



# Reliability analysis of Pole Kheshti historical arch bridge under service loads using SFEM

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## Abstract

The main purpose of this paper is the probabilistic safety analysis of the historical masonry arch bridges (HMAB) and to calculate its reliability index (RI) using the “probabilistic design system” of the ANSYS software. In evaluating the reliability of bridge, the load-resistance model has been used to express the bridge failure functions. Calculating the RI requires the definition of loads effects on the structure and structure resistance. The load and resistance implicit functions are evaluated by stochastic finite element method and the Monte Carlo method has been used for laboratory simulation. The sampling method is the Latin hypercube sampling. The innovations in this paper is to use the functions dependent on parameters, modulus of elasticity, Poisson ratio, density of materials, and traffic load of bridge deck. The number of random parameters is 19. These random parameters are defined by the Log-normal distribution function. In this paper, the reliability status of bridge is investigated in the ultimate limit state under gravitational loading. The constitutive law of the bridge material is considered to be linear elastic. Three types of compressive, tensile, and allowable deflection are considered as limit states of the present research. The case study of the Pole Kheshti Langroud HMAB showed that the required safety is not provided for the ultimate limit state and the bridge is at risk of failure. The RI of bridge in the tensile limit state is lower than the target RI. The sensitivity analysis of random variables of the load and resistance implicit functions to the deflection and tensile responses is investigated, and random parameters with more impact are specified. In the stress limit state and deflection limit state, the modulus of elasticity and weight per unit volume of the sidewalls have the greatest impact on safety, respectively.

**Keywords** Stochastic finite element (SFE) · Deterministic finite element (DFE) · Monte Carlo · Sensitivity analysis (SA) · Epistemic uncertainty · Random variable (RV)

## 1 Introduction

In this paper, the safety status of a HMAB made of bricks has been investigated. The safety in the structure means more resistance of the structure than the applied load. The careful examination of the safety status of the HMAB requires more accurate understanding of the load and resistance implicit functions. In many studies, the popular finite element method has been used in the assessment of the safety of the historical masonry structures (HMS) [1–12].

The loads applied to the structures have a random nature. The structural resistance also depends on the physical and

mechanical properties of the materials and structural geometry. Given the complex geometry and large dimensions of the HMAB, the accurate safety status cannot be achieved using the classical methods of structural analysis. Using statistical studies conducted by research centers and various researchers throughout the world, there are some suggestions for the statistical properties of the parameters affecting the structure behavior [13–17].

Uncertainty in identification of physical and mechanical properties of materials for HMS and old structures which are the heritage of every country is more complicated than new and existing structures with short life and engineering documentations. The deterioration of the physical and mechanical properties of materials due to erosion and moisture during long years and impossibility of destructive and semi-destructive tests in order to identify the material properties are some causes of this problem. Besides, the HMS are the cultural heritage that indicate the civilization and history of each community and

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nation, and in many cases, they are commercially beneficial in the tourism industry and are considered among the highly important structures. The nondestructive tests and more precise analyses such as reliability analysis which have a higher computational cost, are inevitable for such important structures. The reliability analysis in important structures such as arch dams and HMAB has been evaluated in various studies [18–21]. The probabilistic analysis of a HMAB has been carried out by Hacıfendioğlu et al. [22] considering the modulus of elasticity, Poisson ratio, and density of materials as random variable (RV). The safety of the bridge was examined under the single component horizontal ground-motion record. One of the findings of this research is that the changes in modulus of elasticity have the greatest impact on the bridge seismic response, and also, the weight per unit volume of the materials has no significant effect on the bridge seismic response. It was also found that the type of distribution used to determine the random nature of the mechanical properties of materials has no significant effect on the structural response, but in this research, the RI and SA of response to random parameters are not presented [22]. The SA analysis of a historical arch bridge has been made using ANSYS Software by Hardil et al. [5], and the negative effects of the bridge reconstruction and preservation on its structural behavior has been investigated. In this research, the probability of failure and RI of the reconstructed bridge are not provided [5]. Due to the heritage constructions structural variability and complex performance, there are many interesting cases to be considered on structural health monitoring development. Mesquita in his thesis in 2017 developed a new approach for heritage constructions safety assessment based on reliability analysis with recurrence to monitoring of vibrational measurements. He utilized the Santo Antonio de Viana church as the case study [23].

In [24] reliability of the Santo Antonio's church was examined by using health monitoring of the retrofitting process. They concluded that Santo Antonio's church is safe and transverse direction (Y) is more sensitive to environmental excitation than longitudinal direction (X) and the reliability index estimated to the Y direction are lower than the reliability values obtained on the X direction.

The main aim of this paper is to evaluate the probabilistic safety of a historical bridge using a SFEM. The safety index (reliability index) has been used to determine the safety status of the heritage construction. A case study of this paper is the construction of the Pole Kheshti historical arch bridge in northern Iran.

## 2 Theoretical background

### 2.1 Reliability analysis and RI for HMS

Reliability analysis consists of the following steps:

1. Determining limit functions of the structure: for example, the mid-span deflection of a bridge, number of loading in steel bridge joints, and the maximum stress in the bridge material can be examples of limit functions.
2. Detecting structural failure modes that lead to limit states. The final strength of structure, yielding, structural softness, and vibration, structural instability, deflection and fatigue can be considered as the failure mode of the structure. Exceeding the maximum stress in a structure with linear elastic behavior means yield in a part of that structure [25]. Moreover, limiting the structural softness is performed by controlling its maximum deflection in the middle of the span.
3. Identifying important variables and assigning suitable probabilistic properties to them, such that the failure modes can be determined well.
4. The probability of failure of each of the components of the structure and probability of failure of the structure system. As a system, the structure often has various failure modes. The failure of the system can be due to the failure of each of the modes alone, or the failure of all modes or a combination of them, and the structure's system will be parallel, series and hybrid, respectively.

