TECHNICAL PAPER



Nonlinear finite element modeling and experimental investigation of SFRC beams strengthened with GFRP laminate under static loading

L. K. Rex¹ · P. N. Raghunath² · K. Suguna²

Received: 12 May 2021 / Accepted: 2 March 2022 / Published online: 13 April 2022 @ Springer Nature Switzerland AG 2022

Abstract

Various strengthening techniques were adopted to strengthen beams with weaker cross-section. In recent years, Fibre Reinforced Concrete Composites (FRCC) and Fibre Reinforced Polymers (FRP) have led to many potential applications in structural engineering. This study through experimental tests, primarily investigated the performances of Steel Fibre Reinforced Concrete (SFRC) beams bonded externally at the soffit with Glass Fibre Reinforced Polymer (GFRP) laminates inorder to examine their flexural behaviour under static loading conditions. The experimental results were compared in lieu with numerical predictions computed through nonlinear finite element software ANSYS. For the experimental investigation, six concrete beams were externally bonded at beam soffit with GFRP laminates and one concrete beam was treated as control specimen without any external GFRP laminate bonding. The beams were designed considering the under-reinforced condition of the limit state design philosophy. The beams were micro-reinforced with hook-end steel fibres in different fibre volume fractions (Vf) and strengthened at the soffit by GFRP laminates of thicknesses (t) 3 mm and 5 mm. Flexural loading was applied on the beams adopting two-point loading. The loading was applied in increments until the failure of the beams. A 3-D model in ANSYS software was developed and a nonlinear FEA was performed. The load–deflection curves obtained from this nonlinear FEA were plotted and compared with experimental curves. The results of the 3D nonlinear finite element model developed based on the experimentally validated SFRC beam specimens proved to establish a reasonably agreeable predictions.

Keywords Deformation · Ductility · GFRP laminate · Static · Steel fibres · Strengthening · Volume fraction · FEA · ANSYS

Introduction

The behaviour of reinforced concrete elements observed by conducting experiments in a laboratory environment succumbs cost, time and effort. The studies are restricted due to difficulties arising in availability of the materials, scarcity in usage of materials (which are constituted according to certain size and number of elements) and proper conditions to conduct the experimental investigation. In recent years, the theoretical analysis of reinforced concrete involves in creating a model on computer or counting with analytical calculation methods. Modeling in the software takes into

L. K. Rex lkrphd2018@gmail.com

¹ Department of Civil Engineering, AGNI College of Technology, Chennai, Tamil Nadu 600130, India

² Department of Civil & Structural Engineering, Annamalai University, Annamalainagar, Tamil Nadu 608002, India consideration the properties and limitations of materials. Nonlinear finite element program using ANSYS software is chosen for this study. A numerical method can solve complex and difficult physical problems with acceptable approximation. Ever since 971, ANSYS program, a FEA software has found its role in large scale practical applications [1]. As concrete is a material showing nonlinear behaviour during loading, it is modeled in such a way that it will show a nonlinear behaviour with ANSYS finite element program [2].

The parameters involving strength, deflection, ductility and cracking pattern of beams were carefully analyzed in these numerical analyses. These results were compared to test results of full-scale reinforced concrete beams with same specifications cast and experimentally tested in the laboratory.

The load-displacement behaviour of reinforced concrete construction elements was analyzed in finite element program using ANSYS in the nonlinear mode with two different models for the same beam under a low-cycle condition, Barbosa and Riberio [2].

The behaviour of beams externally strengthened with carbon fibre reinforced composite (CFRC) without stirrups were experimentally studied by Kachlakev et al. [3]. Many researchers have strengthened RC beams with FRP and their behaviour was projected [4–6]. Numerical studies on GFRP were also available which gave a general guideline on modelling of the beams [7–10].

The procedure for nonlinear finite element modeling of a RC beam with and without openings in ANSYS to study the parameters of strength, stiffness, deformed shape and crack patterns, presented by Osman M. Ramadan et al. [16] showed an acceptable agreement between the results of FEA study and experimental investigation.

