## **TECHNICAL PAPER**



# **Nonlinear fnite element modeling and experimental investigation of SFRC beams strengthened with GFRP laminate under static loading**

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## **Abstract**

Various strengthening techniques were adopted to strengthen beams with weaker cross-section. In recent years, Fibre Reinforced Concrete Composites (FRCC) and Fibre Reinforced Polymers (FRP) have led to many potential applications in structural engineering. This study through experimental tests, primarily investigated the performances of Steel Fibre Reinforced Concrete (SFRC) beams bonded externally at the soffit with Glass Fibre Reinforced Polymer (GFRP) laminates inorder to examine their fexural behaviour under static loading conditions. The experimental results were compared in lieu with numerical predictions computed through nonlinear fnite element software ANSYS. For the experimental investigation, six concrete beams were externally bonded at beam soffit with GFRP laminates and one concrete beam was treated as control specimen without any external GFRP laminate bonding. The beams were designed considering the under-reinforced condition of the limit state design philosophy. The beams were micro-reinforced with hook-end steel fbres in diferent fbre volume fractions (Vf) and strengthened at the soffit by GFRP laminates of thicknesses (t) 3 mm and 5 mm. Flexural loading was applied on the beams adopting two-point loading. The loading was applied in increments until the failure of the beams. A 3-D model in ANSYS software was developed and a nonlinear FEA was performed. The load–defection curves obtained from this nonlinear FEA were plotted and compared with experimental curves. The results of the 3D nonlinear fnite element model developed based on the experimentally validated SFRC beam specimens proved to establish a reasonably agreeable predictions.

**Keywords** Deformation · Ductility · GFRP laminate · Static · Steel fbres · Strengthening · Volume fraction · FEA · ANSYS

# **Introduction**

The behaviour of reinforced concrete elements observed by conducting experiments in a laboratory environment succumbs cost, time and effort. The studies are restricted due to difculties arising in availability of the materials, scarcity in usage of materials (which are constituted according to certain size and number of elements) and proper conditions to conduct the experimental investigation. In recent years, the theoretical analysis of reinforced concrete involves in creating a model on computer or counting with analytical calculation methods. Modeling in the software takes into

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consideration the properties and limitations of materials. Nonlinear fnite element program using ANSYS software is chosen for this study. A numerical method can solve complex and difficult physical problems with acceptable approximation. Ever since 971, ANSYS program, a FEA software has found its role in large scale practical applications [\[1](#page-12-0)]. As concrete is a material showing nonlinear behaviour during loading, it is modeled in such a way that it will show a nonlinear behaviour with ANSYS fnite element program [\[2](#page-12-1)].

The parameters involving strength, defection, ductility and cracking pattern of beams were carefully analyzed in these numerical analyses. These results were compared to test results of full-scale reinforced concrete beams with same specifcations cast and experimentally tested in the laboratory.

The load–displacement behaviour of reinforced concrete construction elements was analyzed in fnite element program using ANSYS in the nonlinear mode with two diferent

models for the same beam under a low-cycle condition, Barbosa and Riberio [\[2](#page-12-1)].

The behaviour of beams externally strengthened with carbon fbre reinforced composite (CFRC) without stirrups were experimentally studied by Kachlakev et al. [\[3](#page-12-2)]. Many researchers have strengthened RC beams with FRP and their behaviour was projected [[4–](#page-12-3)[6\]](#page-12-4). Numerical studies on GFRP were also available which gave a general guideline on modelling of the beams [[7–](#page-12-5)[10](#page-12-6)].

The procedure for nonlinear fnite element modeling of a RC beam with and without openings in ANSYS to study the parameters of strength, stifness, deformed shape and crack patterns, presented by Osman M. Ramadan et al. [[16\]](#page-12-7) showed an acceptable agreement between the results of FEA study and experimental investigation.

