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RESEARCH PAPER

Simultaneous Fault Detection and Control Design for Linear Fractional-Order Systems

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Abstract

The focus of this paper is the analysis and design scheme of simultaneous fault detection and control (SFDC) for linear continuous-time fractional-order systems assumed to be affected by sensor, actuator and process faults as well as disturbances. In essence, this simultaneous design unifies both the control and the detection modules into a single unit that is called the controller/detector unit. This unit is designed such that it generates two signals, namely the residual and the control signals. The system can be stabilized using the control signal, and the residual signals can detect the fault based on model-based fault detection and isolation algorithms. The SFDC module should be designed so that the effects of faults and disturbances on the residual signals are maximized and minimized, respectively. To this end, the SFDC problem is formulated as the mixed H_-/H_∞ optimization problem. Stability and fault detection are both considered through certain performance indices, and new sufficient conditions in the form of linear matrix inequalities are obtained. Finally, some simulation examples are given to illustrate the effectiveness of the proposed design method.

Keywords Fault detection · Fractional-order system · LMI · Norms

1 Introduction

Fractional calculus, as a branch of mathematical analysis, started a new challenge about traditional integration and differentiation and introduced the concepts of noninteger-order integration and differentiation (Butzer and Westphal 2000; Kenneth and Bertram 1993). It provides powerful mathematical tools whose application in various fields of science and engineering is amazing (Sharma et al. 2019; Ahmed et al. 2019; Sayyaf and Tavazoei 2018; Boukal et al. 2018; Poojary and Gangadharan 2018). In the process of modeling, it has been found that the expression of dynamical equations of systems by the fraction-order (FO) model is very simple, explicit and closer to the real situation (Abdeljawad et al. 2019; Hernandez et al. 2014; Yang et al. 2015). Furthermore, in many systems such as thermal systems (Battaglia et al. 2000) and batteries (Tian et al. 2019; Bankupalli et al. 2018), the FO models have fewer parameters than integer-order systems.

Another interesting topic in the fractional discussion is the design of a fractional controller. The $PI^\lambda D^\mu$ controller (Ren et al. 2019), the fractional-order lead–lag compensator (Raynaud and Zergaınoh 2000) and the CRONE control (Morand et al. 2016) are some illustrious FO controllers, which are proved to have more flexibilities and robustness in terms of their applications compared to integer-order ones. Many studies have been done for the stabilization and stability conditions of fractional-order systems (FOSs). To mention a few, in Zhang et al. (2017), some simplified linear matrix inequality (LMI) stability conditions for linear and nonlinear FOSs can be found. In Khandani et al. (2017), stochastic systems with fractional Gaussian noise (fGn) are stochastically stabilized. Considering the application of norms in robust control, in Malti et al. (2011), for the H_2 norm of FOSs, an analytical computation technique was obtained. Authors of Fadiga et al. (2011), Sabatier et al. (2005) present new methods for the H_∞ norm computation of FOSs. A FOSs bounded real lemma was presented in Moze et al. (2008). In Farges et al.

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(2013), the problem of H_∞ analysis and control of commensurate FOSs were addressed.

On the other hand, it has been widely observed that even a small fault in the system will cause great damage to it. Therefore, the rapid detection of fault and its control can prevent system failure, as well as the damage to the subsystem. According to dynamic systems, a fault is a deviation of the system structure or the system parameters from the nominal situation. Examples for structural changes are the disconnection of a system component, the blocking of an actuator or the loss of a sensor. In recent decades, the development of fault detection algorithms and monitoring of dynamic systems are of paramount importance due to their unique role in ensuring system reliability and system safety (Chadli et al. 2018; Sakthivel et al. 2017; Shi et al. 2015; Zolghadri et al. 2014). A great number of FD algorithms have been developed (Dong et al. 2012; Li and Yang 2015; Meskin and Khorasani 2009; Liu et al. 2019; Luo et al. 2019), but model-based fault diagnosis method has a vital and practical role in the research and engineering domains. Furthermore, the efficiency of this algorithm in dynamic systems for detecting faults has been proven by a great number of successful applications in industry (Ding 2008; Wei et al. 2019; Li et al. 2019). In the first step, model-based fault detection and isolation (FDI) algorithms are based on the design of state observers or filters. In the second step, using the system output and the output of the observer, the residual signal is constructed. In the third step, this signal is compared to the predefined threshold and an alarm is generated if the residual evaluation function has a value larger than the threshold, which means there is a fault in the system. The presence of disturbances as fault false alarms can corrupt the performance of FDI system. Then, the fault detection system should be designed in such a way that it is sensitive to faults and simultaneously robust to disturbances (Wang et al. 2007). As faults may be hidden by control actions and the early detection of low-frequency faults is more troublesome, the design of FDI in closed-loop feedback system is a different argument. To solve this problem, the researchers proposed SFDC. This method creates an integrated unit of feedback controller and FDI unit that is called control/detector unit, rather than design of the detector units and the controller separately. Also, this leads to less complexity (Ding 2009). One of the approaches to solving SFDC problems is to use the theoretical framework of LMIs robust control. In Khosrowjerdi et al. (2004), Davoodi et al. (2012), Zhai et al. (2016), the SCFD problem is addressed by multi-objective H_2/H_∞ framework. Wang and Yang (2009), Davoodi et al. (2012), Li and Yang (2012), Zhong and Yang (2016) and Davoodi et al. (2016) investigated the problem in the mixed H_-/H_∞ optimization method. In Shokouhinejad et al. (2017), Soltani et al. (2015), Wang et al. (2017), the SCFD problem is formulated as an H_∞ filtering problem.

The main purpose of this paper is now simultaneous control and fault detection in FOSs. In recent years, researchers have shown interest in the issue of fault detection problem of FOSs considering the importance of it in both theory and applications. In Aribi et al. (2014), firstly, the thermal system is modeled by a fractional order and then the Luenberger observer is designed for the diagnosis of fault in the systems. The focus in Pisano et al. (2011) has been on the estimation and fault detection by second-order sliding mode in FOS subject to unknown inputs. Discontinuous dynamical observer for FDI in FOSs is presented in Pisano et al. (2014). To the best of our knowledge, the SCFD problem for FOSs has not been investigated yet. The contributions of this paper can be summarized as follows:

(i) In this paper, for the first time in the literature, the problem of SFDC for continuous-time linear fractional-order systems using mixed H_-/H_∞ index is studied.

(ii) Based on the so-called generalized KYP lemma and the bounded real lemma corresponding to $\boldsymbol{H}_\infty$ norm (H-BR) and applying the advantages of classical Luenberger observers, new sufficient conditions in the form of LMI for solving the SFDC problem are obtained.

