



# Displacement of a Hydraulic Jump in a Rectangular Channel: Experimental Study

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## Abstract

The formation of a hydraulic jump is the result of rapid transition from supercritical to subcritical flow regime; this phenomenon is always accompanied by energy dissipation. Knowledge of where hydraulic jumps form is necessary to minimize the erosion risk in hydraulic structures. Protecting the floor of stilling basins, channels and rivers is one of the objectives of studying hydraulic jump characteristics. The present work aims to analyze the displacement of the hydraulic jump in a rectangular channel controlled by a sill. It permitted to find a general equation that allows determining the location of the hydraulic jump in a rectangular channel. This formula was obtained through an analysis of the experimental tests carried out at the LARGHYDE laboratory. Comparison between the displacements of the hydraulic jumps measured experimentally and those calculated by the general equation obtained showed that the absolute relative differences between them are mostly less than 10%, which gives validity to the general equation obtained.

**Keywords** Stilling basins · Hydraulic jump · Displacements · Rectangular channel · Experimental study

## 1 Introduction

The stilling basins are considered the most powerful hydraulic structure for the dissipation of the flow energy (Babaali et al. 2015); they dissipate excess energy downstream of a spillway, sluices or other outlet hydraulic structures by using a hydraulic jump in the stilling basin (Zhang et al. 2017). The location of a hydraulic jump is a key parameter for analyzing the slab stability of the stilling basin in a sluice (Luo et al. 2021). These basins are constituted by some combination of baffle blocks, chute blocks, and sills. Peterka (1984) described many types of stilling basins developed at the United States Bureau of Reclamation (USBR); the first printing of these engineering monographs was in September 1958.

The hydraulic jump phenomenon plays a significant role in dissipating the energy of the upstream current in either

natural or artificial waterways (Baharvand et al. 2021). It can be found in stilling basins, channels, and rivers (Hafnaoui and Debabeche 2021). To shorten the length of the hydraulic jump and stabilize its location, sills and baffle blocks are usually used in these hydraulic structures (Tokyay et al. 2011). Several works have used these techniques to increase the rate of the energy dissipation and reduce the size of the stilling basins. Bejestan and Neisi (2009), Ellayn and Sun (2012), Ibrahim (2017) and Maatooq and Taleb (2018) used different shapes of baffle blocks to study their effect on flow behavior and the design of stilling basins. The effect of the sill on the hydraulic jump formation was investigated by numerous researchers; Hager and Li (1992), Omid et al. (2010), Achour and Khattaoui (2013) described and analyzed the hydraulic jump in a rectangular channel to improve its characteristics. Achour and Debabeche (2003), Debabeche et al. (2009) studied the performance of the hydraulic jump in a horizontal and sloped triangular channel. Demetriou and Dimitriou (2010) studied the mechanical energy losses of the inclined hydraulic jump through a comparative study between a thin wall and a step. Fathi-Moghadam et al. (2011) used a tall sill to reduce the stilling basin length. Kateb et al. (2013) analyzed the effect of the positive step on the hydraulic jump characteristics. Pourabdollah et al. (2019) investigated experimentally and analytically the

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hydraulic jump characteristics on different adverse slopes, bed roughness and positive step heights. The results showed a decrease in the sequent depth ratio of 33% and an increase in the relative energy loss with 27.41% compared to the classical hydraulic jump.

The occurrence of moving hydraulic jump in hydraulic structures such as control gates, weirs and culverts is quite often in an irrigation network of the purpose to generate, accommodate or convey moving hydraulic jump (Rizi Parvaresh et al. 2006). Also, we can find it in the case of emergency gate closures in closed conduits (Mortensen and Kubitschek 2016). The correlation among hydraulic parameters of moving hydraulic jump in a rectangular channel was investigated by Rizi Parvaresh et al. (2006) and Nasvi et al. (2010), through an analysis of experimental tests of unsteady mixed flow regimes. Empirical formulas for the flow arte and pressure force at downstream for moving hydraulic jump were obtained using theoretical expressions and experimental results. Mortensen and Kubitschek (2016) studied the displacement of the hydraulic jump in closed conduits due to an upstream gate closure and its effect on the total air demand. The results indicate that the hydraulic jump displacement is dependent on air vent size as well as gate closure rate.

