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RESEARCH PAPER

Application of Multi-speed Gearbox in Infinitely Variable Transmission

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Abstract One way to reduce vehicles' fuel consumption (FC) is to optimize different parts of its powertrain. Among different modern powertrains, continuously variable transmission (CVT) is one of the common types which changes speed ratio between engine and wheels, continuously. Therefore, the CVT allows the engine to operate at its fuel-optimal speed. Due to the limited power transmission capacity and speed ratio range of the CVT, researchers attempted to combine the CVT with a planetary gear train and a fixed ratio mechanism. This mechanism is called infinitely variable transmission (IVT). In this study, a multi-speed gearbox (MSG) is used in the IVT and its impact on the powertrain performance and the vehicle's FC is investigated. After extracting the governing dynamics of the system, an optimization on the parameters which affect the IVT performance is implemented to minimize the vehicle's FC. It is observed that the vehicle's FC for the case of using the optimized IVT equipped with two-speed MSG is almost 5% less than the case of using the optimized six-speed AT transmission in different driving cycles.

Keywords Infinitely variable transmission · Continuously variable transmission · Fuel consumption · Optimization · Fixed ratio mechanism

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1 Introduction

Continuously variable transmission (CVT) is a kind of power transmission which creates a continuous range of speed ratio. Although the efficiency of the CVT is lower than the conventional powertrains, owing to some beneficial aspects of these powertrains, these powertrains have recently been used in some vehicles. One of the major preferences of these powertrains, in addition to their ability to change the speed ratio continuously, is the fact that engine rpm in case of using this transmission is almost independent of the vehicle speed. Thus, engine is able to operate in its fuel-optimal rpm, where its fuel consumption (FC) is minimal (Delkhosh and Foumani 2013a).

An infinitely variable transmission (IVT) is a type of continuously variable powertrain which is able to create a wide range of speed ratio besides zero and negative values. The core of IVT is a CVT, connected to a planetary gear train (PG) and some fixed ratio mechanisms (FR). Application of the CVT in combination with these elements expands its speed ratio range and increases power transmission capacity.

A number of studies have paid attention to continuously variable powertrains. Beachley et al. (1984) examined a split-path configuration which was able to create a larger speed ratio range than that of the CVT. Yan and Hsieh (1994) examined the maximum mechanical efficiency of an IVT. They also demonstrated that the PG must be placed at the output node in order to reach a zero speed in the output. Mangialardi and Mantriota (1999) studied the power flow types in the IVT as well as its efficiency equation for different power flow types. In addition to the CVT and PG in their mechanism, they considered an FR which can be connected to the CVT in the series or parallel arrangement. Following the studies (Mantriota 2001, 2002a, b), Mantriota revealed that using the CVT with two power flow paths can be more effective than using the CVT, besides creating the same speed ratio range. Mantriota (2001) showed that the third type of power flow (i.e., the power flow without recirculation) is generated only when the speed ratio range of the IVT is less than that of the CVT. Therefore, he studied the first and second types of power flow in his studies. He experimentally examined the efficiency of a series IVT with type 1 power flow and a parallel IVT with type 2 power flow (Mantriota 2002a, b). For this system, he used a belt CVT, a FR and a PG. Pfiffner et al. (2003) and Pfiffner and Guzzella (2001) presented the solution of the fuel-optimal control problem for CVT-based powertrains in transient conditions by using the numerical optimization package direct collocation method (DIRCOL). Frank (2004) emphasized that the fuel economy for conventional vehicle with a CVT is limited. He described an engine control strategy for hybrid electric CVT powertrain with fewer mechanical parts, but many operation modes to obtain the best fuel economy. Bottiglione et al. (2014) investigated the power recirculation and the torque ratio of IVT. They investigated an experimental study and compared the results with a theoretical model in steady-state condition. They demonstrated that the torque transmitted by the CVT in constant speed ratio can be controlled by changing the slip of the CVT belt. Hebbale et al. (2014) simulated a non-circular gear IVT and studied the torque recirculation impact on stress, efficiency and fuel economy. Delkhosh et al. (2014) presented a parallel IVT which has embedded a FR mechanism between IVT and the final drive with two different types of power flow and minimized the vehicle's FC in the New European Driving Cycle (NEDC) by optimizing the powertrain system. Delkhosh and Foumani (2015) considered a power-split CVT including several FRs in all possible places and optimized the speed ratios of the FRs with the aim of minimizing the vehicle's FC in several driving cycles. Kazemzadeh-Parsi (2014) modified the firefly optimization algorithm to improve its performance.

In this study, the IVT is equipped with a multi-speed gearbox (MSG) and an additional FR beside the FR, PG and CVT to increase the efficiency of the IVT. After modeling this transmission, it is attempted to optimize the speed ratios of the IVT components as well as the speed ratio range of the IVT in order to achieve the optimal fuel economy. Then by employing particle swarm optimization (PSO) algorithm, the optimal number of MSG's speed is determined in order to increase achievable fuel economy.