The comparative results of SFRC beams through experimental investigation and finite element analyses using ANSYS in modeling the beam specimen with a 8-noded solid brick element, studied by Mehmet Özcan etal. [17], was found to be in good agreement.

The accuracy of the finite element model (in ANSYS) of RC beams strengthened externally with fiber reinforced polymer (FRP) laminates incorporating a smeared cracking approach for concrete and a three-dimensional layered element for FRP composites developed by Amer M. Ibrahim and Mohammed Sh. Mahmood [18] was found to be in good agreement with the experimental results.

The results of a finite element model developed for CFRP strengthened RC beams using ABAQUS with different material models studied by Yasmeen Taleb Obaidat et al. [19] showed good agreement with the experimental data when the cohesive bond model was used.

The debonding failures in FRP-strengthened concrete beams was examined by G.M.Chen et al. [21] which adopted a dynamic analysis in FE simulations inorder to overcome the problem of convergence criteria when approached by the common solution techniques. As a result, the dynamic analysis provided an effective solution in overcoming the convergence issue by providing accurate prediction of test results.

The different strengthening techniques of RC beams using hybrid FRP proved to be effective in flexure which was substantiated by the numerical predictions of the FE model, Chellpandian et al. [22]

The effect of flexural strengthening in two different modes of reinforced-concrete (RC) beams strengthened with carbon fibre-reinforced polymers (CFRP) was studied by simulating a three-dimensional nonlinear finite-element model, Ahmed Godat et al. [23] Results of the parametric studies indicated that there was a threshold for the contribution of FRP laminates. The finite-element model showed a very reasonable accuracy when validated against published experimental data. An advanced FE simulation on FRP strengthened concrete structures was attempted by M.Z. Naser et al. [24] in order to study its behaviour for various structural parameters under monotonic loading conditions.

Experimental programme

Test materials

In this study, reinforced concrete beams were casted and strengthened with steel fibres [11]. Steel fibres (DURA flex) were used in this investigation in volume fractions of 0%, 0.5%, 1%, 1.5%. For flexural strengthening of SFRC beams, externally reinforced to the soffit of the beam with Glass Fibre Reinforced Polymer laminates (CSS UGF27) of 3 mm and 5 mm thickness using epoxy resin as bonding agent (COROCRETIN IHL-18). The beam specimens to be tested in this study were designed for M20 grade concrete strength conforming to IS 10262:2009.

Test plan

The experimental work was designed in such a way that it suits to investigate GFRP-SFRC beams flexural behaviour and outcome under incremental monotonic loading. The cross-sections of the beams were 150 mm × 250 mm with 3 m length. Totally seven beams were cast, out of that six beams were GFRP laminate strengthened beam and remaining one beam served as control beam. The concrete compressive strength and yield strength of reinforcing steel of the beams were 27.11 MPa and 445.63 MPa respectively. Tension steel designed for the beam consisted of $\rho t = 1.14\%$. In this study, the prime focus of variables to be determined were steel fibre volume fraction 'V_f' and different thicknesses (t) of GFRP laminates. The parameters considered for the study of GFRP strengthened SFRC beams included that of deflection at yield load, deflection at ultimate load, energy ductility, deflection ductility and crack patterns. Among the total seven beams tested for flexural behaviour, one beam was control or reference beam and remaining beams were GFRP strengthened beams. The test beams were loaded under static flexural condition and tested until the failure. The details of tested beams are provided in Table 1 and the reinforcement details are shown in Fig. 1 [11].

Test procedure

The static load testing of beams were carried out using a loading frame of 50 Tons capacity under four-point bending with simple support boundary conditions. Adequate bearing was provided at the beam end supports. The test beams were loaded under two-point loading system and deflections at

mid-span and loading points were measured using deflection gauges. The excessive deflections undergone by the beam during the peak failure loads were measured by a custom designed mechanical dial gauge. The crack widths occurring during the entire loading stage were measured using a precision crack detection microscope. The crack details such as crack widths, number of cracks and spacing of cracks were continuously recorded during the testing process. The readings were measured and recorded until the failure of the beam specimens. Figure 2 shows the complete experimental test set-up and instrumentation adopted in this study.