(iii) The controller/detector unit design is to generate two signals, namely the residual and the control signals. Using the residual signal, based on model-based fault detection, faults can be detected and the control signal can stabilize the system.

(iv) The SFDC module is designed such that the effects of faults and disturbances on the residual signals are maximized and minimized, respectively (for accomplishing the fault detection task), while the effects of disturbances and faults on the specified control outputs are minimized (for accomplishing the state or model reference problems).

This paper is organized as follows. Section 2 presents the definitions and theorems needed to solve the problem. The solutions to the SFDC problem for linear FO system are given in Sect. 3. To demonstrate the validity and effectiveness of the results, a numerical example is presented in Sect. 4. Finally, Sect. 5 presents some concluding remarks of the work.

Notations Throughout this paper, the notation A^T denotes the transpose of the matrix A and the superscript '*' denotes the conjugate transpose of the matrix. For a symmetric matrix, $A > 0$ and $A < 0$ represent positive definiteness and negative definiteness. $\mathrm{Sym}(A)$ is short for $A + A^*$, and $\sigma_{\max}(A)$ denotes the maximum singular value of A. The operator $\otimes$ is the Kronecker's product. $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ denotes n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively.

2 The Problem Statement and Preliminaries

Consider a class of LTI commensurate FOSs that are assumed to be affected by sensor, actuator and process faults as well as disturbances:

$$G: \begin{cases} D^\alpha x(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + Du(t) + D_d d(t) + D_f f(t) \\ z(t) = Ex(t) + F_d d(t) + F_f f(t) \\ x(t) = \phi(t) \quad t \in [-h_1, 0] \end{cases} \tag{1}$$

where $0<\alpha<1$ and α is the fractional commensurate order, $x(t) \in \mathbb{R}^n$ is the pseudo-state vector, $y(t) \in \mathbb{R}^r$ is the measured output and $z(t) \in \mathbb{R}^t$ denotes the regulated output. Likewise, $u(t) \in \mathbb{R}^m$ is the control input, $d(t) \in \mathbb{R}^p$ is the disturbance and $f(t) \in \mathbb{R}^q$ is the fault vector.$\phi(t)$ is the initial function defined on $[-h_1, 0]$ where h_1 is known positive scalar. The appropriately dimensioned matrices $A, B, B_d, B_f, C, E, D_d, F_f$ and F_d are real known constant matrices. D^α is the fractional differentiation operator of order α.

The Caputo's definition of fractional-order derivative can be written as:

$$ {}_aD_t^\alpha \triangleq \frac{1}{\Gamma(k-\alpha)} \int_a^t \frac{f^{(k)}(\tau)}{(t-\tau)^{\alpha+1-k}} \mathrm{d}\tau \tag{2}$$

where K is a positive integer and $(K-1) \le \alpha < K$. If the FOS (1) is relaxed at $t = 0$, it can be displayed by the fractional-order transfer function (FOTF) matrix. The FOTFs from disturbance input and fault input to regulated output, respectively, are

$$G_{zd}(s) = E(S^\alpha I - A)^{-1} B_d + F_d \tag{3}$$

$$G_{zf}(s) = E(S^\alpha I - A)^{-1} B_f + F_f \tag{4}$$

The following controller (state feedback) filter and Luenberger-like state observer for the fault detector are proposed for system (1):

$$\begin{cases} D^\alpha \hat{x}(t) = A\hat{x}(t) + Bu(t) + Lr(t) \\ \hat{y}(t) = C\hat{x}(t) + Du(t) \\ r(t) = y(t) - \hat{y}(t) \\ u(t) = K\hat{x}(t) \\ \hat{x}(t) = \varphi(t) \quad t \in [-h_2, 0] \end{cases} \tag{5}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state vector of detection filter and $\hat{y}(t) \in \mathbb{R}^r$ represents the observer output vectors, $r(t) \in \mathbb{R}^r$ denotes the so-called residual, $K \in \mathbb{R}^{m\times n}$ is the controller gain and $L \in \mathbb{R}^{n\times r}$ is the filter gain. By defining new pseudo-state vector $\xi^T = \left[x(t)^T \quad e(t)^T \right]$ where $e(t) = x(t) - \hat{x}(t)$ and combining the filter unit (5) and system (1) together, the closed-loop FOS is governed by:

$$G: \begin{cases} D^\alpha \xi(t) = \tilde{A}\xi(t) + \tilde{B}_d d(t) + \tilde{B}_f f(t) \\ r(t) = \tilde{C}\xi(t) + D_d d(t) + D_f f(t) \\ z(t) = \tilde{E}\xi(t) + F_d d(t) + F_f f(t) \end{cases} \tag{6}$$

where

$$\begin{aligned} &\tilde{A} = \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, \quad \tilde{B}_d = \begin{bmatrix} B_d \\ B_d - LD_d \end{bmatrix}, \\ &\tilde{B}_f = \begin{bmatrix} B_f \\ B_f - LD_f \end{bmatrix}, \quad \tilde{C} = [0 \quad C], \quad \tilde{E} = [E \quad 0] \end{aligned} \tag{7}$$

2.1 The Distributed SFDC Problem Definition

For the FOS model (1), a detector/controller (5) should be designed such that the closed-loop system (6) is stable, and also to ensure that the fault detection is done correctly. Furthermore, the disturbance should not be assumed as a fault; this will lead to:

$$(i).\ \sup \frac{\|z(t)\|_2}{\|d(t)\|_2} \langle \gamma_1, \gamma_1 \rangle 0 \quad (ii).\ \sup \frac{\|z(t)\|_2}{\|f(t)\|_2} \langle \gamma_2, \gamma_2 \rangle 0$$

$$(iii).\ \sup \frac{\|r(t)\|_2}{\|d(t)\|_2} \langle \gamma_3, \gamma_3 \rangle 0 \quad (iv).\ \sup \frac{\|r(t)\|_2}{\|f(t)\|_2} \langle \beta, \beta \rangle 0$$

The performance indices (i), (ii) and (iii) are H_∞ optimization problems, and the performance index (IV) is H_- optimization problem. The H_∞ is used to reduce the effects of disturbance on the residuals and the control outputs as well as to reduce the fault effects on the control outputs. The H_- index is used to guarantee the sensitivity of the residuals to the faults. The following definitions and lemmas are presented for later developed.