2 Infinitely Variable Transmission

As shown in Fig. 1, the conventional type of IVT includes three main elements: a CVT, a PG and a FR mechanism.

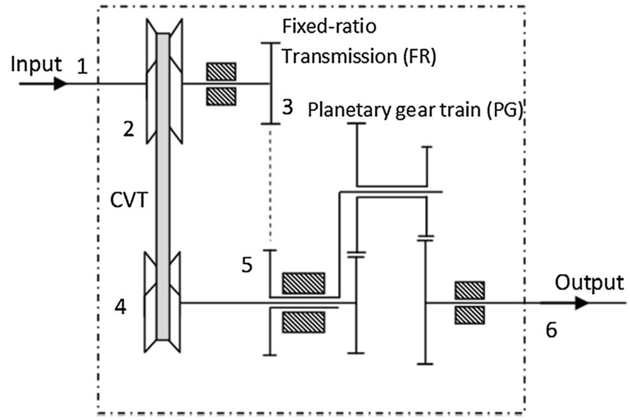


Fig. 1 Schematic arrangement of a typical IVT (Mantriota 2002a)

The IVT efficiency is influenced by the speed ratio and efficiency of its components. The efficiencies of the PG and FR are almost fixed (Mangialardi and Mantriota 1999), while the CVT efficiency is changeable due to variation of its input torque, rotational speed and speed ratio. These parameters can be changed by variation of the PG and FR speed ratio. The speed ratios of PG and FR are definite for a specific range of the CVT and the IVT (Mantriota 2002a). Therefore, if the speed ratio ranges of typical IVT and CVT are fixed, efficiency improvement in this powertrain is not possible.

In this study, an IVT including three FR mechanisms is considered. The schematic arrangement of this powertrain is shown in Fig. 2. It is not necessary to use all FR mechanisms as a single-speed gearbox in the IVT, and some FRs could be in the form of MSG. Embedding additional FRs and MSG adds a number of variables to the efficiency function which enables us to reach a high efficiency and, therefore, low vehicle's FC by changing their speed ratios. However, using a MSG with several speed ratios without remarkable FC reduction may be unnecessary. Thus, it is necessary to find the optimum number of MSG speed ratios. We consider that one of the FR mechanisms is replaced by a MSG and the optimum number of its speeds is determined in the following. According to

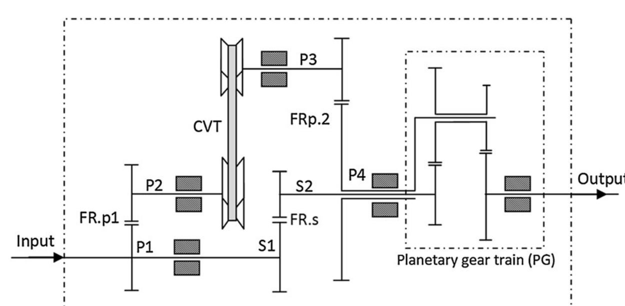


Fig. 2 Schematic arrangement of the proposed IVT

Fig. 2, MSG can be embedded in the place of each of FR.p1, FR.p2 and FR.s. Therefore, there are three arrangements. It is notable that FR.p1 and FR.p2 mean the FR mechanisms connected to the planet of PG, while FR.s means the FR connected to the sun gear of PG.

3 IVT Governing Dynamics

In order to analyze the IVT dynamics, first we suppose that the MSG operates similar to FR (see Fig. 2). Then one of FRs is substituted with a MSG, and the dynamic relations are changed. According to Fig. 2, the rotational speed of each link considering input rotational speed (ω_{In}) and the ratio of the IVT components are calculated according to Eqs. (1) and (2). Moreover, speed ratio of the PG is defined in Eq. (3).

$$\begin{aligned}\omega_{P4} &= \omega_{p3} \times \left(\tau_{\mathrm{FR.p2}}\right) = \omega_{p2} \times \left(\tau_{\mathrm{CVT}} \times \tau_{\mathrm{FR.p2}}\right) \\ &= \omega_{\mathrm{In}} \times \left(\tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{CVT}} \times \tau_{\mathrm{FR.p2}}\right)\end{aligned} \tag{1}$$

$$\omega_{S2} = \omega_{\mathrm{In}} \times \left(\tau_{\mathrm{FR.s}}\right) \tag{2}$$

$$\tau_{\mathrm{PG}} = \frac{\omega_{\mathrm{Out}} - \omega_{\mathrm{Planet}}}{\omega_{\mathrm{Sun}} - \omega_{\mathrm{Planet}}} = \frac{\omega_{\mathrm{Out}} - \omega_{P4}}{\omega_{S2} - \omega_{P4}} \tag{3}$$

where ω_X and τ_X denote the rotational speed and the speed ratio of each component, respectively. From Eqs. (1)–(3), the speed ratio of the IVT as a function of its components' speed ratio is obtained as follows:

$$\begin{aligned}\tau_{\mathrm{IVT}} &= \frac{\omega_{\mathrm{Out}}}{\omega_{\mathrm{In}}} \\ &= \tau_{\mathrm{PG}} \times \tau_{\mathrm{FR.s}} + \tau_{\mathrm{CVT}} \times \left(\tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times (1 - \tau_{\mathrm{PG}})\right)\end{aligned} \tag{4}$$