Calculating the RI of the structure system and its comparison with the decision criterion of the target RI is one of the simplified methods to estimate the RI from the probability of failure using Eq. (1). The RI is presented to make this probability of failure more tangible. The symbol is the standard cumulative distribution function. According to Eq. (2), for the reliability of the structure, the RI should be larger than or equivalent of the target RI. If the existing RI is lower than the target RI, it will indicate the unsafely of the structure, and it is necessary to take steps to solve this problem.

$$\beta = -\Phi^{-1}(p_f) \quad (1)$$

$$\beta \geq \beta_t \quad (2)$$

The target RI can be determined using the reliability assessment references of the structures. In order to evaluate the reliability of the existing cultural heritage bridges that are considered among important structures, the target RI  $\beta_T = 4.3$  is suggested [20, 21, 26]. In evaluating the target RI for existing structures, including historical buildings, the social and cultural importance of these structures should also be considered. In [27], the formula seen in Eq. (3) is presented to calculate the target failure probability [28].

$$P_{fT} = 10^{-4} \times S_C \frac{t_d}{n_p} \times \frac{A_C}{W} \times C_f \quad (3)$$

where  $S_C$  refers to the social criterion factor or value of building protection,  $t_D$  refers to design working life,  $n_p$  refers

to number of people put in danger,  $A_C$  refers to the activity factor,  $W$  refers to warning factor, and  $C_f$  refers to economic factor.

## 2.2 Load-resistant model

The “limit state” criterion is used to decide whether a part or member of a structure has proper function or not. Generally, three extreme, serviceability, and special limit states are investigated. The extreme limit states occur at the time of structure failure or failure of one of its components or members. The serviceability limit state is related to failure occurring during normal use of the structure, and the special limit state occurs in extreme and certain conditions where the structure is under random loading. The aim of this paper is merely to study the reliability of structure in the extreme limit state under gravitational loading. In the probabilistic and reliability analysis of the structures, the load-resistance model is widely used in accordance to Eq. (4) [29, 30]. In Eq. (4),  $R$  represents the resistance of a member or a system and  $S$  represents stress (load effects).

$$g(R, S) = R - S \quad (4)$$

The space  $(R, S)$  is divided into two areas of reliable and failure by failure functions (FF)  $g(R, S) = 0$ . The structure functions in the reliable and failure states are shown in Eq. (5) and Eq. (6), respectively [19].

$$g(R, S) \geq 0 \quad (5)$$

$$g(R, S) < 0 \quad (6)$$

The estimation of FF that is implicit requires a lot of processing time, and with an increase in the RV, the required time increases.

## 2.3 Load and resistance RV

To perform a probabilistic analysis of HMAB, the RV are required to be pre-defined. To define RV, the existing information in the technical literature or the laboratory observations can be used. The weight of structure, traffic load of pedestrians, and vehicles passing on the bridge are among the loads on the structure, which must be defined as random parameters. The main properties of masonry bridge resistance are masonry materials density, compressive and tensile strength of masonry materials, adhesion of mortar joint to masonry unit, density of infill, and internal friction angle of bridge infill material [20]. Given the fact that the HMAB are considered as gravity structures, in this paper, their safety status is only examined under gravitational loading [21]. The reliability analysis of bridge under loads of flood, wind and earthquake can be studied

in another research. The SA of the RV of the implicit functions to the deflection and stress response is examined.

## 3 Pole Kheshti Langroud

The historical construction of the Pole Kheshti Bridge is located in the city of Langroud in northern Iran was made of bricks with the estimated history of 700 years. The Langeroud is divided in two parts by Langeroud River. The Pole Kheshti historical arch bridge across the Langeroud River. When Nader Shah was king of Iran, Langeroud had been one of the most important dockyards in the north of Iran. The most famous archaeological place in Langeroud is Pole Kheshti. The bridge was registered in Iran's Cultural Heritage, Handicrafts and Tourism Organization by the ID 915 in 1973. This heritage construction has survived many natural hazards throughout its life. The dimensions of the bridge bricks are 20 cm with height of 4 cm. The deck width of the bridge is ~4.3 m, span of each of the arches is 10 m, and rise to span ratio is 0.75 (Fig. 1). Exact geometry of the bridge is surveyed by the authors, and the asymmetry of arches at the sidewalls was demonstrated. Today, only pedestrians can walk on the bridge and the vehicles cannot pass on it. There is a proper contraction joint between the buildings adjacent to the bridge and the bridge structure, so that the buildings and bridges are separate from each other and they do not cause any constraint of movement. The case study of reliability assessment has been carried out on this structure.

## 4 DFE and SFE modeling

The linear elastic behavior is considered for bridge material. In the masonry material modeling, the macro modeling and homogeneity of mechanical properties of materials in different directions are used [7, 31]. ANSYS



Fig. 1 Photograph of historic Pole Kheshti bridge in Iran

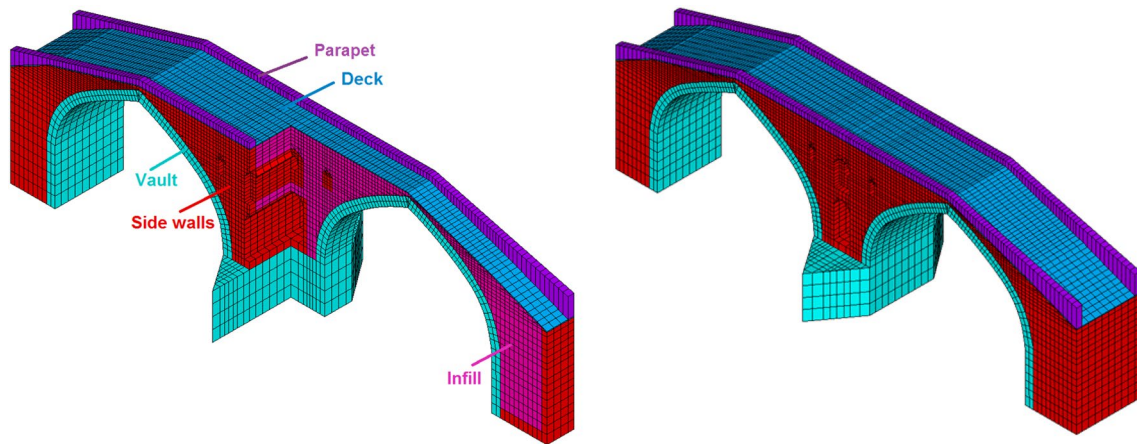


Fig. 2 Discretization of finite element model

Table 1 Deterministic input variables for the bridge material and service loads

Family	Type	Modulus of elasticity (GPa)	Poisson ratio	Density (kg/m <sup>3</sup> )	Load (kN/m <sup>2</sup> )
Bridge material	Deck	2.2	0.2	1800	
	Parapets	1.8	0.2	1800	
	Side Walls	2.2	0.2	1800	
	Vaults	2.2	0.2	1800	
	In filled	0.2	0.22	2000	
Traffic Load	–	–	–	–	3