Definition 1 (Green and Limebeer 1995) The H_∞ norm of $G(s)$ is defined by

$$G(s)_{H_\infty} \triangleq \sup_{\mathrm{Re}(s) \ge 0} \sigma_{\max}(G(s)) \tag{8}$$

Lemma 1 (Sabatier et al. 2010) *The FOS* $D^\alpha x(t) = Ax(t)$, $0<\alpha<1$, *is asymptotically stable if and only if*:

(1) $|\mathrm{Arg}(\mathrm{spec}(A))| > \alpha\frac{\pi}{2}$, *where* $\mathrm{spec}(A)$ *is the set of eigenvalues of A*

or

(2) *There exist* $P > 0$ *and* $Q > 0$ *such that* $\mathrm{sym}(rAP + \bar{r}AQ) < 0$ *where* $r = e^{j(1-\alpha)\frac{\pi}{2}}$.

Lemma 2 (H-BR, Liang et al. 2015) *For the FOS* (1) *with its transfer function* $G_{yu}(s) = C(S^\alpha I - A)^{-1}B + D$. *Then,* $G_{yu}(s)_{H_\infty} < \gamma$ *if there exist* $P > 0$ *and* $Q > 0$ *such that:*

$$\begin{bmatrix} \mathrm{sym}(AX) & * & * \\ CX & -\gamma I & * \\ B^T & D^T & -\gamma I \end{bmatrix} < 0 \tag{9}$$

where

$$X = \begin{cases} e^{j\theta}P + e^{-j\theta}Q, & \text{if} \quad 0 < \alpha < 1 \\ e^{j\theta} & \text{if} \quad 1 \leq \alpha < 1 \end{cases} \quad \theta = \frac{\pi}{2}(1-\alpha).$$

Lemma 3 (Iwasaki and Hara 2005) *Let matrices* $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\Phi \in H_2$ $\Theta \in H_{(n+m)}$ *and* $\psi \in H_2$. *Set* Λ *is defined as*

$$\Lambda(\Phi,\Psi) \triangleq \left\{ \lambda \in C \middle| \begin{bmatrix} \lambda \\ I \end{bmatrix}^* \Phi \begin{bmatrix} \lambda \\ I \end{bmatrix} = 0, \begin{bmatrix} \lambda \\ I \end{bmatrix}^* \Psi \begin{bmatrix} \lambda \\ I \end{bmatrix} \geq 0 \right\}. \tag{10}$$

According to the following two statements, for $H(\lambda) \triangleq (\lambda I_n - A)^{-1}$, there holds

$$\begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix}^* \Theta \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix} < 0, \quad \forall \lambda \in \Lambda \tag{11}$$

there exist $P, Q \in H_n$ and $Q > 0$ such that

$$\begin{bmatrix} A & B \\ I_n & 0 \end{bmatrix}^* (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} A & B \\ I_n & 0 \end{bmatrix} + \Theta < 0 \tag{12}$$

then "(2) $\Rightarrow$ (1)."

If Λ represents a curve in the complex plane, then "(2) $\Leftrightarrow$ (1)."

Lemma 4 (Liang et al. 2015) *If* $\Upsilon(\Phi,\Omega)$ *is defined as*

$$\Upsilon(\Phi,\Psi) \triangleq \left\{ \lambda \in C \middle| \begin{bmatrix} \lambda \\ I \end{bmatrix}^* \Phi \begin{bmatrix} \lambda \\ I \end{bmatrix} \geq 0, \begin{bmatrix} \lambda \\ I \end{bmatrix}^* \Psi \begin{bmatrix} \lambda \\ I \end{bmatrix} \geq 0 \right\}. \tag{13}$$

then condition (11) *holds* $\forall \lambda \in \Upsilon$ *if there exist positive definite symmetric matrices* P *and* Q *such that LMI condition* (12) *holds.*

Lemma 5 (Projection lemma, Gahinet and Apkarian 1994) *For two matrices* U *and* V *of column dimension* m *and a symmetric matrix* $Z \in S_m$, *there exists an unstructured matrix* X *that satisfies*:

$$U^T X V + V^T X^T U + Z < 0 \tag{14}$$

if and only if the following inequalities are satisfied:

$$N_U^T Z N_U < 0 \tag{15a}$$

$$N_V^T Z N_V < 0 \tag{15b}$$

where N_U *and* N_V *are null spaces of* U *and* V, *respectively.*

Assumption 1 Considering $B_i \in \mathbb{R}^{n\times m}$ with full column rank, equality (16) holds:

$$B_i = U_i \begin{bmatrix} \Sigma_i \\ 0 \end{bmatrix} V_i^T \tag{16}$$

where $\Sigma_i \in \mathbb{R}^{m\times m}$ is a diagonal matrix with positive diagonal elements and $U_i \in \mathbb{R}^{n\times n}$ and $V_i \in \mathbb{R}^{m\times m}$ are unitary matrices.

Lemma 6 (Liu et al. 2016) *Consider* $B \in \mathbb{R}^{n\times m}$ *with* $\mathrm{rank}(B) = m$ *and* $X \in \mathbb{R}^{n\times n}$ *is a symmetric matrix; then, there exists an* $\hat{X} \in \mathbb{R}^{m\times m}$ *such that* $XB = B\hat{X}$ *if and only if*

$$X = U \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} U^T \tag{17}$$

where $X_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$ *and* $X_{11} \in \mathbb{R}^{m\times m}$.

Lemma 7 (Matignon 1998) *The fractional-order system* $G(s)$ *is stable if and only if* $G(s)_{H_\infty}$ *is bounded.*

Assumption 2 $\parallel d(t) \parallel 2 < \lambda$ where λ is known positive scalar.

3 Main Results

As stated in the previous section, the SFDC distributed problem can indeed be cast as designing a controller/detector unit such that the augmented system (6) is stable and all multi-objective H_-/H_∞ performance indices (i)–(iv) are satisfied simultaneously. In this section, each performance index will be converted into the LMI conditions in Theorems 1–4. Then, a feasible solution to the problem is obtained by considering all of Theorems 1–4 simultaneously in Corollary 1. First, Theorem 1 proposes an LMI condition for performance index (i), such that the effect of disturbance on regulated output is minimized and fractional-order system (6) is stable.

