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ORIGINAL PAPER

Robust functional observer for stabilising uncertain fuzzy systems with time-delay

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Abstract

This paper presents a new technique for stabilising a Takagi–Sugeno (T-S) fuzzy system with time-delay and uncertainty. A robust fuzzy functional observer is employed to design a controller when the system states are not measurable. The model uncertainty is norm bounded, and the time-delay is time-varying but bounded. The parallel distributed compensation method is applied for defining the fuzzy functional observer to design this controller. The proposed procedure reduces the observer order to the dimension of the control input. Improved stability conditions are provided for the observer compared with the existing results of functional observer-based stabilisation of T-S fuzzy models. Lyapunov–Krasovskii functionals are used to construct delay-dependent stability conditions as linear matrix inequalities. The solution of these inequalities is used for calculating the observer parameters. The sensitivity of the estimation error to the model uncertainty is reduced by minimising the L_2 gain. The new design method developed is illustrated and verified using two examples.

Keywords Takagi–Sugeno fuzzy model · Functional observer · Time-delay · Robust controller design

1 Introduction

A functional observer estimates the function of states directly. The design problem of the functional observer has been an active research field for the last few decades for its ability to estimate the function of states in a single step rather than performing in two steps. It also reduces the observer order. The existence conditions, stability analysis and construction procedure of functional observers for linear systems are well established (Darouach 2000; Ha et al. 2003; Trinh and Fernando 2007; Mohajerpoor et al. 2016); the existence conditions are presented as rank equality conditions while the stability conditions are presented as linear matrix inequalities (LMIs). The effects of parametric uncertainty and time-delay on the functional observer for linear systems are studied in Darouach (2001), Teh and Trinh (2012), Tran et al. (2015) and Boukal et al. (2016). The design and application of functional observers for nonlinear systems represented by fuzzy models, however, received less attention.

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The concept of fuzzy sets proposed by Zadeh (1965) has started a new era in set theories. Fuzzy sets have been successfully applied in classification and system identification problems (Wang and Chen 2008; Chen and Chang 2011; Chen et al. 2012; Wang et al. 2017; Yordanova et al. 2017; Lai et al. 2018; Liu and Zhang 2018). Many modern systems have been modeled by fuzzy reasoning. The fuzzy reasoning comprises fuzzy inference rules described by "IF-THEN" statements. "IF" statements are called premises while "THEN" statements are called consequents. Takagi–Sugeno (T-S) fuzzy modeling is an efficient way of representing a highly nonlinear system in a simple way by applying the fuzzy reasoning. The overall system dynamics is expressed as a fuzzy summation of the linear consequents of fuzzy rules of a T-S fuzzy model (Takagi and Sugeno 1985). The linear consequent models of a T-S fuzzy model are interconnected with each other by membership functions to represent a nonlinear system for any degree of accuracy (Feng 2006). As a consequence, this modeling technique enables the use of existing linear tools and techniques for analysing and synthesising different problems of nonlinear systems. The stability of these model-based systems has been a vibrant research area for a long time.

Controller design problem for nonlinear systems using T-S fuzzy model has been an active research area (Sun et al.

2017; Shojaei et al. 2018; Jiang et al. 2018; Wang et al. 2018). Parallel distributed compensation (PDC) approach is a well accepted technique for designing controller to stabilise a T-S fuzzy system. A PDC controller is a fuzzy blending of the linear feedback controllers designed for the local linear models of a T-S fuzzy model (Wang et al. 1995; Tanaka et al. 1998; Li et al. 1999). The stability analysis methods and design procedures of PDC controllers for T-S fuzzy models are well developed (Tanaka and Sugeno 1992; Wang et al. 1996; Ma and Sun 2001; Nguang and Shi 2003; Guerra and Vermeiren 2004; Zhang et al. 2015). The effect of time-delay and model uncertainty on the stability of T-S fuzzy systems can be minimised by Lyapunov function approach (Liu and Zhang 2003; Chen and Liu 2005; Wu et al. 2012). Due to the recent advancements of numerical techniques for solving convex optimisation problems, most of the recent works transform the stability conditions as LMIs so that the controller gains can be calculated from the solutions of the inequalities. If all states are not accessible, observers are employed for estimating the states and designing a PDC controller. In most of the cases the observers are considered to be full order state observers. Reduced order observers are studied in Ma and Sun (2001), Krokavec and Filasov (2012, 2014) to construct observer-based controllers for T-S fuzzy systems.

A functional observer is inherently a reduced order observer. A fuzzy functional observer can be constructed as a fuzzy summation of the linear functional observers for the subsystems of a T-S fuzzy model. As a PDC controller is a fuzzy summation of feedback controllers, which are linear functions of states, this controller can be obtained using the fuzzy functional observer. This technique reduces the observer order and the real time computational effort of the controller. The main advantages of applying a functional observer for designing a PDC controller are that this observer is of reduced order, and it estimates the control input directly. Considering these advantages, we are interested in studying the existence as well as construction procedure of the robust functional observer-based PDC controller for an uncertain T-S fuzzy system with time-delay.

Ma and Sun (2001) studied the problem of construction of a functional observer for nonlinear systems represented by T-S fuzzy model and provided an observer construction procedure by solving inter connected algebraic equations. Fadali (2005) studied the application of this functional observer for designing a PDC controller. Both procedures, however, require checking the stability of the observer; if the stability conditions are not satisfied the functional observer needs to be redesigned. Improved fuzzy functional observer-based controller design techniques using stability conditions as LMIs are provided by Islam et al. (2018a, b). These works, however, do not consider the effect of model uncertainty on the error dynamics of a fuzzy functional observer. To the best of the authors' knowledge, the problem of controller design using functional observer for uncertain T-S fuzzy model with time-delay is not studied yet fully. This gap motivates us to carry out this work.

In this paper, we study the problem of obtaining a PDC controller using a robust functional observer considering the time-varying time-delay and model uncertainty. The model uncertainty is assumed to be norm bounded, and the time-delay is assumed to be bounded above and below. The proposed functional observer is robust against the model uncertainty. The sensitivity of the estimation error to the model uncertainty is minimised by employing the L_2 gain minimisation technique. The stability condition is formulated as LMIs by using Lyapunov–Krasovskii functional. The functional observer construction procedure includes two steps: first, calculating the PDC gain matrices by solving the stability condition of the plant; second, finding observer matrices by obtaining the solution of a minimisation problem that ensures the asymptotic stability and robustness of the functional observer. The main contributions of this paper are as follows:

1. The functional observer is used to obtain a feedback controller for stabilising an uncertain T-S fuzzy system with time-delay.
2. The functional observer is derived by the proposed method.
3. The existence condition of the proposed controller is presented as rank equality condition.

The rest of the paper is organised as follows. The plant model and problem formulation are described in Sect. 2. It includes the description of a T-S fuzzy model with time-delay and model uncertainty. The mathematical model of the functional observer is also described in this section. Section 3 provides the main results: the existence and stability condition of the proposed observer and the observer construction procedure. Section 4 presents illustrative examples to validate the results. Section 5 summarises this paper with concluding remarks and the scope of future works.

Notation $\mathbb{R}^{n\times m}$ denotes an $n \times m$-dimensional real matrix and $\mathbb{R}^n$ denotes an n-dimensional real vector. I_p represents a $p \times p$ identity matrix. Superscripts $(.)^+$, $(.)^\perp$ and $(.)^-$ mean the Moore–Penrose generalised inverse, orthogonal basis, and inverse of corresponding matrix, respectively. $\star$ denotes the symmetric components of respective blocks of a symmetric matrix while $\mathrm{diag}(X, X, \ldots, X)$ represents a block diagonal matrix.

2 System description and problem formulation

A fuzzy set M on the universe of discourse X may be defined by a membership function ν_{M} such that $\nu_{\mathrm{M}}(\xi) \in [0, 1]$ for all $\xi \in X$. The numeric value of $\nu_{\mathrm{M}}(\xi)$ is called the membership value or degree of membership by which $\xi \in X$ belongs to fuzzy set M. A T-S fuzzy model of a dynamical system is described by a number of fuzzy rules described by "IF-THEN" statements. "IF" statement consists of premise variables that belong to respective fuzzy sets, and are connected to each other by logical operator "AND" to define a particular operating point of the system. The consequent part, "THEN" statement, consists of a linear state-space model of the system at the particular operating point for which the rule has been stated.