software is used for finite element model. In Fig. 2, the discretization model that is utilized in the DFE or SFE analysis can be seen. The isoperimetric 8-node three-dimensional elements are used, each of which has three transitional degrees of freedom. The number of elements and nodes of the discretization model is 4683 and 5901, respectively. In the bridge supports, where the bridge is attached to the ground, the fixed support is assigned. In Table 1, the values of the parameters used in the DFE model are shown. The mechanical and physical properties of the materials are selected according to the technical literature.

Next, the bridge is analyzed under usual loads, bridge self-weight and live load on the bridge deck. The live load is considered to take into account the load of tourists and pedestrians in a way to provide the worst possible state of loading for the structure. The temperature can influence the behavior of masonries structures, but in this case, its effect was not considered. The bridge structure is statically analyzed. The minimum principal stress, maximum principal stress, and maximum deflection of arch key (mid-span of arch) are resistance control in the compression and tension and softness control of bridge. The distribution of stresses and deflection of bridge under the assumed loading are shown in Figs. 3, 4

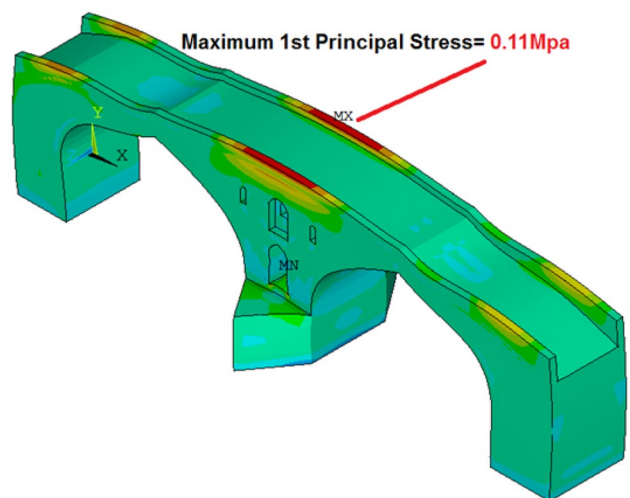


Fig. 3 Maximum principal stress under gravitational loading

and 5, respectively. Under the critical loading, the minimal value of principal stress  $S3^{min} = -0.58$  MPa, maximum principal stress  $S1^{max} = 0.11$  MPa, and maximum deflection  $Y^{max} = 1.5$  mm, are lower than their allowed limits, i.e.  $S3^{min} = -1.8$ ,  $S1^{max} = 0.15$ , and  $Y^{max} = 5$ , respectively. The allowed values of bridge deflection are assumed to

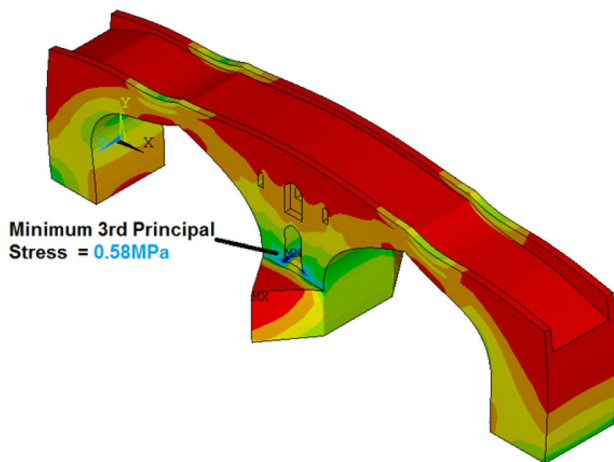


Fig. 4 Minimum principal stress under gravitational loading

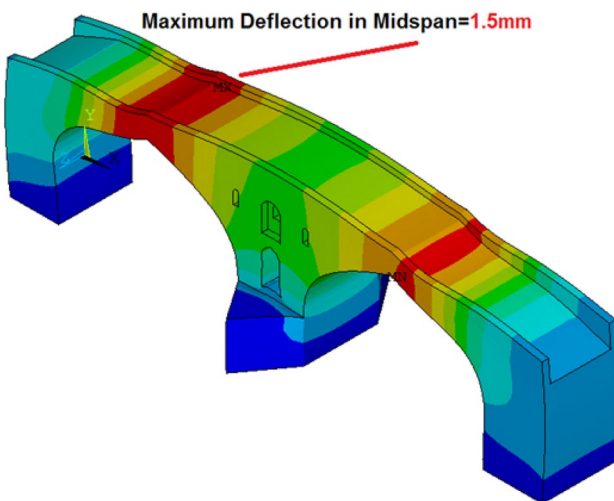


Fig. 5 Maximum deflections under gravitational loading

be  $\gamma^{allow} = S/2000$ , where  $S$  is the length of arch span. The deterministic analysis of the Pole Kheshti Langroud HMAB showed that the required safety is provided for the ultimate limit state and under the worst loadings the bridge

is safe. The most high tensile stress elements are located in the side walls and parapets and in the mid-span of the bridge and at the height of the deck. Also the most high compressive stress elements are located in the bottom of middle pier of the bridge.

