



Time headway distributions for two-lane two-way roads: case study from Dakahliya Governorate, Egypt

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Received: 18 December 2020 / Accepted: 26 April 2021 / Published online: 7 July 2021
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Abstract

The time headway distribution of vehicles plays a significant role in different traffic engineering applications. This paper investigated seven probability distributions that would mimic the distribution of the time headways on the Egyptian two-lane, two-way (TLTW) roads, namely (1) exponential, (2) lognormal, (3) gamma, (4) inverse Gaussian, (5) Pearson type III, (6) shifted exponential, and (7) Schuhl distributions. Two sites from two rural TLTW roads that connect Mansoura city to Damietta and Dikrnis cities are studied. One-hour videotaped data from each site were collected, for both directions. Besides, the Chi-square and the K–S goodness-of-fit (GOF) measures were used to assess which distribution fits the observed headway data better. Based on the observed data, about 75% of travel speed measurements are relatively closer in values (between 60 and 70 km/h) for both directions. Different bin widths (0.50, 1.0, and 2.0 s) along with the recommended bin size (3.0 s) by the Rice rule were used to model the observed headway frequency. The results confirmed that both of the GOF tests reveal the same results in terms of the acceptance and the rejection of the proposed distributions compared to the observed headways. In addition, both of the gamma and the shifted exponential distributions would be good representatives for modeling the time headways in the Dakahliya Governorate rural TLTW roads. Moreover, bin widths less than the recommended by the Rice rule did not affect the acceptance/rejection results of the proposed methods.

Keywords Time headway · Headway distributions · Two-lane two-way roads · Mansoura · Egypt

Introduction

The time headway (i.e. headway) can be defined as the time between two successive vehicles when they pass a single point on a roadway [1]. It is one of the significant microscopic traffic flow parameters that are widely applied in roadway planning, traffic flow analysis, and design of roadway control systems [2]. It can be applied to obtain the

relationship between the 85th percentile speed and the headway [3]. Besides, it can be used in investigating the main reasons for road crashes as well as evaluating policies to improve traffic safety [4, 5]. Furthermore, the mathematical analysis and simulation of traffic operations are based on reliable knowledge of vehicles' headways distribution [6]. Moreover, time headway is considered one of the main performance measures for the two-lane, two-way roads [7]. Hashim and Abdel-Wahed [7] evaluated seven performance measures on eight two-lane, two-way sites in Minoufiya governorate, Egypt. Out of the seven performance measures, they found two performance measures that depend on the time headway, which are the follower density, and the percent followers [7].

Various headway distributions (i.e. models) have been developed over the past decades to represent the distribution of vehicle headways, such as negative exponential, gamma, Erlang (i.e. a special case of the gamma distribution), lognormal, Pearson, log-logistic, Branston's and Pearson type III, inverse Gaussian, Weibull, Schuhl, shifted exponential, semi-Poisson, Weibull lognormal

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(WLN), Weibull extreme value (WEV), and normal distributions [8–12]. The negative exponential distribution is usually applied to represent the headway data [13]. In addition, the negative exponential distribution is more explicit than other models in representing the headway distribution pattern on two-way, two-lane (TLTW) roads, especially where mixed traffic condition is heterogeneous and car-following interaction is frequent [13–15]. It is worth noting that the headway distribution of vehicles is based on the perception reaction time of drivers which is a function of alertness, complexity, and expectation [16]. In addition, the headway distribution of vehicles is influenced by lane location, structural and geometric of the roadway, time of day, weather conditions, traffic flow, and the proportion of heavy vehicles [17].

Headways can be classified into short and long headways. Short headways are those less than 2 s, while headways

larger than 2 s are considered as long headways [16]. Short headways represent closer spacing between vehicles that need to be maintained by drivers who have faster reaction times to avoid collision between vehicles. The gamma, Erlang, lognormal, Pearson type III, and log-logistic distributions have been proposed for short time headways [12]. While the longer headways happen in case of high speeds (≥ 80 km/h), required braking distances will be longer and drivers tend to maintain larger headways for safe stopping sight distances [16].

Al-Ghamdi [10] studied time headways at different traffic flow levels observed at different sites in the city of Riyadh, Kingdom of Saudi Arabia (KSA). Four headway distributions were applied including gamma, Erlang, negative exponential, and shifted exponential distributions. The negative exponential distribution was found to be a good distribution for representing long headways on TLTW roadways at

Table 1 Time headway distributions on TLTW roads

Author	Country	Proposed distributions	Comments
Al-Ghamdi [10]	KSA	Erlang, Gamma, Negative exponential, and Shifted exponential	The negative exponential distribution is the best distribution for long headways at low flows (<400 vehicles/h); Shifted exponential and gamma distributions have a good fit for medium flows (400–1200 vehicle/h); while Erlang distribution provided a good fit for high flows
Dey and Chandra [11]	India	Gamma, and Lognormal	Lognormal distribution was the best-fitted distribution for the steady-state car-following situation
Hoogendorn [8]	The Netherlands	Composite headway distributions: Branston's, and Pearson type III	Pearson type III distribution was the best distribution for the morning, noon, and evening periods
Maurya and Dey [19]	India	Gamma, Pearson type III, Inverse Gaussian, lognormal, and Weibull	Pearson type III distribution was the best for traffic flow between 200 and 600 PCU/h Inverse Gaussian was the best for the flow between 800 and 1200 PCU/h
Khasnabis and Heimbach [18]	USA	Erlang, Negative exponential, Pearson type III, Schuhl, and their combinations	The Schuhl model is the best
Luttinen [6]	Finland	Exponential, Shifted exponential, Gamma, Lognormal, and Semi-Poisson	The gamma distribution is the best model under low to moderate traffic volumes
Haryadi and Narendra [9]	Indonesia	Negative exponential, Normal, and Composite exponential	The normal distribution was the best for two locations The exponential distribution showed a good fit for the third location
Riccardo and Massimiliano [20]	Italy	Pearson type 5, Pearson type 6, Log-logistic, Inverse Gaussian, and Inverse Weibull	Inverse Weibull distribution fits observed headways for the majority of situations analyzed For the highest flow rates (1200–1400 and 1400–1600) Pearson 5 and log-logistic are the best
Yin et al. [14]	China	Lognormal and log-logistic	Lognormal is a good fit during free-flow status The log-logistic is more suitable in congested status