The comparative results of SFRC beams through experimental investigation and finite element analyses using ANSYS in modeling the beam specimen with a 8-noded solid brick element, studied by Mehmet Özcan etal. [[17](#page-12-8)], was found to be in good agreement.

The accuracy of the fnite element model (in ANSYS) of RC beams strengthened externally with fber reinforced polymer (FRP) laminates incorporating a smeared cracking approach for concrete and a three-dimensional layered element for FRP composites developed by Amer M. Ibrahim and Mohammed Sh. Mahmood [[18\]](#page-12-9) was found to be in good agreement with the experimental results.

The results of a fnite element model developed for CFRP strengthened RC beams using ABAQUS with diferent material models studied by Yasmeen Taleb Obaidat et al. [[19\]](#page-12-10) showed good agreement with the experimental data when the cohesive bond model was used.

The debonding failures in FRP-strengthened concrete beams was examined by G.M.Chen et al. [\[21](#page-12-11)] which adopted a dynamic analysis in FE simulations inorder to overcome the problem of convergence criteria when approached by the common solution techniques. As a result, the dynamic analysis provided an efective solution in overcoming the convergence issue by providing accurate prediction of test results.

The diferent strengthening techniques of RC beams using hybrid FRP proved to be effective in flexure which was substantiated by the numerical predictions of the FE model, Chellpandian et al. [[22\]](#page-12-12)

The effect of flexural strengthening in two different modes of reinforced-concrete (RC) beams strengthened with carbon fbre-reinforced polymers (CFRP) was studied by simulating a three-dimensional nonlinear fnite-element model, Ahmed Godat et al. [[23](#page-12-13)] Results of the parametric studies indicated that there was a threshold for the contribution of FRP laminates. The fnite-element model showed a very reasonable accuracy when validated against published experimental data.

An advanced FE simulation on FRP strengthened concrete structures was attempted by M.Z. Naser et al. [\[24](#page-13-0)] in order to study its behaviour for various structural parameters under monotonic loading conditions.

# **Experimental programme**

### **Test materials**

In this study, reinforced concrete beams were casted and strengthened with steel fbres [[11](#page-12-14)]. Steel fbres (DURA fex) were used in this investigation in volume fractions of 0%, 0.5%, 1%, 1.5%. For fexural strengthening of SFRC beams, externally reinforced to the soffit of the beam with Glass Fibre Reinforced Polymer laminates (CSS UGF27) of 3 mm and 5 mm thickness using epoxy resin as bonding agent (COROCRETIN IHL-18). The beam specimens to be tested in this study were designed for M20 grade concrete strength conforming to IS 10262:2009.

### **Test plan**

The experimental work was designed in such a way that it suits to investigate GFRP-SFRC beams fexural behaviour and outcome under incremental monotonic loading. The cross-sections of the beams were  $150 \text{ mm} \times 250 \text{ mm}$  with 3 m length. Totally seven beams were cast, out of that six beams were GFRP laminate strengthened beam and remaining one beam served as control beam. The concrete compressive strength and yield strength of reinforcing steel of the beams were 27.11 MPa and 445.63 MPa respectively. Tension steel designed for the beam consisted of  $pt = 1.14\%$ . In this study, the prime focus of variables to be determined were steel fibre volume fraction ' $V_f'$ ' and different thicknesses (t) of GFRP laminates. The parameters considered for the study of GFRP strengthened SFRC beams included that of defection at yield load, defection at ultimate load, energy ductility, defection ductility and crack patterns. Among the total seven beams tested for fexural behaviour, one beam was control or reference beam and remaining beams were GFRP strengthened beams. The test beams were loaded under static fexural condition and tested until the failure. The details of tested beams are provided in Table [1](#page-2-0) and the reinforcement details are shown in Fig. [1](#page-2-1) [[11\]](#page-12-14).

