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TECHNICAL NOTE

Finite element slope stability analysis of Souk Tleta dam by shear strength reduction technique

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Abstract Slope stability analysis of earth dam is very important to ascertain the stability of the structure. The stability of earth dam depends on its geometry, components materials, water pressure and the forces to which it is subjected. Souk Tleta earth dam, currently under construction, is located on Bougdoura River, in north Algeria. The main purpose of the dam is to store surface water, for irrigation and domestic supply. In this study, reducing gradually the shear strength parameters technique is used to analyze the static slope stability of Souk Tleta zoned embankment dam based on the numerical simulation using Plaxis 2D finite element software. The deformations within the dam and the foundation at the end of construction and reservoir impoundment loading conditions and the corresponding factor of safety have been simulated. The results show that the displacements occurred in the dam body and foundation are the largest at the end of construction and lower during the reservoir filling. The safety factor in the different conditions decreases with the increase in reservoir water level.

Keywords Slope stability · Static analysis · Earth dam · Finite element software · Factor of safety · Shear strength reduction · Displacement

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Introduction

Slope stability is a challenging problem in geotechnical engineering. Evaluation of embankment dam stability and deformation is a crucial aspect that each earth dam structure must be verified.

Slope stability is one of the main and classical problems of embankment dams that have been studied by numerous authors in 2D using variety of methods based on traditional limit equilibrium approach [1, 7].

Limit equilibrium analysis is widely used in practice to calculate the safety factor of slopes. Fellenius [7] and Terzaghi [21] used the method to analyze the problem of soil slope stability at failure using the elastic perfectly plastic Mohr–Coulomb criterion. All thought the principle of the method has been refined and became simplified methods by dividing the soil into slices. These methods include: Bishop's modified method [1], Janbu's generalized procedure of Slices [9], Morgenstern and Price's method [14], and Spencer's method [20].

Due to its simplicity, limit equilibrium method has been for a long time one of the most popular methods in geotechnical engineering.

In the last few decades, with the development of personal computer, finite element method has been increasingly used in slope stability analysis. Comparing with the traditional limit equilibrium approach, the finite element method has many advantages such as no advance assumption about location or shape of the failure surface, slice side forces and their directions, and a good application for complex soil profiles.

With numerical methods using finite element method, there are two basic approaches for calculating the factor of safety "shear strength reduction technique" [30] and "enhanced limit method" introduced by [16].

The shear strength reduction technique is one of the important numerical methods for dam slope stability analysis and successfully used in the study of geotechnical structures (excavations, embankments, earth dams and landfills). It can get the safety factor, displacement and deformation of the dam slope.

Strength reduction means reducing gradually the shear strength parameters (cohesion C and internal friction angle φ) of soil slope by applying finite element [8, 13, 28] or finite difference programs [3–5] until the first indication of failure appears.

The calculation of factor of safety (FOS) based on finite element is possible using the strength reduction method and it is defined as the ratio of soil or rock at failure to the shear strength on the failure surface.

The concept of shear strength reduction method was first introduced by Zienkiewiez et al. [30] to evaluate the performance of slopes. This technique has been subsequently used as a common approach in several numerical codes for slope stability analysis, such as [6, 8, 10, 13, 15, 16, 18, 19, 22, 23, 28, 29] and many other researchers.

The case study is Souk Tleta zoned embankment dam, located on Bougdoura River immediately downstream of the confluence of Tleta and Tala Imedrane Rivers, 20 km West of Tizi-Ouzou city in north Algeria (Fig. 1). Souk Tleta dam has a crest length of about 200 m, a maximum height above the river bed level of about 95 m and an estimated total storage capacity of about 96 million m^3. The stored water will be used for various purposes, including irrigation and domestic supply.

Earth dams need to be checked for a number of different load cases of slope failure such as during and at the end of construction, steady-state seepage conditions, operational conditions which include rapid drawdown conditions, construction modification and earthquake loading.

According to the Algerian Earthquake Regulations RPA99 Version 2003 of Algeria, the site is situated in a zone known as moderate to weak seismicity. Therefore, this study discusses the static slope stability analysis.

The aim of the present study is to examine numerically the slope stability of Souk Tleta earth dam using the finite element method with strength reduction technique in two-dimensional computer program Plaxis Version 2010.

In this paper, it is proposed to evaluate the displacements and the corresponding factor of safety of the Souk Tleta dam for four different essential conditions in the service life of a dam: the stage at the end of construction where large deformations are expected in the dam and its foundation, the minimum water level condition, where the pore pressures appear in the dam, the normal water level condition, which corresponds to the most frequent state and the maximum water level condition caused by an exceptional flood.

This study also allows checking the geometric design of the dam and making the right choice of the embankment construction materials through the determination, in the laboratory and in situ tests, of the various geotechnical parameters affecting the slope stability.