Numerical model

In order to simulate a 3D finite element model to study the flexural behaviour of the experimentally tested beams, ANSYS (R.2016) finite element software was implemented

 Table 1
 Details of tested beams [11]

Beam ID	Beam type	Steel fibre volume fraction ' V_{f} '	el fibre volume GFRP laminate Tension steel C_{f} thickness 't'		Compression steel	Spacing of 2L—8φ Stir- rups 'Sv'	
		%	mm			mm c/c	
NSC	Control beam	0	0	3-12φ	2-10φ	200	
NSF-A3	Strengthened beam	0.5	3				
NSF-A5	Strengthened beam	0.5	5				
NSF-B3	Strengthened beam	1.0	3				
NSF-B5	Strengthened beam	1.0	5				
NSF-C3	Strengthened beam	1.5	3				
NSF-C5	Strengthened beam	1.5	5				



Fig. 1 Reinforcement details of tested beams [11]



a Schematic Representation

Fig. 2 Test set-up and instrumentation [11]

b Testing under Progress

 Table 2
 Element types for working model

Material Type	Element
Concrete	SOLID 65
Steel Reinforcement	BEAM 188
Laminate	SHELL 181



Fig. 3 SOLID 65. [Source: ANSYS Theory Reference R16.0]

by incorporating workbench and mechanical APDL adaptations.

Element types

Geometry of the concrete beam was created as solid block. Reinforcement was created as 1-D beam model with crosssection specified in sections. The laminate was created as a shell element with specified thickness. The elements and its types used in the FEA of the beams using ANSYS code are presented in Table 2 and are shown through Figs. 3, 4 and 5.

Material properties

Material properties defined to the material models are presented in Tables 3, 4 and 5. Material Model Number 1 refers to the SOLID 65 element. The SOLID 65 element requires linear isotropic and multi-linear isotropic material properties to properly model concrete and they are presented in Table 3. Concrete was modeled in ANSYS by an eight-node solid element, Solid65, that consists of 8 nodes with 3 degrees of freedom (translations in *x*, *y*, and *z* directions) at each node. This element has the capabilities of cracking, crushing and deforming plastically [26, 27].

Material Model Number 2 refers to the BEAM 188 element. The BEAM 188 element requires linear isotropic



Fig. 4 BEAM 188. [Source: ANSYS Theory Reference R16.0]



Fig. 5 SHELL 181. [Source: ANSYS Theory Reference R16.0]

and multi-linear isotropic material properties to model steel reinforcement and they are presented in Table 4. It is suitable for analyzing slender to moderately stubby/thick beam structures. The element is based on Timoshenko beam theory which includes shear-deformation effects. The element provides options for unrestrained warping and restrained warping of cross-sections. The element is a linear, quadratic, or cubic two-node beam element in 3-D. BEAM188 has 6 to 7 degrees of freedom of translations in x, y, and z directions and rotations about x, y, and z directions at every node.

Table 3 Material models for SOLID 65



Table 4Material models forBEAM 188

Linear isotropic					
EX	1.67E05				
PRXY	0.30				
Bi-linear isotropic					
Yield stress	445.63 Mpa				
Tang. mod	0				

Table 5 Material models for SHELL 181

Linear orthotropic	
Young's modulus Ex	39 GPa
Young's modulus Ey	8.6 GPa
Young's modulus Ez	8.6 GPa
Poisson's ratio V1	0.28
Poisson's ratio $(V2 = V3)$	0.09
Shear modulus $G12 = G23 = G31$	3.8 GPa

Material Model Number 3 refers to the SHELL 181 element. SHELL 181 element requires linear orthotropic material properties to model laminate and they are presented in Table 5. It is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. (If the membrane option is used, the element has translational degrees of freedom only). The degenerate triangular option should only be used as filler elements in mesh generation. SHELL181 is well-suited for linear, large rotation, and/or large strain nonlinear applications.

In this study, SOLID 65 element, which has been used as material model number 1, requires linear isotropic and multi-linear isotropic material properties to properly model concrete and they are presented in Table 3. EX is the modulus of elasticity of the concrete (Ec) and PRXY is the Poisson's ratio (μ). The material model in ANSYS requires that different constants be defined.