### **Test procedure**

The static load testing of beams were carried out using a loading frame of 50 Tons capacity under four-point bending with simple support boundary conditions. Adequate bearing was provided at the beam end supports. The test beams were loaded under two-point loading system and defections at mid-span and loading points were measured using defection gauges. The excessive defections undergone by the beam during the peak failure loads were measured by a custom designed mechanical dial gauge. The crack widths occurring during the entire loading stage were measured using a precision crack detection microscope. The crack details such as crack widths, number of cracks and spacing of cracks were continuously recorded during the testing process. The readings were measured and recorded until the failure of the beam specimens. Figure [2](#page-2-2) shows the complete experimental test set-up and instrumentation adopted in this study.

# **Numerical model**

In order to simulate a 3D fnite element model to study the flexural behaviour of the experimentally tested beams, ANSYS (R.2016) fnite element software was implemented

<span id="page-2-0"></span>**Table 1** Details of tested beams [\[11\]](#page-12-14)





<span id="page-2-1"></span>**Fig. 1** Reinforcement details of tested beams [[11](#page-12-14)]



a Schematic Representation

<span id="page-2-2"></span>**Fig. 2** Test set-up and instrumentation [\[11\]](#page-12-14)

**b** Testing under Progress

<span id="page-3-0"></span>**Table 2** Element types for working model

Element
SOLID <sub>65</sub>
<b>BEAM 188</b>
SHELL 181



<span id="page-3-1"></span>**Fig. 3** SOLID 65. [ *Source*: ANSYS Theory Reference R16.0]

by incorporating workbench and mechanical APDL adaptations.

#### **Element types**

Geometry of the concrete beam was created as solid block. Reinforcement was created as 1-D beam model with crosssection specifed in sections. The laminate was created as a shell element with specifed thickness. The elements and its types used in the FEA of the beams using ANSYS code are presented in Table [2](#page-3-0) and are shown through Figs. [3,](#page-3-1) [4](#page-3-2) and [5.](#page-3-3)

#### **Material properties**

Material properties defned to the material models are presented in Tables [3](#page-4-0), [4](#page-4-1) and [5.](#page-4-2) Material Model Number 1 refers to the SOLID 65 element. The SOLID 65 element requires linear isotropic and multi-linear isotropic material properties to properly model concrete and they are presented in Table [3.](#page-4-0) Concrete was modeled in ANSYS by an eight-node solid element, Solid65, that consists of 8 nodes with 3 degrees of freedom (translations in *x*, *y*, and *z* directions) at each node. This element has the capabilities of cracking, crushing and deforming plastically [[26,](#page-13-1) [27\]](#page-13-2).

Material Model Number 2 refers to the BEAM 188 element. The BEAM 188 element requires linear isotropic



<span id="page-3-2"></span>**Fig. 4** BEAM 188. [ *Source*: ANSYS Theory Reference R16.0]



<span id="page-3-3"></span>**Fig. 5** SHELL 181. [ *Source*: ANSYS Theory Reference R16.0]

and multi-linear isotropic material properties to model steel reinforcement and they are presented in Table [4.](#page-4-1) It is suitable for analyzing slender to moderately stubby/thick beam structures. The element is based on Timoshenko beam theory which includes shear-deformation efects. The element provides options for unrestrained warping and restrained warping of cross-sections. The element is a linear, quadratic, or cubic two-node beam element in 3-D. BEAM188 has 6 to 7 degrees of freedom of translations in *x*, *y*, and *z* directions and rotations about *x*, *y*, and *z* directions at every node.

# <span id="page-4-0"></span>**Table 3** Material models for SOLID 65



### <span id="page-4-1"></span>**Table 4** Material models for BEAM 188



#### <span id="page-4-2"></span>**Table 5** Material models for SHELL 181



Material Model Number 3 refers to the SHELL 181 element. SHELL 181 element requires linear orthotropic material properties to model laminate and they are presented in Table [5.](#page-4-2) It is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node: translations in the *x*, *y*, and *z* directions, and rotations about the *x*, *y*, and *z*-axes. (If the membrane option is used, the element has translational degrees of freedom only). The degenerate triangular option should only be used as fller elements in mesh generation. SHELL181 is well-suited for linear, large rotation, and/or large strain nonlinear applications.