Geology

Geological factor plays a major role in designing and constructing a dam. The regional geology of Souk Tleta dam and its reservoir are located on the meridional bank of

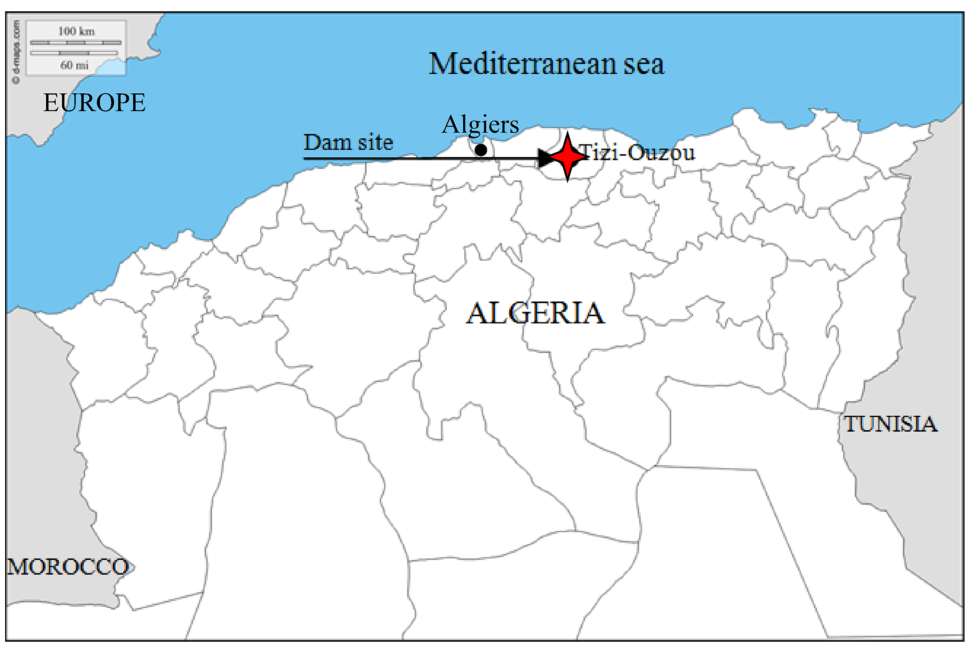

Fig. 1 Location of Souk Tleta dam

sedimentary basin of Lower Miocene (Burdigalian) of Tizi-Ouzou city. In the area of the dam, Burdigalian sediments cover the site of under-Miocene with a transgressive facies which crops out at the upstream of the gorge of Souk Tleta valley, and covers a large surface right up to the foot of limestone chain of Djurdjura.

Geological overview of Fig. 2 shows the outcrops extension of lithofacies in the dam site. The foundation of Souk Tleta dam is composed of two layers: first, an alluvial sediments layer at the top, having an average thickness of about 21 m; second: Burdigalian sandstone which forms the two supports of the dam, it is a transgressive sandstone formation on phylladic basement. The transgression is characterized by a series of conglomerate, composed of few meters of puddingstone locally coarse, and then follows fine sandstone, siltstone/pelites in alternation. The conglomerate series thickness ranges between 15 and 20 m. Above conglomerate, Burdigalian becomes fairly homogenous sandstone and stratified in layers; the joints of stratification dip toward the North, toward downstream according to the flow direction of the river with a dip of 15°–20°.

Geometry and soil properties of the studied earth dam

Every slope soil is subjected to shear stresses on internal surfaces and its stability is highly correlated with its geometry. The influence of geometry variation on stability of earth dam has been the subject of many studies; Yuzhen and Zhang [27] investigated the 3D slope stability under effect of geometrical characteristics and topography of the canyon site of core thickness on stability, Vrubel and Říha [26] evaluated the safety factor for several small dams shape and Khanna et al. [11] studied the influence of geometry of an earth dam on its slope stability.

Figure 3 shows schematic cross section of the Souk Tleta earth dam and its foundation layers considered in this paper. Table 1 illustrates the geometrical characteristics of the dam. The study was conducted at the cross section of the dam with the largest section, i.e. with the greatest height.

Souk Tleta dam will be constructed with locally available materials. Material deposits of five different areas near the dam site are investigated for construction purpose. The volume of aggregates required for the construction of the Souk Tleta dam is approximately 1,173,000 m^3. The construction materials need to have the correct physical properties to permit the dam safety. Laboratory and in situ tests were performed to discover the physical and mechanical properties of 62 selected core samples taken from boreholes. To achieve this, 38 boreholes, with a total length of 977 m were drilled on the 5 areas.

To assess the condition of the dam foundations, Laboratory experiments were carried out on 32 samples to determine the physical and mechanical properties. In this work, 19 boreholes with 870 m length were drilled on the dam axis.

The computational parameters φ and C of Mohr–Coulomb model and elastic modulus E for all dam materials are determined through the triaxial compression test. Poisson's ratio ν is determined through engineering experience.