For the probabilistic analysis of the system (PDS), the PDS capability in ANSYS software was used. In numerous studies, the PDS has been used to evaluate the reliability of structures [5, 13, 29]. The probabilistic analysis is performed using the Monte Carlo simulation method. The Monte Carlo method provides the possibility of generating  $N$  samples of the vector of RV ( $X$ ) to evaluate the FF  $G(X)$  [29, 30]. The LHS is used for sampling. For probabilistic analysis, it is necessary to define the resistance and load parameters (modulus of elasticity, Poisson ratio, density and traffic load), which can be seen in Table 2 [22, 32]. In the probabilistic analysis, the probabilistic distribution that has best fitting in accordance to various references for probability density function (PDF) of the input variables is selected. The log-normal Gaussian function (LN) is considered in the present study. To generate RV with the mentioned distribution, there is a need to define mean value and standard deviation of each variable. In Table 2, the type of variable, PDF, mean value, and standard deviation are also shown. The values in the table are selected according to technical literature. Moreover, the values of stresses and allowed deflection of the bridge is considered with LN distribution, mean values reported in the Sect. 4 (These values achieved from deterministic analysis result that shown in Figs. 3, 4 and 5), and coefficient of variation 10%.

### 5 Estimation of failure probability of system

In order to accelerate the evaluation of FF value, the Monte Carlo and LHS are used. The number of sample evaluations is independent of the number of RV and depends on the type of output parameter required by the analyst.  $N_{sim} = 50 \sim 200$  is suggested to prepare the histogram of variables and  $N_{sim} = 30/p_f \sim 100/p_f$  is suggested to obtain the probability. In this paper, 1200 simulations are considered for histograms

Table 2 Random input variables for the bridge material and service loads

Family	Type	Modulus of elasticity (GPa)	Poisson ratio	Density (kg/m <sup>3</sup> )	Live load (kN/m <sup>2</sup> )
Bridge material	Deck	LN (2.2, 0.22)	LN (0.2, 0.02)	LN (1800, 180)	–
	Parapets	LN (1.8, 0.18)	LN (0.2, 0.02)	LN (1800, 180)	–
	Side walls	LN (2.2, 0.22)	LN (0.2, 0.02)	LN (1800, 180)	–
	Vaults	LN (2.2, 0.22)	LN (0.2, 0.02)	LN (1800, 180)	–
	In filled	LN (0.2, 0.02)	LN (0.22, 0.022)	LN (2000, 200)	–
Load	–	–	–	–	LN (3, 0.03)

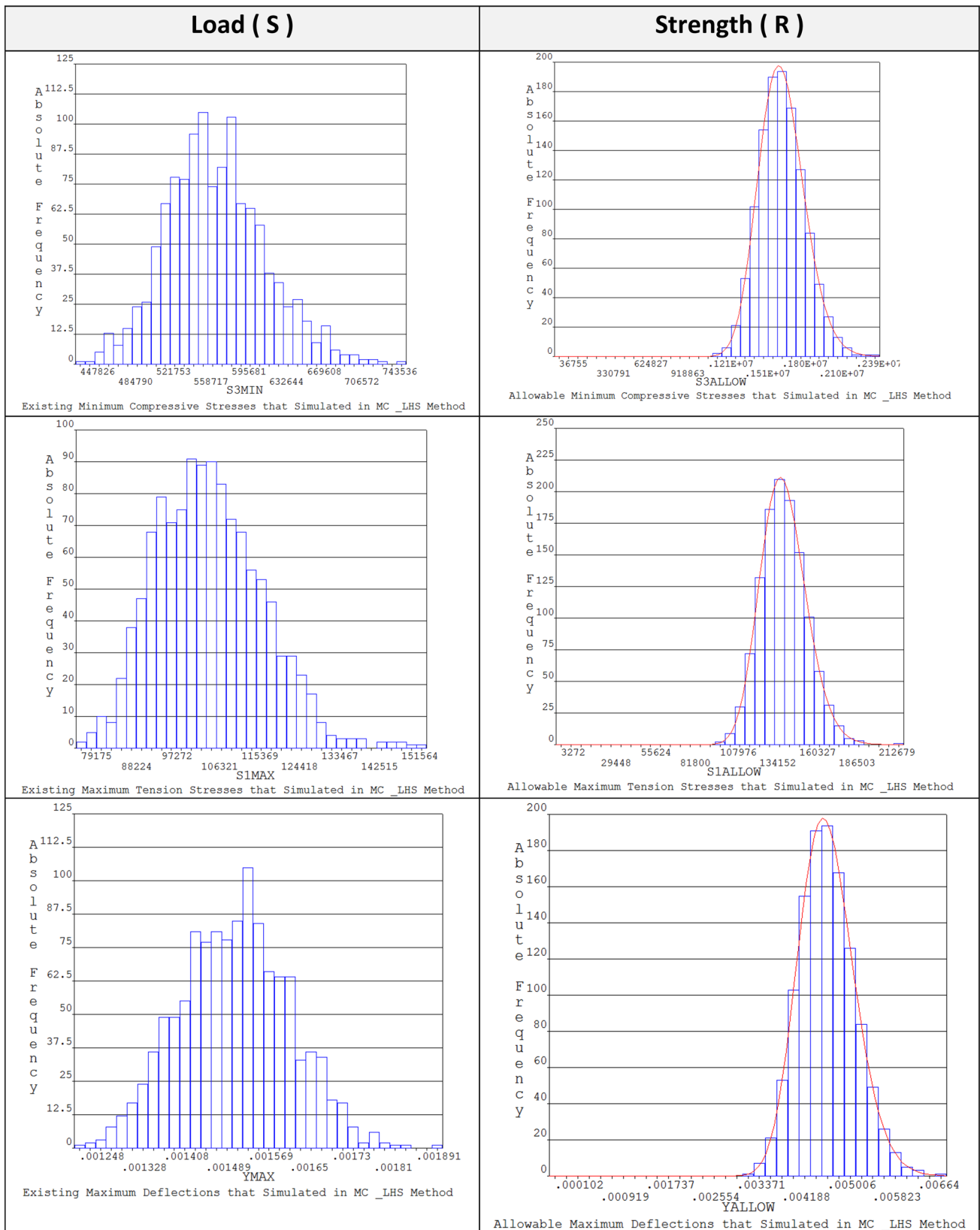


Fig. 6 Histogram of load and strength

and 12,000 simulations are considered for probability of failure. In Fig. 6, on the right, the absolute histogram and fitted curve of the allowed stress and deflection RV can be seen, and the overlap area of histogram on the left and right side of the Fig. 6 indicates the failure probability in that particular FF.