The ith rule of a T-S fuzzy model with model uncertainty representing a nonlinear system with time-delay can be expressed as

$$\begin{aligned}
&\text{IF } \xi_1(t) \text{ is } M_i^1 \text{ AND } \cdots \text{ AND } \xi_l(t) \text{ is } M_i^l \\
&\text{THEN } \dot{x}(t) = (A_i + \Delta A_i(t))x(t) \\
&\qquad + (A_{di} + \Delta A_{di}(t))x(t-\tau(t)) + B_i u(t) \\
&\qquad y(t) = Cx(t) \\
&\qquad x(t) = \phi(t), t \in [-\tau(t), 0], \quad i = 1, \dots, r,
\end{aligned} \tag{1}$$

where $u(t) \in \mathbb{R}^m$ is the input, $y(t) \in \mathbb{R}^p$ is the output, $x(t) \in \mathbb{R}^n$ is the state, $\tau(t)$ is the time-varying time-delay, and $\xi_1(t), \dots, \xi_l(t)$ are the premise variables. $\xi_k(t)$ belongs to fuzzy set M_i^k in the ith rule with the degree of membership defined by membership function $M_i^k(\xi_k(t))$. $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ represent the ith linear subsystem. $\Delta A_i(t)$ and $\Delta A_{di}(t)$ represent the model uncertainty. Taking $\xi(t) = [\xi_1(t), \dots, \xi_l(t)]$, the fuzzy summation,

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t)) \Big\{ (A_i + \Delta A_i(t))x(t) \\
&\quad + (A_{di} + \Delta A_{di}(t))x(t-\tau(t)) + B_i u(t) \Big\} \\
y(t) &= Cx(t),
\end{aligned} \tag{2}$$

where

$$\mu_i(\xi(t)) = \frac{\prod_{k=1}^{l} M_i^k(\xi_k(t))}{\sum_{i=1}^{r} \prod_{k=1}^{l} M_i^k(\xi_k(t))}$$

with $\mu_i(\xi(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$,

represents the overall nonlinear plant dynamics.

The upper and lower bounds of the time-delay are denoted by τ_{M} and τ_{m}, respectively. The time derivative of time-delay $\tau(t)$ is bounded above; $\dot{\tau}(t) \leq \rho < 1$. The time-varying model uncertainties are assumed to be

$$\Delta A_i(t) = R_i U_i(t) S_i, \quad i = 1, 2, \dots, r \quad \text{and}$$
$$\Delta A_{di}(t) = R_{di} U_{di}(t) S_{di}, \quad i = 1, 2, \dots, r,$$

such that time-varying uncertain parameters $U_i(t)$ and $U_{di}(t)$ of proper dimensions satisfy

$$U_i^T(t) U_i(t) \leq I, \quad i = 1, 2, \dots, r \quad \text{and} \tag{3a}$$

$$U_{di}^T(t) U_{di}(t) \leq I, \quad i = 1, 2, \dots, r, \tag{3b}$$

where R_i, S_i, R_{di} and S_{di} are known real constant matrices of proper dimensions.

Remark 1 The plant model uncertainties, which in many cases may not be exactly modeled by mathematical expressions, can be generally treated as uncertainties over-bounded by the condition $U_i^T(t)U_i(t) < I$. While $U_i(t)$ carries the actual information of the uncertain nature of the systems, matrices R_i and S_i link this uncertainty with the nominal system (Shi et al. 2003).

A PDC controller for system (1) can be expressed as

$$\begin{aligned}
u(t) &= \sum_{j=1}^{r} \mu_j(\xi(t)) u_j(t) \\
&= \sum_{j=1}^{r} \mu_j(\xi(t)) K_j x(t),
\end{aligned}$$

where K_j corresponds to the linear feedback gain of the respective subsystem. The procedure for calculating stabilising feedback controller gain K_j is described in the next section. Considering all states are not accessible, our main focus is to employ a linear functional observer to estimate $u_j(t)$ for each linear subsystem, and to obtain $u(t)$ using the fuzzy summation. The proposed functional observer-based PDC controller is as described below:

$$\begin{aligned}
\dot{w}_j(t) &= \sum_{i=1}^{r} \mu_i(\xi(t)) \Big\{ N_{ij} w_j(t) + N_{dij} w_j(t-\tau) \\
&\quad + J_{ij} y(t) + J_{dij} y(t-\tau) + H_{ij} \hat{u}(t) \Big\},
\end{aligned} \tag{4a}$$

$$\hat{u}_j(t) = w_j(t) + F_j y(t), \tag{4b}$$

$$\hat{u}(t) = \sum_{j=1}^{r} \mu_j(\xi(t)) \{\hat{u}_j(t)\}, \tag{4c}$$

$$w_j(t) = 0 \text{ for all } t \in [-\tau, 0], \tag{4d}$$

where $w_j(t) \in \mathbb{R}^m$, $F_j \in \mathbb{R}^{m\times p}$, $N_{ij} \in \mathbb{R}^{m\times m}$, $N_{dij} \in \mathbb{R}^{m\times m}$, $J_{ij} \in \mathbb{R}^{m\times p}$, $J_{dij} \in \mathbb{R}^{m\times p}$, and $H_{ij} \in \mathbb{R}^{m\times m}$. Here $\hat{u}_j(t)$ is the estimated function of states, where $u_j(t)$ is a linear combination of the states $x(t)$ and is defined by $u_j(t) = K_j x(t)$. The estimation error can be expressed as

$$\begin{aligned}
e_j(t) &= u_j(t) - \hat{u}_j(t) \\
&= T_j x(t) - w_j(t),
\end{aligned}$$

where $T_j = K_j - F_jC$.

Remark 2 In existing observer-based PDC controller design techniques, an observer estimates system state $x(t)$. Estimated state $\hat{x}(t)$ is used to obtain the control input as $\hat{u}(t) = \sum_{j=1}^{r} \mu_j(\xi(t))\{K_j\hat{x}(t)\}$. The observer is constructed by using known system matrices and an unknown observer gain matrix that is obtained so that estimation error, $e_x(t) = x(t) - \hat{x}(t)$, converges to zero. The observer dynamics is described by $\hat{x}(t)$ which is of the same order of system state $x(t)$. The proposed functional observer, on the other hand, estimates control input $u_j(t)$ directly as a function of states. Estimation error $e_j(t)$ of the functional observer-based PDC controller is different from estimation error $e_x(t)$ of existing observers. The order of observer state $w_j(t)$ of the functional observer may be different from the order of system state $x(t)$. Unlike existing observer construction procedures, the proposed functional observer employs different observer parameters, N_{ij}, N_{dij}, J_{ij}, J_{dij}, H_{ij}, and F_j for the observer dynamics. These observer parameters are unknown and are to be constructed so that the estimation error approaches zero.

The error dynamics can be expressed as

$$\begin{aligned}\dot{e}_j(t) = \sum_{i=1}^{r} \mu(\xi)\Big\{ & N_{ij}e_j(t) + N_{dij}e_j(t-\tau(t)) \\ & + (T_jA_i - N_{ij}T_j - J_{ij}C + T_jR_iU_i(t)S_i)x(t) \\ & + (T_jA_{di} - N_{dij}T_j - J_{dij}C \\ & + T_jR_{di}U_{di}(t)S_{di})x(t-\tau(t)) \\ & + (T_jB_i - H_{ij})\hat{u}(t)\Big\}. \end{aligned} \tag{5}$$

Error dynamics (5) reduces to

$$\begin{aligned}\dot{e}_j(t) = \sum_{i=1}^{r} \mu(\xi)\Big\{ & N_{ij}e_j(t) + N_{dij}e_j(t-\tau(t)) \\ & + T_jR_iU_i(t)S_ix(t) + T_jR_{di}U_{di}(t)S_{di}x(t-\tau(t))\Big\} \end{aligned} \tag{6}$$

if we have

$$T_jA_i - N_{ij}T_j - J_{ij}C = 0, \tag{7a}$$

$$T_jA_{di} - N_{dij}T_j - J_{dij}C = 0, \tag{7b}$$

$$T_jB_i - H_{ij} = 0. \tag{7c}$$

Therefore, the controller design problem for the uncertain T-S fuzzy system with time-delay using the functional observer turns into obtaining matrices N_{ij}, N_{dij}, J_{ij}, J_{dij}, H_{ij}, and F_{ij}, such that error system (6) is asymptotically stable and conditions (7) hold.

3 Main results

PDC gain matrices K_j can be calculated by solving the LMIs presented in the following lemma.

Lemma 1 *The fuzzy time-delay system described by* (2) *is stable if, for some given constants* $\bar{\sigma}_1, \bar{\sigma}_2, \tau_m, \tau_M$ *and* ρ, *there exist positive definite symmetric matrices* $\bar{P}_1$ *and* $\bar{P}_2$, *and matrices* $\bar{Y}_j$ *of proper dimension such that*

$$\begin{bmatrix} \bar{P}_2 + A_i\bar{P}_1 + \bar{P}_1A_i^T + B_i\bar{Y}_j + \bar{Y}_j^TB_i^T & A_{di}\bar{P}_1 & \frac{1}{2}\kappa(\bar{P}_1A_i^T + \bar{Y}_j^TB_i^T) & R_i & R_{di} & 0 & 0 & \bar{P}_1S_i^T & 0 \\ \star & -(1-\bar{\rho})\bar{P}_2 & \frac{1}{2}\kappa\bar{P}_1A_{di}^T & 0 & 0 & 0 & 0 & 0 & \bar{P}_1S_{di}^T \\ \star & \star & -\tau_m\bar{\sigma}_2\bar{P}_1 & 0 & 0 & \kappa R_i & \kappa R_{di} & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\frac{1}{2}I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -\frac{1}{2}I \end{bmatrix} < 0 \tag{8}$$

for $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, r$, *where $\kappa = \tau_{\mathrm{M}}(\bar{\sigma}_1 + \bar{\sigma}_2)$. PDC controller gain matrices K_j can be obtained from the relation $\bar{Y}_j = K_j\bar{P}_1$.*

Proof Proof is given in the appendix. □

Having obtained gain matrices K_j, our goal is to construct robust functional observer (4) that estimates the fuzzy summation of the function of states $K_jx(t)$ directly. The error dynamics of the observer is sensitive to the model uncertainty. Therefore, our goal includes two aspects: first, to ensure that the estimation error approaches zero asymptotically if there is no model uncertainty; second, to minimise the sensitivity of estimation error to uncertainty. The sensitivity minimisation problem is formulated in the form of minimising a cost function subject to L_2 gain bound constraint. We say that the functional observer is robust if there exists a positive scalar γ such that

$$\frac{\left||u_j(t) - \hat{u}_j(t)\right||_2}{\left||u_j(t)\right||_2} = \frac{\left||e_j(t)\right||_2}{\left||u_j(t)\right||_2} < \gamma,$$

where $||\cdot||_2$ is an L_2 norm as expressed below:

$$\left||u_j(t)\right||_2^2 = \int_0^{\infty} u_j^T(t)u_j(t)\mathrm{d}t.$$

The following theorem describes the stability condition of the functional observer.