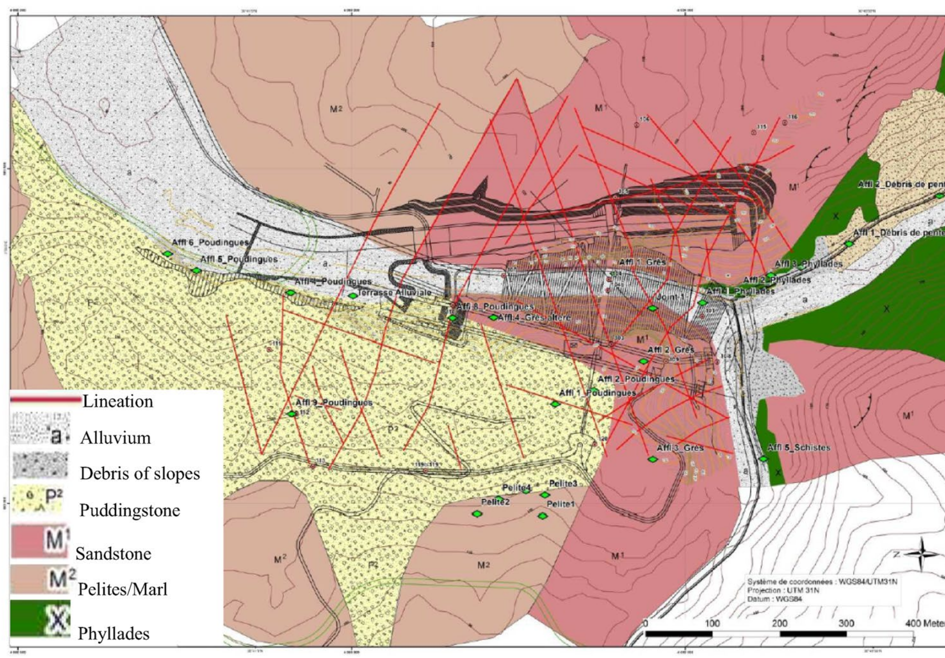

Fig. 2 Outcrops extension of lithofacies in the dam site

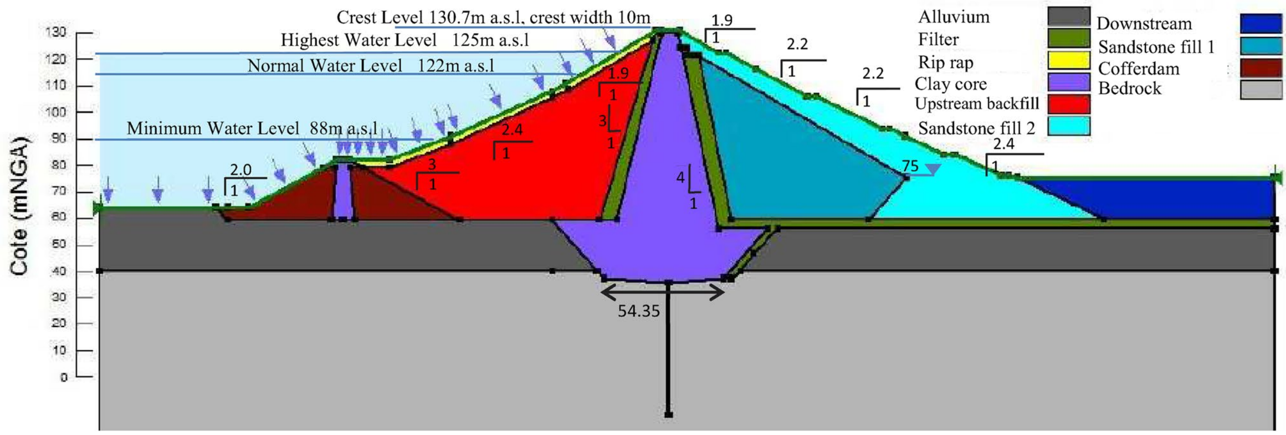

Fig. 3 Geometry of the cross section of the Souk Tleta Earth Dam

Table 1 Geometrical characteristics of Souk Tleta dam

Parameter	Value
Dam's height	95 m
Crest level	137.5 m above sea level
Crest length	200 m
Crest width	10 m
Minimum water level	88 m ASL
Normal water level	122 m ASL
Highest water level	125 m ASL
Area of the watershed	465 km^2
Storage capacity	96 M m^3

The permeability k of dam materials is measured in laboratory from falling head permeability tests. In dam foundation, Water Pressure Tests (WPT) are used to determine the permeability of the rock mass. The laboratory tests are carried out in the Laboratory of Geomaterials, Environment and Development (LGED).

The investigations were performed to characterize the materials and determine the relevant parameters for slope stability analyses. The necessary input material properties used for the slope stability evaluation are summarized in Table 2. Only the soil mechanics parameters and properties concerning the slope stability analysis of the body of the Souk Tleta earth embankment dam are given.

Stability analysis and loading conditions

Finite element method has been widely accepted for the analysis of slope stabilities. To reflect the behavior of Souk Tleta dam in this study, PLAXIS 2D finite element software is used with Elastic-perfectly plastic Mohr–Coulomb model for all layers of dike and foundation materials.

PLAXIS computer program is applicable to many geotechnical problems, including stability analysis calculations and was used for the slope stability analysis of earth dams in numerous publications [12, 17, 24, 26] and others.