Modeling

The beam model generated is shown in Fig. 6. Cohesive Zone Model coupled with FE analyses is the most used method to analyze adhesive joints considering failure of an adhesive (Da Silva & Campilho, 2012). CZM model uses a combination of stress and fracture mechanics. Its application is based on continuum assumptions of thin adhesive bonds that join structural members (Campilho, Moura, & J.J.M.S., 2008). The concept of cohesive zone was proposed by Barenblatt and Dugdale in early 1950s through late 1960s in their separate research. The technique consists of an established traction–separation laws to model interfaces or finite regions. CZM shape/laws are applied between paired nodes of contact elements representing different materials adhered together (Da Silva & Campilho, 2012). CZM laws distinguish between normal/tension and tangential/shear forces in the cohesive region. Thus, tension and shear in an adhesive joint may be analyzed separately by a computer software [25]. The variable considered in the Cohesive Zone Material (CZM) are.

C1-maximum normal contact stress (tension) in GPa.

C2—critical fracture energy for normal separation in kN/mm.

C3-maximum shear stress in GPa.

C4—critical fracture energy for tangential slip in kN/mm.

C5—artificial damping coefficient in sec.

C6—allow tangential slip.

Meshing

The reinforcement was created as 1-D model with crosssections applied to it (Fig. 7). Rectangular mesh offers good results when SOLID 65 element is used. Therefore, the mapped meshing was done for the concrete beam (Fig. 8).

Loads and boundary conditions

In order to obtain relatively convergent solutions with the experimental results, it is always necessary to ensure that appropriate loading and boundary conditions similar to that of experimental testing were applied in the numerical finite element simulation model. Loading and boundary conditions are shown in Fig. 9.

Results and discussion

The seven numbers of beam specimens experimentally tested under static loading were analyzed by nonlinear Finite Element Analysis (FEA) using the ANSYS finite element code. The ANSYS outputs for load–deflection curves and stress contour of GFRP laminate strengthened SFRC beams are shown through Figs. 10, 11 and 12. Failure mechanisms



Fig. 6 Modeling of concrete with reinforcement steel and GFRP laminate

Fig. 7 Meshing of reinforcement steel



Fig. 8 Meshing of concrete beam section







of adhesive bonding can be studied considering structural failure, adhesive failure and cohesive failure. The comparative figure (Expt vs. FEA) of the crack patterns and failure



Fig. 11 Stress contour at failure of tested beams



of the specimens under incremental monotonic loading is shown in Fig. 12. The comparison of experimental and FEA load-deflection responses of GFRP laminate strengthened SFRC beams are shown through Fig. 13a–f.

Strength and deformation

Even though several Codes and Standards are accessible for the analysis and design of FRP composites, the designer's forum could not outweigh the complexity and tediousness involved in the design when dealing with unique loading conditions. This situation in particular forced the designers to opt for the numerical model studies on FRP strengthened structural members. The significance of finite element simulation studies on models has narrowed down the knowledge gap of functional behaviour of FRP strengthened structures/ members and proved to be a better alternative to substantiate the theoretical validations. This part of the study deals with the adhesives/bonding agents, properties of constitutive materials and its plasticity and FRP laminate bonding systems under static loading conditions. All these FE simulation results were compared to the outcomes of experimentally tested beams to find out the close convergence of results.

The principles and findings of this work can be of great interest to researchers, practitioners, and students. The comparative test results of beams obtained from experimental investigation and numerical nonlinear finite element analysis were tabulated in Table 6.

The percentage variation between the experimental and FEA results of control beam NSC in deflection at yield load and ultimate load was 3.75% and 5.32% respectively. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-A5, NSF-B3, NSF-B5, NSF-C3 and NSF-C5 in deflection at yield load was 2.72%, 8.81%, 12.03%, 3.19%, 3.83% and 3.4% respectively. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-B5, NSF-C3 and NSF-C5 in deflection at ultimate load was 1.41%, 3.72%, 6.03%, 2.23%, 3.05% and 6.41%, respectively. The comparison plots for deflection response of beams through Experiment Vs Finite Element Analysis at yield and ultimate loads are shown in Figs. 14 and 15.