The location of the hydraulic jump was treated by Achour et al. (2002), Debabeche and Achour (2007), Alikhani et al. (2010) through a study of the sill position in a rectangular channel, relations was developed between sill height and position, sequent depth ratio, and length of stilling basin. Behrouzi-Rad et al. (2013) evaluated the effect of a sill with circular holes on the location of the hydraulic jump and its length. Luo et al. (2021) proposed a new method to locate the toe of a hydraulic jump in sloping channels.

Displacement of the hydraulic jump was analyzed by Hafnaoui et al. (2016, 2018) in triangular and rectangular channels. Formulas were obtained through a numerical analysis of the displacement of the hydraulic jump can be used to predict the location of the hydraulic jump.

Because of the limited information in the literature dealing with hydraulic jump displacement, it is necessary to give importance to additional studies to obtain sufficient knowledge about hydraulic jump displacement.

Knowledge of the hydraulic jump location in hydraulic structures is of great importance since it allows reducing the erosion risk and controlling flow regimes. This study is considered as a contribution to the research related to moving hydraulic jump by finding equations that help determine the hydraulic jump location in hydraulic structures such as stilling basins, control structures in a channel and artificial structures designed for surfing that rely on creating a hydraulic jump to make waves.

The objective of this work is to find an experimental formula, through which we can determine the location of the hydraulic jump. The experimental study was based on the

analysis of the hydraulic jump displacement in a rectangular channel controlled by a sill.

## 2 Material and Methods

### 2.1 Experimental Setup

The tests were carried out at the Research Laboratory of Civil Engineering, Hydraulics, Environment and Sustainable Development (LARGHYDE) of Biskra University. The experimental model comprises an 11 m long main channel, in which is inserted a measuring rectangular flume with a 0.2 m wide  $\times$  0.2 m high cross section. Water is supplied from a tank and controlled by a pressure convergent placed at the upstream of the flume Hafnaoui (2018). Figures 1 and 2 show the simplified diagram of the experimental model and a photograph of the measurement channel.

The initial depth  $h_1$  of the hydraulic jump is assimilated into the opening of the pressure convergent, this convergent placed at the entrance of the flume and stretches over a distance of 0.3 m. To assimilate the values of the initial depths, three pressures convergent were fabricated taking the same values of the initial depth  $h_1$ . The purpose of fabricating these pressures convergent is to produce a high flow velocity in the upstream of the flume to obtain a large Froude numbers. The experimental tests were conducted at three initial depths:  $h_1$  (m) = 0.032; 0.037 and 0.041. The values of the flow velocity obtained varied between 1.76 and 3.06 m/s for  $h_1 = 0.032$  m; 2.03 to 2.66 m/s for  $h_1 = 0.037$  m and 1.83 to 2.67 m/s for  $h_1 = 0.041$  m. Figure 3 shows the exit of water through the pressure convergent used for these experimental tests.

A wide range of the Froude number was obtained, corresponding to:  $2.9 < F_1 < 5.5$ . The range of the Froude number varied between 2.94 to 5.47, 2.92 to 4.42 and 2.78 to 4.21 for the initial depths  $h_1$  (m) = 0.032; 0.037 and 0.041, respectively.

The hydraulic jump was created by a sill placed downstream the flow, and the distance between the sill and the opening of the pressure convergent is equal to 4 m. The measurement of the displacement distance of the hydraulic jump  $L_c$  in the flume is based on increasing the flow  $Q$  sometimes and raising the sill height  $h_s$  at other times. For this reason, different sill heights  $h_s$  for the range varied between 0.028 m and 0.071 m were used, the choice of the sill heights was selected according to the flow value and the location of the hydraulic jump from upstream to downstream of the channel, starting with the highest value of the sill that controlled the hydraulic jump in the upstream of the channel, while ensuring the maximum of the displacement by reducing the value of the sill height until the downstream of the channel. The final depth of the hydraulic jump  $h_2$  is