Table 2 Parameters used for the Slope Stability Analysis of Souk Tleta dam

Material type	Unit weight γ_h (kN/m^3)	Saturated unit weight γ_{sat} (kN/m^3)	Internal angle of friction φ (°)	Cohesion of soil C (kPa)	Poisson's ratio ν	Young modulus E (MPa)	Permeability k (m/day)
Filter	19	21	34	0	0.25	100	8.64
Clay core	18	19	18	10	0.3	25	0.864×10^{-3}
Sandstone fill 1	19.5	20	30	0	0.23	90	0.0864
Sandstone fill 2	21	21.5	34	0	0.23	90	0.0864
Upstream backfill	16	19	10	0	0.3	90	0.0864
Rip rap	21	22	30	0	0.3	55	0.0864
Alluvium	20	21.5	32	0	0.23	35	0.864
Bedrock	21.3	21.8	28	20	0.3	150	8.6×10^{-3}

Stage of construction has a significant effect on stress distributions and deformations; therefore, analysis is carried out in different stages to reflect the construction conditions.

In the finite element method, a continuum is divided into a number of elements; each element consists of a number of nodes. The slope was modeled in the input module of PLAXIS, based on 15-nodded elements in a plane strain model. The finite element mesh generated in the simulation of slope stability analysis is a very fine mesh to obtain the least possible factor of safety (FOS), and consists of (12983) 15-nodded triangular elements. The vertical boundaries are free to move, whereas the horizontal boundary is considered to be fixed.

The finite element mesh generated in the simulation of slope stability analysis is a very fine mesh to obtain the least possible factor of safety (FOS). It consists of (12,983) 15-nodded triangular elements and 1587 elements which have an average size of 8.915 m. The vertical boundaries are free to move, whereas the horizontal boundary is considered to be fixed.

The 2D view for the finite element mesh of the dam and its surrounded soil mass is illustrated in Fig. 4.

Two conditions of reservoir filling were examined, as follows:

Right after construction condition

The calculation for the analysis of the dam after construction is divided into alternate construction phases until the end of construction. The critical condition to be analyzed is at the completion of embankment dam construction but prior to filling with water.

Cases of steady-state seepage conditions

The FE stability analysis with Plaxis 2D and steady state seepage analysis was conducted for Souk Tleta embankment dam for low and high water levels in the river. Reservoir impoundment is carried out gradually over several phases until:

- The minimum water level, corresponding to the minimum stored water level (88 m ASL).
- Normal water level (122 m ASL).
- The highest water level (flood level: 125 m ASL).

Slope stability analysis results and discussion

The safety factor was assessed for the dam cross section of the greater height and computed using the '$c - \varphi$ reduction' procedure. For each load case of steady state condition, appropriate slope stability analysis is computed based on shear strength reduction method employed with the use of finite element method.

Two-dimensional program Plaxis 2010 [2] computes the factor of safety as the ratio of the available shear strength to the strength at failure by summing up the incremental multiplier (Msf) as defined by:

$$\mathrm{FS} = \frac{\text{available strengh}}{\text{shear strength at failure}} = \text{value of } \sum M_{\mathrm{sf}} \text{at failure}$$

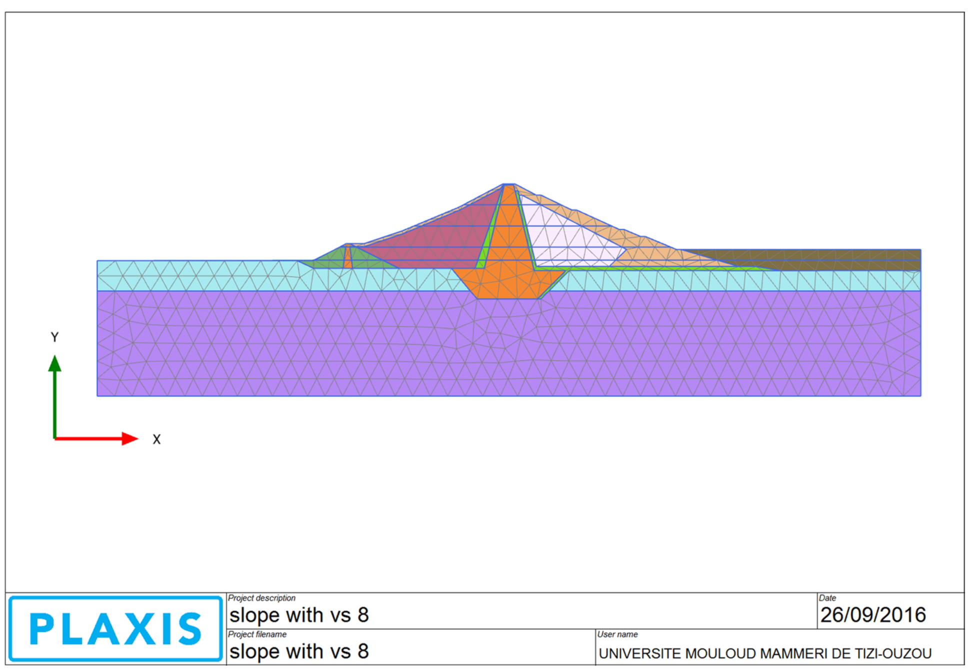

Fig. 4 Finite element mesh

Typical minimum acceptable values of factor of safety are about 1.3 for the end of construction and multistage loading, and 1.5 for normal long-term loading conditions.

The values of factor of safety listed in Slope Stability engineer manual from US Army Corp EM 1110-2-1902 [25] provide guidance but are not prescribed for slopes other than the slopes of new embankment dams.