From the above discussions, it was observed that the results obtained through ANSYS modeling for deflection at yield load varied by 3.75% in control beam and 2.72 to



Fig. 12 Crack patterns at failure of tested beams

12.03% in strengthened beams and the percentage variation for deflection at ultimate load was 5.32% in control beam and 1.41 to 6.41% in strengthened beams. Parthiban et al. (2014) [12], reported that the numerical results (ANSYS) for the concrete beams reinforced with different discrete micro fibres and strengthened by GFRP lamination varied from 6 to 12.5% for yield deflection and 8.8-11.76% for ultimate deflection. Alper Buyukkaragoz (2010) [13], modeled and analyzed the beams strengthened with prefabricated reinforced concrete plate using ANSYS finite element program. The author reported a percentage variation of 12% in control beam and 6% in strengthened beam for ultimate load. Maghsoudi et al., 2008 [14], reported that the results obtained for HSC flexural beams modeled using nonlinear finite element software ANSYS varied from 12.92 to 18.55% for yield load, 8.56% to 17.72% for ultimate load, 10.85% to 24.02% for yield deflection and 10% to 34.82% for ultimate deflection.

Ductility indices

The ductility indices of the beam specimens analysed by static nonlinear Finite Element Analysis using ANSYS and test results obtained experimentally were compared and are presented in Table 7.

The percentage variation between the experimental and FEA results of control beam NSC in energy ductility was 34.11% and 1.63% in deflection ductility. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A3, NSF-A5, NSF-B3, NSF-B5, NSF-C3 and NSF-C5 in energy ductility was 1.65%, 1.64%, 2.67%, 1.58%, 1.54% and 0.96% respectively. The percentage variation between the experimental and FEA results of GFRP laminate strengthened SFRC beams NSF-A5, NSF-C3, NSF-C3, NSF-C3, NSF-C5, NSF-C3 and NSF-C5 in deflection ductility was 1.36%, 5.59%,



Fig. 13 Static load-deflection response of tested beams

6.82%, 0.99%, 0.82% and 3.12%, respectively. The comparison plots for ductility indices of beams through experiment Vs finite element analysis are shown in Figs. 16 and 17.

From the above discussions, it was observed that the energy ductility varied by 34.11% in control beam and from 0.96 to 1.65% in strengthened beams and the

☑ Springer

deflection ductility varied by 1.63% in control beam and from 0.82 to 6.82% in strengthened beams. Parthiban et al. (2014) [5], reported that the finite element model simulation results (ANSYS) for the RC beams reinforced with various micro fibre reinforcements and bonded with GFRP lamination varied from 6.2 to 9.9% for deflection

Beam ID	Yield stage				Ultimate stage			
	Load	Expt. deflection	FEA deflection	Variation	Load	Expt. deflection	FEA deflection	Variation
	kNN N	mm	mm	%	kN	mm	mm	%
NSC	25	9.60	9.24	3.75	35	12.40	11.74	5.32
NSF-A3	100	12.85	12.50	2.72	125	18.50	18.24	1.41
NSF-A5	105	13.50	12.31	8.81	135	19.90	19.16	3.72
NSF-B3	105	13.80	12.14	12.03	135	20.90	19.64	6.03
NSF-B5	115	14.10	13.65	3.19	150	23.30	22.78	2.23
NSF-C3	110	14.35	13.80	3.83	145	25.60	24.82	3.05
NSF-C5	120	15.90	15.36	3.40	160	32.00	29.95	6.41

15

10

Variation between

\$

Expt. and FEA

6

4

2

Û

5.32

NSC



a Deflection at Yield Load





Fig. 15 Experiment versus FEA Comparison of Deflection at Ultimate Load

ductility. Maghsoudi et al. (2008) [7], reported that the

results obtained for HSC flexural beams modeled using

nonlinear finite element software ANSYS varied from 0.25

to 4.98% for deflection ductility and 0.15% to 13.53% for

energy ductility.