After determining the failure probability in each of the limit states, it is possible to calculate the failure probability of the bridge system. It is assumed that bridge failure probability is due to the failure of compressive  $P_f^c$  and tensile  $P_f^t$  stresses and non-allowed deflection  $P_f^d$ . The stresses and deflection FF are three series components of the bridge system. This means that the safety of bridge will only be provided with simultaneous safety of all three limit states, or in other words, the positivity of all three FF. In Eq. (7), the failure probability of bridge system is shown. In Eq. (7), it is assumed that the three FF are independent of each other. In Eqs. (8)–(10), the FF are also shown for the compressive and tensile stress and bridge deflection.

$$P_f^{System} = P\left[P_f^c \cup P_f^t \cup P_f^d\right] = P\left[GF_{S3} < 0 \cup GF_{S1} < 0 \cup GF_Y < 0\right] \tag{7}$$

$$GF_{S3}(\sigma) = S3^{allow} - S3^{min} \tag{8}$$

$$GF_{S1}(\sigma) = S1^{allow} - S1^{max} \tag{9}$$

$$GF_Y(\delta) = Y^{allow} - Y^{max} \tag{10}$$

Monte Carlo method is used to calculate the probability of failure. The allowed values of control parameters (R) are presented in the Table 2. The calculations determined the values for probability of failure for control parameters equal to  $p_f^c = 0$ ,  $p_f^t = 1.55 \times 10^{-2}$  and  $p_f^d = 0$ . In Fig. 7, the system failure tree and failure probability of system components and bridge system are shown. Given the low samples of Monte Carlo simulations, the validity of failure probability values is between 2 and 98% [33].

In Fig. 8, the cumulative distribution function (CDF) of values of load effects and FF are shown. According to the left column of Fig. 8, the probability of compressive stress, tensile stress, and deflection in the deterministic analysis of bridge is less than the values shown in Figs. 3, 4 and 5, 51, 58 and 34%, respectively. The right column of Fig. 8 indicates that only the probability of tensile stress FF has negative values ( $GF_{S1}(\sigma) < 0$ ). The probabilistic analysis of the Pole Kheshti Langroud HMAB showed that the required safety is not provided for the ultimate limit state and the bridge is at risk of failure. The most vulnerable elements in tensile stress are located in the side walls and parapets in the mid-span of the bridge and at the height of the bridge deck. The worst value for tension FF is obtained  $GF_{S1}(\sigma) = -27,515$  Pa for the MCS number 11105. Maximum and Minimum stresses are  $S1^{max} = 0.165$  MPa and

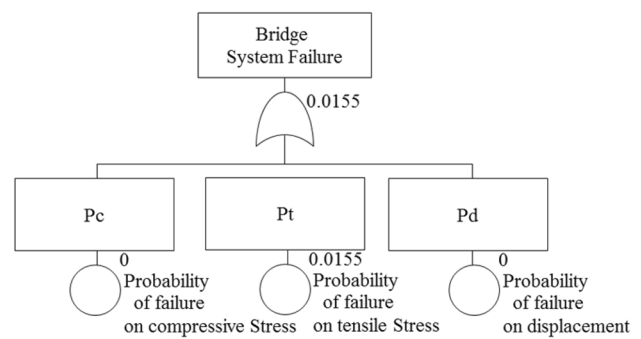


Fig. 7 Fault tree of masonry arch bridge system

$S3^{min} = 0.71$  MPa respectively for this case. The negative value of FF means that cracking will occur in some elements. Random variables in MCS number 11105 is shown in Table 3.

### 6 SA of responses to RV

The SA has been made to determine the effect of different parameters such as mechanical and physical properties of materials and loading on the reliability of bridge. The reliability analysis process also requires a high computational cost. The SA is presented to overcome this problem. In Fig. 9, the sensitivity of limit function of minimum principal stress  $GF_{S3}$  to the input RV is shown. The major changes in the limit function  $GF_{S3}$  are caused by variation in 6 input variables, including modulus of elasticity of vault, density of sidewall, modulus of elasticity of sidewalls, density of parapet, density of vault, and live load; with an increase in all parameters except for modulus of elasticity of sidewall, the maximum compressive stress is reduced. Increasing the modulus of elasticity of sidewall leads to an increase in the maximum compressive stress. The rest of the random parameters of the model have small impact (Insignificant) on the limit function.

The sensitivity of the limit function of the maximum tensile stress  $GF_{S1}$  to the random input variables are shown (Fig. 9). The major changes in the limit state function  $GF_{S1}$  are caused by variation in 7 input variables including modulus of elasticity of deck, modulus of elasticity of sidewalls, density of sidewall, density of parapet, modulus of elasticity of vault, density of vault, and modulus of elasticity of parapet. Thus, with an increase in the values of parameters for modulus of elasticity of deck, density of sidewall, density of parapet, and density of vault, the maximum tensile stress is reduced. Also, an increase in the other parameter leads to an increase in maximum tensile stress. The rest of the random parameters of model have small impact on the limit function.