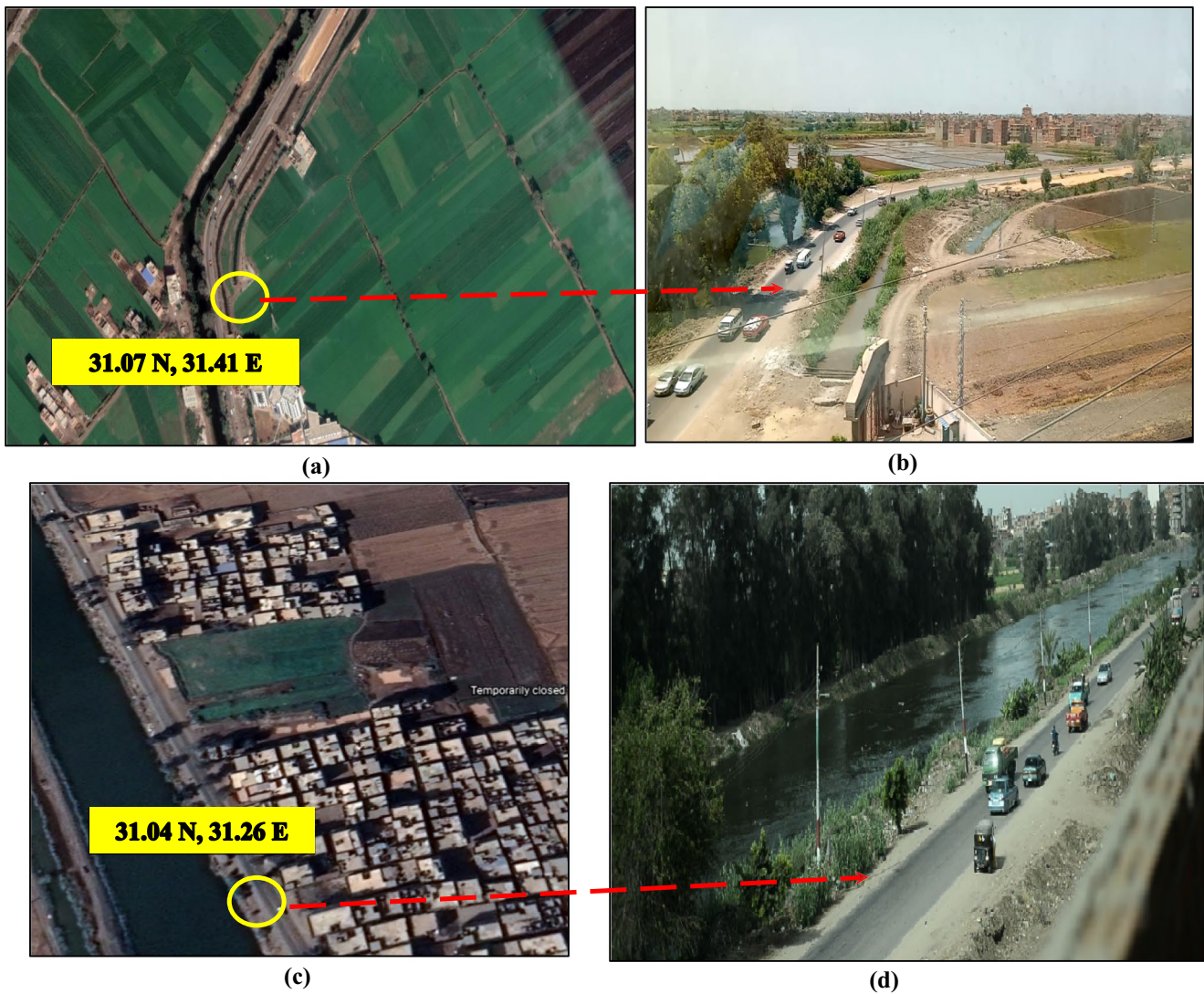


Fig. 1 Study locations: Mansoura–Damietta road (site S1): **a** location from Google Earth, and **b** a snap shot of site S1; Mansoura–Dikrnis road (site S2): **c** location from Google Earth, and **d** a snap shot of site S2

Table 2 Geometric characteristics of the two studied sites

Site	Site S1	Site S2
Classification according to HCM [22]	Class II	Class II
Road width (m)	9	8
Shoulder width (m)	1.5	1.5

flow rates less than 400 vehicles/h, while the shifted exponential and the gamma distributions were found to better fit for traffic flows between 400 and 1200 vehicle/h [10]. In addition, the Erlang distribution was found to be a good representative of the observed headways at locations with high traffic flows (> 1200 vehicles/h) [10]. Furthermore,

Dey and Chandra [11] applied the gamma and lognormal distributions for representing time headway in a steady car-following state on TLTW roads under mixed traffic conditions in India. They found that lognormal distribution was the best-fitted distribution in representing time headways in a steady-state car-following situation. In addition, hasnabis and Heimbach [18] studied headway distribution models for a two-lane rural highway under low and medium traffic flow conditions (80–630 vehicle/h/lane) in North Carolina, USA. They applied various headway distributions including Erlang, negative exponential, Pearson type III, Schuhl models, and their combinations, and concluded that the Schuhl model is the best representative model for the headway distributions. Table 1 summarizes the studies that investigated the time headway distributions on TWTL roads.

Table 3 Traffic composition for the two studied sites

Site	Site S1		Site S2	
	NB direction	SB direction	EB direction	WB direction
<i>Traffic volume (veh./h)</i>	182	265	201	278
% pickup vehicle	45%	40%	40%	40%
% microbus	20%	23%	23%	23%
% private car	15%	16%	17%	16%
% truck	10%	11%	9%	11%
% bus	5%	4%	6%	3%
% three wheel motorcycle	3%	4%	3%	5%
% taxi	2%	2%	2%	2%

Table 4 Summary statistics of travel speed (in km/h) for the studied sites

Site	Site S1		Site S2	
	NB direction	SB direction	EB direction	WB direction
Sample size	182	265	201	278
Minimum	23.70	26.40	21.70	24.40
Maximum	94.33	90.00	94.77	91.21
Mean	59.42	57.19	56.42	57.21
Standard deviation	9.75	10.57	8.09	9.57
Mode	64.28	56.25	54.54	57.25
Median	60.00	56.25	56.25	57.25

Table 5 Summary statistics of time headway (in seconds) for the studied sites

Site	Site S1		Site S2	
	NB direction	SB direction	EB direction	WB direction
Sample size	182	265	201	278
Minimum	0.10	0.11	0.10	0.11
Maximum	87.01	63.001	84.01	65.00
Mean	19.96	13.200	19.77	13.24
Standard deviation	20.19	12.471	20.49	12.47
Median	15.02	8.00	14.57	8.57

Table 1 shows that the negative exponential and the log-normal distributions are the best fit to represent time headway for most of the studied locations, especially at free-flow conditions [10, 11, 14]. In addition, the gamma and the log-logistic distributions are more suitable for moderate and congested traffic flow conditions, respectively [14].

In Egypt, Elkafoury et al. [17] studied the time headway frequency for private cars and heavy vehicles on four lanes divided roads (two lanes for each direction) at the entrance of the New Borg El-Arab city. They indicated that a major portion of vehicles (about 80% of HVs and 78% of PCs) tend to keep headway of fewer than 5 s. In addition, Hashim [3] studied the relationship between 85th percentile speed and headway to estimate a headway value corresponding to vehicles in the free-moving situation on rural TLTW

highways in Minoufiya Governorate, Egypt. He concluded that the 85th percentile speed takes a constant value at headway ≥ 5 s, so free-flow speeds can be defined for time headway between consecutive vehicles ≥ 5 s [3]. Furthermore, the results showed that most of the vehicles traveling on TLTW roads are traveling at speeds close to their desired speeds, as more than 60% of vehicles have headway values of greater than or equal to 5 s [3]. Moreover, Sabry [21] collected data from the Cairo-Alexandria agricultural road in Egypt and proposed five headway distributions to model the time headway, namely Pearson type III, Schuhl, negative exponential, lognormal, and Erlang distributions. The lognormal and the negative exponential distributions were found to be adequate in fitting the observed headways in Egyptian conditions [21].

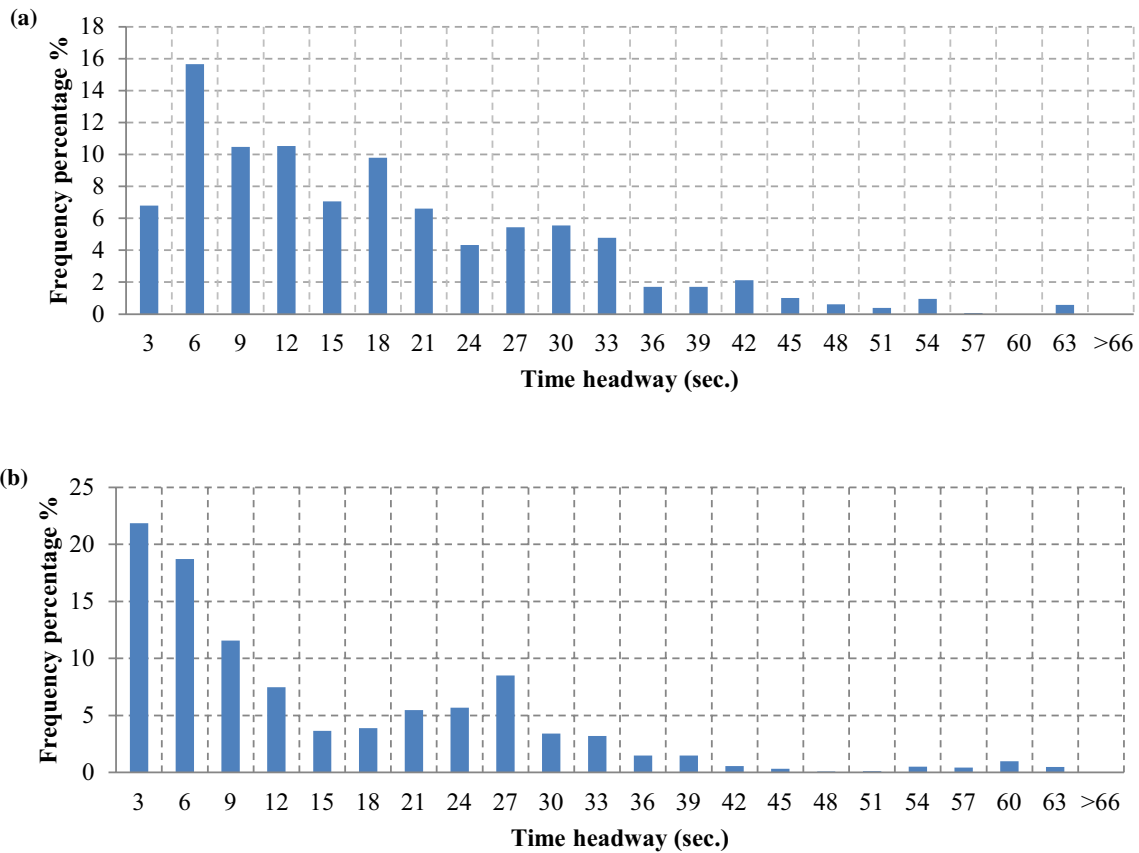


Fig. 2 Time Headway Frequency for Site S1: **a** NB direction, **b** SB direction

The time headway distribution studies on TLTW roads in Egypt are limited. The only study that investigated the time headway distributions on Egyptian roads was in 1989 (i.e. more than 30 years ago) [21]. Hence, the main objective of this study is to find the best statistical distributions that represent the time headways on the Egyptian TLTW roads, specifically for the Dakahliya Governorate rural TLTW roads. In this analysis, seven different statistical distributions, namely the lognormal, gamma, Pearson type III, inverse Gaussian, exponential, shifted exponential, and Schuhl distributions were investigated. The Chi-square and the Kolmogorov–Smirnov (K–S) goodness-of-fit measures were applied to select the best appropriate model of headway distributions.

