC (2012) Fuzzy risk analysis based on ranking generalized fuzzy numbers with different left heights and right heights. Expert Syst Appl 39(7):6320–6334

Chen SM, Tanuwijaya K (2011) Fuzzy forecasting based on high-order fuzzy logical relationships and automatic clustering techniques. Expert Syst Appl 38(12):15425–1543

Davvaz B, Sadrabadi EH (2016) An application of intuitionistic fuzzy sets in medicine. Int J Biomath 9(3):1650037–1-15

De SK, Biswas R, Roy AR (2001) An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Set Syst 117(2):209–213

Dick S, Yager RR, Yazdanbakhsh O (2016) On pythagorean and complex fuzzy set operations. IEEE Trans Fuzzy Syst 24(5):1009–1021

Du Y, Hou F, Zafar W, Yu Q, Zhai Y (2017) A novel method for multi-attribute decision making with interval-valued pythagorean fuzzy linguistic information. Int J Intell Syst 32(10):1085–1112

Ejegwa PA (2015) Intuitionistic fuzzy sets approach in appointment of positions in an organization via max–min–max rule. Global J Sci Front Res F Math Decis Sci 15(6):1–6

Ejegwa PA (2018) Distance and similarity measures for pythagorean fuzzy sets. Granul Comput. https://doi.org/10.1007/s41066-018-00149-z

Ejegwa PA, Akubo AJ, Joshua OM (2014a) Intuitionistic fuzzzy sets in career determination. J Inf Comput Sci 9(4):285–288

Ejegwa PA, Modom ES (2015) Diagnosis of viral hepatitis using new distance measure of intuitionistic fuzzy sets. Int J Fuzzy Math Arch 8(1):1–7

Ejegwa PA, Onoja AM, Chukwukelu SN (2014b) Application of intuitionistic fuzzy sets in research questionnaire. J Global Res Math Arch 2(5):51–54

Ejegwa PA, Onoja AM, Emmanuel IT (2014c) A note on some models of intuitionistic fuzzy sets in real life situations. J Global Res Math Arch 2(5):42–50

Ejegwa PA, Tyoakaa GU, Ayenge AM (2016) Application of intuitionistic fuzzy sets in electoral system. Int J Fuzzy Math Arch 10(1):35–41

Gao H, Wei GW (2018) Multiple attribute decision making based on interval-valued pythagorean fuzzy uncertain linguistic aggregation operators. Int J Knowl Based Intell Eng Syst 22:59–81

Garg H (2016) A novel correlation coefficients between pythagorean fuzzy sets and its applications to decision making processes. Int J Intell Syst 31(12):1234–1252

Garg H (2017) Generalized pythagorean fuzzy geometric aggregation operators using einstein t-norm and t-conorm fo multicriteria decision making process. Int J Intell Syst 32(6):597–630

Garg H (2018a) Generalized pythagorean fuzzy geometric interactive aggregation operators using einstein operations and their application to decision making. J Exp Theor Artif Intell 30(6):763–794

Garg H (2018b) A linear programming method based on an improved score function for interval-valued pythagorean fuzzy numbers and its application to decision-making. Int J Uncert Fuzz Knowl Based Syst 29(1):67–80

Garg H (2018c) Linguistic pythagorean fuzzy sets and its applications in multiattribute decision making process. Int J Intell Syst 33(6):1234–1263

Garg H (2018d) A new exponential operational laws and their aggregation operators of interval-valued pythagorean fuzzy information. Int J Intell Syst 33(3):653–683

Garg H (2018e) Some methods for strategic decision-making problems with immediate probabilities in pythagorean fuzzy environment. Int J Intell Syst 33(4):687–712

Gou XJ, Xu ZS, Ren PJ (2016) The properties of continuous pyhagorean fuzzy information. Int J Intell Syst 31(5):401–424

Hadi-Venchen A, Mirjaberi M (2014) Fuzzy inferior ratio method for multiple attribue decision making problems. Inf Sci 277:263–272