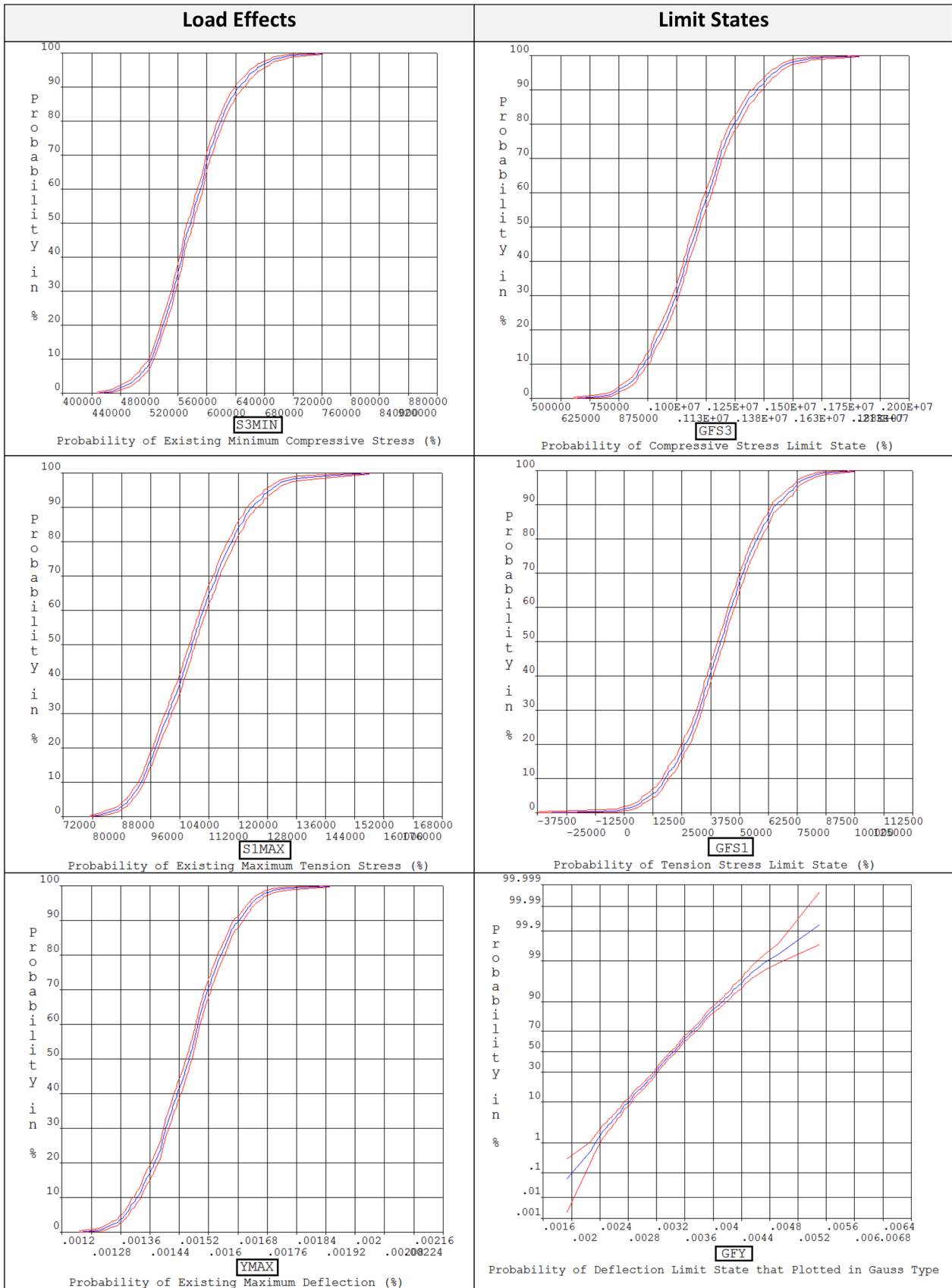
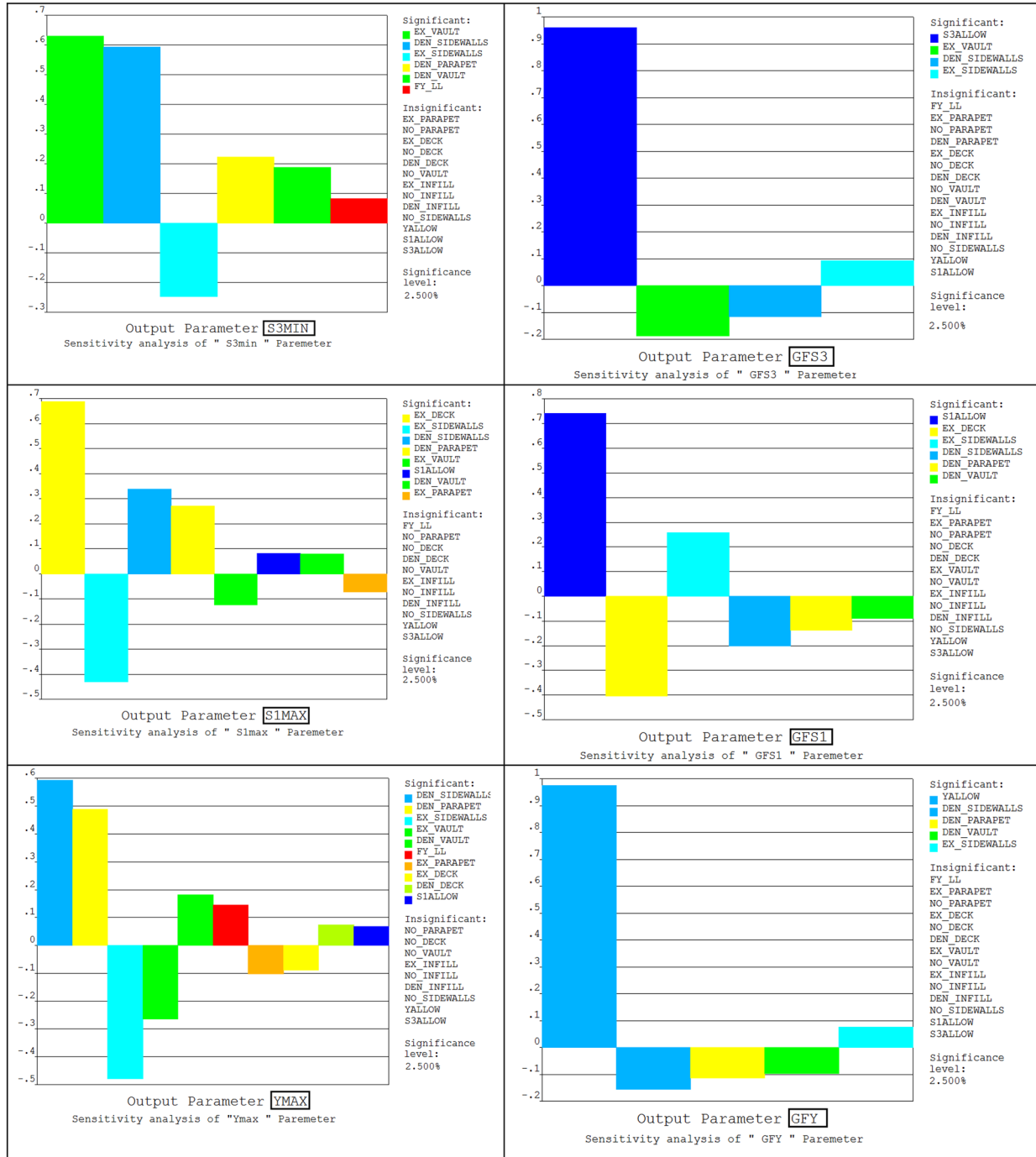