End of construction condition

The end of Construction Condition is a critical stage for the upstream and downstream slopes. In this first stage, the stability analyses have been performed for different upstream and downstream slopes for the purpose of optimizing the volume of the dam body and materials. The analyses have been performed for the most critical cross-section taken along the highest part of the dam.

The result of upstream slope analysis under end-of-construction condition is similar to that for downstream slope.

Numerical analysis of unsaturated dam in the end of construction gives a safety factor of 1.62.

The total displacements are very important when studying slope stability problems. For this study, the total settlement which is computed for the end of construction stage can be seen in Fig. 5.

The dam body undergoes a maximum displacement of 24 cm which is due to dead load of the dam and the settlement of different zones and foundation of the structure. The calculation of the displacements during this step is necessary because the imposed loads are very high and can lead to instability and even to a slope failure.

As the dam rests on alluvial sediments, the settlements of the crest increased by the compression of the foundation materials produced by the weight of the dam. Usually, only the dead load of the dam acts during the construction period until the end of its construction. The sequence of constructing the embankment dam, by placing compacted layers of material, causes settlements of its crest, slopes and foundation (Fig. 5).

The minimum water level (88 m ASL)

Before reservoir filling analysis, all displacements obtained from the construction model were set to zero. This was done to exclude construction related deformations and obtain accurate displacement created by the first filling.

Following the construction of Souk Tleta embankment dam, deformations occur during the first reservoir impoundment period. The study of minimum water level condition is justified by the fact that during the first impounding, the dam experiences small deformations (1 cm). Deformations occur at the upstream toe of the dam at the level of 88 m and correspond to an establishment of a new steady state of the dam led by both water and pore pressures at the upstream.

When the water in the reservoir is at its lowest level, total displacements in the dam are illustrated in Fig. 6. Numerical analysis of dam for the minimum water level shows a factor of safety equal to 1.62.

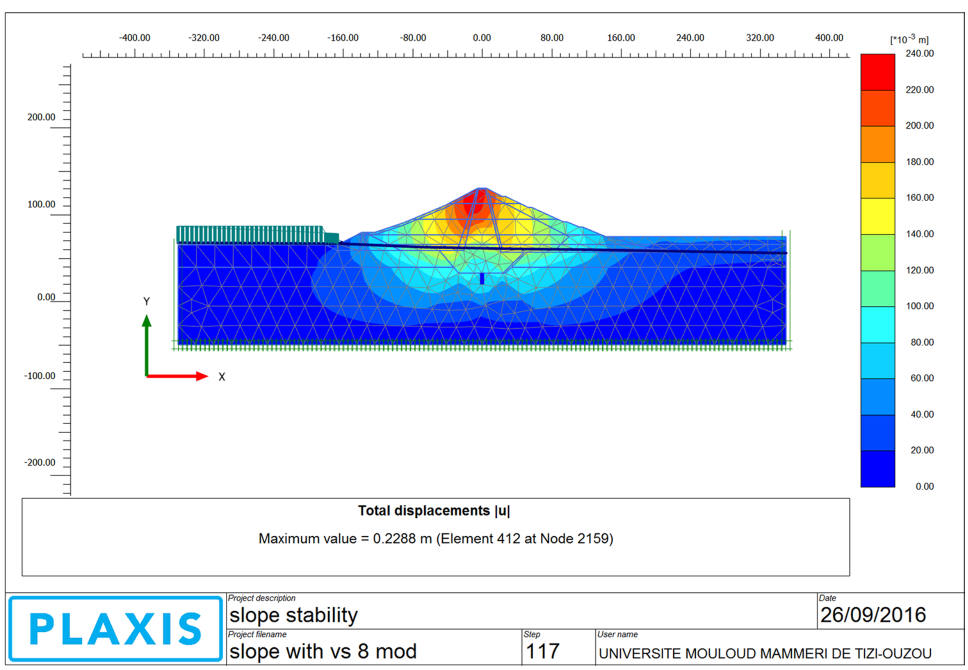

Fig. 5 Total displacements in the dam right after construction, FS = 1.62

Normal water level (122 m ASL)

Throughout the service life of a dam, the Normal water level is the level that is frequently reached and deformations can occur in the embankment and its foundation especially when the level undergoes fluctuations over the seasons alternating wet and dry periods.

The impoundment modeling is divided into smaller substages to enable a gradual increase of water level to be simulated. When the storage reservoir filling reaches the normal water level of 122 m ASL, the seepage line is at the same level, and this is considered as a critical condition.

The rise of water level in the reservoir and seepage through the dam body increase both pore water pressure and weight of the dam. Displacements increase from the bottom to the crest, the total displacements obtained for both upstream and downstream are illustrated in Fig. 7, and the factor of safety obtained for this condition is 1.54.

The highest water level (125 m ASL)

The highest water level can be caused by heavy rainfalls and waves action. The Souk Tleta dam must be designed to withstand exceptional floods that would cause the rising of water level in the reservoir. This condition is considered as critical in the service life of a dam, although for Algerian dams this level is rarely reached.

To evaluate how resistant materials are to water flow, total displacements obtained in case of steady state condition with the highest water level are illustrated in Fig. 8.