Theorem 1 *For given scalars* $\gamma_1 > 0$ *and* $\lambda > 0$, *the augmented fractional-order system* (6) *is stable and guarantees the performance index (i) if there exist positive definite symmetric matrices* P_1, Q_1 *and matrices* X_1, X_2, $\hat{X}_1$, N *and* M *that satisfy the following LMI*:

$$\begin{bmatrix} Her(\Pi) + \Xi_1 & \Xi_2 & \Omega + \Xi_3 F_d \\ * & -\lambda(X + X^T) & \lambda\Omega \\ * & * & F_d^T F_d - \gamma_1^2 I \end{bmatrix} < 0 \tag{18}$$

where

$$\Xi_1 = \begin{bmatrix} E^T & 0 \end{bmatrix} \begin{bmatrix} E \\ 0 \end{bmatrix}, \quad \Xi_3 = \begin{bmatrix} E & 0 \end{bmatrix}^T, \quad \Omega = \begin{bmatrix} X_1^T B_d \\ X_2^T B_d - N^T D_d \end{bmatrix}$$

$$\Pi = \begin{bmatrix} A^T X_1 + M^T B^T & 0 \\ -M^T B^T & A^T X_2 - C^T N^T \end{bmatrix}, \quad X = \mathrm{diag}(X_1, X_2),$$

$$\Xi_2 = \lambda\Omega - X^T + \bar{r}P_1 + rQ_1, \quad r = e^{j\theta}, \quad \theta = (1-\alpha)\frac{\pi}{2}.$$

The filter gains L and the controller gains K are now specified as follows:

$$L = X_2^{-T} N \quad K = \hat{X}_1^{-T} M \tag{19}$$

Proof Based on Definition 1, $\|G(s)_{zd}\|_{H_\infty}$ can be written as

$$\begin{aligned}\|G(s)_{zd}\|_{H_\infty} &\triangleq \sup_{\mathrm{Re}(s)\geq 0} \bar{\sigma}\left(G(s)_{zd}\right) \\ &= \sup_{\mathrm{Re}(s)\geq 0} \bar{\sigma}\left(E(S^{\alpha}I - A)^{-1}B_d + F_d\right)\end{aligned} \tag{20}$$

By some basic matrix calculations, we have

$$\begin{aligned}\|G(s)\|_{H_\infty} < \gamma &\Leftrightarrow G(s)G(s)^{*} - \gamma^2 I < 0 \quad \forall Re(s) \geq 0 \\ &\Leftrightarrow \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix}^{*} \Theta \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix} < 0, \quad \forall \lambda \in \Lambda\end{aligned} \tag{21}$$

where $H(\lambda) \triangleq \left(\lambda I_n - \tilde{A}\right)^{-1}\tilde{B}_d$ and $\Lambda(\Phi, \Psi)$ is defined in Eq. (10), also:

$$\Theta = \begin{bmatrix} \tilde{E}^T\tilde{E} & \tilde{E}^T F_d \\ F_d^T \tilde{E} & F_d^T F_d - \gamma_1^2 I \end{bmatrix} \tag{22}$$

By invoking Lemma 3, the last part of (21) is also equivalent to the statement that $\exists P, Q \in H_n, P > 0$ and $Q > 0$ such that the following LMI holds:

$$\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \end{bmatrix}^{*} (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \end{bmatrix} + \Theta < 0 \tag{23}$$

Similar to Davoodi et al. (2012),

$$\Phi = \begin{bmatrix} 0 & \bar{r} \\ r & 0 \end{bmatrix} \quad \Psi = \begin{bmatrix} 0 & r \\ \bar{r} & 0 \end{bmatrix} \tag{24}$$

Now inequality (23) can be reformulated as $N_U^T Z N_U < 0$ where N_U and Z are given by:

$$Z = \begin{bmatrix} \tilde{E}^T\tilde{E} & \bar{r}P_1 + rQ_1 & \tilde{E}^T F_d \\ rP_1 + \bar{r}Q_1 & 0 & 0 \\ F_d^T\tilde{E} & 0 & F_d^T F_d - \gamma_1^2 I \end{bmatrix} \quad N_U = \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \\ 0 & I \end{bmatrix} \tag{25}$$

By designing N_V and V as:

$$N_V = \begin{bmatrix} \lambda I & 0 \\ -I & 0 \\ 0 & I \end{bmatrix} \rightarrow V = \begin{bmatrix} I & \lambda I & 0 \end{bmatrix} \tag{26}$$

Also, according to Lemma 5, the inequality $N_U^T Z N_U < 0$ is equivalent to:

$$Z + \begin{bmatrix} \tilde{A}^T \\ -I \\ \tilde{B}_d^T \end{bmatrix} \begin{bmatrix} X & \lambda X & 0 \end{bmatrix} + \begin{bmatrix} X^T \\ \lambda X^T \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{A} & -I & \tilde{B}_d \end{bmatrix} < 0 \tag{27}$$

Partitioning X into $X = \mathrm{diag}(X_1, X_2)$, $X_i \in R^{n\times n}$, $i = 1, 2$ and substituting (25)–(26) in (27) result in:

$$\begin{bmatrix} \mathrm{Her}(\Delta) + \Xi_1 & \Xi_2 & \Omega + \Xi_3 F_d \\ * & -\lambda(X + X^T) & \lambda\Omega \\ * & * & F_d^T F_d - \gamma_1^2 I \end{bmatrix} < 0 \tag{28}$$

where

$$\Xi_2 = \lambda\Omega - X^T + \bar{r}P_1 + rQ_1 \tag{29}$$

$$\Delta = \begin{bmatrix} A^T X_1 + K^T B^T X_1 & 0 \\ -K^T B^T X_1 & A^T X_2 + C^T L^T X_2 \end{bmatrix} \tag{30}$$

Let partition X_1 as:

$$X_1 = U \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} U^T \tag{31}$$

Then, from Lemma 6, there exists $\hat{X}_1 = V\Sigma^{-1}X_{11}\Sigma V^T$ such that $B^T X_1 = \hat{X}_1 B^T$ where $\hat{X}_1^{-1} = V\Sigma^{-1}X_{11}^{-1}\Sigma V^T$ and by substituting $N^T = L^T X_2$, $M^T = K^T \hat{X}_1$, in Eq. (30) inequality (18) is obtained, and the proof is completed.

Remark 1 LMI (23) is equivalent to LMI (9), and the feasibility of LMI (9) implies $\mathrm{sym}(\tilde{A}X) < 0$ which is sufficient LMI condition for the FOSs stability based on Lemma 1.

Remark 2 Lemma 7 shows that H_∞ norm can guarantee the stability of FOS. To minimize the effect of fault on regulated output, Theorem 2 proposes an LMI condition such that the overall system (6) is stable and performance index (ii) holds.