Study area and data collection

Two sites from TLTW roads within Mansoura city in Egypt were used in this analysis. The first site (i.e. site S1) is from the Mansoura–Damietta TLTW road that connects Mansoura to Damietta city. The second site (i.e. site S2)

is from the Mansoura–Dikirnis TLTW road that connects Mansoura to Dikirnis city.

Figure 1 shows the studied locations. A 1 h video (12 p.m. to 1 p.m.) of the traffic at the first location is captured for both directions on June 17, 2019. The camera was placed at a high vantage location to capture the moving traffic of the entire width in both directions. Another 1-h (12 p.m. to 1 p.m.) traffic video for the second site was captured for both directions on April 19, 2019. Time headway and speed data were then extracted from the videos. Table 2 summarizes the geometric characteristics of the two study locations.

According to the Highway Capacity Manual (HCM) [22], the TLTW roads are classified into three classes based on their functionality. Both of the study sites are rural collector roads passing through different cities which can be classified as “Class II” TLTW roads, according to HCM [22].

Traffic composition for the two studied sites is shown in Table 3. From Table 3, the traffic composition is similar to both of the studied sites. In addition, the pickup vehicles represent about 40%–45% of the total traffic volume while taxi represents the least percentage of total traffic volume for both studied sites.

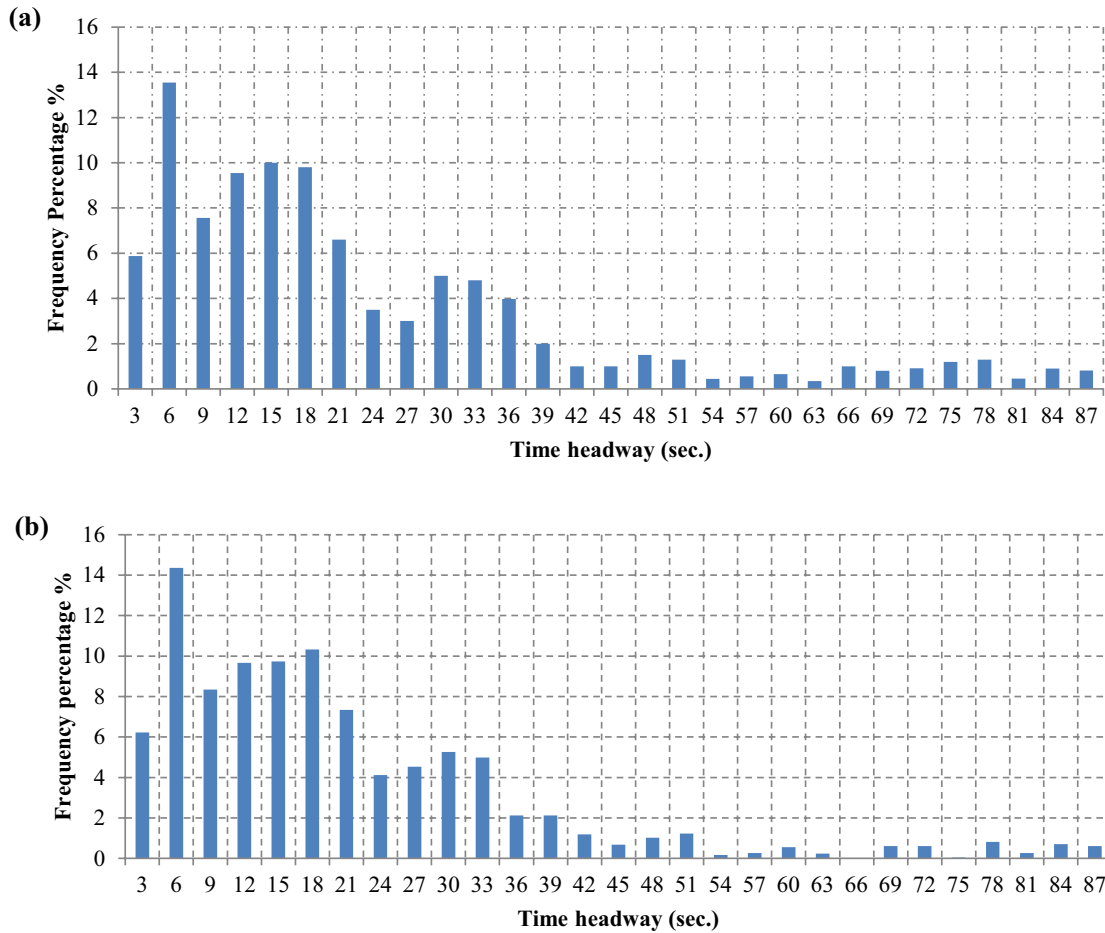


Fig. 3 Time headway frequency for Site S2: a EB direction, b WB direction

Table 6 Probability density functions (PDFs) for the proposed distributions

References	Distribution	Probability density function (PDF)
Mark and Pinsky [27]	Exponential	$P(t) = \lambda \times e^{(-\lambda t)} = \lambda \times e^{(-t/\bar{t})}$
Roy and Saha [13]	Lognormal	$P(t) = \frac{1}{t \times \sigma \sqrt{2\pi}} \exp\left(-\frac{(\ln(t)-\mu)^2}{2\sigma^2}\right)$
Singh et al. [28]	Pearson type III	$P(t) = \frac{1}{a \times t \times \Gamma(b)} \times \left(\frac{\ln(t)-c}{a}\right)^{b-1} \times \exp\left(-\frac{\ln(t)-c}{a}\right)$
Luttinen [6]	Gamma	$p(t) = \frac{1}{\beta^\alpha \times \Gamma(\alpha)} \times t^{\alpha-1} \times \exp\left(-\frac{t}{\beta}\right)$
Kong and Guo [29]	Inverse Gaussian	$p(t) = \sqrt{\frac{z}{2\pi t^2}} \times \exp\left(-\frac{z(t-\mu)^2}{2t \times \mu^2}\right)$
Al-Ghamdi [30]	Shifted exponential	$P(t) = \lambda' \times \exp\left(-\frac{(t-t_m)}{(m-t_m)}\right)$
Haryadi and Narendra [9] Gerlough et al. [31]	Schuhl	$p(t) = 1 - e^{-\frac{t}{\bar{t}'}} + (1-\alpha) \times \exp\left(\frac{-(t-t_m)}{(t_b-t_m)}\right)$

Where, exp=exponential function; t =time headway, seconds; λ =traffic flow, vehicle/h; μ =the mean value of observed time headways, seconds; σ =standard deviation of observed time headways, seconds; σ^2 =variance of observed time headways, seconds; Γ =Gamma function; a , b , and c =Pearson III distribution parameters; β =scale parameter of the gamma distribution; α =shape parameter of the gamma distribution; z =Inverse Gaussian distribution parameter; m =mean of the shifted exponential distribution ($1/m$ is the parameter of the distribution), seconds; λ' =traffic flow, vehicle/h; α' =the number of vehicles in the free-flow state; $(1 - \alpha')$ =number of vehicles in the car-following (constrained) state; t_m =minimum value of time headway; \bar{t} =the average headway for free vehicles, seconds, seconds; and \bar{t}' =the average headway for constrained vehicles, seconds