In this equation, the changes of IVT speed ratio can be in proportion or inverse proportion to the changes of the CVT speed ratio, depending on the sign of the τ_{CVT} coefficient $\left(\tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times (1 - \tau_{\mathrm{PG}})\right)$. Equations (5) and (6) reveal this relationship.

$$\begin{cases} \tau_{\mathrm{IVTmin}} = \tau_{\mathrm{PG}} \times \tau_{\mathrm{FR.s}} + \tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times \tau_{\mathrm{CVTmin}} \times (1 - \tau_{\mathrm{PG}}) \\ \tau_{\mathrm{IVTmax}} = \tau_{\mathrm{PG}} \times \tau_{\mathrm{FR.s}} + \tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times \tau_{\mathrm{CVTmax}} \times (1 - \tau_{\mathrm{PG}}) \end{cases} \text{if } \tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times (1 - \tau_{\mathrm{PG}}) > 0 \tag{5}$$

$$\begin{cases} \tau_{\mathrm{IVTmin}} = \tau_{\mathrm{PG}} \times \tau_{\mathrm{FR.s}} + \tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times \tau_{\mathrm{CVTmax}} \times (1 - \tau_{\mathrm{PG}}) \\ \tau_{\mathrm{IVTmax}} = \tau_{\mathrm{PG}} \times \tau_{\mathrm{FR.s}} + \tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times \tau_{\mathrm{CVTmin}} \times (1 - \tau_{\mathrm{PG}}) \end{cases} \text{if } \tau_{\mathrm{FR.p1}} \times \tau_{\mathrm{FR.p2}} \times (1 - \tau_{\mathrm{PG}}) < 0 \tag{6}$$

If one of FRs is considered as a MSG, each speed of the MSG results in a definite speed ratio range for the IVT. For each speed ratio of the MSG and speed ratio range of the IVT, Eq. (5) or (6) must be satisfied. There should be an overlap between speed ratio ranges of the IVT, which eventually cover the overall IVT ratio range. For this purpose Eqs. (7) and (8) should be established among the IVT ratio ranges.

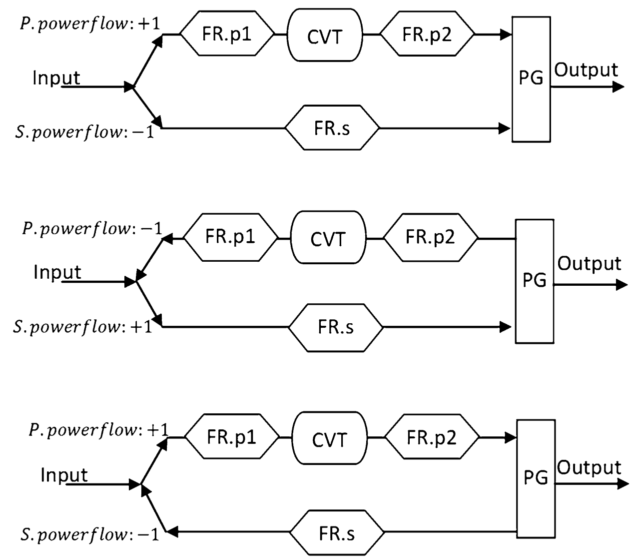


Fig. 3 Feasible power flow directions in the IVT system

$$\tau^{j+1}_{\mathrm{IVTmin}} \le \tau^{j}_{\mathrm{IVTmax}}; \quad j = 1, 2, \ldots, n-1 \tag{7}$$

$$\tau^{1}_{\mathrm{IVTmin}} \le \tau_{\mathrm{IVTmin\ Desired}} \quad \text{and} \quad \tau_{\mathrm{IVTmax\ Desired}} \le \tau^{n}_{\mathrm{IVTmax}} \tag{8}$$

In these equations, the index j represents the number of the IVT ratio ranges and n denotes the total number of the MSG's speeds. The desired speed ratio range of the IVT ($[\tau_{\mathrm{IVTmin\ Desired}}, \tau_{\mathrm{IVTmax\ Desired}}]$) is determined with respect to the intended performance of the vehicle.

In order to establish the governing dynamics of the proposed IVT, it is necessary to determine the possible power flow paths. The power flow direction is determined with respect to the speed ratio and efficiency of the IVT components (Bottiglione and Mantriota 2008). There are three feasible power flow directions shown in Fig. 3. Regarding operating conditions, the power flow direction is matched to one of the shown directions.