Theorem 2 *For a given positive real number γ_2 and $\lambda > 0$, the augmented FOS (6) is stable and guarantees the performance index (ii) if there exist positive definite symmetric matrices P_2, Q_2 and matrices $X_1, X_2, \hat{X}_1, N$ and M such that the following LMIs hold:*

$$\begin{bmatrix} \mathrm{Her}(\Pi) + \Xi_1 & \Xi_2 & \Omega + \Xi_3 F_f \\ * & -\lambda(X + X^T) & \lambda\Omega \\ * & * & F_f^T F_f - \gamma_2^2 I \end{bmatrix} < 0 \tag{32}$$

where

$$\Xi_1 = \begin{bmatrix} E^T & 0 \end{bmatrix}\begin{bmatrix} E \\ 0 \end{bmatrix}, \quad \Xi_3 = \begin{bmatrix} E & 0 \end{bmatrix}^T, \quad \Omega = \begin{bmatrix} X_1^T B_f \\ X_2^T B_f - N^T D_f \end{bmatrix}$$
$$\Pi = \begin{bmatrix} A^T X_1 + M^T B^T & 0 \\ -M^T B^T & A^T X_2 - C^T N^T \end{bmatrix}, \quad X = \mathrm{diag}(X_1, X_2)$$
$$\Xi_2 = \lambda\Omega - X^T + \bar{r}P_2 + rQ_2, \quad r = e^{j\theta}, \quad \theta = (1-\alpha)\frac{\pi}{2}.$$

The filter and controller gains L and K are specified by Eq. (19).

Proof The proof of this theorem is similar to that of Theorem 1, so it is omitted for the sake of brevity.

In the following theorem, the LMI feasibility condition to achieve stability of the augmented system (6) by considering H_∞ performance given in (iii) is obtained such that the effect of disturbance $d(t)$ on residual signal $r(t)$ is minimized.

Theorem 3 *For a given positive real number γ_3 and $\lambda > 0$, the augmented FO system (6) is stable and guarantees performance index (iii) if there exist positive definite symmetric matrices P_3, Q_3 and matrices $X_1, X_2, \hat{X}_1, N$ and M that satisfy the following LMI*:

$$\begin{bmatrix} \mathrm{Her}(\Pi) + \Xi_1 & \Xi_2 & \Omega + \Xi_3 D_d \\ * & -\lambda(X + X^T) & \lambda\Omega \\ * & * & D_d^T D_d - \gamma_2^2 I \end{bmatrix} < 0 \tag{33}$$

where

$$\Xi_1 = \begin{bmatrix} 0 & C^T \end{bmatrix}\begin{bmatrix} C \\ 0 \end{bmatrix}, \quad \Xi_3 = \begin{bmatrix} 0 & C \end{bmatrix}^T,$$
$$\Omega = \begin{bmatrix} X_1^T B_d \\ X_2^T B_d - N^T D_d \end{bmatrix}$$
$$\Pi = \begin{bmatrix} A^T X_1 + M^T B^T & 0 \\ -M^T B^T & A^T X_2 - C^T N^T \end{bmatrix},$$
$$X = \mathrm{diag}(X_1, X_2)$$
$$\Xi_2 = \lambda\Omega - X^T + \bar{r}P_2 + rQ_2, \quad r = e^{j\theta}, \quad \theta = (1-\alpha)\frac{\pi}{2}.$$

The filter and controller gains L and K, are specified by Eq. (19).

Proof The proof of this theorem is similar to that of Theorem 1, so it is omitted for the sake of brevity.

Now, the sufficient conditions in the form of LMI are obtained such that the effect of fault on residual is maximized according to H_- performance defined in (iv) and the effect of fault on residual is maximized.

Theorem 4 *For a given positive real number β and $\lambda > 0$, the augmented FOS (6) is stable and guarantees the performance index (iv) if there exist positive definite symmetric matrices P_4, Q_4, and matrices $X_1, X_2, \hat{X}_1, N$ and M such that*

$$\begin{bmatrix} \mathrm{Her}(\Pi) - \Xi_1 & \Xi_2 & \Omega - \Xi_3 D_f \\ * & \lambda(X + X^T) & \lambda\Omega \\ * & * & -D_f^T D_f + \beta^2 I \end{bmatrix} < 0 \tag{34}$$

where

$$\Xi_1 = \begin{bmatrix} 0 & C^T \end{bmatrix}\begin{bmatrix} C \\ 0 \end{bmatrix}, \quad \Xi_3 = \begin{bmatrix} 0 & C \end{bmatrix}^T, \quad \Omega = \begin{bmatrix} X_1^T B_f \\ X_2^T B_f - N^T D_f \end{bmatrix}$$
$$\Pi = \begin{bmatrix} A^T X_1 + M^T B^T & 0 \\ -M^T B^T & A^T X_2 - C^T N^T \end{bmatrix}, \quad X = \mathrm{diag}(X_1, X_2)$$
$$\Xi_2 = \lambda\Omega - X^T + \bar{r}P_2 + rQ_2, \quad r = e^{j\theta}, \quad \theta = (1-\alpha)\frac{\pi}{2}.$$

The filter and controller gains L and K are specified by Eq. (19).

Proof By changing Θ and Z as follows:

$$\Theta = \begin{bmatrix} -\tilde{E}^T\tilde{E} & -\tilde{E}^T D_f \\ -D_f^T\tilde{E} & -D_f^T D_f + \beta^2 I \end{bmatrix} \tag{35}$$

$$Z = \begin{bmatrix} -\tilde{E}^T\tilde{E} & \bar{r}P_1 + rQ_1 & -\tilde{E}^T D_d \\ rP_1 + \bar{r}Q_1 & 0 & 0 \\ -D_d^T\tilde{E} & 0 & -D_f^T D_f + \beta^2 I \end{bmatrix} \tag{36}$$

The rest of the proof is similar to that of Theorem 1, and the inequality in Eq. (34) is satisfied.

At this point, all LMI feasibility solutions of the proposed detection and control objectives in Theorems 1–4 are obtained. The next corollary gives a procedure for solving the optimization SFDC problem.

Corollary 1 *Given γ_1, γ_2 and γ_3, a feasible solution to the SFDC problem for system (6) is obtained by solving the following convex optimization problem*:

$$\max_{X_1, X_2, P_1, P_2, P_3, P_4, Q_1, Q_2, Q_3, Q_4, N, M} \beta \quad \text{s.t} \quad (18), (32), (33), (34) \tag{37}$$

Proof By invoking Theorems 1, 2, 3 and 4 in previous sections, Corollary 1 can be proved without difficulty.