Theorem 1 *The functional observer described by* (4) *with time-varying time-delay having the upper bound τ_{M} and the lower bound τ_{m} is robustly asymptotically stable if, for given scalars σ_1 and σ_2, there exist positive definite symmetric matrices P_1, P_2^1, P_2^2, P_2^3 and P_2^4, and matrices W_{11}, W_{12}, W_{21}, W_{22}, W_{31}, W_{33} and Y_j of proper dimensions such that the following optimisation problem has a solution*:

minimise $\gamma > 0$

subject to

$$\begin{bmatrix}
\Xi_{ij}^{1,1} & \Xi_{ij}^{1,2} & -W_{21} & W_{31} & \Xi_{ij}^{1,5} & 0 & 0 & \tau_{\mathrm{M}}W_{11} & \tau W_{21} & \tau W_{31} & \Xi_{ij}^{1,11} & \Xi_{ij}^{1,12} & 0 & 0 \\
\star & \Xi_{ij}^{2,2} & -W_{22} & W_{32} & \Xi_{ij}^{2,5} & 0 & 0 & \tau_{\mathrm{M}}W_{11} & \tau W_{22} & \tau W_{32} & 0 & 0 & 0 & 0 \\
\star & \star & -P_2^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & -P_2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \Xi_{ij}^{5,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Xi_{ij}^{5,13} & \Xi_{ij}^{5,14} \\
\star & \star & \star & \star & \star & \Xi_{ij}^{6,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \Xi_{ij}^{7,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \Xi_{ij}^{8,8} & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \Xi_{ij}^{9,9} & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \Xi_{ij}^{10,10} & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -I & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -I & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -I & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -I
\end{bmatrix} < 0, \tag{9}$$

where

$$\begin{aligned}
&\Xi_{ij}^{1,1} = I_p + P_1N_{ij}^1 + Y_jN_{ij}^2 + (N_{ij}^1)^TP_1 + (N_{ij}^2)^TY_j \\
&\quad + P_2^1 + P_2^2 + P_2^3 + W_{11} + W_{11}^T, \\
&\Xi_{ij}^{1,2} = P_1N_{dij}^1 + Y_jN_{dij}^2 - W_{11} + W_{12}^T + W_{21} - W_{31}, \\
&\Xi_{ij}^{1,5} = \tfrac{1}{2}\tau_{\mathrm{M}}(\sigma_1 + \sigma_2)((N_{ij}^1)^TP_1 + (N_{ij}^2)^TY_j), \\
&\Xi_{ij}^{1,11} = (P_1T_j^1 + Y_jT_j^2)R_i, \quad \Xi_{ij}^{1,12} = (P_1T_j^1 + Y_jT_j^2)R_{di}, \\
&\Xi_{ij}^{2,2} = -(1-\rho)P_2^1 - W_{12} - W_{12}^T + W_{22} + W_{22}^T - W_{32} - W_{32}^T \\
&\Xi_{ij}^{2,5} = \tfrac{1}{2}\tau_{\mathrm{M}}(\sigma_1 + \sigma_2)((N_{dij}^1)^TP_1 + (N_{dij}^2)^TY_j), \\
&\Xi_{ij}^{5,5} = -\tau_{\mathrm{m}}\sigma_2P_1, \quad \Xi_{ij}^{5,13} = \tau_{\mathrm{M}}(\sigma_1 + \sigma_2)(P_1T_j^1 + Y_jT_j^2)R_i, \\
&\Xi_{ij}^{5,14} = \tau_{\mathrm{M}}(\sigma_1 + \sigma_2)(P_1T_j^1 + Y_jT_j^2)R_{di}, \\
&\Xi_{ij}^{6,6} = -\gamma^2K_j^TK_j + \tfrac{3}{2}S_i^TS_i, \quad \Xi_{ij}^{7,7} = -(1-\rho)P_2^4 + \tfrac{3}{2}S_{di}^TS_{di}, \\
&\Xi_{ij}^{8,8} = -\tau_{\mathrm{M}}\sigma_1P_1, \quad \Xi_{ij}^{9,9} = -\tau(\sigma_1 + \sigma_2)P_1, \quad \Xi_{ij}^{10,10} = -\tau\sigma_2P_1, \\
&T_j^1 = K_j - F_j^1C, \quad T_j^2 = -F_j^2C, \quad Y_j = P_1Z_j, \quad \tau = \tau_{\mathrm{M}} - \tau_{\mathrm{m}},
\end{aligned}$$

and N_{ij}^1, N_{ij}^2, N_{dij}^1, N_{dij}^2, F_j^1 and F_j^2 are defined in (12a), (12b) *and* (12e).

Proof Using the definition, $T_j = K_j - F_jC$, and considering all values of i, (7a) and (7b) can be expressed as

$$
\begin{aligned}
&F_j \begin{bmatrix} C\mathcal{A} & C\mathcal{A}_d \end{bmatrix} + \begin{bmatrix} \mathcal{N}_j & \mathcal{N}_{dj} \end{bmatrix} \begin{bmatrix} \mathcal{K}_j & 0 \\ 0 & K_j \end{bmatrix} \\
&\quad + \begin{bmatrix} \mathcal{M}_j & \mathcal{M}_{dj} \end{bmatrix} \begin{bmatrix} \mathcal{C} & 0 \\ 0 & \mathcal{C} \end{bmatrix} \\
&\quad = \begin{bmatrix} K_j\mathcal{A} & K_j\mathcal{A}_d \end{bmatrix},
\end{aligned} \tag{10}
$$

where

$$
\mathcal{K}_j = \begin{bmatrix} K_j & 0 & \dots & 0 \\ 0 & K_j & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & K_j \end{bmatrix}, \qquad \mathcal{C} = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{bmatrix},
$$

$$
\begin{aligned}
&\mathcal{N}_j = \begin{bmatrix} N_{1j} & N_{2j} & \dots & N_{rj} \end{bmatrix}, && \mathcal{N}_{dj} = \begin{bmatrix} N_{d1j} & N_{d2j} & \dots & N_{drj} \end{bmatrix}, \\
&M_{ij} = J_{ij} - N_{ij}F_j, && M_{dij} = J_{dij} - N_{dij}F_j, \\
&\mathcal{M}_j = \begin{bmatrix} M_{1j} & M_{2j} & \dots & M_{rj} \end{bmatrix}, && \mathcal{M}_{dj} = \begin{bmatrix} M_{d1j} & M_{d2j} & \dots & M_{drj} \end{bmatrix}, \\
&\mathcal{A}_j = \begin{bmatrix} A_1 & A_2 & \dots & A_r \end{bmatrix}, && \mathcal{A}_{dj} = \begin{bmatrix} A_{d1} & A_{d2} & \dots & A_{dr} \end{bmatrix}.
\end{aligned}
$$

The general solution of the unknown matrices of (10) can be given as

$$
\begin{bmatrix} F_j & \mathcal{N}_j & \mathcal{M}_j & \mathcal{N}_{dj} & \mathcal{M}_{dj} \end{bmatrix} = \boldsymbol{\Phi}_j\boldsymbol{\Psi}_j^{+} - Z_j(I - \boldsymbol{\Psi}_j\boldsymbol{\Psi}_j^{+}), \tag{11}
$$

where $\boldsymbol{\Psi}_j = \begin{bmatrix} C\mathcal{A} & C\mathcal{A}_d \\ \mathcal{K}_j & 0 \\ \mathcal{C} & 0 \\ 0 & \mathcal{K}_j \\ 0 & \mathcal{C} \end{bmatrix}$ and $\boldsymbol{\Phi}_j = \begin{bmatrix} K_j\mathcal{A} & K_j\mathcal{A}_d \end{bmatrix}$ are known,

and Z_j is an arbitrary matrix of proper dimension. N_{ij}, N_{dij}, M_{ij}, M_{dij} and F_j can be expressed as

$$
N_{ij} = N_{ij}^1 + Z_jN_{ij}^2, \tag{12a}
$$

$$
N_{dij} = N_{dij}^1 + Z_jN_{dij}^2, \tag{12b}
$$

$$
M_{ij} = M_{ij}^1 + Z_jM_{ij}^2, \tag{12c}
$$

$$
M_{dij} = M_{dij}^1 + Z_jM_{dij}^2, \tag{12d}
$$

$$
F_j = F_j^1 + Z_jF_j^2, \tag{12e}
$$

where N_{ij}^1, N_{ij}^2, N_{dij}^1, N_{dij}^2, M_{dij}^1, M_{dij}^2, F_j^1 and F_j^2 are extracted from (11) by partitioning $\boldsymbol{\Phi}_j$ and $\boldsymbol{\Psi}_j$ properly.