In this system, the input power is split into two separate power flows in the IVT input, and the power flows will be unified in the PG after passing through different elements, and get out of the IVT. As shown in Fig. 3, if the input power flows from the input node to the PG, the sign of the power flow direction (P.power flow or S.power flow) will be positive and if it flows from the PG to the input node, the sign of direction will be negative.

To determine the efficiency of the IVT, it is necessary to derive dynamic relations among IVT elements. Equations (9) and (10) are obtained regarding the power balance in the input node and the PG.

$$T_{\text{In}} = T_{p1} + T_{s1} \tag{9}$$

$$T_{\text{Output}} = T_{\text{Planet}} + T_{\text{Sun}} = T_{p4} + T_{s2} \tag{10}$$

where T_X denotes the torque of links. By multiplying speed by the torque, the power flow through each element (P_X) is calculated. Due to the equality of the speeds in the input node, Eq. (11) is established.

$$P_{\text{In}} = P_{p1} + P_{s1} \tag{11}$$

Since the aim is to find an overall equation for all power flows, we used the power flow direction parameters (i.e., S.power flow and P.power flow) in order to obtain a general equation shown in Eqs. (12) and (13).

$$P_{\text{Planet}} = P_{p4} = \left(\eta_{\text{FR.p1}} \times \eta_{\text{CVT}} \times \eta_{\text{FR.p2}}\right)^{(\text{P.powerflow})} \times P_{p1} \tag{12}$$

$$P_{\text{Sun}} = P_{s2} = \left(\eta_{\text{FR.s}}\right)^{(\text{S.powerflow})} \times P_{s1} \tag{13}$$

where η_X denotes the efficiency of each element. Then the IVT output power is obtained by:

$$P_{\text{Out}} = P_{p4} \times \eta_{\text{PG}}^{\frac{\text{P.powerflow}+|\text{P.powerflow}|}{2}} + P_{s2} \times \eta_{\text{PG}}^{\frac{\text{S.powerflow}+|\text{S.powerflow}|}{2}} \tag{14}$$

In Eq. (14), when the sign of power flow is positive (i.e., the link is related to the power input of the planetary gear), the efficiency exponent will be equal to one and when it is negative (i.e., the link is related to the power output), it is equal to zero. Therefore, the efficiency of the PG is applied only to the power of input links. Equation (15) represents this relation:

$$\begin{aligned} \frac{\text{P.powerflow} + |\text{P.powerflow}|}{2} &= \begin{cases} 1 & \text{if P.powerflow} = +1 \\ 0 & \text{if P.powerflow} = -1 \end{cases} \\ \frac{\text{S.powerflow} + |\text{S.powerflow}|}{2} &= \begin{cases} 1 & \text{if S.powerflow} = +1 \\ 0 & \text{if S.powerflow} = -1 \end{cases} \end{aligned} \tag{15}$$

Finally, the efficiency of the IVT is obtained by Eq. (16).

$$\eta_{\text{IVT}} = \left(\frac{\left(\eta_{\text{FR.s}}\right)^{(\text{S.powerflow})} \times \eta_{\text{PG}}^{\frac{\text{S.powerflow}+|\text{S.powerflow}|}{2}}}{1+M}\right) + \left(\frac{\left(\eta_{\text{FR.p1}} \times \eta_{\text{CVT}} \times \eta_{\text{FR.p2}}\right)^{(\text{P.powerflow})} \times \eta_{\text{PG}}^{\frac{\text{P.powerflow}+|\text{P.powerflow}|}{2}}}{1+\frac{1}{M}}\right) \tag{16}$$

The parameter M is calculated by:

$$M = \frac{\left(\frac{\left(\eta_{\text{FR.s}}\right)^{(\text{S.powerflow})}}{\tau_{\text{S.total}}}\right) \times \left(\tau_{\text{S.total}} \times \eta_{\text{PG}}^{\frac{\text{S.powerflow}+|\text{S.powerflow}|}{2}} - \tau_{\text{IVT}}\right)}{\left(\frac{\left(\eta_{\text{FR.p1}} \times \eta_{\text{CVT}} \times \eta_{\text{FR.p2}}\right)^{(\text{P.powerflow})}}{\tau_{\text{P.total}}}\right) \times \left(\tau_{\text{IVT}} - \tau_{\text{P.total}} \times \eta_{\text{PG}}^{\frac{\text{P.powerflow}+|\text{P.powerflow}|}{2}}\right)} \tag{17}$$

Equations (16) and (17) are useable for each of the three types of power flows. Regarding these equations, to calculate the IVT efficiency, it is necessary to determine the efficiency of its components.

Due to small changes in the efficiency of the FRs and PG, their efficiencies are assumed 98% as a fixed value (Mangialardi and Mantriota 1999). A V-belt CVT is used in this IVT. The efficiency of the V-belt CVT is obtained as a function of its input torque, rotational speed and the maximum values of these parameters (Nair and Singh 1992).