Remark 3 Now, after all the works that were done to detect the fault in this article, the last and most vital stage in the SFDC techniques is to define threshold J_{th} and evaluation function $J(t)$ for developing an FD deductive logic. In this work, the residual evaluation function is defined as Frank and Ding (1997)

$$J(t) = \left(\theta^{-1}\int_0^\theta r^T(s)r(s)\mathrm{d}s\right)^{1/2} \tag{38}$$

where θ represents the detection time range. The upper threshold values are calculated as

$$J_{th} = \sup_{\substack{f(t)=0 \\ d(t)\in L_2}} J(t) \tag{39}$$

The following logic relationship can diagnose whether there is a fault or not:

$$J(t) > J_{th} \Rightarrow \text{Alarm (a fault is detected)}$$
$$J(t) \le J_{th} \Rightarrow \text{No alarm (fault free)}$$

4 Numerical Example

To show the effectiveness and capabilities of the proposed methodology and solution in this paper, consider the FO-LTI system (1) with the following parameters where $0<\alpha<1$:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}, \quad C = [0.1 \quad 0], \quad E = [00.1],$$
$$B = [0.2, 0]^T, \quad D = 0, \quad \alpha = 0.5, \quad \lambda = 1$$

The disturbance and fault models are supposed as:

$$B_d = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$
$$D_d = D_f = 0.2, F_f = F_d = 0.1,$$

$d(t)$ is assumed to be $0.1\exp(-0.4k)\cos(0.03\pi k)$. The fault signal $f(t)$ is a square wave of unit amplitude that occurred from 40 to 60 steps. It is favorable to detect the fault $f(t)$ in the presence of the disturbance $d(t)$. For given $\gamma_1 = 1.7017, \gamma_2 = 1.7874$ and $\gamma_3 = 1.1944$, we solve the optimization problem (37) by YALMIP toolbox in MATLAB and obtain $\beta = 0.0836$. Furthermore, the observer and controller gains were obtained, respectively, as follows:

$$L = \begin{bmatrix} 50.1036 \\ 13.6360 \end{bmatrix}, \quad K = \begin{bmatrix} -61.3024 \\ -10 \end{bmatrix}$$

Given the initial condition $x(0) = [00]^T$, the simulation results are shown in Figs. 1, 2, 3, 4, 5 and 6. In Fig. 1, the eigenvalues of matrix A are depicted. It is easy to see that the open-loop system is unstable. Figure 2 shows the step response of an open-loop system. The state trajectories ($x_1(t)$ and $x_2(t)$) are depicted in Figs. 3 and 4. These step responses confirm that the closed-loop system is stable with our proposed control strategy in this paper. The residual signal $r(t)$ is shown in Fig. 5. It can be concluded that the robustness against disturbance and the fault sensitivity are both amplified, and the fault is well separated from disturbance. Hence, by using a threshold test, the fault $f(t)$ can be effectively detected. The regulated output of the closed-loop system is depicted in Fig. 6. It can be understood that the effects of disturbance and fault on the regulated output have been weakened. At this point, by selecting $d(t) = 0.01w(t)$, where $w(t)$ is an energy-limited white noise, the effect of noise on the system will be investigated. Figure 7 shows the residual response to noise and fault