Consider a Lyapunov–Krasovskii functional

$$
\begin{aligned}
V(t) &= e_j^T(t)P_1e_j(t) + \int_{t-\tau(t)}^{t} e_j^T(s)P_2^1e_j(s)\mathrm{d}s \\
&\quad + \int_{t-\tau_{\mathrm{M}}}^{t} e_j^T(s)P_2^2e_j(s)\mathrm{d}s \\
&\quad + \int_{t-\tau_{\mathrm{m}}}^{t} e_j^T(s)P_2^3e_j(s)\mathrm{d}s \\
&\quad + \int_{t-\tau(t)}^{t} x^T(s)P_2^4x(s)\mathrm{d}s \\
&\quad + \int_{-\tau_{\mathrm{M}}}^{0}\int_{t+\theta}^{t} \dot{e}_j^T(s)P_3\dot{e}_j(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + \int_{-\tau_{\mathrm{M}}}^{-\tau_{\mathrm{m}}}\int_{t+\theta}^{t} \dot{e}_j^T(s)P_4\dot{e}_j(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned} \tag{13}
$$

where P_1, P_2, P_3, and P_4 are positive definite symmetric matrices, and τ_{m} and τ_{M} are lower and upper bounds of delay $\tau(t)$, respectively. Taking the derivative of (13) along the error dynamics, we have

$$
\begin{aligned}
\dot{V}(t) &= 2e_j^T(t)P_1\dot{e}_j(t) + e_j^T(t)P_2^1e_j(t) \\
&\quad - (1-\dot{\tau}(t))e_j^T(t-\tau(t))P_2^1e_j(t-\tau(t)) \\
&\quad + e_j^T(t)P_2^2e_j(t) - e_j^T(t-\tau_{\mathrm{M}})P_2^2e_j(t-\tau_{\mathrm{M}}) \\
&\quad + e_j^T(t)P_2^3e_j(t) \\
&\quad - e_j^T(t-\tau_{\mathrm{m}})P_2^3e_j(t-\tau_{\mathrm{m}}) + x^T(t)P_2^4x(t) \\
&\quad - (1-\dot{\tau}(t))x^T(t-\tau(t))P_2^4x(t-\tau(t)) \\
&\quad + \dot{e}_j^T(t)(\tau_{\mathrm{M}}P_3 + (\tau_{\mathrm{M}}-\tau_{\mathrm{m}})P_4)\dot{e}_j(t) \\
&\quad - \int_{t-\tau_{\mathrm{M}}}^{t} \dot{e}_j^T(s)P_3\dot{e}_j(s)\mathrm{d}s - \int_{t-\tau_{\mathrm{M}}}^{t-\tau_{\mathrm{m}}} \dot{e}_j^T(s)P_4\dot{e}_j(s)\mathrm{d}s \\
&\leq 2e_j^T(t)P_1\dot{e}_j(t) + e_j^T(t)(P_2^1 + P_2^2 + P_2^3)e_j(t) \\
&\quad - (1-\rho)e_j^T(t-\tau(t))P_2^1e_j(t-\tau(t)) \\
&\quad - e_j^T(t-\tau_{\mathrm{M}})P_2^2e_j(t-\tau_{\mathrm{M}}) \\
&\quad - e_j^T(t-\tau_{\mathrm{m}})P_2^3e_j(t-\tau_{\mathrm{m}}) + x^T(t)P_2^4x(t) \\
&\quad - (1-\rho)x^T(t-\tau(t))P_2^4x(t-\tau(t)) \\
&\quad + \dot{e}_j^T(t)(\tau_{\mathrm{M}}P_3 + (\tau_{\mathrm{M}}-\tau_{\mathrm{m}})P_4)\dot{e}_j(t) \\
&\quad - \int_{t-\tau(t)}^{t} \dot{e}_j^T(s)P_3\dot{e}_j(s)\mathrm{d}s \\
&\quad - \int_{t-\tau_{\mathrm{M}}}^{t-\tau(t)} \dot{e}_j^T(s)(P_3+P_4)\dot{e}_j(s)\mathrm{d}s \\
&\quad - \int_{t-\tau(t)}^{t-\tau_{\mathrm{m}}} \dot{e}_j^T(s)P_4\dot{e}_j(s)\mathrm{d}s.
\end{aligned}
$$

Using the Leibniz–Newton formula, we can obtain the following identities:

$$2(e_j^T(t)W_{11} + e_j^T(t-\tau(t))W_{12})\left(e_j(t) - e_j(t-\tau(t) - \int_{t-\tau(t)}^{t} \dot{e}_j(s)\mathrm{d}s\right) = 0 \tag{15a}$$

$$2(e_j^T(t)W_{21} + e_j^T(t-\tau(t))W_{22})\left(e_j(t-\tau(t)) - e_j(t-\tau_\mathrm{M}) - \int_{t-\tau_\mathrm{M}}^{t-\tau(t)} \dot{e}_j(s)\mathrm{d}s\right) = 0 \tag{15b}$$

$$2(e_j^T(t)W_{31} + e_j^T(t-\tau(t))W_{32})\left(e_j(t-\tau_\mathrm{m}) - e_j(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_\mathrm{m}} \dot{e}_j(s)\mathrm{d}s\right) = 0, \tag{15c}$$

where W_{kl} with $k = 1, \ldots 3$ and $l = 1, 2$ are matrices of proper dimensions with real entries. By using the identities in (15) and defining augmented vector

$$\zeta_j^T(t) = e_j^T(t)e_j^T(t-\tau(t))e_j^T(t-\tau_\mathrm{M}) e_j^T(t-\tau_\mathrm{m})\dot{e}^T(t)x^T(t)x^T(t-\tau(t)),$$

we obtain

$$\begin{aligned}-\int_{t-\tau(t)}^{t} \dot{e}_j^T(s)P_3\dot{e}_j(s)\mathrm{d}s &= 2\zeta_j^T(t)W_1e_j(t) - 2\zeta_j^T(t)W_1e_j(t-\tau(t)) + \tau(t)\zeta_j^T(t)W_1P_3^{-1}W_1^T\zeta_j(t)\\ &- \int_{t-\tau(t)}^{t} (\zeta_j^T(t)W_1 + \dot{e}_j^T(s)P_3)P_3^{-1}\left(W_1^T\zeta_j(t) + P_3\dot{e}_j(s)\right)\mathrm{d}s,\end{aligned} \tag{16a}$$

$$\begin{aligned}-\int_{t-\tau_\mathrm{M}}^{t-\tau(t)} \dot{e}_j^T(s)(P_3+P_4)\dot{e}_j(s)\mathrm{d}s &= 2\zeta_j^T(t)W_2e_j(t-\tau(t)) - 2\zeta_j^T(t)W_2e_j(t-\tau_\mathrm{M})\\ &+ (\tau_\mathrm{M}-\tau(t))\zeta_j^T(t)W_2(P_3+P_4)^{-1}W_2^T\zeta_j(t)\\ &- \int_{t-\tau_\mathrm{M}}^{t-\tau(t)} (\zeta_j^T(t)W_2 + \dot{e}_j^T(s)(P_3+P_4))(P_3+P_4)^{-1}(W_2^T\zeta_j(t)\\ &+ (P_3+P_4)\dot{e}_j(s))\mathrm{d}s \text{ and}\end{aligned} \tag{16b}$$

$$\begin{aligned}-\int_{t-\tau(t)}^{t-\tau_\mathrm{m}} \dot{e}_j^T(s)P_3\dot{e}_j(s)\mathrm{d}s &= 2\zeta_j^T(t)W_3e_j(t-\tau_\mathrm{m}) - 2\zeta_j^T(t)W_3e_j(t-\tau(t))\\ &+ (\tau(t)-\tau_\mathrm{m})\zeta_j^T(t)W_3P_4^{-1}W_3^T\zeta_j(t) - \int_{t-\tau(t)}^{t-\tau_\mathrm{m}} (\zeta_j^T(t)W_3\\ &+ \dot{e}_j^T(s)P_4)P_4^{-1}(W_3^T\zeta_j(t) + P_4\dot{e}_j(s))\mathrm{d}s,\end{aligned} \tag{16c}$$

where

$$W_1 = \begin{bmatrix} W_{11}\\ W_{12}\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}, \quad W_2 = \begin{bmatrix} W_{21}\\ W_{22}\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}, \quad \text{and } W_3 = \begin{bmatrix} W_{31}\\ W_{32}\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}.$$

Using assumptions $U_i^T(t)U_i(t) < I$ and $U_{di}^T(t)U_{di}(t) < I$, the following inequalities can be obtained:

$$2e_j^T(t)P_1T_jR_iU_i(t)S_ix(t) \le e_j^T(t)(P_1T_jR_i)(P_1T_jR_i)^Te_j(t) + x^T(t)S_i^TS_ix(t) \tag{17a}$$

$$2e_j^T(t)P_1T_jR_{di}U_{di}(t)S_{di}x(t) \le e_j^T(t)(P_1T_jR_{di})(P_1T_jR_{di})^Te_j(t) + x^T(t)S_{di}^TS_{di}x(t). \tag{17b}$$