Table 7 Parameters estimation based on the maximum likelihood method

References	Distribution	Parameter	Equation
Transportation Research Board [1]	Exponential	λ	$\lambda = \frac{1}{\bar{t}}$
	Shifted exponential	λ'	$\lambda' = \frac{1}{\bar{t}-\eta}$
Singh et al. [28]	Pearson type III	a	$a = \sqrt{\frac{\sigma^2}{b}}$
		b	$b = \frac{4}{skewness^2}$
		c	$c = (\mu - (b \times a))$
Wilks and Daniel [32]	Gamma	α	$\alpha = \frac{\mu}{\beta}$
		β	$\beta = \frac{\sigma^2}{\mu}$
Folks and Chhikara [33]	Inverse Gaussian	Z	$z = \frac{\mu^3}{\sigma^2}$
Roy and Saha [13]	Lognormal	μ	$\mu = \frac{\sum_{i=1}^n \ln x_i}{n}$
		σ^2	$\sigma^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$
Gerlough et al. [31]	Schuhl	\bar{t}_f	$\bar{t}_f = \frac{\sum_{i=1}^n t_{fi}}{n}$
		\bar{t}_b	$\bar{t}_b = \frac{\sum_{i=1}^n t_{bi}}{n}$

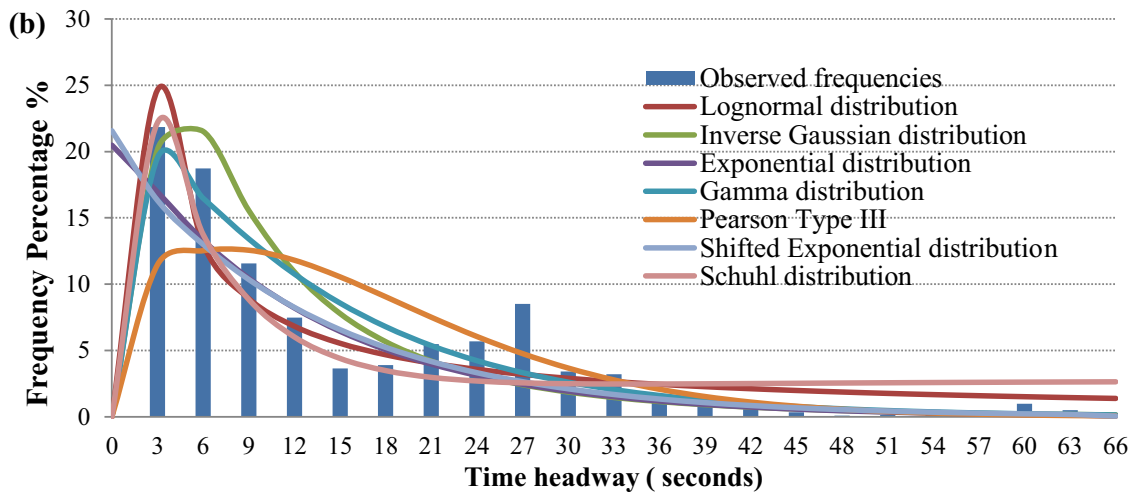
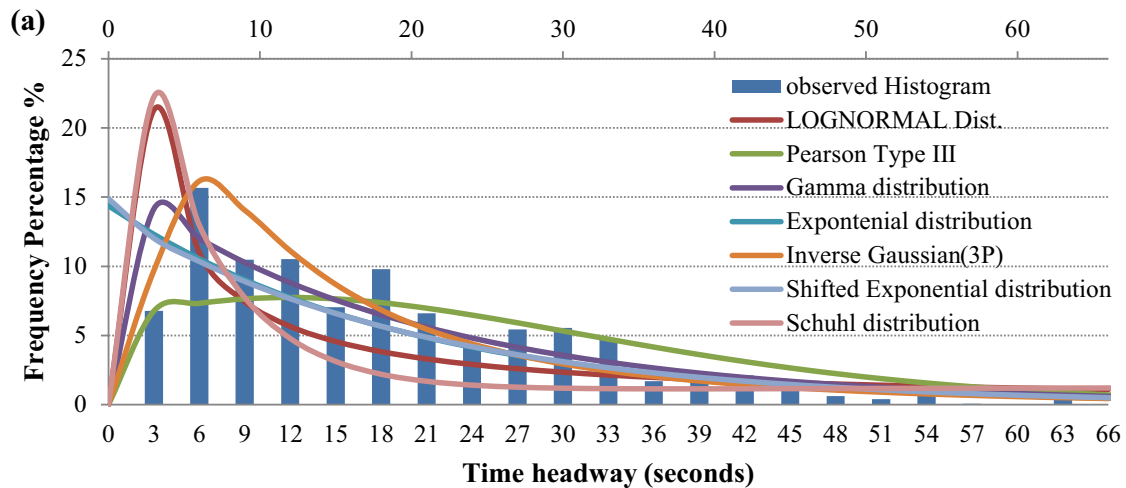


Fig. 4 Proposed distributions for site S1: **a** NB direction, **b** SB direction

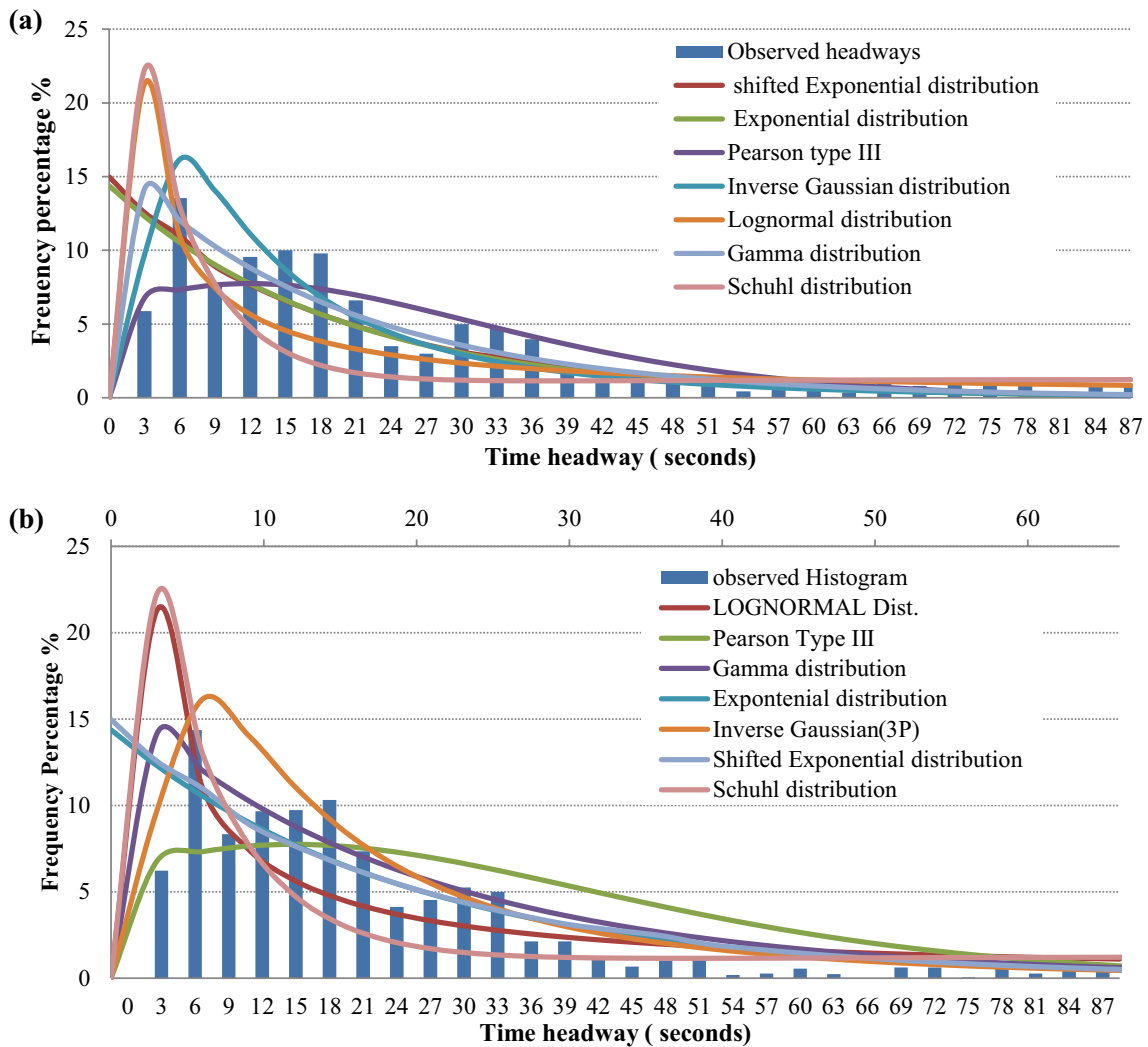


Fig. 5 Proposed distributions for site S2: **a** EB direction, **b** WB direction

It is worth noting that, based on the observed headways at both sites, trucks and private cars maintain higher time headway at low flow conditions for both directions but minibuses and pickup vehicles maintain lower time headway values.