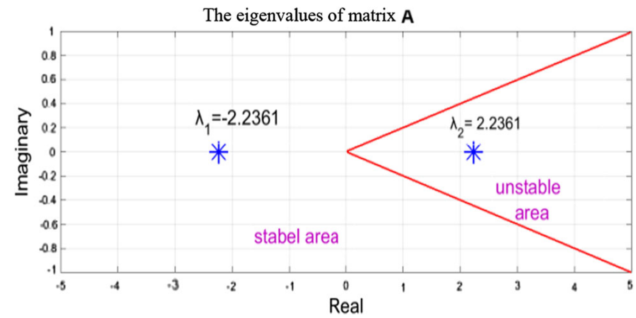

Fig. 1 Eigenvalues of matrix A

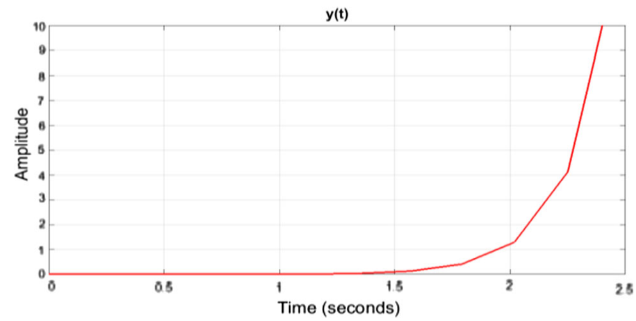

Fig. 2 Step response of the open-loop system

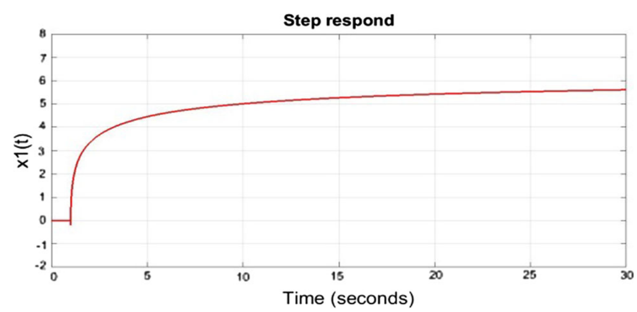

Fig. 3 Step response of the closed-loop system

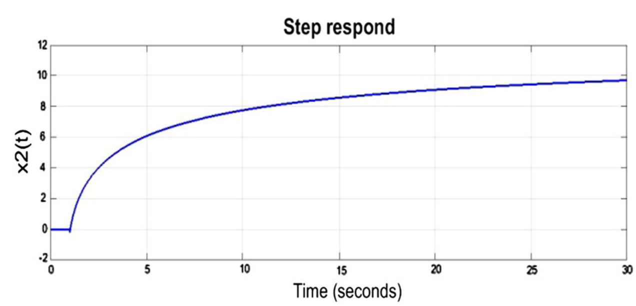

Fig. 4 Step response of the closed-loop system

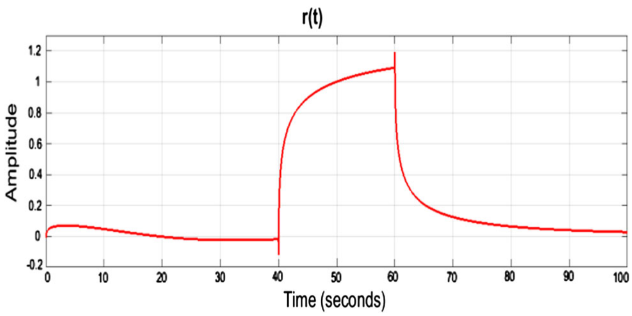

Fig. 5 Residual signal $r(t)$

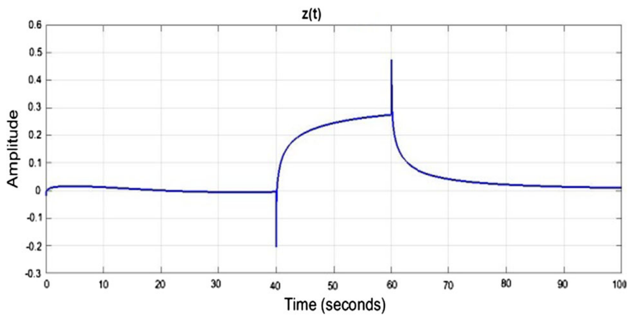

Fig. 6 Regulated output $z(t)$

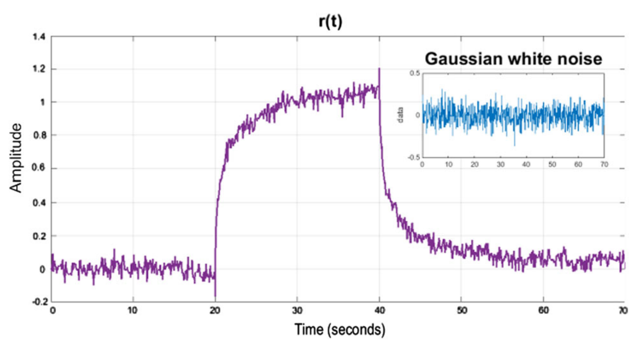

Fig. 7 Residual signal $r(t)$

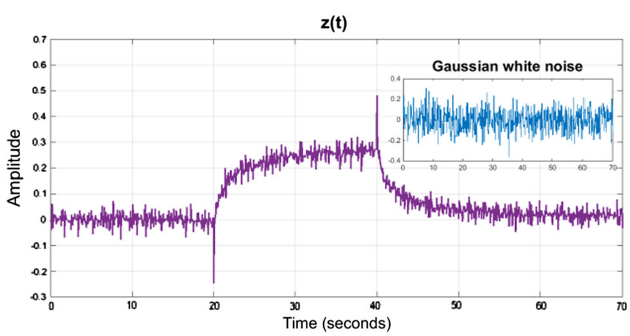

Fig. 8 Regulated output $z(t)$

input. It demonstrates that the system is robust against noise, and moreover, the fault sensitivities are enhanced. The regulated output $z(t)$ is shown in Fig. 8. From Fig. 8, it can be concluded that the effects of noise and fault on the regulated output have been attenuated. For a given $\gamma_1 = \gamma_2 = \gamma_3 = 1$, we solve the optimization problem (37) and obtain $\beta = 0.0282$. This result shows a larger value of β is obtained in comparison with the study performed by Davoodi et al. (2012). It shows more sensitivity of residual generator to fault signal. Note that, we only compared the results, systems and methods were different.

5 Conclusion

In this paper, a robust distributed simultaneous fault detection and control (SFDC) problem for fractional-order systems using observer detector and state feedback controller is proposed and developed. An LMI approach for SFDC design in fractional-order system has been introduced in order to stabilize the closed-loop system and guarantee some control and detection objectives. Finally, some simulation results are given to illustrate the effectiveness and capabilities of the proposed approach. In future work, this problem will be solved by considering other types of filters and also the use of fuzzy algorithms can develop this issue.

References

Abdeljawad T, Al-Mdallal QM, Jarad F (2019) Fractional logistic models in the frame of fractional operators generated by conformable derivatives. Chaos Solitons Fract 119:94–101

Ahmed S, Wang H, Tian Y (2019) Adaptive high-order terminal sliding mode control based on time delay estimation for the robotic manipulators with backlash hysteresis. IEEE Trans Syst Man Cybern Syst. https://doi.org/10.1109/TSMC.2019.2895588

Aribi A, Farges C, Aoun M, Melchior P, Najar S, Abdelkrim MN (2014) Fault detection based on fractional order models: application to diagnosis of thermal systems. Commun Nonlinear Sci Numer Simul 19(10):3679–3693

Bankupalli PT, Ghosh S, Kumar L (2018) Fractional order modeling and two loop control of PEM fuel cell for voltage regulation considering both source and load perturbations. Int J Hydrog Energy 43:6294–6309

Battaglia JL, Lay LL, Bastale JC, Oustaloup A, Cois O (2000) Heat flux estimation through inverted non-integer identification models. Int J Therm Sci 39(3):374–389

Boukal Y, Darouach M, Zasadzinski M (2018) Large-scale fractional-order systems: stability analysis and their decentralised functional observers design. IET Control Theory Appl 12(3):359–367. https://doi.org/10.1049/iet-cta.2017.0264

Butzer P, Westphal U (2000) An introduction to fractional calculus. World Scientific, Singapore

Chadli LL, Ding M, Qiu SX, Yang J, Diagnositic Y (2018) Observer design for *t–s* fuzzy systems: application to real-time weighted fault detection approach. IEEE Trans Fuzzy Syst 26:805–816

Davoodi MR, Talebi HA, Momeni HR (2012a) A novel simultaneous fault detection and control approach based on dynamic observer. Int J Innov Comput Inf Control 8:4915–4930

Davoodi MR, Golabi A, Talebi HA, Momen HR (2012b) Simultaneous fault detection and control design for switched linear systems: a linear matrix inequality approach. J Dyn Syst Meas Control 134:061010

Davoodi MR, Meskina N, Khorasani Kh (2016) Simultaneous fault detection and consensus control design for a network of multi-agent systems. Automatica 66:185–194

Ding SX (2008) Model based fault diagnosis techniques-design schemes, algorithms and tools. Springer, Berlin

Ding SX (2009) Integrated design of feedback controllers and fault detectors. Annu Rev Control 33:124–135

Dong HL, Wang ZD, Ding SX, Gao HJ (2012) Finite-horizon estimation of randomly occurring faults for a class of nonlinear time-varying systems. Automatica 50(12):3182–3189

Fadiga L, Farges C, Sabatier J, Moze M (2011) On computation of $H\propto$ norm for commensurate fractional order systems. In: Proceedings of the 50th IEEE conference on decision and control and European control conference. IEEE, Orlando, pp 8231–8236

Farges C, Fadiga L, Sabatier J (2013) H_∞ analysis and control of commensurate fractional order systems. Mechatronics 23(7):772–780

Frank P, Ding X (1997) Survey of robust residual generation and evaluation methods in observer-based fault detection systems. J Process Control 7(6):403–424

Gahinet P, Apkarian P (1994) A linear matrix inequality approach to $H\propto$ control. Int J Robust Nonlinear Control 4(4):421–448

Green M, Limebeer DJN (1995) Linear robust control. Prentice-Hall Inc, Upper Saddle River

Hernandez O, Ortiz P, Herrera J (2014) Cement rotary kiln model using fractional identification. IEEE Latin Am Trans 12(2):87–92

Iwasaki T, Hara S (2005) Generalized KYP lemma: unified frequency domain inequalities with design applications. IEEE Trans Autom Control 50(1):41–59

Kenneth S, Bertram R (1993) An introduction to the fractional calculus and fractional differential equations'. Wiley, Hoboken