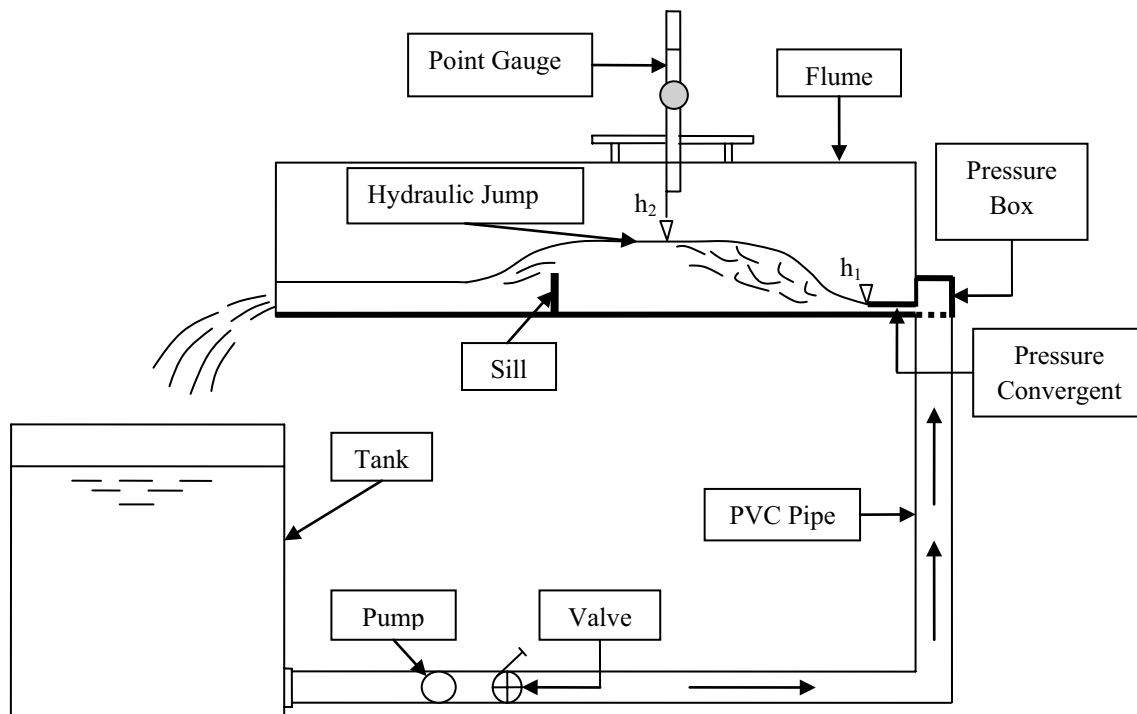


Fig. 1 Simplified diagram of the experimental model



Fig. 2 Photograph of the measurement channel

measured at the maximum free surface flow by a vernier height gauge. Figure 4 shows the formation and displacement of the hydraulic jump in the flume.

The hydraulic and geometric characteristics of this study are: the flow  $Q$ , the initial depth  $h_1$ , the final depth  $h_2$ , the length between the opening of the pressure convergent and the sill  $L$ , the height of the sill  $h_s$  and the displacement of the hydraulic jump  $L_c$ .

Because it is difficult to accurately determine the position of the hydraulic jump due to the turbulent flow regime. The measurement of the hydraulic jump location was based on the observation that the formation of the hydraulic jump is in most of the cross section of the channel. Figure 5 shows the method to measure the location of the hydraulic jump.