When the reservoir filling reaches the highest level, the lowest factor of safety of 1.52 is observed.

During construction, settlement rapidly increased with a continuous increase in dam weight load. During reservoir filling, settlement increased with water pressure, but the deformation rate was slower than that during dam construction. The settlement increased slowly and tended to stabilize after reservoir filling. However, the displacement appeared by the fill load can be compensated by filling more materials (Fig. 8).

For the behavior of the dam after impounding, displacements occurred by water pressure in the dam body and alluvium foundation are observed. The maximum displacements are 1, 6 and 2 for minimum, normal and highest water level conditions, respectively.

Factor of safety for all examined analysis conditions are summarized in Table 3. The results show that the safety factor of the dam decreased along with water level increasing in the reservoir. The Factor of Safety values under static condition are found to be greater than 1.5, which means the slope is safe under static condition.

Conclusions

The aim of the present study was to examine numerically the slope stability of Souk Tleta earth dam via the shear strength reduction method with the use of PLAXIS two dimensional software. The stability for each load case combination of the dam was expressed via a safety factor.

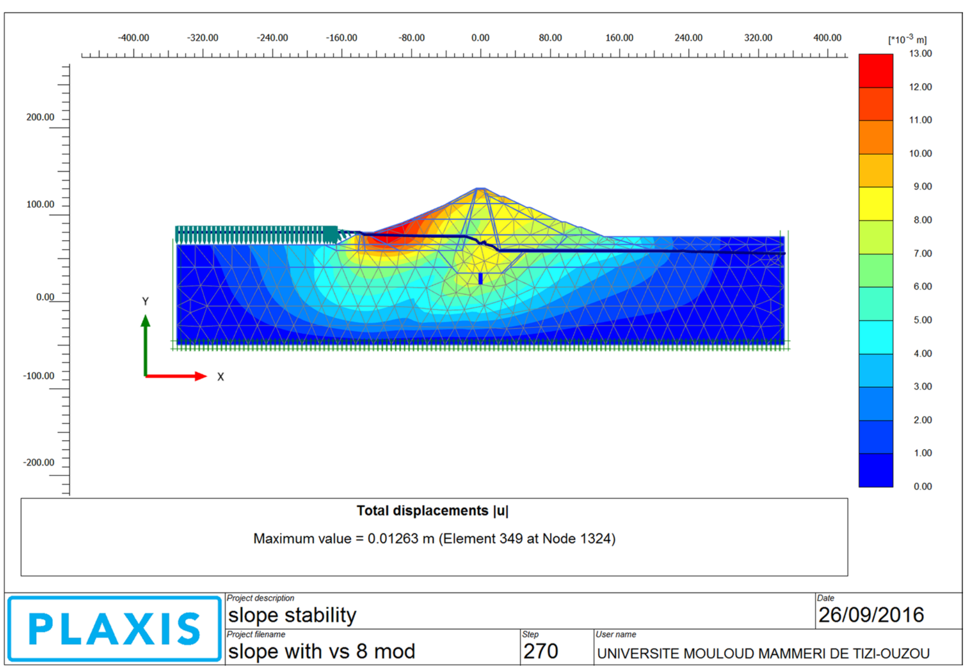

Fig. 6 Total displacements in the dam in minimum steady-state water level of 88 m ASL, FS = 1.62

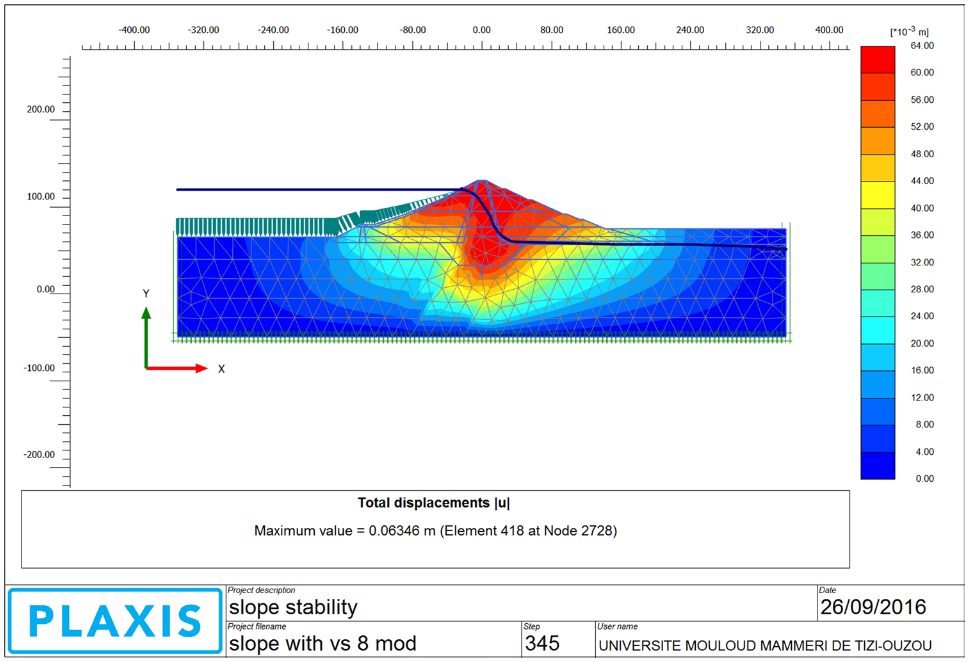

Fig. 7 Total displacements in the dam in normal steady-state water level of 122 m ASL, FS = 1.54