Travel speeds and time headways

Based on the video tape recording, a section of 5.0 m was selected, taking into consideration the scale between the video and the real world. This is done by drawing two lines that represent the 5.0 m on a transparent paper attached to the computer screen. For each vehicle, two times are extracted, at the beginning (t_1) and at the end (t_2) of the

5.0 m section, as the front bumper of the vehicle passes through them. The vehicle speed was then estimated by dividing the distance (i.e., 5.0 m) by the time differences (i.e. " $t_2 - t_1$ "), for each vehicle.

The time headway for each vehicle’s pair (i.e. leader and following vehicles) was estimated based on the time difference between the time recorded for the front bumper of the leader vehicle and the front bumper of the following vehicle.

A summary statistics of the travel speed and time headways for the two studied locations in both directions are shown in Tables 4 and 5 respectively.

To plot time headway frequencies, the best bin size should be selected, as the selection of a very small bin width would result in a jagged histogram, while very large

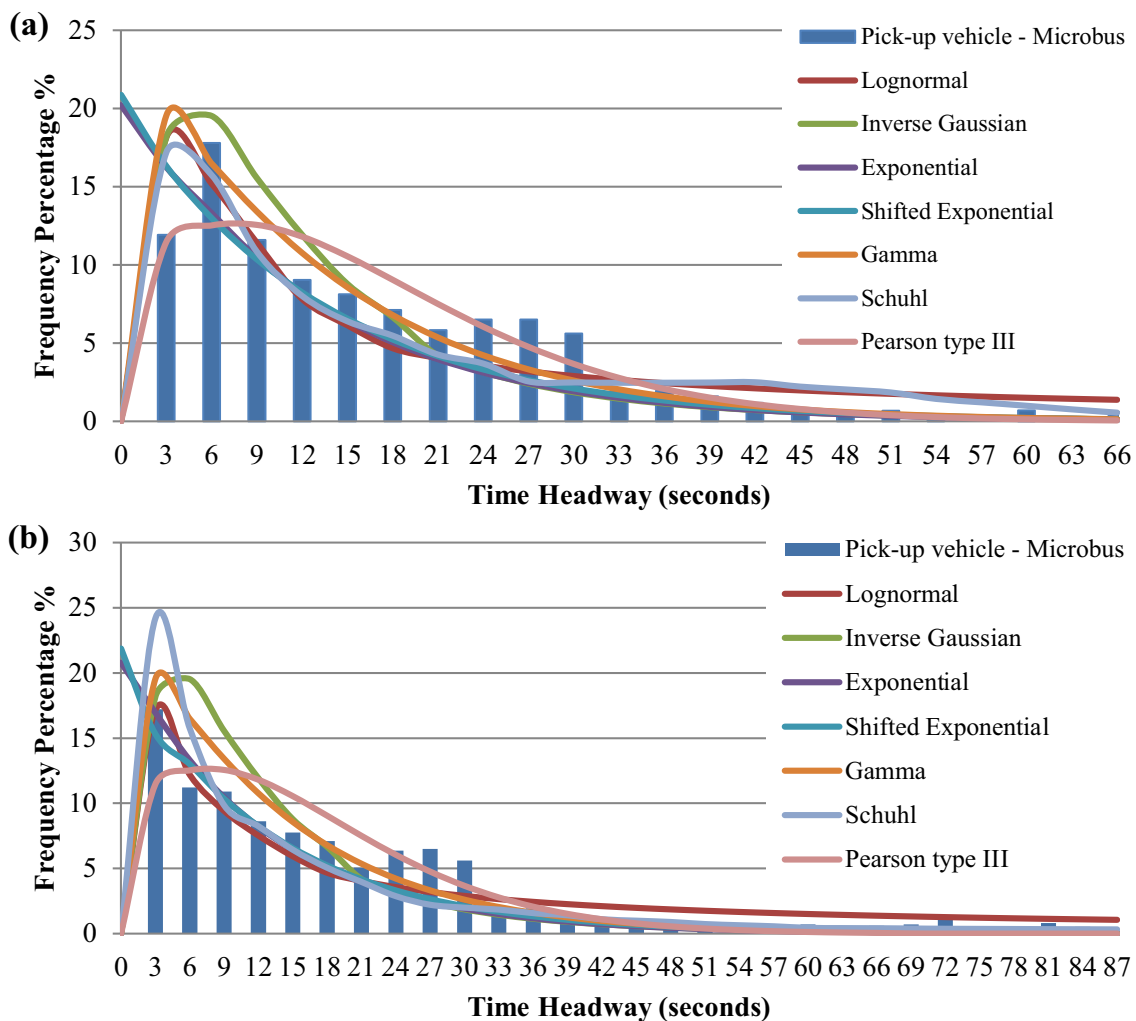


Fig. 6 Proposed distributions for “pickup-microbus” category: **a** at sites S1 and **b** at site S2

bin width would result in a histogram with a single block. To obtain a diversity of histogram shapes between these two extremes, intermediate bin widths should be used. Lohaka [23] recommended that the optimal number of bins is between 20 and 30 [23]. Dogan et al. [24] recommended that the Rice rule is suitable for classifying a series of “*n*” items that produce reasonable histograms. The mathematical form of the Rice rule is as follows:

$$\text{Bin width} = \frac{\text{Data range}}{\text{Number of classes}} \tag{1}$$

$$\text{Number of classes} = 2 \times (\text{Number of observations})^{1/3} \tag{2}$$

The best bin width for the two sites (for both directions) was found to be 3 s. The time headway frequencies

and their corresponding percentages on each time headway interval were calculated for both directions of the two studied sites as shown in Figs. 2 and 3. For both sites, about 38% of the observed headways are within the range of 0 and 6 s, while $\geq 50\%$ is more than 6 s and less than 30 s, while the rest of the observed headways (about 12%) are ≥ 30 s.

Time headway modeling

Probability distribution models are usually used to model time headways, to find the suitable probability distribution that is better represents the observed time headways. Seven probability distributions were considered in this study to model the time headway, on the Egyptian TLTW roads,