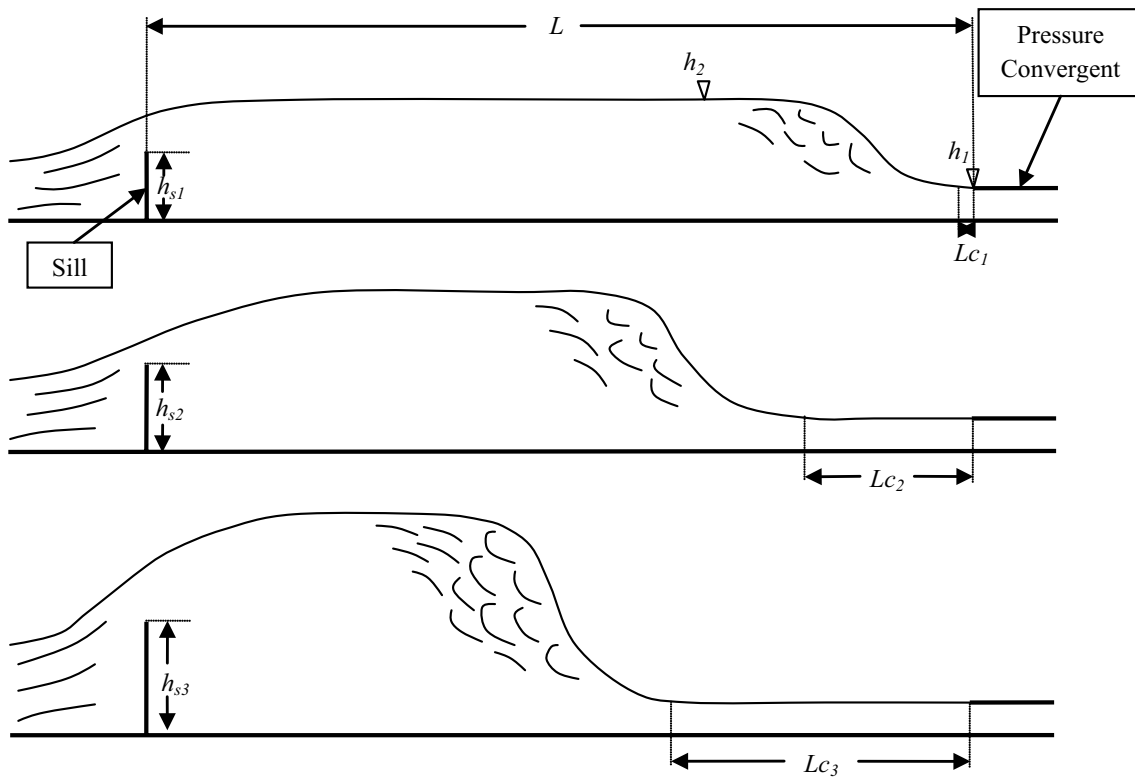
### 3 Results and Discussion

Analysis of the experimental tests aims to find a relationship that determines the location of the hydraulic jump. In this context, several factors that could have an impact on the hydraulic jump position were analyzed, such as the sequent depth ratio  $Y = h_2/h_1$ , the length of the channel  $L$  and the height of the sill  $h_s$ .

To find the experimental approach that represents the displacement of the hydraulic jump, the analysis of the experimental tests was carried out in two stages:

- The first stage depends on searching for a relation that represents the displacement of the hydraulic jump  $L_c$  according to its characteristics.
- The second stage based on determining a relation which represents the variation of the sequent depth ratio  $Y = h_2/h_1$  as a function of the Froude number  $F_1$ .

**Fig. 3** Photograph of pressure convergent used in the experimental tests



**Fig. 4** Simplified diagram of the formation and displacement of the hydraulic jump

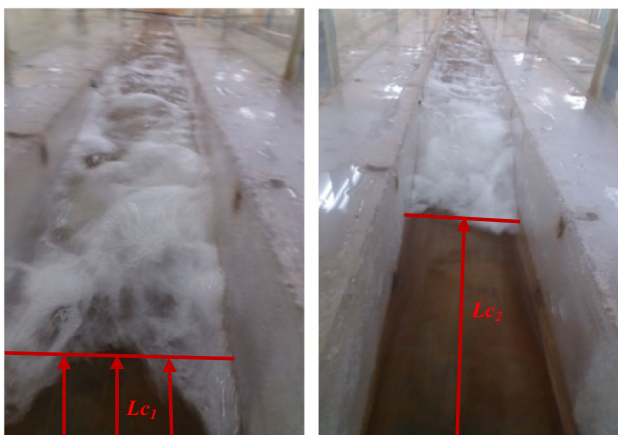


Fig. 5 Photograph of the formation and the displacement of the hydraulic jump

### 3.1 Displacement of the Hydraulic Jump Lc According to its Characteristics

To determine the location of the hydraulic jump, we studied the ratio of: the multiplication of the sequent depth ratio  $Y_{exp}$  with the height of the sill  $h_s$ , and the difference between the length of the channel  $L$  and the displacement of the hydraulic jump  $Lc$ ; depending on the Froude number  $F_1$  (Fig. 6).