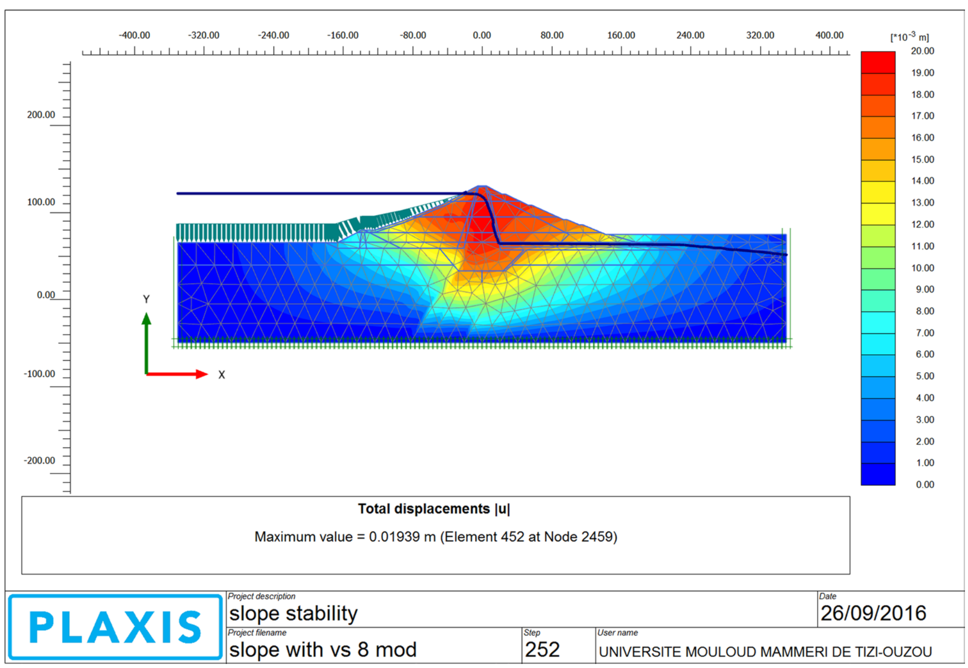

Fig. 8 Total displacements in the dam in steady-state high water level of 125 m ASL, FS = 1.52

Table 3 Factor of safety summary

Loading conditions	Factor of safety
Right after construction condition	1.62
Minimum water level	1.62
Normal water level	1.54
High water level	1.52

Analyzing the static simulation of Souk Tleta dam using the strength reduction principle, displacements and corresponding safety factor are obtained for both end of construction and reservoir filling conditions.

The study was performed on the highest profile of the zoned embankment dam. The dam body and its foundation parameters used for the analysis were determined based on laboratory and in situ test results. Using these parameters, slope stability was conducted.

The total displacements are very important when studying most of geotechnical problems. The analysis results indicated that the displacements occurred in the dam body and in the alluvium foundation layer are the largest at the end of the construction. The displacements are mainly caused by the fill materials load which can be compensated by filling more materials.

The stability of the dam body at the end of construction is highly related to fill strength parameters obtained from site geotechnical investigations. It is particularly recommended that laboratory tests to be performed in accordance with real conditions.

The construction materials must be selected very carefully. This selection of materials allows composing the dam of homogeneous zones. It is also necessary to mention that maximum compaction of material layers gives maximum strength to the dam and minimum permeability to the sealing elements.

On the other hand, the analysis results revealed that after the dam construction completion, lower displacements occurred in the dam body and foundation by impounding.

The reservoir filling makes the upstream side of the dam saturated, and causes its uplift according to Archimedes' principle.

The process of filling induces also settlements, as far as water causes movements of individual particles and further compaction of the dam materials.

Finally, it is worth mentioning that the behavior of Souk Tleta dam in its largest cross section was reasonable in term of displacements at the different conditions. The slope stability is controlled by the shear strength of the dam materials and the foundation.

Factor of safety presents the main key for the slope stability analysis. The safety factor evaluated using PLAXIS is found to be greater than 1.5 for all studied conditions which means that the dam slopes are stable under static conditions.

It can be concluded that the soil properties chosen for the construction of the dam are adequate and provide relatively high factor of safety which means less risk of failure.

The stability criteria have been well satisfied with the designed shape of the dam body during and at the end of construction and also for different water level conditions. The numerical results show that the strength reduction method used in 2D finite element method is very effective in capturing the progressive failure induced by reservoir dead load and water level fluctuations.

In view of the obtained results, when studying the problems related to slope stability in earth dams, it is recommended that inspections should be carried out on the dam body during its service life, so that the results serve to assess the accuracy and reliability of the calculation approaches.

According to the Algerian Earthquake Regulations RPA99 version 2003 of Algeria, the site of Souk Tleta dam is situated in a zone known as moderate to weak seismicity. Based on this information, a static study of the slope stability of the dam was carried out. This does not prevent the completion of this work by a dynamic study taking into consideration the data of the largest earthquake recorded in north Algeria.