Khandani K, Majd VJ, Tahmasebi M (2017) Robust stabilization of uncertain time-delay systems with fractional stochastic noise using the novel fractional stochastic sliding approach and its application to stream water quality regulation. IEEE Trans Autom Control 62(4):1742–1751

Khosrowjerdi MJ, Nikoukhah R, Safari-Shad N (2004) A mixed H_2/H_∞ approach to simultaneous fault detection and control. Automatica 40:261–267

Li XJ, Yang GH (2012) Dynamic observer-based robust control and fault detection for linear systems. IET Control Theory Appl 6:2657–2666

Li XJ, Yang GH (2015) Adaptive $H\propto$ control in finite frequency domain for uncertain linear. Inf Sci 314(1):14–27

Li L, Luo H, Ding SX, Yang Y, Peng K (2019) Performance-based fault detection and fault-tolerant control for automatic control systems. Automatica 99:308–316

Liang S, Wei Y-H, Pan J-W, Gao Q, Wang Y (2015) Bounded real lemmas for fractional order systems. Int J Autom Comput 12(2):192–198

Liu L, Liu X, Man Ch (2016) Delayed observer-based H_∞ control for networked control systems. Neurocomputing 179:101–109

Liu X, Su X, Shi P, Nguang SK, Shen C (2019) Fault detection filtering for nonlinear switched systems via event-triggered communication approach. Automatica 101:365–376

Luo B, Wang H, Liu H, Li B, Peng F (2019) Early fault detection of machine tools based on deep learning and dynamic identification. IEEE Trans Ind Electron 66(1):509–518

Malti R, Aoun M, Levron F, Oustaloup A (2011) Analytical computation of the H_2 norm of fractional commensurate transfer functions. Automatica 47(11):2425–2432

Matignon D (1998) Stability properties for generalized fractional differential systems. ESAIM Proc 5:145–158

Meskin N, Khorasani K (2009) Robust fault detection and isolation of time-delay systems using a geometric approach. Automatica 45(6):1567–1573

Morand A, Moreau X, Melchior P, Moze M, Guillemard F (2016) CRONE cruise control system. IEEE Trans Veh Technol 65(1):15–28

Moze M, Sabatier J, Oustaloup A (2008) On bounded real lemma for fractional systems. In: Proceedings of the 17th IFAC world congress. IFAC, COEX, pp 15267–15272

Pisano A, Rapaic MR, Usai E, Jelici Z (2011) Second-order sliding mode approaches to disturbance estimation and fault detection in fractional-order systems. IFAC Proc Vol 44(1):2436–2441

Pisano A, Rapaic MR, Usai E (2014) Discontinuous dynamical systems for fault detection: a unified approach including fractional and integer order dynamics. Math Comput Simul 95:111–125

Poojary UR, Gangadharan KV (2018) Integer and fractional order-based viscoelastic constitutive modelling to predict the frequency and magnetic field induced properties of magneto rheological elastomer. J Vib Acoust 140(4):041007

Raynaud HF, Zergaınoh A (2000) State-space representation for fractional order controllers. Automatica 36(7):1017–1021

Ren HP, Fan JT, Kaynak O (2019) Optimal design of a fractional-order proportional-integer-differential controller for a pneumatic position servo system. IEEE Trans Ind Electron 66(8):6220–6229

Sabatier J, Moze M, Oustaloup A (2005) On fractional systems $H\propto$ norm computation. In: Proceedings of the 44th IEEE conference on decision and control and European control conference. IEEE, Seville, pp 5758–5763

Sabatier J, Moze M, Farges C (2010) LMI stability conditions for fractional order systems. Comput Math Appl 59(5):1594–1609

Sakthivel R, Selvaraj P, Lim Y, Karimi HR (2017) Adaptive reliable output tracking of networked control systems against actuator faults. J Frankl Inst 354(9):3813–3837

Sayyaf N, Tavazoei MS (2018) Robust control for time fractional diffusion processes: application in temperature control of an alpha silicon carbide cutting tool. IET Control Theory Appl 12(15):2022–2030

Sharma R, Bhasin Sh, Gaur P, Joshi D (2019) A switching-based collaborative fractional order fuzzy logic controllers for robotic manipulators. Appl Math Model 73:228–246

Shi P, Liu M, Zhang L (2015) Fault-tolerant sliding-mode-observer synthesis of markovian jump systems using quantized measurements. IEEE Trans Ind Electron 62(9):5910–5918

Shokouhinejad H, Ghiasi AR, Badamchizadeh MA (2017) Robust simultaneous finite-time control and fault detection for uncertain linear switched systems with time-varying delay. IET Control Theory Appl 11(7):1041–1052

Soltani H, Naoui S, Aitouche A (2015) Robust simultaneous fault detection and control approach for time-delay systems. IFAC 48:1244–1249

Tian J, Xiong R, Yu Q (2019) Fractional-order model-based incremental capacity analysis for degradation state recognition of lithium-ion batteries. IEEE Trans Ind Electron 66(2):1579–1584

Wang H, Yang GH (2009) Simultaneous fault detection and control for uncertain linear discrete-time systems. IET Control Theory Appl 3:583–594

Wang JL, Yang GH, Liu J (2007) An LMI approach to H_{-} index and mixed H_{-}/H_{∞} fault detection observer design. Automatica 43(9):1656–1665

Wang Sh, Jiang Y, Li Y (2017) Fault detection and control co-design for discrete-time delayed fuzzy networked control systems subject to quantization and multiple packet dropouts. Fuzzy Sets Syst 306:1–25

Wei J, Dong G, Chen Z (2019) Model-based fault diagnosis of Lithium-ion battery using strong tracking extended Kalman Filter. Energy Proc 158:2500–2505

Yang C, Yu H, Shang Y, Fei W (2015) Characterization of CMOS metamaterial transmission line by compact fractional-order equivalent circuit model. IEEE Trans Electron Devices 62(9):3012–3018

Zhai D, An L, Dong J (2016) Simultaneous H_2/H_{∞} fault detection and control for networked systems with application to forging equipment. Signal Process 125:203–215

Zhang S, Yu Y, Yu J (2017) LMI conditions for global stability of fractional-order neural networks. IEEE Trans Neural Netw Learn Syst 28(10):2423–2433

Zhong GX, Yang GH (2016) Simultaneous control and fault detection for discrete-time switched delay systems under the improved persistent dwell time switching. IET Control Theory Appl 10:814–824

Zolghadri A, Henry D, Cieslak JO, Efimov D, Goupil P (2014) Fault diagnosis and fault-tolerant control and guidance for aerospace vehicles, from theory to application. Springer, London