According to Fig. 6, the results show three curves that have a relationship with the values of the initial depths  $h_1$ . The variation of the ratio  $(Y_{exp} * h_s)/(L - Lc)$  as a function of the Froude number  $F_1$  can be written with the following equation:

$$\frac{Y_{exp} * h_s}{L - Lc} = a * e^{b * F_1} \tag{1}$$

Fig. 6 Variation of  $(Y_{exp} * h_s)/(L - Lc)$  according to  $F_1$

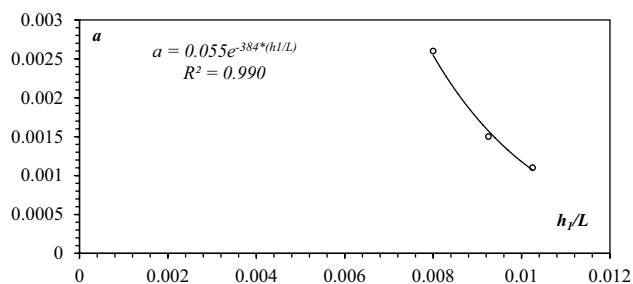
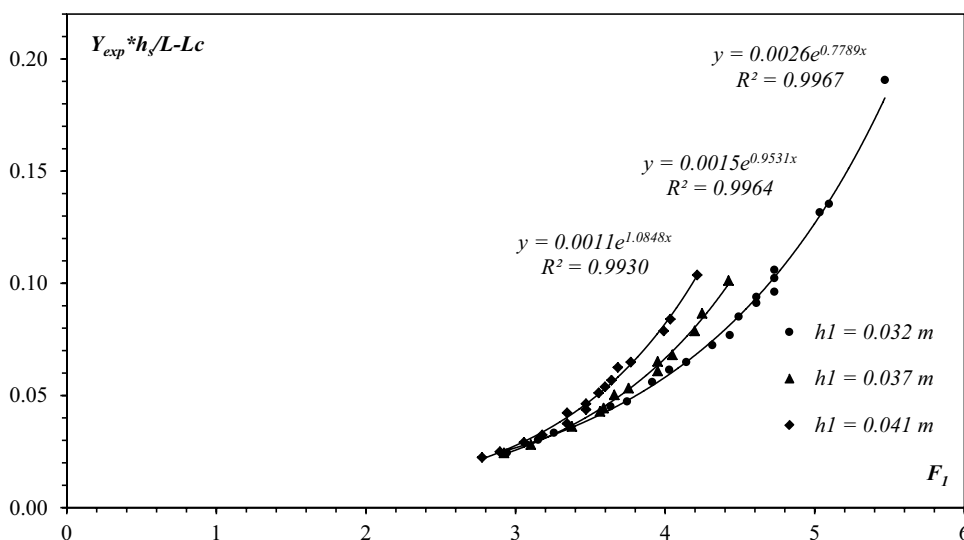


Fig. 7 Variation of the variable  $a$  according to the ratio  $h_1/L$

where,  $a$  and  $b$  represent the variables resulting from the changes of the initial depth  $h_1$ . These variables can be represented by equations as a function of the ratio  $h_1/L$  (Figs. 7 and 8). Table 1 shows the values of the variables  $a$ ,  $b$  and the ratio  $h_1/L$ .

Replacing variables  $a$  and  $b$  by their equations shown in Figs. 7 and 8, Eq. (1), which represents the variation of the ratio  $(Y_{exp} * h_s)/(L - Lc)$  as a function of the Froude number  $F_1$ , can be written as follows:

$$\frac{Y_{exp} * h_s}{L - Lc} = 0.055e^{-384 * (\frac{h_1}{L})} * e^{(504.2 * (\frac{h_1}{L})^{1.34}) * F_1} \tag{2}$$

The adjustment curve of the experimental measurements using Eq. (2) is shown in Fig. 9. The results show a good agreement between the experimental measurements and the curve traced using Eq. (2).

In Eq. (2), the adjustment curve was based on the experimental measurement values of the sequent depth ratio  $Y_{exp}$  which makes Eq. (2) implicit. To make it explicit, the sequent depth ratio  $Y_{exp}$  must be replaced by a new relation.

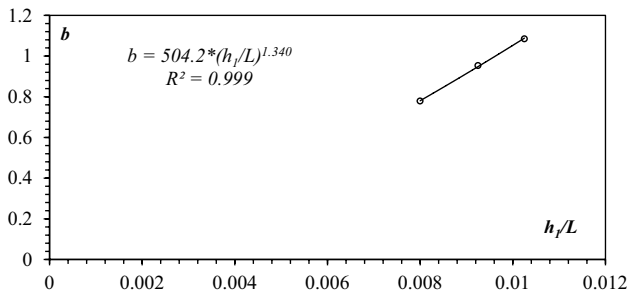


Fig. 8 Variation of the variable  $b$  according to the ratio  $h_1/L$

Table 1 Values of the variables  $a$ ,  $b$  and the ratio  $h_1/L$

$h_1$	$a$	$b$	$h_1/L$
0.032	0.0026	0.7789	0.00800
0.037	0.0015	0.9531	0.00925
0.041	0.0011	1.0848	0.01025