Applying inequalities (17), it can be shown that

$$\begin{aligned}2e_j^T(t)P_1\dot{e}_j(t) \le \sum_{i=1}^{r}\mu_i(\xi(t))\Big\{& e_j^T(t)\Big(P_1N_{ij} + N_{ij}^TP_1\Big)e_j(t)\\ &+ 2e_j^T(t)P_1N_{dij}e_j(t-\tau(t))\\ &+ e_j^T(t)P_1T_jR_i(P_1T_jR_i)^Te_j(t)\\ &+ e_j^T(t)P_1T_jR_{di}(P_1T_jR_{di})^Te_j(t)\\ &+ x^T(t)S_i^TS_ix(t) + x^T(t-\tau(t))\\ &S_{di}^TS_{di}x(t-\tau(t))\Big\}\end{aligned} \tag{18}$$

and

$$\begin{aligned}&\dot{e}_j^T(t)(\tau_\mathrm{M}P_3 + (\tau_\mathrm{M}-\tau_\mathrm{m})P_4)\dot{e}_j(t)\\ &= \sum_{i=1}^{r}\mu_i(\xi(t))\Big\{\dot{e}_j^T(t)\Lambda(\bar{N}_{ij}\bar{e}_j(t)\\ &\quad + \left[T_jR_iU_i(t)S_i \;\; T_jR_{di}U_{di}(t)S_{di}\right]\bar{x}(t)) - \tau_\mathrm{m}\dot{e}_j^T(t)P_4\dot{e}_j(t)\Big\}\\ &\le \sum_{i=1}^{r}\mu_i(\xi(t))\Big\{\dot{e}_j^T(t)\Lambda\bar{N}_{ij}\bar{e}_j(t)\\ &\quad + \frac{1}{2}\dot{e}_j^T(t)\Lambda\left[T_jR_i \;\; T_jR_{di}\right]\left[T_jR_i \;\; T_jR_{di}\right]^T\Lambda\dot{e}_j(t)\\ &\quad + \frac{1}{2}\bar{x}^T(t)\begin{bmatrix} S_i^TS_i & 0\\ 0 & S_{di}^TS_{di}\end{bmatrix}\bar{x}(t) - \tau_\mathrm{m}\dot{e}_j^T(t)P_4\dot{e}_j(t)\Big\},\end{aligned} \tag{19}$$

where $\bar{e}_j^T(t) = [e_j^T(t)\ e_j^T(e-\tau(t))]$, $\bar{x}^T(t) = [x^T(t)\ x^T(e-\tau(t))]$, $\bar{N}_{ij} = [N_{ij}\ N_{dij}]$ and $\Lambda = \tau_M(P_3 + P_4)$. Using the identities in (16), and the inequalities in (18) and (19), we can write

$$\dot{V}(t) \le \sum_{i=1}^{r} \zeta_j^T(t)\big(\mathcal{G}_{ij} + \tau_M W_1 P_3^{-1} W_1^T \\ +(\tau_M - \tau_m) W_2 (P_3+P_4)^{-1} W_2^T \\ +(\tau_M - \tau_m) W_3 P_4^{-1} W_3^T + \Gamma_{ij}\Gamma_{ij}^T\big)\zeta_j(t),$$

where

$$\mathcal{G}_{ij} = \begin{bmatrix} \mathcal{G}^{11} & \mathcal{G}^{12} & -W_{21} & W_{31} & \mathcal{G}_{ij}^{15} & 0 & 0 \\ \star & \mathcal{G}^{23} & -W_{22} & W_{32} & \mathcal{G}_{ij}^{25} & 0 & 0 \\ \star & \star & -P_2^2 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -P_2^3 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\tau_m P_4 & 0 & 0 \\ \star & \star & \star & \star & \star & \mathcal{G}_{ij}^{66} & 0 \\ \star & \star & \star & \star & \star & \star & \mathcal{G}_{ij}^{77} \end{bmatrix},$$

$$\Gamma_{ij} = \begin{bmatrix} P_1 T_j R_i & P_1 T_j R_{di} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\Lambda T_j R_i & \frac{1}{2}\Lambda T_j R_{di} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

with

$$\mathcal{G}_{ij}^1 = P_1 N_{ij} + N_{ij}^T P_1 + P_2^1 + P_2^2 + P_2^3 + W_{11} + W_{11}^T,$$
$$\mathcal{G}_{ij}^{12} = P_1 N_{dij} - W_{11} + W_{12}^T + W_{21} - W_{31},$$
$$\mathcal{G}_{ij}^{15} = \tfrac{1}{2} N_{ij}^T \Lambda,$$
$$\mathcal{G}_{ij}^{22} = -(1-\rho)P_2^1 - W_{12} - W_{12}^T + W_{22} + W_{22}^T - W_{32} - W_{32}^T,$$
$$\mathcal{G}_{ij}^{25} = \tfrac{1}{2} N_{dij}^T \Lambda,$$
$$\mathcal{G}_{ij}^{66} = \tfrac{3}{2} S_i^T S_i + P_2^4, \quad \mathcal{G}_{ij}^{77} = -(1-\rho)P_2^4 + \tfrac{3}{2} S_{di}^T S_{di}.$$

Now, to minimise the effect of parameter uncertainties on the error dynamics, we assume a positive scalar γ and consider

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} + e_j^T(t)e_j(t) - \gamma^2 x^T(t) K_j^T K_j x(t) < 0. \tag{20}$$

By integration we can write

$$V(\infty) - V(0) < \int_0^\infty \left(-e_j^T(s)e_j(s) + \gamma^2 x^T(s) K_j^T K_j x(s)\right)\mathrm{d}s. \tag{21}$$

Under zero initial condition, (21) eventually implies

$$\int_0^\infty \left(e_j^T(s)e_j(s) - \gamma^2 x^T(s) K_j^T K_j x(s)\right)\mathrm{d}s < 0 \\ \iff \frac{\|e_j(t)\|_2}{\|u_j(t)\|_2} < \gamma. \tag{22}$$

Therefore, a sufficient condition for the error dynamics to approach zero asymptotically with minimised effect of parameter uncertainty on the convergence of error can be given as minimising γ subject to (20). Considering $P_3 = \sigma_1 P_1$ and $P_4 = \sigma_2 P_1$ and applying the Schur complement, it can be shown that the inequalities in (20) hold if the inequalities in (9) hold, where N_{ij} and N_{dij} are obtained from (12a) and (12b), respectively. This completes the proof. □

The robust functional observer-based controller construction procedure for stabilising a T-S fuzzy system with model uncertainty and time-delay is outlined below.

Synthesising steps for the robust functional observer:

Step 1: calculate K_j from the solution of (8). Obtain N_{ij}^1, N_{ij}^2, N_{dij}^1, N_{dij}^2, M_{ij}^1, M_{ij}^2, M_{dij}^1, M_{dij}^2, F_j^1 and F_j^2 from (12).
Step 2: specify the ranges of σ_1 and σ_2, and increments $\Delta\sigma_1$ and $\Delta\sigma_2$. Take the minimum value of the ranges of σ_1 and σ_2.
Step 3: solve the minimising problem in (9). If no solution is obtained increase σ_1 and σ_2 by their respective increments and repeat Step 3, else follow the next step.
Step 4: calculate Z_j using the values of Y_j. Then, calculate N_{ij}, N_{dij}, F_j, M_{ij}, and M_{dij} as defined in (12).
Step 5: calculate J_{ij} and J_{dij} using the relations $J_{ij} = M_{ij} + N_{ij}F_j$ and $J_{dij} = M_{dij} + N_{dij}F_j$, respectively.
Step 6: obtain H_{ij} using (7c).

Remark 3 It is evident that (10) requires to have a solution for some Z_j such that N_{ij}, N_{dij}, M_{ij}, M_{dij} and F_j can be expressed by (12). Therefore, one necessary condition for the existence of the functional observer is given as the rank equality as below (Rao and Mitra 1971):

$$\operatorname{rank}\begin{bmatrix} K_j\mathcal{A} & K_j\mathcal{A}_d \\ C\mathcal{A} & C\mathcal{A}_d \\ \mathcal{K}_j & 0 \\ C & 0 \\ 0 & \mathcal{K}_j \\ 0 & C \end{bmatrix} = \operatorname{rank}\begin{bmatrix} C\mathcal{A} & C\mathcal{A}_d \\ \mathcal{K}_j & 0 \\ C & 0 \\ 0 & \mathcal{K}_j \\ 0 & C \end{bmatrix}.$$

Remark 4 The solution of the optimisation problem in (9) depends on choosing two scalars σ_1 and σ_2. This solution depends on the ranges and the increments of these two scalars. We can increase the solution domain by choosing smaller increments and larger ranges.

Remark 5 This paper considers an uncertain T-S fuzzy model of a delayed nonlinear system for obtaining the functional observer-based PDC controller. If there is no model uncertainty in the system, this problem reduces to the problem investigated in Islam et al. (2018a). Therefore, this paper investigates a more generalised problem compared with existing results. Moreover, the equality constraints of the stability conditions for the observer presented in Islam et al. (2018a) are eliminated in the proposed stability condition in this paper.