In this study, SOLID 65 element, which has been used as material model number 1, requires linear isotropic and multi-linear isotropic material properties to properly model concrete and they are presented in Table [3.](#page-4-0) EX is the modulus of elasticity of the concrete (E*c*) and PRXY is the Poisson's ratio  $(\mu)$ . The material model in ANSYS requires that diferent constants be defned.

#### **Modeling**

The beam model generated is shown in Fig. [6](#page-5-0). Cohesive Zone Model coupled with FE analyses is the most used method to analyze adhesive joints considering failure of an adhesive (Da Silva & Campilho, 2012). CZM model uses a combination of stress and fracture mechanics. Its application is based on continuum assumptions of thin adhesive bonds that join structural members (Campilho, Moura, & J.J.M.S., 2008). The concept of cohesive zone was proposed by Barenblatt and Dugdale in early 1950s through late 1960s in their separate research. The technique consists of an established traction–separation laws to model interfaces or fnite regions. CZM shape/laws are applied between paired nodes of contact elements representing diferent materials adhered together (Da Silva & Campilho, 2012). CZM laws distinguish between normal/tension and tangential/shear forces in the cohesive region. Thus, tension and shear in an adhesive joint may be analyzed separately by a computer software [[25\]](#page-13-3). The variable considered in the Cohesive Zone Material (CZM) are.

C1—maximum normal contact stress (tension) in GPa.

C2—critical fracture energy for normal separation in kN/ mm.

C3—maximum shear stress in GPa.

C4—critical fracture energy for tangential slip in kN/mm.

C5—artificial damping coefficient in sec.

C6—allow tangential slip.

#### **Meshing**

The reinforcement was created as 1-D model with cross-sections applied to it (Fig. [7\)](#page-6-0). Rectangular mesh offers good results when SOLID 65 element is used. Therefore, the mapped meshing was done for the concrete beam (Fig. [8\)](#page-6-1).

#### **Loads and boundary conditions**

In order to obtain relatively convergent solutions with the experimental results, it is always necessary to ensure that appropriate loading and boundary conditions similar to that of experimental testing were applied in the numerical fnite element simulation model. Loading and boundary conditions are shown in Fig. [9](#page-6-2).

# **Results and discussion**

The seven numbers of beam specimens experimentally tested under static loading were analyzed by nonlinear Finite Element Analysis (FEA) using the ANSYS fnite element code. The ANSYS outputs for load–defection curves and stress contour of GFRP laminate strengthened SFRC beams are shown through Figs. [10,](#page-7-0) [11](#page-7-1) and [12](#page-8-0). Failure mechanisms



<span id="page-5-0"></span>**Fig. 6** Modeling of concrete with reinforcement steel and GFRP laminate

<span id="page-6-0"></span>**Fig. 7** Meshing of reinforcement steel



<span id="page-6-1"></span>**Fig. 8** Meshing of concrete beam section



<span id="page-6-2"></span>



of adhesive bonding can be studied considering structural failure, adhesive failure and cohesive failure. The comparative fgure (Expt vs. FEA) of the crack patterns and failure

<span id="page-7-0"></span>

<span id="page-7-1"></span>



of the specimens under incremental monotonic loading is shown in Fig. [12.](#page-8-0)The comparison of experimental and FEA load–defection responses of GFRP laminate strengthened SFRC beams are shown through Fig. [13a](#page-9-0)–f.