In the process of efficiency calculation, at first, the input torque and speed of the CVT are not known, and therefore, the IVT efficiency is unknown. Thus, a rough estimate is considered for the efficiency of the CVT. After determining the input torque and rotational speed of the CVT, its efficiency is revised. Again, the transmitting torque of each part is set using the revised efficiency of the CVT. The procedure will continue until the IVT efficiency converges.

In this model, the most important outcome is IVT efficiency, which includes the most complicated parameters of the system. Therefore, the model accuracy is confirmed if the efficiency of the IVT obtained from the model is approximately equal to the efficiency of the experimental model. For this purpose, the efficiency extracted from the model is compared with Mantriota's experimental results (2002a). This comparison is provided in Fig. 4.

As shown in Fig. 4, the obtained results match the experimental results (Mantriota 2002a) perfectly. Therefore, the model can be used to calculate the IVT efficiency.

4 IVT Control Strategy

In order to reduce the vehicle's FC, the IVT speed ratio should be controlled to reach this aim. According to this strategy, the required power of the vehicle is determined at every moment of the driving with regard to the vehicle speed and acceleration.

The value of transmission efficiency is needed to calculate the required power of the engine. Since the efficiency of the IVT is unknown, at first a rough estimate is considered for its efficiency and then the required power of engine is obtained. Then, the fuel-optimal rpm of the

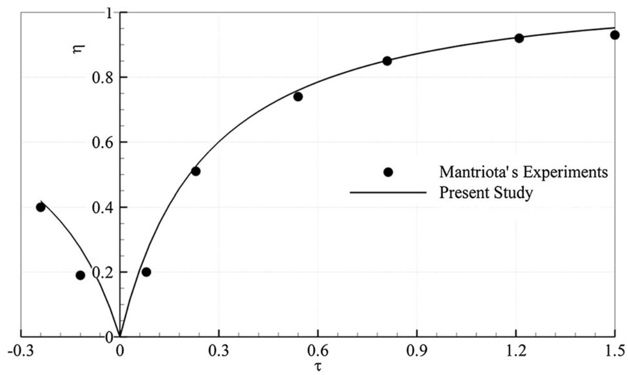


Fig. 4 Comparison of calculated IVT efficiency versus its speed ratio, with experimental data (Mantriota 2002a)

engine in this power is determined from the experimental data of the engine brake specific fuel consumption (bsfc).

Knowing the engine rpm and power, the input torque and rpm of the powertrain is determined, and consequently, the IVT speed ratio is defined with respect to the vehicle speed. In addition, the efficiency of the powertrain could be specified based on the IVT operating condition. Following the calculation of the IVT efficiency, the required power of the engine is determined. This algorithm is repeated until the efficiency of the powertrain is converged. The block diagram of this control strategy is presented in Fig. 5.

The FC of the vehicle equipped with this powertrain can be calculated using the methodology presented in Delkhosh and Foumani (2013b). In order to calculate the vehicle's FC, it is necessary to consider its motion in a driving cycle. In this paper, NEDC is chosen as the driving schedule. The overall FC in the NEDC can be obtained by Delkhosh and Foumani (2013b):

$$\mathrm{FC}\left(\frac{\mathrm{L}}{100\ \mathrm{km}}\right) = \left(\int \frac{\mathrm{bsfc} \times P}{3600\rho_{\mathrm{Fuel}}}\mathrm{d}t\right) \times \frac{100}{10.98} \cong \left(\sum \frac{\mathrm{bsfc}_i(\omega_i, P_i) \times P_i}{3600\rho_{\mathrm{Fuel}}}\Delta t_i\right) \times \frac{100}{10.98} \quad (18)$$

where bsfc is the engine brake specific fuel consumption presented by the engine manufacturer. These data are shown in Fig. 6. In this equation, ρ_{Fuel} is the fuel density, Δt is the simulation time step and 10.98 is the overall distance (in km) traveled in NEDC cycle. Some details of the considered vehicle are listed in Table 1.