4 Illustrative examples

4.1 Example 1

In this subsection, we apply the proposed method to a two-rule T-S fuzzy model for illustrating the main results presented in Sect. 3. The matrices of the linear systems representing the two rules are as below:

$$A_1 = \begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0 & -0.2 \\ 0.2 & 0 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 0 & -0.1 \\ 0.1 & 0 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$R_1 = R_2 = R_{d1} = R_{d1} = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$S_1 = S_2 = \begin{bmatrix} -0.05 & 0.02 \\ 0 & 0.04 \end{bmatrix},$$
$$S_{d1} = S_{d2} = \begin{bmatrix} -0.05 & -0.05 \\ 0.08 & -0.05 \end{bmatrix}.$$

State variable $x_1(t)$ is considered to be the premise variable. Membership functions for the $x_1(t)$ are displayed in Fig. 1.

We consider $\tau_M = 0.85$, $\tau_m = 0.05$, and $\rho = 0.95$. By following the steps given in Sect. 3, we obtain the observer parameters. SOSTOOLS toolbox in MATLAB has been used to obtain the results of the optimisation problem of (9). The observer parameters are as follows:

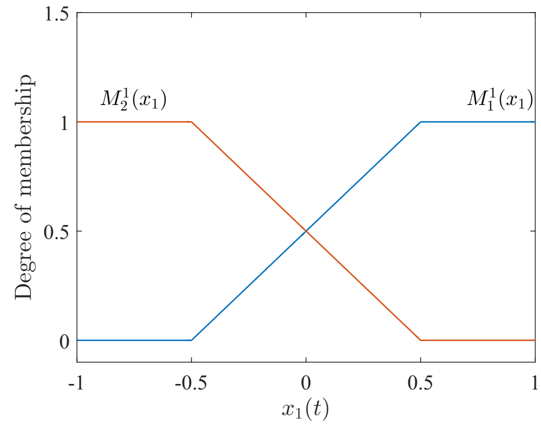


Fig. 1 Membership functions of fuzzy sets M_2^1 and M_1^1

$$\begin{aligned}
&K_1 = \begin{bmatrix} -9.7763 & -2.4474 \end{bmatrix}, && N_{11} = -0.3084, && N_{21} = -0.7320, \\
&J_{11} = 2.3363, && J_{21} = 5.5453, && F_1 = -7.5760, \\
&N_{d11} = -0.0799, && N_{d21} = -0.0400, && J_{d11} = 0.6054, \\
&H_{11} = -2.2003, && H_{21} = -1.3202, && J_{d21} = 0.3027 \\
&K_2 = \begin{bmatrix} -9.7763 & -2.4474 \end{bmatrix}, && N_{12} = -0.3084, && N_{21} = -0.7320, \\
&J_{12} = 2.3363, && J_{22} = 5.5453, && F_2 = -7.5760, \\
&N_{d12} = -0.0799, && N_{d22} = -0.0400, && J_{d12} = 0.6054 \\
&H_{12} = -2.2003, && H_{22} = -1.3202 && J_{d22} = 0.3027.
\end{aligned}$$

Considering two input conditions $\phi(t) = [4\ 2]^T$ and $\phi(t) = [-2\ -4]^T$, simulations are run in MATLAB. Figures 2 and 3 display the state responses of the system with the proposed functional observer-based PDC controller and the conventional PDC controller (Wang et al. 1995). It can be observed that the proposed functional observer-based PDC controller stabilises the system asymptotically. Figures 4 and 5 compare the control signals for these two methods under two initial conditions. In these figures $u(t)$ is the desired control input generated using the conventional PDC controller considering all states are measurable while $\hat{u}(t)$ is the estimated control input obtained by the functional observer. It can be seen that $\hat{u}(t)$ converges with desired $u(t)$ as expected. The convergence is depicted with enlarged graphs in Fig. 4. In comparison with the conventional PDC controller, the proposed method can stabilise the fuzzy system satisfactorily. Nevertheless, its performance can be enhanced by choosing suitable stabilisation conditions. Future work may consider this point.

4.2 Example 2

In this subsection, we apply the proposed method to the benchmark problem of truck trailer system represented by the delayed uncertain T-S fuzzy model (Chen and Liu 2005) for verifying its applicability. The system is expressed as a two-rule T-S fuzzy model with the following matrices:

Fig. 2 Time response of state $x_1(t)$

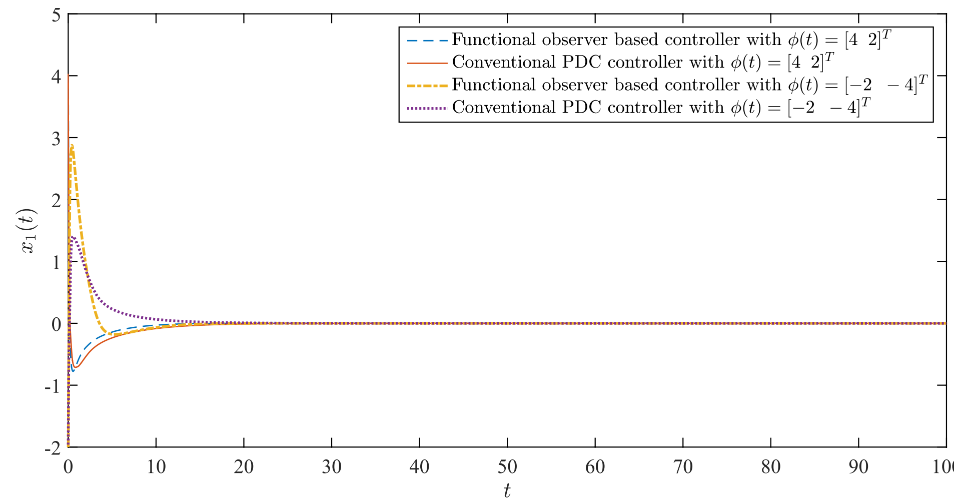


Fig. 3 Time response of state $x_2(t)$

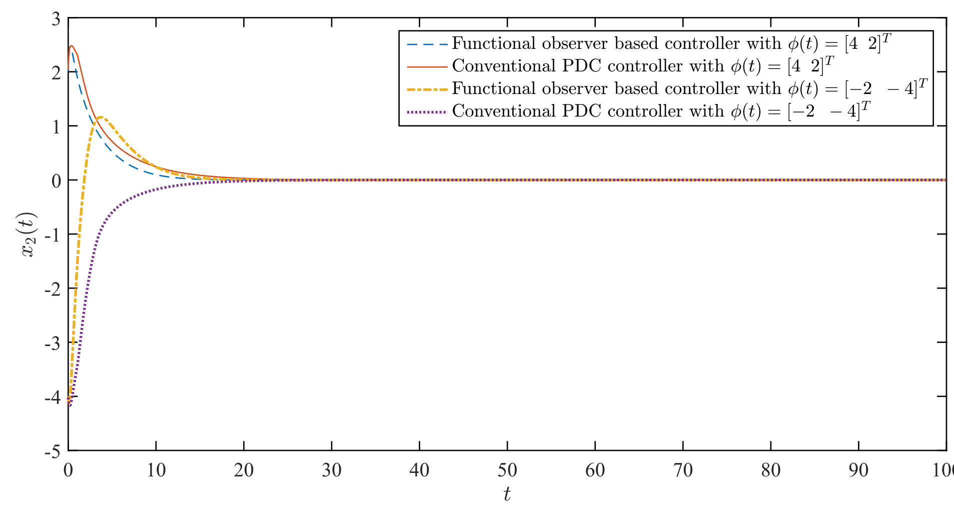
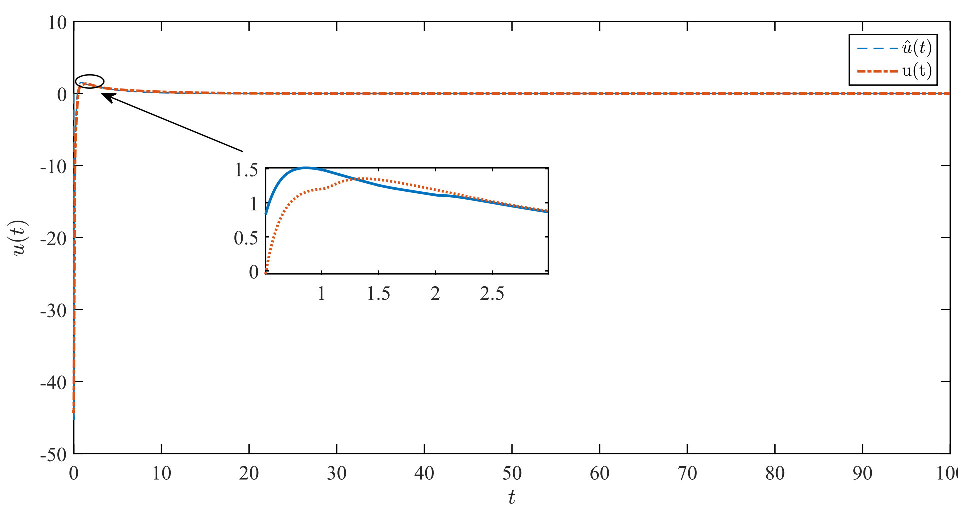