# **Strength and deformation**

Even though several Codes and Standards are accessible for the analysis and design of FRP composites, the designer's forum could not outweigh the complexity and tediousness involved in the design when dealing with unique loading conditions. This situation in particular forced the designers to opt for the numerical model studies on FRP strengthened structural members. The signifcance of fnite element simulation studies on models has narrowed down the knowledge gap of functional behaviour of FRP strengthened structures/ members and proved to be a better alternative to substantiate the theoretical validations. This part of the study deals with the adhesives/bonding agents, properties of constitutive materials and its plasticity and FRP laminate bonding systems under static loading conditions. All these FE simulation results were compared to the outcomes of experimentally tested beams to fnd out the close convergence of results.

The principles and fndings of this work can be of great interest to researchers, practitioners, and students. The comparative test results of beams obtained from experimental investigation and numerical nonlinear fnite element analysis were tabulated in Table [6](#page-10-0).

The percentage variation between the experimental and FEA results of control beam NSC in defection at yield load and ultimate load was 3.75% and 5.32% respectively. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-A5, NSF-B3, NSF-B5, NSF-C3 and NSF-C5 in defection at yield load was 2.72%, 8.81%, 12.03%, 3.19%, 3.83% and 3.4% respectively. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-A5, NSF-B3, NSF-B5, NSF-C3 and NSF-C5 in defection at ultimate load was 1.41%, 3.72%, 6.03%, 2.23%, 3.05% and 6.41%, respectively. The comparison plots for defection response of beams through Experiment Vs Finite Element Analysis at yield and ultimate loads are shown in Figs. [14](#page-10-1) and [15.](#page-10-2)

From the above discussions, it was observed that the results obtained through ANSYS modeling for defection at yield load varied by 3.75% in control beam and 2.72 to



<span id="page-8-0"></span>**Fig. 12** Crack patterns at failure of tested beams

12.03% in strengthened beams and the percentage variation for defection at ultimate load was 5.32% in control beam and 1.41 to 6.41% in strengthened beams. Parthiban et al. (2014) [[12\]](#page-12-15), reported that the numerical results (ANSYS) for the concrete beams reinforced with diferent discrete micro fbres and strengthened by GFRP lamination varied from 6 to 12.5% for yield defection and 8.8–11.76% for ultimate defection. Alper Buyukkaragoz (2010) [[13\]](#page-12-16), modeled and analyzed the beams strengthened with prefabricated reinforced concrete plate using ANSYS fnite element program. The author reported a percentage variation of 12% in control beam and 6% in strengthened beam for ultimate load. Maghsoudi et al., 2008 [\[14](#page-12-17)], reported that the results obtained for HSC flexural beams modeled using nonlinear finite element software ANSYS varied from 12.92 to 18.55% for yield load, 8.56% to 17.72% for ultimate load, 10.85% to 24.02% for yield defection and 10% to 34.82% for ultimate defection.

### **Ductility indices**

The ductility indices of the beam specimens analysed by static nonlinear Finite Element Analysis using ANSYS and test results obtained experimentally were compared and are presented in Table [7.](#page-11-0)

The percentage variation between the experimental and FEA results of control beam NSC in energy ductility was 34.11% and 1.63% in defection ductility. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-A5, NSF-B3, NSF-B5, NSF-C3 and NSF-C5 in energy ductility was 1.65%, 1.64%, 2.67%, 1.58%, 1.54% and 0.96% respectively. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-A5, NSF-B3, NSF-B5, NSF-C3 and NSF-C5 in defection ductility was 1.36%, 5.59%,



<span id="page-9-0"></span>**Fig. 13** Static load–defection response of tested beams

6.82%, 0.99%, 0.82% and 3.12%, respectively. The comparison plots for ductility indices of beams through experiment Vs fnite element analysis are shown in Figs. [16](#page-11-1) and [17.](#page-11-2)

From the above discussions, it was observed that the energy ductility varied by 34.11% in control beam and from 0.96 to 1.65% in strengthened beams and the