Fig. 8 CDF of load effects and FF



**Table 3** Random values for variables in MCS number 11105

Family	Type	Modulus of elasticity (GPa)	Poisson ratio	Density (kg/m <sup>3</sup> )	Live load (kN/m <sup>2</sup> )
Bridge material	Deck	2.606	0.1936	1773.7	—
	Parapets	1.7797	0.1945	1865.3	—
	Side walls	1.834	0.2177	1774.7	—
	Vaults	2.0559	0.21779	1631.27	—
	In filled	0.21709	0.2206	1964.84	—
Load	—	—	—	—	112.11



**Fig. 9** SA of loads and FF

From the results of deflection limit state sensitivity analysis (SA), it was specified that four RV including density of sidewalls, density of parapet, modulus of elasticity of sidewalls, and modulus of elasticity of vault have the greatest effect on the limit state of deflection ( $GF_Y$ ); an increase in the density of sidewall and parapet reduces the maximum deflection, while an increase in modulus of elasticity of sidewall and vault increases the bridge deflection. The effectiveness of input parameters on  $GF_Y$  has been shown in Fig. 9. The rest of the input parameters have insignificant on the deflection limit state.

In order to reduce the computational time (CT), 7 significant parameters affecting on limit functions, including modulus of elasticity of deck, density of parapet, modulus of elasticity of vault, density of sidewall, live load, density of vault, and modulus of elasticity of sidewalls have been selected. The probabilistic model based on seven parameters is analyzed and results of failure probability and RI, which is shown in Table 4, are obtained. The CT was reduced from 55 to 39 min, which is equivalent to 30% savings in CT.

In Fig. 10, the sensitivity to probability of failure of the limit state of tensile stress is shown compared to the changes in number of Monte Carlo simulations. In 12,000 simulations, the values of  $p_f^\delta = 0$  and  $p_f^c = 0$  are obtained. The analysis time for 12,000 samples is 40 h, and due to the long CT required, there is a large of limitation for complex models. Besides, the structures of great importance such as historical monuments have the target probability failure in order of  $10^{-4}$ – $10^{-5}$ , and to estimate with this accuracy using the Monte Carlo method, the samples with a few hundred

thousand simulations are required. In Table 4, a summary of probability values with number of samples and different number of variables is shown.

### 7 RI of Pole Kheshti

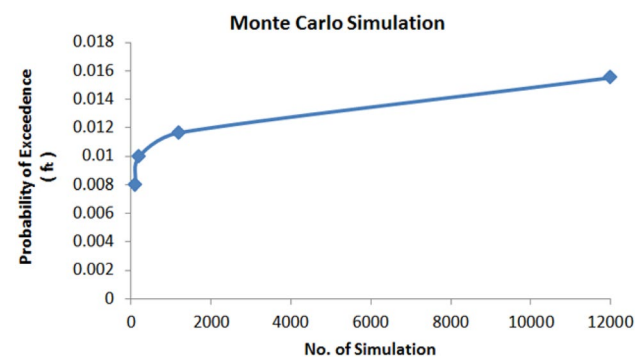
The values of RI of the limit state of tensile stress is equal to  $\beta^t = 2.156$ . In the Excel software, the inversed CDF can be calculated with the following command:  $\beta = -NORMSINV(p_f)$

The target RI in ISO13822 standard of  $\beta^{T\ arg\ et} = 4.2$  is suggested. The target probability of failure, in accordance to the formula presented in Eq. (3), is also obtained. The parameters of Eq. (3),  $S_C = 0.05$  as the social criterion factor or value of building protection,  $t_d = 50$  as design working life,  $n_p = 5$  as number of people put in danger,  $A_C = 3$  as activity factor,  $W = 0.3$  as warning factor, which can be observed with assumption of failure as gradual degradation without observable signs, and  $C_f = 1$  is considered as an economic factor. Where the target RI corresponding to  $P_{FT}$  is equal to  $\beta^{T\ arg\ et} = 3.29$ . In the present paper, as the target RI in the limit state of tensile stress is not provided, the bridge structure is considered to be unsafe.

The values of FF in the simulation iterations of Monte Carlo are depicted as a scattered in Fig. 11. The pairwise correlation of FF in Fig. 11 shows that with the probability of > 99.5%, these functions are not closely related, and their value of R2 is 0.57% at most. Therefore, the assumption in Eq. (7) holds.