Fig. 4 Control signals generated for initial condition $\phi = [4 \quad 2]^T$



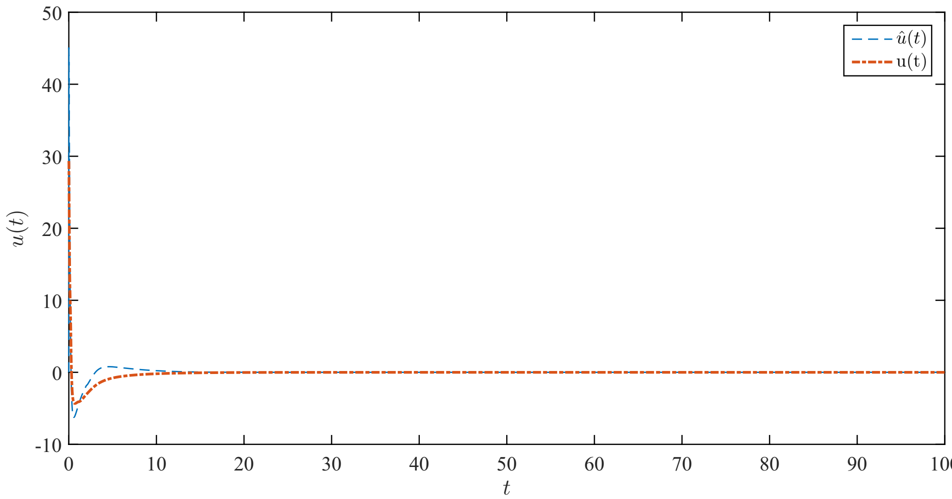


Fig. 5 Control signals generated for initial condition $\phi = [-2 \quad -4]^T$

$$A_1 = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ -a\frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ -ad\frac{v^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$R_1 = R_2 = R_{d1} = R_{d2} = \begin{bmatrix} 0.255 \\ 0.255 \\ 0.255 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix},$$

$$S_1 = S_2 = S_{d1} = S_{d2} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$

where $a = 0.7$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, $L = 5.5$, $l = 2.8$, $d = \frac{10t_0}{\pi}$. By solving the LMIs in Lemma 1 for $\tau_M = 0.5$, $\tau_m = 0.1$, $\rho = 0.4$, $\bar{\sigma}_1 = 0.01$, and $\bar{\sigma}_2 = 0.02$, we find PDC gain matrices $K_1 = K_2 = \begin{bmatrix} 7.0648 & -30.1913 & 0.7873 \end{bmatrix}$. This PDC controller uses state vector $x(t)$ to obtain control law $u(t) = \sum_{j=1}^{r} \mu_i(\theta(t))K_j x(t)$, where

$$\mu_1(\theta(t)) = \left(1 - \frac{1}{1 + exp(3(-\theta(t) - 0.5\pi))}\right) \frac{1}{1 + exp(3(-\theta(t) + 0.5\pi))},$$
$$\mu_2(\theta(t)) = 1 - \mu_1(\theta(t)).$$

Our objective is to design a functional observer-based PDC controller so that the control input is estimated directly without estimating the states. Considering $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and following the steps described in Sect. 3, we find the functional observer parameters as below:

$$\begin{aligned}
N_{11} &= -0.8062, N_{12} &&= -0.8062,\\
N_{21} &= 0.4950, N_{22} &&= 0.4950,\\
N_{d11} &= -2.2310, N_{d12} &&= -2.2310,\\
N_{d21} &= -2.2310, N_{d22} &&= -2.2310,\\
F_1 &= \begin{bmatrix} -78.9713 & 31.3140 \end{bmatrix}, F_2 &&= \begin{bmatrix} -78.9713 & 31.3140 \end{bmatrix},\\
J_{11} &= \begin{bmatrix} 161.4356 & -24.6120 \end{bmatrix}, J_{12} &&= \begin{bmatrix} 161.4356 & -24.6120 \end{bmatrix},\\
J_{21} &= \begin{bmatrix} 170.1920 & 15.1114 \end{bmatrix}, J_{21} &&= \begin{bmatrix} 170.1920 & 15.1114 \end{bmatrix},\\
J_{d11} &= \begin{bmatrix} 108.8304 & -68.1065 \end{bmatrix}, J_{d12} &&= \begin{bmatrix} 108.8304 & -68.1065 \end{bmatrix},\\
J_{d21} &= \begin{bmatrix} 108.8304 & -68.1065 \end{bmatrix}, J_{d22} &&= \begin{bmatrix} 108.8304 & -68.1065 \end{bmatrix},\\
H_{11} &= H_{12} = H_{21} = H_{22} = -10.0926.
\end{aligned}$$

A simulation is run in MATLAB for an initial condition $\phi(t) = \begin{bmatrix} 3 & -2 & 5 \end{bmatrix}^T$. The time-varying delay is considered to be $\tau(t) = 0.4 + 0.1\sin(t)$, and the the model uncertainty is emulated by MATLAB random number generator. Both conventional fuzzy control law and proposed functional

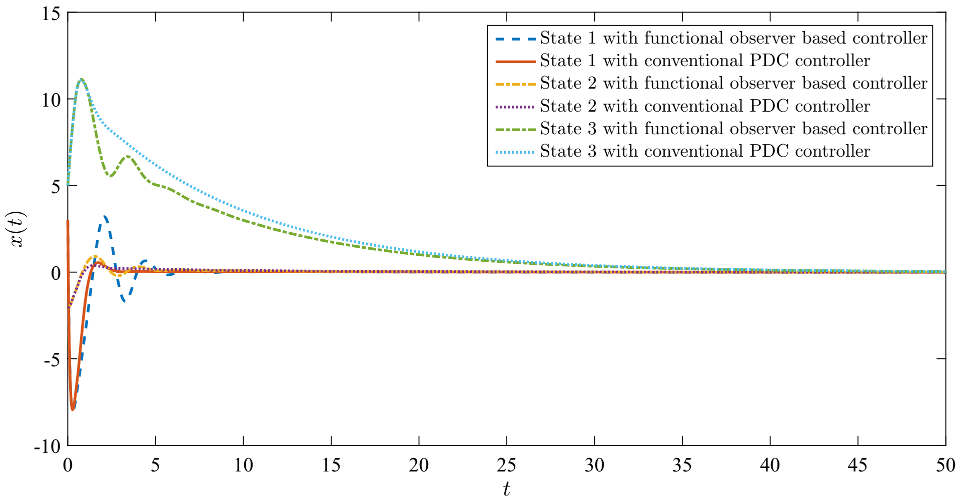


Fig. 6 State responses of the close loop truck–trailer system

observer-based fuzzy control law are applied to the closed-loop system. The state responses for both cases are displayed in Fig. 6. The system is stable under the effect of model uncertainty and time-delay. The functional observer-based controller is compared with the conventional PDC controller in Fig. 7. $\hat{u}(t)$ obtained by the functional observer converges with the desired control input $u(t)$ as expected.

5 Conclusion

A systematic synthesis procedure for obtaining a robust fuzzy functional observer for an uncertain fuzzy system with time-varying time-delay is presented. This functional observer is employed for designing a fuzzy controller that stabilises the system asymptotically. The proposed observer is inherently a reduced order observer compared with existing observer-based fuzzy controllers. More importantly, the observer estimates the control input vector directly. The stability of the observer is guaranteed in the sense that the estimation error approaches zero asymptotically. The sensitivity of the estimation error to the uncertainty of the model is minimised using a performance index. Lyapunov–Krasovskii functionals are used to ensure asymptotic stability of the observer and the system; the stability conditions are formulated as LMIs. Solutions of these LMIs are used to construct the observer. The proposed design methodology is illustrated using two examples. Future work may consider improving the stability conditions to guarantee the finite-time convergence of the observer. Application of this fuzzy functional observer for fault diagnosis scheme of nonlinear systems can also be an interesting topic of research.

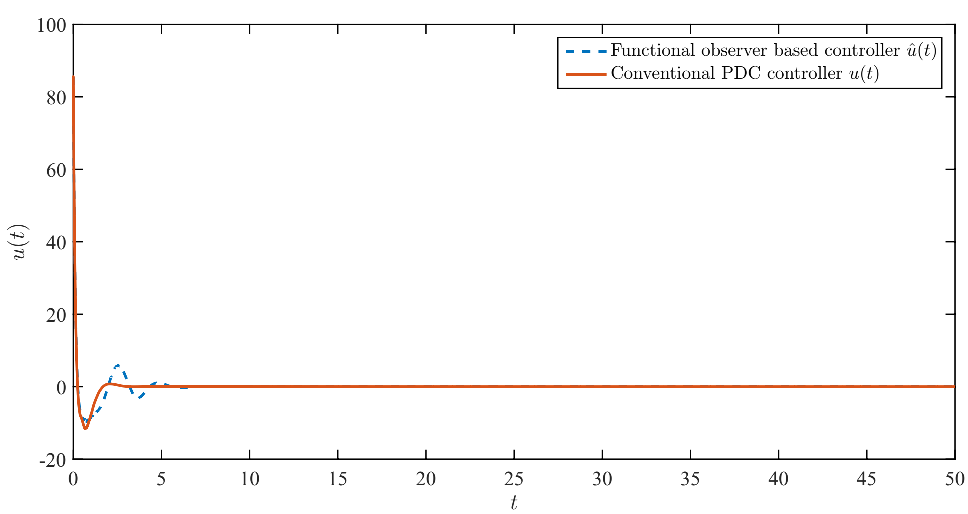


Fig. 7 Control law for the closed-loop truck–trailer system

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Appendix

Proof of Lemma 1

By applying PDC controller $u = \sum_{j=1}^{r} \mu_j(\xi(t))K_j x(t)$, the dynamics of closed-loop system (2) can be expressed as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))\Big\{(A_i + \Delta A_i + B_i K_j)x(t) + (A_{di} + \Delta A_{di})x(t-\tau(t))\Big\}. \tag{23}$$