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defection ductility varied by 1.63% in control beam and from 0.82 to 6.82% in strengthened beams. Parthiban et al. (2014) [[5](#page-12-18)], reported that the fnite element model simulation results (ANSYS) for the RC beams reinforced with various micro fbre reinforcements and bonded with GFRP lamination varied from 6.2 to 9.9% for defection



a Deflection at Yield Load

<span id="page-10-1"></span>



<span id="page-10-2"></span>

ductility. Maghsoudi et al. (2008) [[7](#page-12-5)], reported that the results obtained for HSC fexural beams modeled using nonlinear fnite element software ANSYS varied from 0.25 to 4.98% for defection ductility and 0.15% to 13.53% for energy ductility.

In overall, it was observed that the results obtained from the experimental test specimens and numerical fnite element model are quite similar but partially difer from each other due to various associated reasons. Though concrete behaved in heterogeneous nature in all the directions of



**Beam ID b** % Variation in Deflection at Ultimate Load



NSF-B3

12.03

3.19

NSF-B5

**Beam ID** 

3.83

NSF-C3

3.40

**NSF-C5** 

8.81





<span id="page-10-0"></span>

15

 $10$ 

 $\mathcal{S}_{\mathbf{0}}$ 

3.75  $\overline{5}$ 

NSC

2.72

NSF-A3 NSF-A5

<span id="page-11-0"></span>**Table 7** Ductility indices of tested beams

Beam ID	Energy ductility			Deflection ductility			
	Experiment	<b>FEA</b>	% Variation	Experiment	<b>FEA</b>	% Variation	
NSC	1.29	1.73	34.11	1.29	1.27	1.63	
NSF-A3	1.82	1.85	1.65	1.44	1.46	1.36	
NSF-A5	1.83	1.86	1.64	1.47	1.56	5.59	
NSF-B3	1.87	1.92	2.67	1.51	1.62	6.82	
NSF-B5	1.90	1.93	1.58	1.65	1.67	0.99	
NSF-C3	1.95	1.98	1.54	1.78	1.80	0.82	
NSF-C5	2.08	2.10	0.96	2.01	1.95	3.12	



 $\overline{a}$ 



a Energy Ductility



<span id="page-11-1"></span>**Fig. 16** Experiment versus FEA comparison of energy ductility





**b** % Variation in Deflection Ductility

<span id="page-11-2"></span>**Fig. 17** Experiment versus FEA comparison of defection ductility

the experimentally tested beams, it is always inputted with homogeneous material properties in the numerical analysis and hence a material non-linearity always existed between the two systems. Similarly, the loading and support points of the beams in the experimental conditions may not exactly match with the simulation of the numerical model due to geometrical non-linearity's and eccentricities occurring in the contact surface points (lack of symmetry) in contrast with the beam test specimen. These factors play and efective role in the correlation and accuracy of results obtained from experimental tests and numerical predictions.

# **Conclusions**

The static fexural performance of GFRP strengthened RC beams were numerically examined by simulating a 3D nonlinear fnite element model using ANSYS software in terms of fexural strength, defection, ductility and cracking. The numerical predictions were correlated with the experimental outcomes and the results were found to be falling closely in the line of convergence. Thus, it can be concluded that the finite element model showed reasonable agreement with experimental outputs. The results obtained establish that glass fber reinforced polymer (GFRP) laminates act as an efficient strengthening material for reinforced concrete beams under static-fexural conditions.

# **Recommendations for future study**

As a futuristic study, this fnite element model shall be considered to develop various parametric studies and design philosophies in strengthening of RC members using FRP laminates in line with studies carried out by earlier researchers. The flexural effect of reinforcement diameter was examined using a validated FE model in the parametric study of Rami A. Hawileh et al. (2013). Threshold limitations in some parameters of GFRP laminates were found out in the parametric studies of Ahmed Godat et al. (2020).

## **Declarations**

**Conflict of interest** The authors have no conficts of interest to declare that are relevant to the content of this article.

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