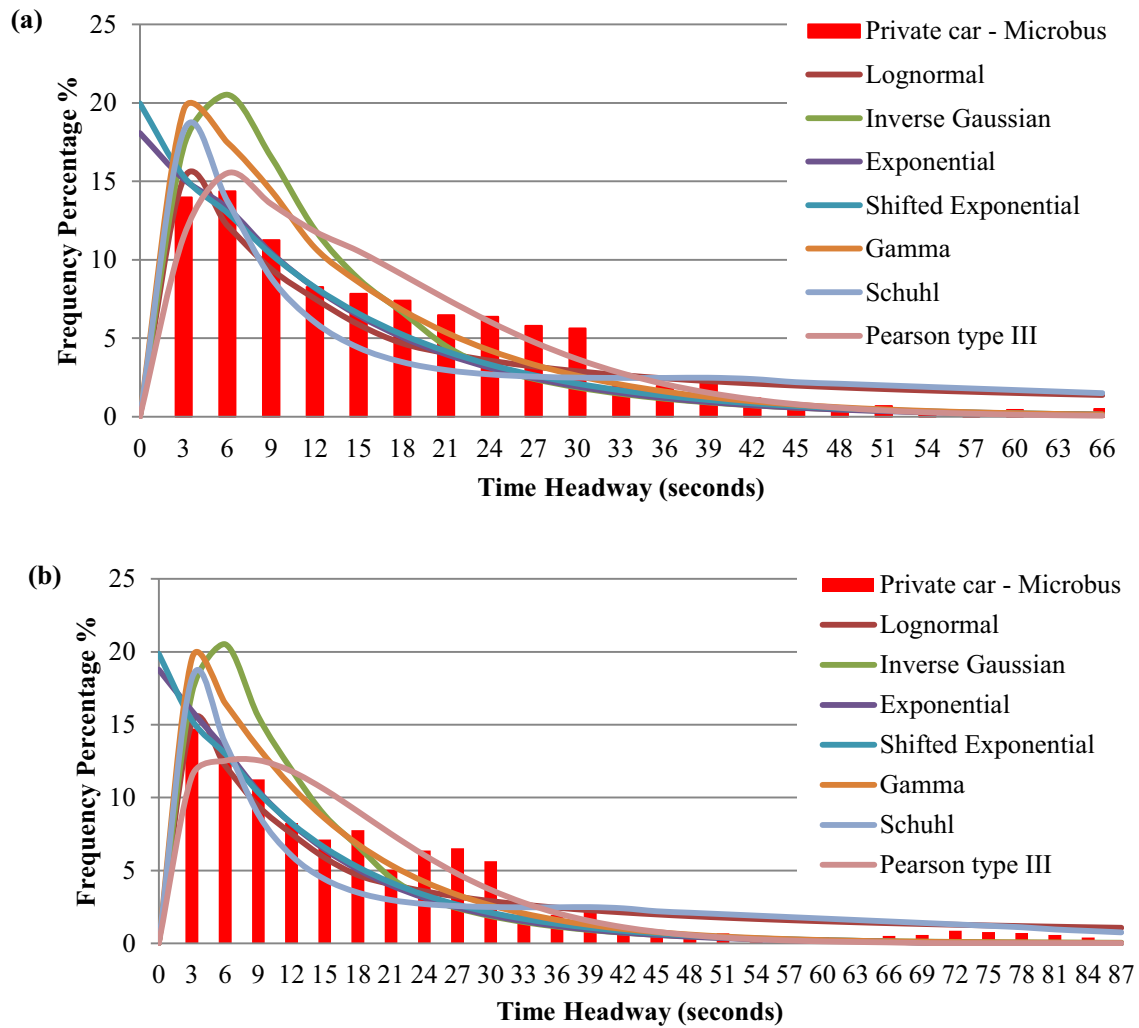


Fig. 7 Proposed distributions for “private car-microbus” category: **a** at sites S1 and **b** at site S2

during the 1 h period at both sites for both directions, namely (1) exponential, (2) lognormal, (3) gamma, (4) inverse Gaussian, (5) Pearson type III, (6) shifted exponential, and (7) Schuhl distributions.

Table 6 summarizes probability density functions (PDFs) for all proposed distributions for modeling observed headway distributions for two directions. In addition,

Table 7 summarizes the formulas used to estimate the parameters of the proposed distributions based on the maximum likelihood estimation method.

It is worth noting that to estimate the PDF for Schuhl distribution, the average time headways for the free-moving vehicles (t_f) and the constrained vehicles (t_b) are needed.

Tolle [25] observed that the peak frequencies of headways of constrained vehicles are usually somewhere in the range of 0.5 to 2.0 s. While Ayres et al. [26] found that the average time headway varies between 1 to 2.0 s for constrained vehicles during rush hours. Both authors found that the average time headway is more than 2.0 s for free-moving vehicles [25, 26].

Figures 4 and 5 show the proposed distribution plots compared to the observed headway frequency for both directions of the studied sites. For the northbound direction of site S1 (refer to Fig. 4-a), the inverse Gaussian distribution has a better match to the observed headway frequencies up to headways of 18.0 s than other distributions, after which it underestimates the observed headways (in the range 18–33 s), while the

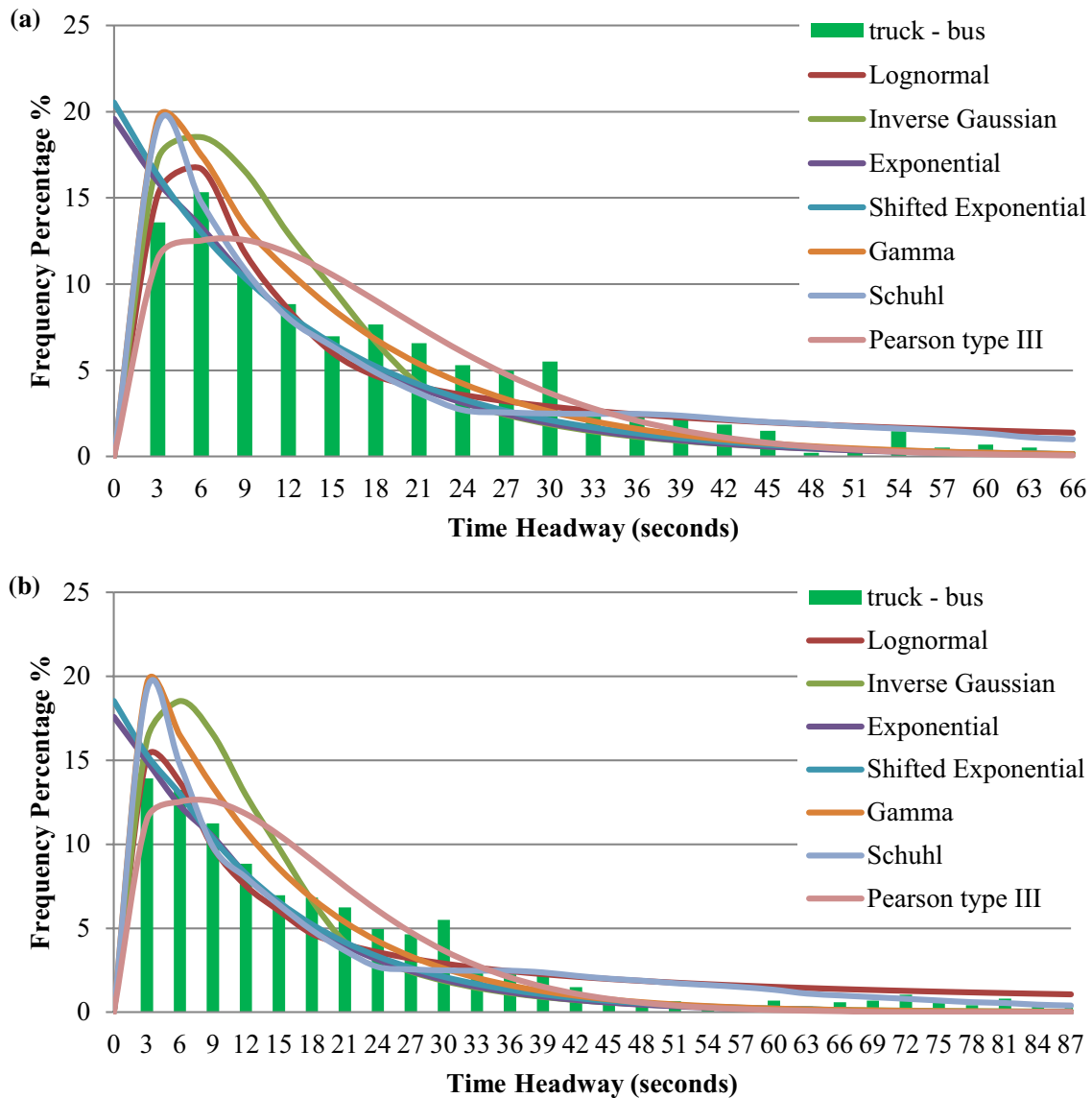


Fig. 8 Proposed distributions for “truck-bus” category: **a** at sites S1 and **b** at site S2

gamma distribution has a better match for the observed headways than other distributions, for headways on the southbound direction of site S1, as shown in Fig. 4-b. Furthermore, for site S2, the gamma distribution has a better match to the observed headway frequencies, for both directions, as shown in Fig. 5.

The impact of follower–leader vehicle type on time headway distributions

Three categories of follower–leader vehicles are analyzed based on their percentage in the observed traffic flow (refer to Table 3), namely (1) pickup-microbus, in which the

pickup vehicle is the follower and the microbus is the leader; (2) private car-microbus; and (3) truck-bus.