Similarly, to get closer to reality, a numerical analysis with plaxis 3D can be proposed to understand the effect of the third dimension and compare its FOS and deformations results to those obtained from the 2D analyses.

References

1. Bishop AW (1955) The use of the slip circle in the stability analysis of slopes. Géotechnique 5(1):7–17
2. Brinkgreve RBJ, Engin E, Swolfs WM (2010) PLAXIS 2D 2010 user manual, Plaxis bv. Deft, the Netherland. ISBN-13: 978-90-76016-08-5
3. Cala M, Flisiak J (2003) Slope stability analysis with numerical and limit equilibrium methods. In: 15th international conference computer methods in mechanics, Gliwice, Poland, pp 1–4
4. Cheng YM, Lansivaara T, Wei WB (2007) Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods. Comput Geotech 34:137–150
5. Dawson EM, Roth WH, Drescher A (1999) Slope stability analysis by strength reduction. Geotechnique 49(6):835–840
6. Donald IB, Giam SK (1988) Application of the nodal displacement method to slope stability analysis. In: Proceedings of the fifth Australia-New Zealand conference on geomechanics Sydney, Australia, pp 456–460
7. Fellenius W (1927) Erdstatische Berechnungen Mit Reibung Und Kohaesion. Ernst, Berlin
8. Griffiths DV, Lane PA (1999) Slope stability analysis by finite elements. Geotechnique 49(3):387–403
9. Janbu N (1973) Slope stability computations. In: Hirschfield E, Poulos S (eds) Embankment dam—engineering. Wiley, New York, pp 47–86
10. Kainthola A, Singh PK, Wasnik AB, Sazid M, Singh TN (2012) Finite element analysis of road cut slopes using Hoek and Brown failure criterion. Int J Earth Sci Eng 5(5):1100–1109
11. Khanna R, Datta M, Ram GV (2017) Influence of core thickness on stability of downstream slope of earth and rockfill dams under end-of-construction and steady-state-seepage: a comparison. Int J Geotech Eng. doi:10.1080/19386362.2017.1318230
12. Krishna PA (2006) Slope stability evaluations by limit equilibrium and finite element methods. Doctoral Thesis at NTN. ISBN:82-471-7881-8
13. Matsui T, San KC (1992) Finite element slope stability analysis by shear strength reduction technique. Soils Found 32(1):59–70
14. Morgenstern NR, Price VE (1965) The analysis of the stability of general slip surfaces. Geotechnique 15(1):79–93
15. Maji VB (2017) An insight into slope stability using strength reduction technique. Geol Soc India 89(1):77–81. doi:10.1007/s12594-017-0561-7
16. Naylor DJ (1982) Finite elements and slope stability. Numerical Methods in Geomechanics. Springer, Dordrecht, pp 229–244. doi:10.1007/978-94-009-7895-9_10

17. Rawat S, Gupta AK (2016) Analysis of a nailed soil slope using limit equilibrium and finite element methods. Int J Geosynth Ground Eng. doi:10.1007/s40891-016-0076-0
18. Roosta RM, Sadaghyani MH, Pak A (2005) Strength reduction technique in stability analysis of jointed rock slopes. Int J Civ Eng 3(3&4):152–165
19. Rachez X, Billaux D, Hart R (2002) Slope stability analysis with integrated shear strength reduction algorithm. Numerical Methods in Geomechanical Engineering. Presses de I'ENPC/LCPC, Paris, pp 731–736
20. Spencer EE (1967) A method of the analysis of the stability of embankments assuming parallel interslice forces. Géotechnique 17:11–26
21. Terzaghi K (1943) Theoretical soil mechanics. Wiley, New York
22. Ugai K (1989) A method of calculation of global safety factor of slopes by elasto-plastic FEM. Soils Found 29(2):190–195
23. Ugai K, Leshchinsky D (1995) Three-dimensional limit equilibrium and finite element analyses: a comparison of results. Soils Found 35:1–7
24. Uromeihy A, Barzegari G (2007) Evaluation and treatment of seepage problems at Chapar-Abad Dam. Iran Eng Geol 91:219–228
25. US Army Corps of Engineers (2003) Slope stability engineer manual No. 1110-2-1902 in: EM 1110-2-1902 US Army Corps of Engineers Washington, DC 20314-1000
26. Vrubel J, Říha J (2017) Discussion on the safety factors of slopes recommended for small dams. Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis 65(2):569–576. doi:10.11118/actaun201765020569
27. Yuzhen Yu, Zhang B (2005) Stability of earth–rockfill dams: influence of geometry on the three-dimensional effect. Comput Geotech 32:326–339
28. Zheng H, Liu DF, Li CG (2005) Slope stability analysis based on elastoplastic finite element method. Int J Numer Methods Eng 64:1871–1888
29. Zheng H, Sun G, Liu D (2009) A practical procedure for searching critical slip surfaces of slopes based on the strength reduction technique. Comput Geotech 36:1–5
30. Zienkiewicz OC, Humpheson C, Lewis RW (1975) Associated and non-associated viscoplasticity and plasticity in soil mechanics. Géotechnique 25(4):671–689