NSF-A3 NSF-A5 NSF-B3 NSF-B5 NSF-C3 NSF-C5 Beam ID

6.03

3.72

1.41

6.41

3.05

2.23

b % Variation in Deflection at Ultimate Load



12.03

8.81

Table 7 Ductility indices of tested beams

Beam ID	Energy ductilit	у		Deflection duc	ility	
	Experiment	FEA	% Variation	Experiment	FEA	% Variation
NSC	1.29	1.73	34.11	1.29	1.27	1.63
NSF-A3	1.82	1.85	1.65	1.44	1.46	1.36
NSF-A5	1.83	1.86	1.64	1.47	1.56	5.59
NSF-B3	1.87	1.92	2.67	1.51	1.62	6.82
NSF-B5	1.90	1.93	1.58	1.65	1.67	0.99
NSF-C3	1.95	1.98	1.54	1.78	1.80	0.82
NSF-C5	2.08	2.10	0.96	2.01	1.95	3.12





a Energy Ductility



Fig. 16 Experiment versus FEA comparison of energy ductility





b % Variation in Deflection Ductility

Fig. 17 Experiment versus FEA comparison of deflection ductility

the experimentally tested beams, it is always inputted with homogeneous material properties in the numerical analysis and hence a material non-linearity always existed between the two systems. Similarly, the loading and support points of the beams in the experimental conditions may not exactly match with the simulation of the

Conclusions

The static flexural performance of GFRP strengthened RC beams were numerically examined by simulating a 3D nonlinear finite element model using ANSYS software in terms of flexural strength, deflection, ductility and cracking. The numerical predictions were correlated with the experimental outcomes and the results were found to be falling closely in the line of convergence. Thus, it can be concluded that the finite element model showed reasonable agreement with experimental outputs. The results obtained establish that glass fiber reinforced polymer (GFRP) laminates act as an efficient strengthening material for reinforced concrete beams under static-flexural conditions.

Recommendations for future study

As a futuristic study, this finite element model shall be considered to develop various parametric studies and design philosophies in strengthening of RC members using FRP laminates in line with studies carried out by earlier researchers. The flexural effect of reinforcement diameter was examined using a validated FE model in the parametric study of Rami A. Hawileh et al. (2013). Threshold limitations in some parameters of GFRP laminates were found out in the parametric studies of Ahmed Godat et al. (2020).

Declarations

Conflict of interest The authors have no conflicts of interest to declare that are relevant to the content of this article.

References

- 1. Lawrance KL (2002) ANSYS tutorial release 7.0 and 6.1, SDC Publications, Canonsburg, 1.1- 2.25
- Barbosa AF, Riberio GO (2004) Analysis of reinforced concrete structures using ANSYS nonlinear concrete model. Comput Mech 1(8):1–7
- 3. Kachlakev DI, Miller T, Yim S, Chansawat K, Potisuk T (2001). Finite element modelling of reinforced concrete structures strengthened with FRP laminates. Ph.D. dissertation, California Polytechnic State University, San Luis Obispo, USA
- (1998) Prediction of failure load of R/C beams strengthened with FRP plate due to stress concentration at the plate end. ACI Struct J 95(2). https://doi.org/10.14359/534
- Parthiban B, Suguna K, Raghunath PN (2015) Flexural behaviour of hybrid fibre reinforced concrete beams strengthened with FRP laminates. Int J Eng Sci Innov Technol 4(2):328–334
- Maalej M, Goh WH, Paramasivam P (2001) Analysis and design of FRP externally-reinforced concrete beams against debondingtype failures. Mater Struct 34(7):418–425. https://doi.org/10. 1007/bf02482288