Figure 6, 7 and 8 show the proposed distribution plots compared to the observed headway frequency for each follower–leader at the two sites for both directions together. For the “pickup-microbus” category, all proposed distributions are accepted for both sites except Schuhl distribution that is rejected for site S2. Furthermore, Pearson type III is the best representative distribution for site S1 while exponential distribution is the best representative model for site S2. For the “private car-microbus” category, all proposed distributions are accepted for both sites. Exponential distribution is found

Table 8 Cumulative distribution function (CDF) for all proposed distributions

References	Distribution	Cumulative distribution function (CDF)
Mark and Pinsky [27]	Exponential	$F(t) = 1 - e^{-\lambda t}$
Ananth and Shah [34]	Lognormal	$F(t) = \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\ln(t) - \mu_t}{\sigma_t \sqrt{2}} \right) \right)$ $\operatorname{erf} =$ The complementary error function $\sigma_t^2 = \ln(1 + (\sigma/\mu)^2)$ $\mu_t = \ln \mu - \frac{1}{2} \sigma_t^2$
Luttinen [6]	Gamma	$F(t) = \frac{1}{\Gamma(\alpha)} \times \gamma(\alpha, \beta t)$
Singh [35]	Pearson type III	$F(t) = \frac{1}{a\Gamma(b)} \int_0^\infty \left(\frac{t-c}{a} \right)^{b-1} \exp \left(-\frac{t-c}{a} \right) dt$
Folks and Chhikara [33] Weisstein [36]	Inverse Gaussian	$F(t) = \frac{1}{2} \left(1 + \operatorname{erf} \left(\sqrt{\frac{z}{2t}} \left(\frac{t}{\mu} - 1 \right) \right) \right) + \frac{1}{2} \times e^{2z/\mu} \left(1 - \operatorname{erf} \left(\sqrt{\frac{z}{2t}} \left(\frac{t}{\mu} + 1 \right) \right) \right)$
Teodorescu and Vernic [37]	Schuhl	$F(t) = 0.574(1 - e^{-\frac{1.35t}{\theta}})$, if $0 < t < \theta$, $F(t) = 1 - 0.574 \times \left(\frac{\theta}{t} \right)^{1.35}$, if $\theta < t < \infty$ “ θ can be calculated from: $\frac{1}{2 - e^{-1.35\theta}} = 0.574$ ”
Al-Ghamdi [30]	Shifted exponential	$F(t) = 1 - \exp \left(-\frac{(t-t_m)}{(m-t_m)} \right)$

to be the best representative distribution for both sites. For the “truck-bus” category, five proposed distributions are accepted except gamma and Schuhl distributions

Goodness-of-fit (GOF) tests

To check whether the studied distributions are a statisti-

Table 9 Parameters’ estimates of the proposed distributions

Distribution	Parameter	Site S2		Site S2	
		NB direction	SB direction	EB direction	WB direction
Exponential	λ	182	265	201	278
Pearson type III	a	7.970	4.960	9.128	5.845
	b	6.410	5.590	5.050	4.550
	c	- 31.140	- 14.590	- 26.321	- 13.374
Gamma	α	0.976	1.119	0.932	1.123
	β	20.420	11.780	21.210	11.762
Lognormal	μ	19.96	13.200	19.77	13.24
	σ^2	407.63	155.50	419.84	155.50
Inverse Gaussian	Z	19.500	14.780	18.413	14.850
Shifted exponential	m	19.960	13.200	19.770	13.220
	t_m	0.100	0.110	0.101	0.112
Schuhl	$\hat{\alpha}$	101	150	116	161
	t_m	0.100	0.110	0.101	0.112
	\bar{t}_f	20.070	21.101	21.672	22.001
	\bar{t}_b	1.310	1.601	1.421	1.695

which are rejected for both sites. Furthermore, lognormal distribution is found to be the best representative distribution for both sites.

cally good fit for the observed data, two GOF tests were applied, namely (1) the Chi-square test and (2) Kolmogorov–Smirnov (K–S) test.

Table 10 Chi-square test results for both directions of the two studied sites

Direction	Distribution	Number of parameters “P”	The calculated value of “ χ^2 ”	The critical value of “ χ^2 ”	Decision
<i>Site S1</i>					
SB direction	Exponential	1	29.25	31.41	Accepted
	Lognormal	2	30.81	30.14	Rejected
	Gamma	2	28.01	30.14	Accepted
	Inverse Gaussian	2	28.55	30.14	Accepted
	Pearson type III	3	29.79	28.87	Rejected
	Shifted exponential	2	28.11	30.14	Accepted
	Schuhl	3	29.77	28.87	Rejected
NB direction	Exponential	1	32.30	31.41	Rejected
	Lognormal	2	31.41	30.14	Rejected
	Gamma	2	29.74	30.14	Accepted
	Inverse Gaussian	2	30.98	30.14	Rejected
	Pearson type III	3	31.88	28.87	Rejected
	Shifted exponential	2	30.08	30.14	Accepted
	Schuhl	3	31.67	28.87	Rejected
<i>Site S2</i>					
EB direction	Exponential	1	29.88	31.41	Accepted
	Lognormal	2	30.56	30.14	Rejected
	Gamma	2	28.34	30.14	Accepted
	Inverse Gaussian	2	30.87	30.14	Rejected
	Pearson type III	3	29.34	28.87	Rejected
	Shifted exponential	2	27.97	30.14	Accepted
	Schuhl	3	29.11	28.87	Rejected
WB direction	Exponential	1	32.77	31.41	Rejected
	Lognormal	2	33.34	30.14	Rejected
	Gamma	2	30.07	30.14	Accepted
	Inverse Gaussian	2	33.45	30.14	Rejected
	Pearson type III	3	32.23	28.87	Rejected
	Shifted exponential	2	29.87	30.14	Accepted
	Schuhl	3	32.21	28.87	Rejected

Bold represents the accepted distributions

Chi-square test

The Chi-square test was used to test whether the observed time headway follows a certain distribution or not. For example, to check whether the exponential distribution is a good fit for the data, the estimated Chi-square value is compared with the critical Chi-square from the Chi-square statistical table at the desired level of significance. In our analysis, the 5% significance level has been used. To calculate the Chi-square value for each proposed distribution, the cumulative distribution function (CDF) must be

obtained. Table 8 summarizes the CDF for each proposed distribution.

Kolmogorov–Smirnov (K–S) test

The K–S test is a nonparametric GOF test of the equality of continuous, one-dimensional probability distributions. It is usually applied in comparing a proposed distribution with the null hypothesis assumption being that the two samples are drawn from the same distribution for a given desired level of significance [38]. The K–S test tries to estimate the distance “D” between the empirical

Table 11 K–S test results for proposed distributions for the two studied locations

Direction	Distribution	K–S critical value	“D” value	Decision
<i>Site S1</i>				
NB direction	Exponential	0.100	0.075	Accepted
	Lognormal		0.152	Rejected
	Gamma		0.074	Accepted
	Inverse Gaussian		0.092	Accepted
	Pearson type III		0.130	Rejected
	Shifted exponential		0.071	Accepted
SB direction	Schuhl	0.083	0.196	Rejected
	Exponential		0.097	Rejected
	Lognormal		0.130	Rejected
	Gamma		0.083	Accepted
	Inverse Gaussian		0.144	Rejected
	Pearson type III		0.100	Rejected
<i>Site S2</i>	Shifted exponential	0.081	0.079	Accepted
	Schuhl		0.211	Rejected
	Exponential		0.071	Accepted
	Lognormal		0.155	Rejected
	Gamma		0.072	Accepted
	Inverse Gaussian		0.095	Rejected
WB direction	Pearson type III	0.084	0.100	Rejected
	Shifted exponential		0.065	Accepted
	Schuhl		0.196	Rejected
	Exponential		0.091	Rejected
	Lognormal		0.132	Rejected
	Gamma		0.080	Accepted
WB direction	Inverse Gaussian	0.076	0.101	Rejected
	Pearson type III		0.920	Rejected
	Shifted exponential		0.076	Accepted
	Schuhl		0.211	Rejected

Bold represents the accepted distributions

distribution functions of the two samples. Hashim [3] showed that the K–S test is depended on the maximum difference “D” between the sample and the hypothesized cumulative distributions. The K–S test examines whether the observations could reasonably have obtained from the specified distribution. If the “D” statistic, the most extreme absolute difference, is significant, then the null hypothesis should be rejected. If the maximum difference “D” statistic has \leq a corresponding value at the desired significance level, then the null hypothesis should be accepted.

Results

The estimated parameters for all the proposed distributions are shown for the studied sites are shown in Table 9. Table 10 summarizes the results of the Chi-square test for the studied sites, while Table 11 summarizes the results of the K–S test. In the later test, the cumulative percentages of all distributions have been compared to the corresponding observed time headway percentages, and then the K–S critical value is computed and compared to the maximum “D” value.

Based on the results (refer to Tables 10, 11), both GOF tests yielded the same results in terms of the acceptance/rejection of the proposed distribution. In addition, based on the results of the GOF tests (Tables 10, 11), the accepted distributions for site S1 are the exponential, the gamma, the inverse Gaussian, and shifted exponential for the NB direction, while the gamma and the shifted exponential distributions are accepted for the SB direction. For site S2, the shifted exponential, the gamma, and the exponential distributions are accepted for the EB direction, while the gamma and the shifted exponential distributions are accepted for the WB direction.

Effect of the Bin size

To check the effect of the bin size on the accuracy of the studied distributions, the time headway frequencies were calculated for different bin widths, namely 0.50, 1.00, and 2.00 s. Table 12 summarizes the K–S test results for the different bin widths (W). Table 12 shows that using bin sizes lower than the recommended by Rice rule did not affect the results.