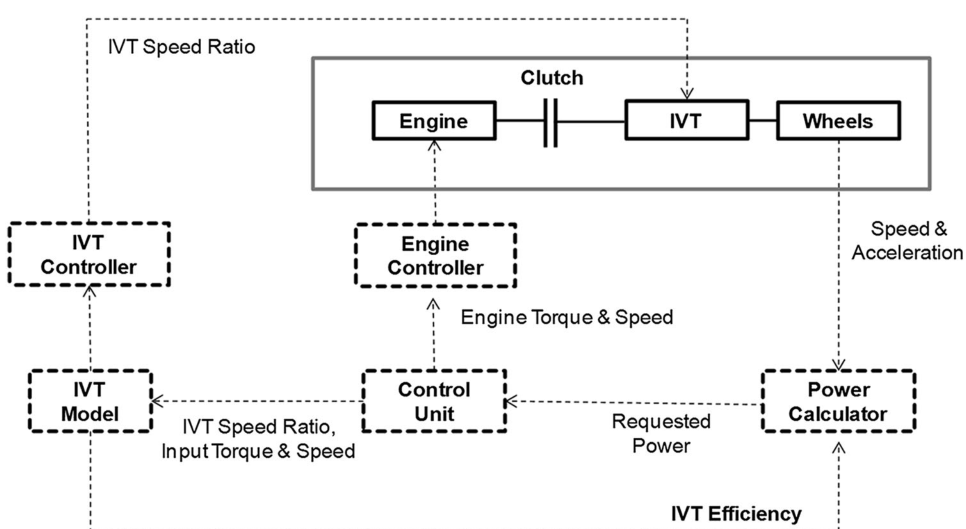
5 The Optimization of IVT

As shown earlier, the IVT efficiency changes due to variations of the components' speed ratios, the location and number of the MSG speed ratios, and also the efficiency of the CVT. Furthermore, the IVT speed ratio range affects the performance of the engine. Therefore, the speed ratio ranges of the IVT and its components can be optimized to achieve the optimal vehicle's FC.

For this purpose, particle swarm optimization algorithm (PSO) is employed. This method is completely described in Delkhosh et al. (2011).

As noted earlier, MSG can be placed in three different positions in the proposed IVT. These three different arrangements will be studied separately. Moreover, the number of MSG speeds could be variable. It is notable that the increase in MSG speeds could improve the IVT efficiency and, meanwhile, cause more complexity in the system. First, the number of MSG speeds is supposed, and then, the optimization procedure is done. Finally, the

Fig. 5 Block diagram of fuel control strategy



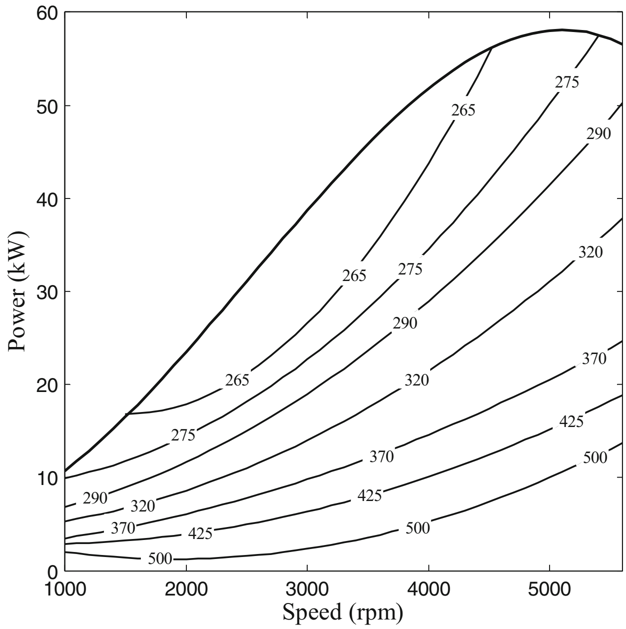


Fig. 6 bsfc curve of the vehicle's engine

Table 1 Detailed parameters of the vehicle

Parameters		Value
m	Vehicle mass	1100 kg
A	Frontal area	2.09 m^2
C_D	Drag coefficient	0.355
f_r	Rolling resistance coefficient	0.013
n_d	Final drive speed ratio	3.895
R_d	Tire dynamic radius	0.3279 m
η_d	Final drive efficiency	90%
$P_{\text{engine max}}$	Engine max. power	58 kW
$T_{\text{engine max}}$	Engine max. torque	126 N m
$\omega_{P \text{ max}}$	Engine speed at engine max. power	5500 rpm
$\omega_{T \text{ max}}$	Engine speed at engine max. torque	3500 rpm

results of the different arrangements will be compared with each other.

The optimization parameters and their variation ranges are presented in Table 2. The variation range of FRs, MSG and PG are defined considering real cases. Furthermore, the minimum lower bound and the maximum upper bound of the CVT ratio range are derived from Gauthier and Micheau (2010). The maximum lower bound and the minimum upper bound of the IVT ratio range ($[\tau_{\text{IVTmin Desired}}, \tau_{\text{IVTmax Desired}}]$) are defined with regard to the desired dynamic performance of the considered vehicle.

The maximum upper bound for the IVT ratio is assumed to be 3 according to the speed ratio of powertrain which is currently used in this vehicle. As discussed, the CVT used in this study is of V-belt type, though its transmitting torque and disk rpm are limited due to the limitation of friction coefficient and centrifugal forces (Srivastava and Haque 2008).