Consider a Lyapunov–Krasovskii functional for the stability analysis of closed-loop system (23)

$$V(t) = x^T(t)P_1x(t) + \int_{t-\tau(t)}^{t} x^T(s)P_2x(s)\mathrm{d}s + \int_{-\tau_M}^{0}\int_{t+\theta}^{t} x^T(s)P_3x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} x^T(s)P_4x(s)\mathrm{d}s\mathrm{d}\theta.$$

Taking derivative along the state dynamics and applying the assumption $\dot{\tau}(t) \le \rho$, we can obtain

$$\begin{aligned}\dot V(t) &= 2x^T(t)P_1\dot x(t) + x^T(t)P_2x(t) \\ &\quad - (1-\dot\tau(t))x^T(t-\tau(t))P_2x(t-\tau(t)) \\ &\quad + \dot x^T(t)(\tau_M P_3 + (\tau_M - \tau_m)P_4)\dot x(t) \\ &\quad - \int_{t-\tau_M}^{t} \dot x^T(s)P_3\dot x(s)\mathrm{d}s \\ &\quad - \int_{t-\tau_M}^{t-\tau_m} \dot x^T(s)P_4\dot x(s)\mathrm{d}s \\ &\le 2x^T(t)P_1\dot x(t) + x^T(t)P_2x(t) \\ &\quad - (1-\rho)x^T(t-\tau(t))P_2x(t-\tau(t)) \\ &\quad + \dot x^T(t)(\tau_M P_3 + (\tau_M - \tau_m)P_4)\dot x(t).\end{aligned} \tag{24}$$

By (23) and the assumption of (3), it can be shown that

$$\begin{aligned}&\dot x^T(t)(\tau_M P_3 + (\tau_M - \tau_m)P_4)\dot x(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))\Big\{\dot x^T(t)\Lambda\begin{bmatrix}A_i + B_iK_j & A_{di}\end{bmatrix}\bar x(t) \\ &\quad + \dot x^T(t)\begin{bmatrix}\Lambda R_i & \Lambda R_{di}\end{bmatrix}\begin{bmatrix}U_i(t)S_i & 0 \\ 0 & U_{di}(t)S_{di}\end{bmatrix}\bar x(t) - \tau_m \dot x^T(t)P_4\dot x(t)\Big\} \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))\Big\{\dot x^T(t)\Lambda\begin{bmatrix}A_i + B_iK_j & A_{di}\end{bmatrix}\bar x(t) \\ &\quad + \dot x^T(t)\begin{bmatrix}\Lambda R_i & \Lambda R_{di}\end{bmatrix}\begin{bmatrix}\Lambda R_i & \Lambda R_{di}\end{bmatrix}^T\dot x(t) + \bar x^T(t)\begin{bmatrix}S_i^TS_i & 0 \\ 0 & S_{di}^TS_{di}\end{bmatrix}\bar x(t) \\ &\quad - \tau_m \dot x^T(t)P_4\dot x(t)\Big\},\end{aligned} \tag{25}$$

where $\bar x^T(t) = \begin{bmatrix}x^T(t) & x^T(t-\tau(t))\end{bmatrix}$ and $\Lambda = \tau_M(P_3 + P_4)$. It can also be shown that

$$\begin{aligned}&2x^T(t)P_1\dot x(t) \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))\bar x^T(t)\Big\{\begin{bmatrix}P_1(A_i + B_iK_j) + (A_i + B_iK_j)^TP_1 & P_1A_{di} \\ \star & 0\end{bmatrix} \\ &\quad + \begin{bmatrix}P_1R_i & P_1R_{di} \\ 0 & 0\end{bmatrix}\begin{bmatrix}(P_1R_i)^T & 0 \\ (P_1R_{di})^T & 0\end{bmatrix} + \begin{bmatrix}S_i^TS_i & 0 \\ 0 & S_{di}^TS_{di}\end{bmatrix}\Big\}\bar x(t).\end{aligned} \tag{26}$$

Therefore, using (24), (25) and (26) we get

$$\begin{aligned}\dot V(t) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))\eta^T(t)\Bigg(&\begin{bmatrix}P_2 + P_1A_i + A_i^TP_1 + P_1B_iK_j + K_j^TB_i^TP_1 & P_1A_{di} & \frac{1}{2}(A_i + B_iK_j)^T\Lambda \\ \star & -(1-\rho)P_2 & \frac{1}{2}A_{di}^T\Lambda \\ \star & \star & -\tau_m P_4\end{bmatrix} \\ &+ 2\begin{bmatrix}S_i^TS_i & 0 & 0 \\ 0 & S_{di}^TS_{di} & 0 \\ 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}P_1R_i & P_1R_{di} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Lambda R_i & \Lambda R_{di}\end{bmatrix}\begin{bmatrix}(P_1R_i)^T & 0 & 0 \\ (P_1R_{di})^T & 0 & 0 \\ 0 & 0 & R_i^T\Lambda \\ 0 & 0 & R_{di}^T\Lambda\end{bmatrix}\Bigg)\eta(t),\end{aligned}$$

where $\eta^T(t) = \left[x^T(t)\ x^T(t-\tau(t))\ \dot{x}^T(t)\right]$. As a consequence, an asymptotic stability condition of the fuzzy system can be given as

$$\begin{bmatrix} P_2 + P_1A_i + A_i^TP_1 + P_1B_iK_j + K_j^TB_i^TP_1 & P_1A_{di} & \frac{1}{2}(A_i + B_iK_j)^T\Lambda \\ \star & -(1-\rho)P_2 & \frac{1}{2}A_{di}^T\Lambda \\ \star & \star & -\tau_m P_4 \end{bmatrix} + 2\begin{bmatrix} S_i^TS_i & 0 & 0 \\ 0 & S_{di}^TS_{di} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} P_1R_i & P_1R_{di} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Lambda R_i & \Lambda R_{di} \end{bmatrix} \times \begin{bmatrix} (P_1R_i)^T & 0 & 0 \\ (P_1R_{di})^T & 0 & 0 \\ 0 & 0 & R_i^T\Lambda \\ 0 & 0 & R_{di}^T\Lambda \end{bmatrix} < 0. \tag{27}$$

By the Schur complement, (27) can be expressed as

$$\begin{bmatrix} P_2 + P_1A_i + A_i^TP_1 + P_1B_iK_j + K_j^TB_i^TP_1 & P_1A_{di} & \frac{1}{2}(A_i^T + K_j^TB_i^T)\Lambda & P_1R_i & P_1R_{di} & 0 & 0 & S_i^T & 0 \\ \star & -(1-\rho)\bar{P}_2 & \frac{1}{2}A_{di}^T\Lambda & 0 & 0 & 0 & 0 & 0 & S_{di}^T \\ \star & \star & -\tau_m P_4 & 0 & 0 & \Lambda R_i & \Lambda R_{di} & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\frac{1}{2}I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -\frac{1}{2}I \end{bmatrix} < 0. \tag{28}$$

Considering $P_3 = \bar{\sigma}_1P_1$ and $P_4 = \bar{\sigma}_2P_1$, and pre-multiplying and post-multiplying (28) by block-diagonal matrix

$\mathrm{diag}(P_1^{-1}\ P_1^{-1}\ P_1^{-1}\ I\ I\ I\ I\ I\ I)$,

we obtain

$$\begin{bmatrix} \bar{P}_2 + A_i\bar{P}_1 + \bar{P}_1A_i^T + B_i\bar{Y}_j + \bar{Y}_j^TB_i^T & A_{di}\bar{P}_1 & \frac{1}{2}\kappa(\bar{P}_1A_i^T + \bar{Y}_j^TB_i^T) & R_i & R_{di} & 0 & 0 & \bar{P}_1S_i^T & 0 \\ \star & -(1-\rho)\bar{P}_2 & \frac{1}{2}\kappa\bar{P}_1A_{di}^T & 0 & 0 & 0 & 0 & 0 & \bar{P}_1S_{di}^T \\ \star & \star & -\tau_m\bar{\sigma}_2\bar{P}_1 & 0 & 0 & \kappa R_i & \kappa R_{di} & 0 & 0 \\ \star & \star & \star & -I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\frac{1}{2}I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -\frac{1}{2}I \end{bmatrix} < 0,$$

where $\bar{P}_1 = P_1^{-1}$, $\bar{P}_2 = \bar{P}_1P_2\bar{P}_1$, $\bar{Y}_j = K_j\bar{P}_1$, $\kappa = \tau_M(\bar{\sigma}_1 + \bar{\sigma}_2)$ with some given scalars $\bar{\sigma}_1$ and $\bar{\sigma}_2$.

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