Fig. 9 Variation of  $(Y_{exp} * h_s) / (L - Lc)$  according to  $F_1$  using Eq. (2)

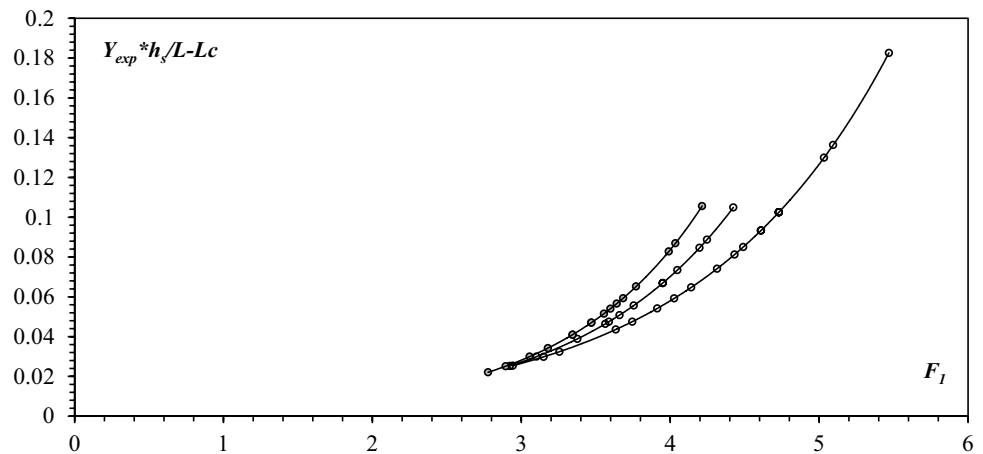
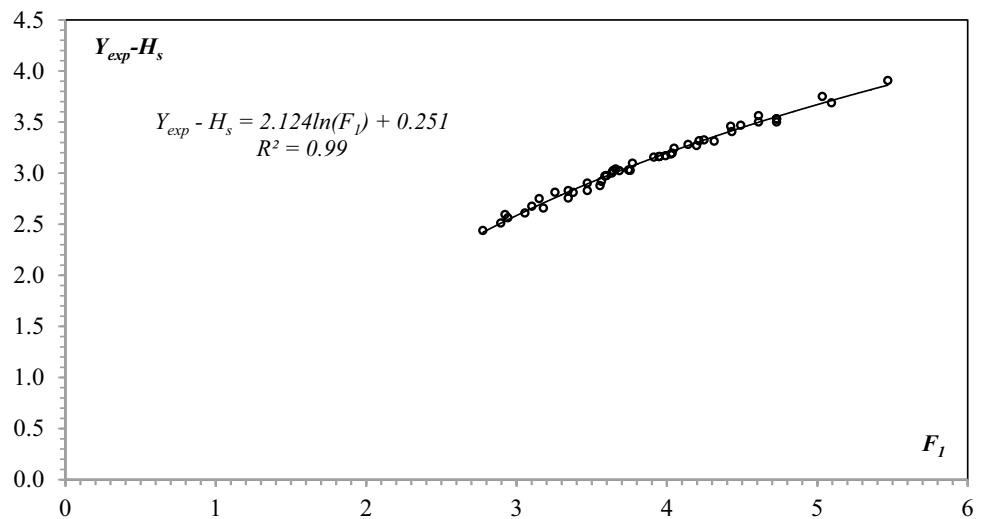


Fig. 10 Variation of  $Y_{exp} - H_s$  according to  $F_1$



### 3.2 Variation of the Sequent Depth Ratio $Y_{exp}$ as a Function of the FROUDE Number $F_1$

To find a relation that represents the ratio  $Y_{exp} = h_2/h_1$  of the sequent depths as a function of the Froude number  $F_1$ , we studied the variation of the difference between the sequent depth ratio  $Y_{exp}$  and the relative height of the sill  $H_s = h_s/h_1$  as a function of the Froude number  $F_1$  (Fig. 10).