**Table 4** Safety Index

Number of RV	$N_{Sim}$	Probability of failure $p_f^t$	Reliability Index $\beta^t$
19	12,000	$1.55 \times 10^{-2}$	2.156
19	1200	$1.17 \times 10^{-2}$	2.27
7	1200	$1.549 \times 10^{-2}$	2.16

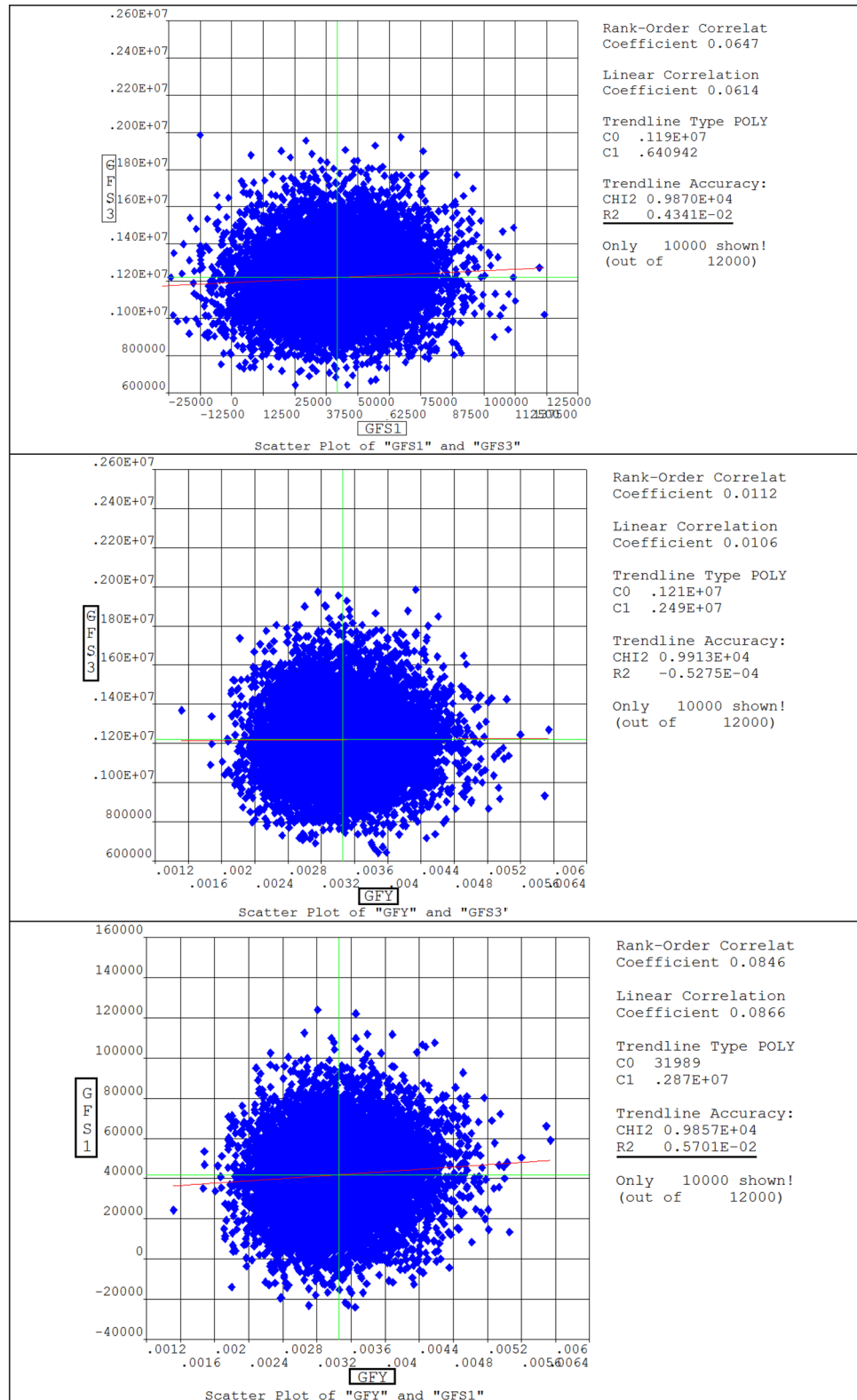


**Fig. 10** Sensitivity of probability of failure to number of simulation

### 8 Conclusions

The historical arch bridges are considered as cultural heritage and are among the important structures, thus, the accurate reliability evaluation of bridge structure should be investigated for their preservation. In the present paper, a systematic method is presented to determine the RI of HMAB using the SFE analysis. A case study has been carried out on the Pole Kheshti HMAB. Examining the reliability status of HMAB requires a more accurate understanding of the implicit functions of load effects and structure resistance. Thus, the load and resistance functions are considered to be probabilistic, due to the probabilistic nature of independent variables of these functions, including traffic load and the physical properties of bridge body materials. 19 random parameters are considered in the probability analysis of bridge safety. Given that the HMAB are regarded as gravity structures, their reliability status is only examined in this paper at the extreme limit state subjected to gravitational loading. Temperature loading is not considered in

**Fig. 11** Correlations between FF



structural analysis. The PDS tool of ANSYS software is used to calculate the probability of the FF. The value of probability of failure for the FF is also obtained by Monte Carlo

simulation, and then the value of RI is calculated. The RI of the studied bridge is less than the target RI suggested by various references, and it is obvious that the reliability

margin of bridge is not sufficient under the loads of its own weight and traffic. In the deterministic analysis of the bridge, safety has been provided, while in probabilistic analysis has been at risk of failure. This failure is the cracking of side walls and parapets elements in the mid-span at the height of the deck. These are vulnerable areas of the bridge. The worst value for tension FF is obtained  $GF_{S1}(\sigma) = -27,515$  Pa for the MCS number 11105. Maximum and Minimum stresses are  $S1^{\max} = 0.165$  MPa and  $S3^{\min} = 0.71$  MPa respectively for this case. The negative value of FF means that cracking will occur in vulnerable areas.

The random parameters are obtained with the greater impact on the FF by the SA. It became obvious that the modulus of the elasticity of vault, modulus of elasticity of deck, and density of sidewalls have the greatest impact on the reliability. From the results of the present statistical study, important decisions can be made to reconstruction, preservation and retrofit these structures, so in case of low reliability index of structure, it can reach the target RI.

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