- Kaushal P, Modhera CD (2012) Application of GFRP on preloaded retrofitted beam for enhancement in flexural strength. Int J Civil Struct Eng 2(4):1070–1080
- 8. Ritchie PA, Thomas DA, Lu LW, Connelly GM (1990) External reinforcement of concrete beams using fiber-reinforced plastics. ATLSS Report 90(6)
- Saadatmanesh H, Ehsani MR (1991) RC beams strengthened with GFRP plates-I: experimental study. ASCE J Struct Eng 117(11):3417–3433
- Saadatmanesh H, Ehsani MR (1991) RC beams strengthened with GFRP plates. I: experimental study. J Struct Eng 117(11):3417-3433. https://doi.org/10.1061/(asce)0733-9445(1991)117:11(3417)
- Rex LK, Premalatha PV (2018) Structural performance of concrete beams with micro-reinforcement strengthened with GFRP laminates under monotonic loading. J Test Eval 48(5):20170714. https://doi.org/10.1520/jte20170714
- Parthiban B, Suguna K, Raghunath PN (2014) Hybrid fibre reinforced concrete beams strengthened with externally bonded GFRP laminates. Asian J Eng Technol 02(05):415–423
- Büyükkaragöz A (2010) Finite element analysis of the beam strengthened with prefabricated reinforced concrete plate. Sci Res Essays 5(6):533–544
- Masti K, Maghsoudi AA, Rahgozar R (2008) Nonlinear models and experimental investigation of lifetime history of HSC flexural beams. Am J Appl Sci 5(3):248–262. https://doi.org/10.3844/ajassp.2008.248.262
- Lu XZ, Ye LP, Teng JG, Jiang JJ (2005) Meso-scale finite element model for FRP sheets/plates bonded to concrete. Eng Struct 27(4):564–575. https://doi.org/10.1016/j.engstruct.2004. 11.015
- Ramadan M, O., M. ABDELBAK, S., M. SALEH, A., & Y. ALKHATTABI, A. (2009) Modeling of reinforced concrete beams with and without opening by using ANSYS. JES J Eng Sci 37(4):845–858. https://doi.org/10.21608/jesaun.2009. 127752
- Özcan DM, Bayraktar A, Şahin A, Haktanir T, Türker T (2009) Experimental and finite element analysis on the steel fiber-reinforced concrete (SFRC) beams ultimate behavior. Constr Build Mater 23(2):1064–1077. https://doi.org/10.1016/j.conbuildmat. 2008.05.010
- Ibrahim AM, Mahmood MSh (2009) Finite element modeling of reinforced concrete beams strengthened with FRP laminates. Eur J Sci Res 30(4):526–541
- Obaidat YT, Heyden S, Dahlblom O (2010) The effect of CFRP and CFRP/concrete interface models when modelling retrofitted RC beams with FEM. Compos Struct 92(6):1391–1398. https:// doi.org/10.1016/j.compstruct.2009.11.008
- Hawileh RA, Naser MZ, Abdalla JA (2013) Finite element simulation of reinforced concrete beams externally strengthened with short-length CFRP plates. Compos B Eng 45(1):1722–1730. https://doi.org/10.1016/j.compositesb.2012.09.032
- Chen GM, Teng JG, Chen JF, Xiao QG (2015) Finite element modeling of debonding failures in FRP-strengthened RC beams: a dynamic approach. Comput Struct 158:167–183. https://doi. org/10.1016/j.compstruc.2015.05.023
- 22. Chellapandian M, Prakash SS, Sharma A (2019) Experimental and finite element studies on the flexural behavior of reinforced concrete elements strengthened with hybrid FRP technique. Compos Struct 208:466–478. https://doi.org/10.1016/j.comps truct.2018.10.028
- Godat A, Chaallal O, Obaidat Y (2020) Non-linear finite-element investigation of the parameters affecting externally-bonded FRP flexural-strengthened RC beams. Results Eng 8:100168. https://doi.org/10.1016/j.rineng.2020.100168

- 24. Naser MZ, Hawileh RA, Abdalla JA (2021) Modeling strategies of finite element simulation of reinforced concrete beams strengthened with FRP: a review. J Compos Sci 5:19
- 25. Kabaluk M (2019) Adhesive joint analyses using ANSYS CZM modeling of a prefabricated hybrid concrete-GFRP-CFRP unit. A

Master of Science Thesis, Faculty of College of Engineering and Computer Science, Florida Atlantic University, Boca Raton, FL

- 26. Theory reference for the mechanical APDL and mechanical applications
- 27. https://faculty.up.edu/lulay/ME304/BeamElements.pdf