Hatzimichailidis AG, Papakostas AG, Kaburlasos VG (2012) A novel distance measure of intuitionistic fuzzy sets and its application to pattern recognition problems. Int J Intell Syst 27:396–409

He X, Du Y, Liu W (2016) Pythagorean fuzzy power average operators. Fuzzy Syst Math 30(6):116–124

Khan MSA, Abdullah S, Ali A, Amin F (2018a) An extension of vikor method for multiattribute decision making under pythagorean hesitant fuzzy setting. Granul Comput. https://doi.org/10.1007/s41066-018-0102-9

Khan MSA, Abdullah S, Ali A, Amin F (2018b) Pythagorean fuzzy prioritized aggregation operators and their application to multiattribute group decision making. Granul Comput. https://doi.org/10.1007/s41066-018-0093-6

Liang D, Xu Z (2017) The new extension of topsis method for multiple criteria decision making with hesitant pythagorean fuzzy sets. Appl Soft Comput 60:167–179

Liu P, Chen SM (2017) Group decision making based on heronian aggregation operators of intuitionistic fuzzy numbers. IEEE Trans Cybern 47(9):2514–2530

Mohagheghi V, Mousavi SM, Vahdani B (2017) Enhancing decision-making flexibility by introducing a new last aggregation evaluating approach based on multi-criteria group decision making and pythagorean fuzzy sets. Appl Soft Comput 61:527–535

Peng X, Selvachandran G (2017) Pythagorean fuzzy set: state of the art and future directions. Artif Intell Rev. https://doi.org/10.1007/s10462-017-9596-9

Peng X, Yang Y (2015) Some results for pythagorean fuzzy sets. Int J Intell Syst 30:1133–1160

Perez-Dominguez L, Rodriguez-Picon LA, Alvarado-Iniesta A, Cruz DL, Xu Z (2018) Moora under pythagorean fuzzy sets for multiple criteria decision making. Complex. https://doi.org/10.1155/2018/2602376

Rahman K, Abdullah S (2018) Generalized interval-valued pythagorean fuzzy aggregation operators and their application to group decision making. Granul Comput. https://doi.org/10.1007/s41066-018-0082-9

Rahman K, Abdullah S, Ali A (2018a) Some induced aggregation operators based on interval-valued pythagorean fuzzy numbers. Granul Comput. https://doi.org/10.1007/s41066-018-0091-8

Rahman K, Abdullah S, Shakeel M, Khan MSA, Ullah M (2017) Interval-valued pythagorean fuzzy geometric aggregation operators and their application to group decision making problem. Cogent Math. https://doi.org/10.1080/23311835.2017.1338638

Rahman K, Ali A, Abdullah S, Amin F (2018b) Approaches to multi-attribute group decision making based on induced interval-valued pythagorean fuzzy einstein aggregation operator. New Math Natural Comput 14(3):343–361

Szmidt E, Kacprzyk J (2001) Intuitionistic fuzzy sets in some medical applications. Note IFS 7(4):58–64

Szmidt E, Kacprzyk J (2004) Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets. Note IFS 10(4):61–69

Wang HY, Chen SM (2008) Evaluating students' answerscripts using fuzzy numbers associated with degrees of confidence. IEEE Trans Fuzzy Syst 16(2):403–415

Yager RR (2013a) *Pythagorean fuzzy subsets*. In: Proceedings of the Joint IFSAWorld Congress NAFIPS Annual Meeting, pp 57–61

Yager RR (2013b) DPythagorean membership grades in multicriteria decision making. Technical Report MII-3301 Machine Intelligence Institute, Iona College, New Rochelle, NY

Yager RR (2014) Pythagorean membership grades in multicriteria decision making. IEEE Trans Fuzzy Syst 22(4):958–965

Yager RR (2016) Properties and applications of Pythagoean fuzzy sets. Springer, Berlin

Yager RR, Abbasov AM (2013) Pythagorean membership grades, complex numbers and decision making. Int J Intell Syst 28(5):436–452

Zadeh LA (1965) Fuzzy sets. Inform Control 8:338–353