It is evident that the conditions of the problems are not satisfied unless the relationships among parameters are established. Therefore, these conditions must be considered as the constraints of the optimization problem. The maximum torque and transmitting rpm by CVT are set to be, respectively, $T_{\max(\text{CVT})_{\text{allowed}}} = 100$ N m and $\omega_{\max(\text{CVT})_{\text{allowed}}} = 7000$ rpm regarding the experimental model (Gauthier and Micheau 2010). The optimization problem is formulated below:

$$\begin{aligned}
&\text{Minimized FC}(X)\\
&\text{Subject to:}\\
&\quad \text{if}\, \tau_{\text{FR.p1}} \times \tau_{\text{FR.p2}} \times (1-\tau_{\text{PG}}) > 0:\\
&\qquad \begin{cases} h_1(X) = \tau_{\text{IVTmin}} - \tau_{\text{PG}} \times \tau_{\text{FR.s}} - \tau_{\text{FR.p1}} \times \tau_{\text{FR.p2}} \times \tau_{\text{CVTmin}} \times (1-\tau_{\text{PG}}) = 0\\ h_2(X) = \tau_{\text{IVTmax}} - \tau_{\text{PG}} \times \tau_{\text{FR.s}} - \tau_{\text{FR.p1}} \times \tau_{\text{FR.p2}} \times \tau_{\text{CVTmax}} \times (1-\tau_{\text{PG}}) = 0 \end{cases}\\
&\quad \text{if}\, \tau_{\text{FR.p1}} \times \tau_{\text{FR.p2}} \times (1-\tau_{PG}) < 0:\\
&\qquad \begin{cases} h_3(X) = \tau_{\text{IVTmin}} - \tau_{\text{PG}} \times \tau_{\text{FR.s}} - \tau_{\text{FR.p1}} \times \tau_{\text{FR.p2}} \times \tau_{\text{CVTmax}} \times (1-\tau_{\text{PG}}) = 0\\ h_4(X) = \tau_{\text{IVTmax}} - \tau_{\text{PG}} \times \tau_{\text{FR.s}} - \tau_{\text{FR.p1}} \times \tau_{\text{FR.p2}} \times \tau_{\text{CVTmin}} \times (1-\tau_{\text{PG}}) = 0 \end{cases}\\
&\quad g_j(X) = \tau^{j+1}_{\text{IVTmin}} - \tau^{j}_{\text{IVTmax}} \le 0; \quad j = 1, 2, \ldots, n-1\\
&\quad g_n(X) = T_{\max(\text{CVT})} - T_{\max(\text{CVT})_{\text{allowed}}} \le 0\\
&\quad g_{n+1}(X) = \omega_{\max(\text{CVT})} - \omega_{\max(\text{CVT})_{\text{allowed}}} \le 0\\
&\quad \text{where} \quad X = \left[\tau_{\text{FRp1}}, \tau_{\text{FRp2}}, \tau_{\text{FRs}}, \tau_{\text{PG}}, \tau_{\text{IVTmin Desired}}, \tau_{\text{IVTmax Desired}}, \tau_{\text{CVTmin}}, \tau_{\text{CVTmax}}\right]^{\text{T}}
\end{aligned}$$

Table 2 Optimization parameters and their variation ranges

Optimization parameters	Variation ranges
τ_{FR}	[0.25–4]
τ_{MSG}	[0.25–4]
τ_{PG}	$\left[\frac{1}{8} - 8\right]$
τ_{CVTmin}	[0.53–1]
τ_{CVTmax}	[1–1.7]
τ^1_{IVTmin}	$[0 - \tau_{IVTmin\ Desired}]$
τ^n_{IVTmax}	$[\tau_{IVTmax\ Desired} - 3]$
$\tau^i_{IVT(min,max)}$	$\left[\tau^1_{IVTmin} - \tau^n_{IVTmax}\right]$

In these equations n denotes the number of MSG speeds.

During the optimization procedure, if a set of optimization parameters violates the mentioned constraints, the value of the objective function is considered infinite. In this way, those values for the parameters would be eliminated.

5.1 Optimization Results

In order to minimize the vehicle's FC, optimization of the speed ratios is carried out with respect to:

- Different arrangements and number of MSG speeds.
- How the ratios of CVT and IVT are related according to Eqs. (5) and (6).

The optimization results are shown in Fig. 7.

As shown in Fig. 7, the FC decreases when the number of MSG speeds grows. Moreover, while the CVT coefficient in Eq. (4) $\left(\tau_{FR.p1} \times \tau_{FR.p2} \times (1 - \tau_{PG})\right)$ is negative, lower FC is expected compared to its positive values.

It should be noticed that difference between the optimal states of IVT through using two-speed and one-speed MSG in terms of the vehicle's FC is almost 5%, while this difference between the case of using two-speed and three-speed MSG is less than 1%. Consequently, it seems that more increase in the number of speeds has no significant effects on the vehicle's FC. On the other hand, as discussed above, increasing the speeds of MSG will raise the powertrain complexity. Due to these facts, the two-speed MSG is determined as the optimal MSG in the presented IVT.

As stated by the vehicle manufacturer, the FC of the vehicle equipped with a conventional powertrain (i.e., 5-speed manual gearbox) is 7.16 L/100 km. It is obvious that in a mode that all ratios are as one-speed and the coefficient of CVT ratio is positive, the vehicle's FC is greater than the case of using the conventional powertrain of the vehicle. Therefore, it is imperative to design the IVT accurately.

As a result, among different arrangements, the minimum FC will be obtained when MSG is placed between the CVT and PG. The optimized values of parameters for the mentioned condition are provided in Table 3.