Effect of vehicle type on time headway distributions

To check the effect of vehicle type on the studied distributions, the time headway frequencies were calculated for the three predefined categories. Table 13 summarizes the K–S test results for the different categories. Table 13 shows that, for “pickup–microbus” category, all the seven distributions are accepted to be used to model the headways at site S1, where at site S2, all the distributions but the Schuhl distribution are accepted to model the headways for this category. For the “private car–microbus” category, all the seven distributions are accepted to be used to model the headways at both sites.

Table 12 K–S test results at lower bin sizes for both studied sites

Direction	Distribution	K–S critical value	D value				Decision
			W=3.0S	W=2.0S	W=1.0S	W=0.50S	
<i>Site S1</i>							
NB direction	Exponential	0.100	0.075*	0.073*	0.073*	0.075*	Accepted
	Lognormal		0.152	0.155	0.163	0.166	Rejected
	Gamma		0.074*	0.077*	0.079*	0.081*	Accepted
	Inverse Gaussian		0.092*	0.092*	0.093*	0.093*	Accepted
	Pearson type III		0.130	0.130	0.130	0.131	Rejected
	Shifted exponential		0.071*	0.072*	0.078*	0.078*	Accepted
	Schuhl		0.196	0.199	0.201	0.201	Rejected
SB direction	Exponential	0.083	0.097	0.098	0.098	0.098	Rejected
	Lognormal		0.130	0.130	0.139	0.142	Rejected
	Gamma		0.083*	0.083*	0.083*	0.083*	Accepted
	Inverse Gaussian		0.144	0.155	0.157	0.157	Rejected
	Pearson type III		0.100	0.102	0.102	0.102	Rejected
	Shifted exponential		0.079*	0.080*	0.080*	0.082*	Accepted
	Schuhl		0.211	0.213	0.213	0.214	Rejected
<i>Site S2</i>							
EB direction	Exponential	0.081	0.071*	0.072*	0.076*	0.077*	Accepted
	Lognormal		0.155	0.162	0.168	0.171	Rejected
	Gamma		0.072*	0.074*	0.074*	0.075*	Accepted
	Inverse Gaussian		0.095	0.096	0.096	0.097	Rejected
	Pearson III		0.100	0.104	0.110	0.111	Rejected
	Shifted exponential		0.065*	0.066*	0.068*	0.068*	Accepted
	Schuhl		0.196	0.197	0.197	0.198	Rejected
WB direction	Exponential	0.084	0.091	0.092	0.092	0.092	Rejected
	Lognormal		0.132	0.133	0.134	0.135	Rejected
	Gamma		0.080*	0.080*	0.081*	0.082*	Accepted
	Inverse Gaussian		0.101	0.102	0.102	0.103	Rejected
	Pearson type III		0.920	0.921	0.921	0.922	Rejected
	Shifted exponential		0.076*	0.077*	0.079*	0.079*	Accepted
	Schuhl		0.211	0.212	0.212	0.221	Rejected

Bold represents the accepted distributions

*Accepted distribution, as the calculated “D” value is less than or equal to the critical value

Finally, for the “truck-bus” category, all the distributions except the gamma and the Schuhl distributions are accepted to model the headways of this category.

Conclusion

This paper presents the results of studying the best time headway distribution for Egyptian two-way, two-lane roads. The headway data from two sites from two rural TLTW roads, from Dakahliya Governorate, are used in this study. The first site (site S1) from the road that connects Mansoura city to

Damietta city, while the second site (site S2) is from the road that connects Mansoura to Dikirnis city. One-hour videotaped data from each site were collected around noon, for both directions. Based on the observed data, about 75% of travel speed measurements are relatively closer in values (between 60 to 70 km/h) for both directions. In addition, for both sites, about 38% of the observed headways are within the range of 0 and 6 s, while $\geq 50\%$ is more than 6 s and less than 30 s, while the rest of the observed headways (about 12%) are ≥ 30 s.

Seven probability distributions were considered in our study to model the observed time headways, during the 1 h period at both sites for both directions, namely (1) exponential, (2) lognormal, (3) gamma, (4) inverse Gaussian, (5) Pearson type III, (6) shifted exponential, and (7) Schuhl

Table 13 K–S test results at vehicle type categories for both studied sites

Vehicle	Distribution	K–S critical value	“D” value	Decision
<i>Site S1</i>				
Pickup–microbus	Exponential	0.083	0.073	Accepted
	Lognormal		0.064	Accepted
	Gamma		0.074	Accepted
	Inverse Gaussian		0.082	Accepted
	Pearson type III		0.052*	Accepted
	Shifted exponential		0.083	Accepted
Private car –microbus	Exponential	0.085	0.058	Accepted
	Lognormal		0.056*	Accepted
	Gamma		0.075	Accepted
	Inverse Gaussian		0.082	Accepted
	Pearson type III		0.063	Accepted
	Shifted exponential		0.074	Accepted
Truck-bus	Exponential	0.087	0.074	Accepted
	Lognormal		0.059*	Accepted
	Gamma		0.088	Rejected
	Inverse Gaussian		0.075	Accepted
	Pearson type III		0.064	Accepted
	Shifted exponential		0.075	Accepted
Truck-bus	Schuhl	0.089	0.068	Accepted
	Exponential		0.074	Accepted
	Lognormal		0.059*	Accepted
	Gamma		0.088	Rejected
	Inverse Gaussian		0.075	Accepted
	Pearson type III		0.064	Accepted
<i>Site S2</i>	Pickup vehicle–microbus	0.82	0.053*	Accepted
			0.056	Accepted
			0.074	Accepted
			0.072	Accepted
			0.068	Accepted
			0.061	Accepted
Private car–microbus	0.084	Schuhl	0.094	Rejected
		Exponential	0.038	Accepted
		Lognormal	0.036*	Accepted
		Gamma	0.078	Accepted
		Inverse Gaussian	0.080	Accepted
		Pearson type III	0.065	Accepted
Truck-bus	0.086	Shifted exponential	0.073	Accepted
		Schuhl	0.070	Accepted
		Exponential	0.064	Accepted
		Lognormal	0.053*	Accepted
		Gamma	0.087	Rejected
		Inverse Gaussian	0.077	Accepted
Truck-bus	0.086	Pearson type III	0.054	Accepted
		Shifted exponential	0.065	Accepted
		Schuhl	0.087	Rejected
		Schuhl	0.087	Rejected

Bold represents the accepted distributions

*Lowest “D” value compared to critical value

distributions. In addition, the Chi-square and the K–S goodness-of-fit measures were used to assess which distribution fits the data better.

Based on the results of this research paper, some conclusions can be considered as follow:

1. Both of the GOF tests reveal the same results in terms of the acceptance and the rejection of the proposed distributions compared to the observed headways;
2. For site S1, the accepted distributions are the exponential, the gamma, the inverse Gaussian, and the shifted exponential distributions for the NB direction; while the accepted distributions for the SB direction are the gamma and the shifted exponential;
3. For site S2, the accepted distributions are the exponential, the gamma, and the shifted exponential distribution for the EB direction, the accepted distributions for the SB direction are the gamma and the shifted exponential distributions;
4. Based on the K–S test, the shifted exponential distribution yielded the lowest difference in both directions of the site S1, while the gamma distribution yielded the lowest difference (i.e. D-value) in both directions of the site S2. Thus, both the gamma and the shifted exponential distributions would be good representatives for modeling the time headways for the Dakahliya Governorate rural TLTW roads. These distributions were considered based on traffic flow rate between 182 and 278 vehicles/h/direction
5. Based on the vehicle type, three selected categories, pickup vehicle–microbus, Pearson type III and Schuhl distributions were found to be the best representative distributions for site S1 whereas the exponential and lognormal distributions were proved to be the best representative distributions for site S2. For private car–microbus, the exponential and lognormal distributions were found to be the best representative distributions for the two sites. Finally, for truck-bus, Pearson type III and lognormal distributions were found to be the best representative distributions for the two sites
6. Bin widths (0.50, 1.0, and 2.0 s) lower than the recommended (3.0 s) by the Rice rule did not affect the acceptance/rejection results of the proposed methods. Hence, the Rice rule would be adequate to determine the recommended bin width for the observed time headways.

It is worth noting that the data used in this analysis are for uncongested traffic flow condition and only for 2 h from only two sites. Hence, further research using more data that represent both the congested and uncongested traffic flow conditions is recommended to support the findings of this

research and to validate the headway distribution for different traffic flow conditions.

Declarations

Conflict of interest This is to confirm that the authors of this paper certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge, or beliefs) in the subject matter or materials discussed in this manuscript.

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