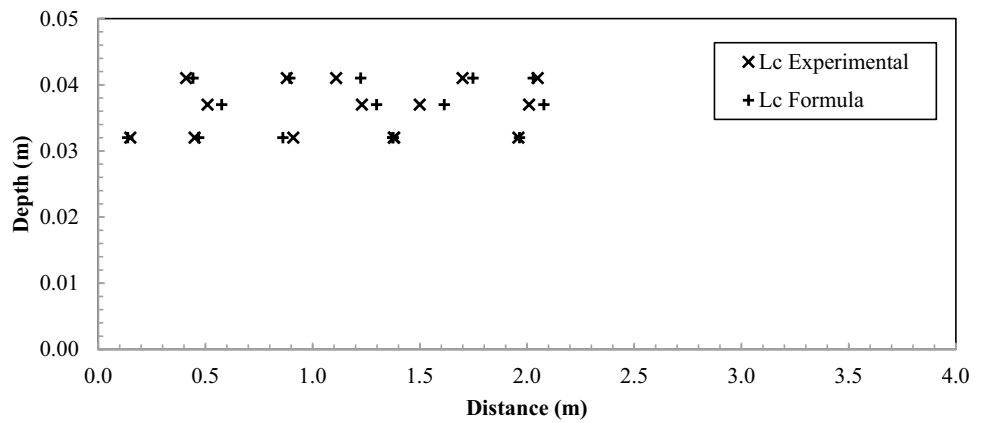
After adjusting the experimental measures, a relation that allows calculating the sequent depth ratio was found. The latter can be expressed as follows:

$$Y_{form} = H_s + 2.124 * \ln(F_1) + 0.251 \tag{3}$$

Replacing the sequent depth ratio  $Y_{exp}$  by the  $Y_{form}$  of Eq. (3), Eq. (2) can be written as follows:

$$Lc = L - \frac{\left(\frac{h_s}{h_1} + 2.124 * \ln(F_1) + 0.251\right) * h_s}{0.055e^{-384 * \left(\frac{h_1}{L}\right)} * e^{\left(504.2 * \left(\frac{h_1}{L}\right)^{1.34}\right) * F_1}} \tag{4}$$

**Fig. 11** Difference between the displacements of the hydraulic jumps measured and calculated



**Table 2** important equations used to calculate the displacement of the hydraulic jump in rectangular channel

Sequent depth ratio $Y_{form}$ as a function of the Froude number $F_1$	$Y_{form} = H_s + 2.124 * \ln(F_1) + 0.251$
Location of the hydraulic jump $Lc$ in a rectangular channel	$Lc = L - \frac{(\frac{h_2}{h_1} + 2.124 * \ln(F_1) + 0.251) * h_2}{0.055e^{-384 * (\frac{h_1}{L})} * e^{504.2 * (\frac{h_1}{L})^{1.34}} * F_1}$

The general Eq. (4) obtained represents the location of the hydraulic jump  $Lc$  in a rectangular channel. The absolute relative differences between the displacements of the experimental hydraulic jumps  $Lc_{exp}$  and those calculated by the general Eq. (4)  $Lc_{form}$  are in majority less than 10%. Figure 11 shows a comparison between these displacements for different initial depths  $h_1$ .

This study was carried out in a rectangular channel of fixed width; the general Eq. (4) obtained is therefore not valid for channels of different width.

For the initial depths  $h_1 = 0.032$  m, there is a great agreement between the displacements measured experimentally and the those calculated by the general equation obtained, this agreement remained good for the initial depths  $h_1 = 0.041$  m and decrease for the initial depths  $h_1 = 0.037$  m.

Generally, the difference between the compared displacements is minimal, which gives validity to the general equation obtained (4).

To summarize the results, Table 2 represents the most important equations that were found to determine the location of the hydraulic jump rectangular channel.

### 4 Conclusion

In this work we experimentally studied the displacement of the hydraulic jump in a rectangular channel controlled by a sill. A general equation was obtained from the analysis of the experimental tests carried out at the LARGHYDE laboratory. This equation allows calculating the location of the hydraulic jump at any point in the channel. The difference between the displacements of the hydraulic jumps measured experimentally

and those calculated by the general equation obtained is minimal and does not exceed 10%, which gives validity to this equation.

This work is a contribution that helps engineers and designers to determine the location of the hydraulic jump in a rectangular channel.

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### Declarations

**Conflict of interest** The authors declare that no conflict of interest regarding the publication of this paper, and all authors have agreed to publish this paper.

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