As it can be seen, the parameters of the lower limits of the IVT and the CVT ratio ranges and speed ratios of some other elements are placed in the boundaries of their variation ranges. Therefore, the constraints of these parameters are of active types, and consequently, there is a possibility for obtaining lower FC by changing the boundaries of the constraints. However, as mentioned, the IVT ratio range is defined concerning the expected capabilities of the vehicle, and therefore, it is not changeable. Furthermore, the ranges of the PG, FR and CVT ratio are unchangeable, considering the difficulties of construction. Therefore, the active constraints of the optimization problem seem to be of rigid type.

In order to investigate the impact of using different FRs and MSG in the IVT (shown in Fig. 2), comparison of the proposed IVT with the typical IVT (shown in Fig. 1) seems to be essential. However, as discussed above, the IVT shown in Fig. 1 has no parameters which can be changed to optimize this transmission. On the other hand, it does not satisfy the constraints on the maximum rpm and torque of the CVT. Thus, this powertrain cannot be a benchmark. Therefore, in the present study, it is compared just with the optimized form of a 6-speed AT transmission. Hence, the speed ratios and gear shifting speeds of this transmission are optimized with the aim of minimizing the vehicle's FC, and the results are compared in different driving cycles, namely Federal Test Procedure (FTP), Urban Dynamometer Driving Schedule (UDDS) and NEDC. These values are shown in Table 4.

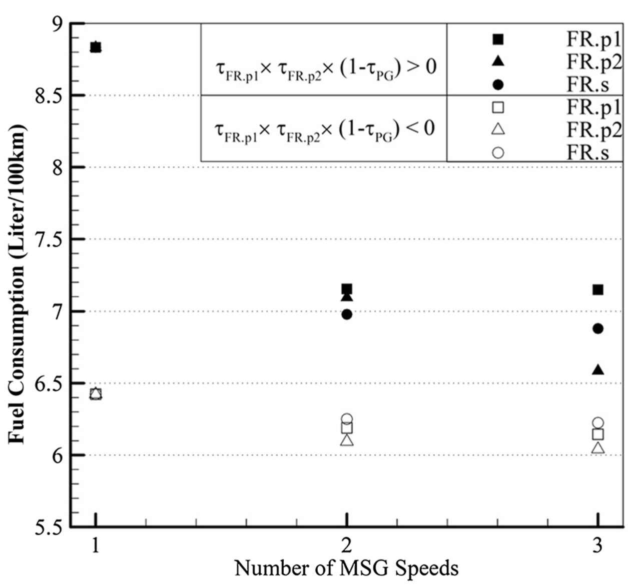


Fig. 7 Vehicle's FC in the NEDC cycle in different IVT arrangement and MSG speeds

Table 3 Optimized parameters value when the two-speed MSG placed on the FR.p2's position

Parameters	IVT min	IVT max	FR.p1	FR.p2	FR.s	PG	CVT
Parameters variation ranges	[0–0.2895]	[1.2755–3]	[1/4–4]	[1/4–4]	[1/4–4]	[1/8–8]	[0.53–1.7]
Parameters optimized value	0.2895	1.2802	2.234	0.3932 and 1.1265	1.2478	1.2935	0.53–1.59

Table 4 FC of the vehicle equipped with optimized 6-speed AT transmission and the optimized IVT in different driving cycles

Powertrain system type	Driving cycles and FC (L/100 km)		
	FTP	UDDS	NEDC
Optimized 6-speed AT transmission	6.269	6.577	6.498
Optimized IVT	5.923	6.184	6.094
Percent difference (%)	5.52	5.98	6.22

As shown in Table 4, FC of the vehicle equipped with the optimized IVT is less than the optimized 6-speed AT transmission for all considered driving cycles. Thus, this powertrain has better performance in different driving conditions.

6 Conclusion

In the present study, a novel IVT equipped with two FRs and one MSG was proposed. In the typical IVT, the MSG transmissions are not used, and also, only one FR mechanism is employed. This modification is proposed to improve the IVT performance and also decrease the vehicle's FC. In order to investigate the effectiveness of modifications, first, the governing equations of the IVT were extracted in different conditions. Then the control method of the IVT speed ratio during driving was presented. It was shown that a forward/backward facing approach is used to simulate the IVT application in the vehicle. Also, the method of FC calculation for the IVT-equipped vehicle was described. By defining the constructions of the optimization problem, all the parameters which impact the vehicle's FC were optimized. It was found that through increasing the number of MSG speeds, the FC decreases. However, the complexity of the system rises up. Consequently, it was concluded that using the two-speed MSG is more economical than the three-speed one. In case of the IVT layout, it was demonstrated that placing MSG following the CVT makes the best arrangement among possible arrangements. Finally, the optimized IVT was compared with the optimized 6-speed AT transmission in terms of the vehicle's FC. It was demonstrated that the optimized IVT yields better results in different driving cycles.

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