



# Deterministic Seismic Stability Analysis of Reinforced Slopes using Pseudo-Static Approach

Mahtab Fatehi<sup>1</sup> · Iman Hosseinpour<sup>1</sup> · Reza Jamshidi Chenari<sup>1</sup> · Meghdad Payan<sup>1</sup> · Sina Javankhoshdel<sup>2</sup>

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## Abstract

Limit equilibrium analyses were conducted using pseudo-static approach to investigate the behavior of a typical reinforced slope under seismic loading. Slope height, slope face inclination, internal friction angle of the fill materials and seismic acceleration coefficients were all varied so that the length and tensile strength requirements of the geosynthetic reinforcement layers were determined for different factor of safety (FS) values. The influence of failure mechanism (i.e. external and internal) on geosynthetic required length and tensile strength for a certain value of FS was also discussed. According to the results, for a particular FS value, there was a minimum geosynthetic length and tensile strength to ensure that the internal or external failure mechanism will occur. Regardless of the failure mechanism, increasing the slope height was shown to critically increase the minimum safety demands in terms of the geosynthetic tensile strength as well as its length. Moreover, the seismic acceleration coefficients were found to substantially alter the pullout capacity of the reinforcement layers in the passive zone, thus changing the geosynthetic strength requirement. Nevertheless, this impact has been less significant when the slope was subjected to a vertical downward seismic acceleration.

**Keywords** Geosynthetic-reinforced slopes · Deterministic slope stability analysis · Failure mechanism · Limit equilibrium method · Pseudo-static approach

## 1 Introduction

Soil reinforcement with geosynthetic materials is increasingly used in geotechnical engineering practice to ensure satisfactory performance of either natural or manmade slopes under static and seismic loadings (Hosseinpour et al.,

2010; 2017; Fathipour et al., 2021; Mirmoazen et al., 2021a, 2021b). Stability analysis using limit equilibrium method (LEM) has been the conventional approach to estimate the factor of safety (FS) for a reinforced slope considering different failure surfaces including circular (Kitch, 1994; Kitch et al., 2011), log-spiral (Leshchinsky and Boedeker, 1989), two-part wedge geometries (Jewell, 1991; Bathurst and Jones, 2001) and non-circular (Dastpak et al., 2021).

Because of catastrophic failures of geotechnical structures during earthquakes, seismic stability and efficiency of reinforced soil slopes require considerable attentions. Performance assessment of a reinforced slope subjected to the seismic loading is commonly conducted by (a) pseudo-static slope stability analysis (Leshchinsky and San, 1994; Macedo and Candia, 2020), (b) Newmark-based slope displacement analysis (Michalowski and You, 2000) and (c) advanced numerical procedures including finite element or finite difference methods (Arvin et al., 2019; Sharma et al., 2019). In a pseudo-static analysis, the weight of the sliding mass, the shear and normal soil resisting forces along the sliding surface together with the constant horizontal and vertical forces caused by the seismic loading are taken into account for the

✉ Iman Hosseinpour  
imanhp@guilan.ac.ir

Mahtab Fatehi  
fatehi.mahtab@gmail.com

Reza Jamshidi Chenari  
jamshidi\_reza@guilan.ac.ir

Meghdad Payan  
payan@guilan.ac.ir

Sina Javankhoshdel  
sina.javankhoshdel@rocscience.com

<sup>1</sup> Civil Engineering Department, Faculty of Engineering, University of Guilan, Persian Gulf Highway, P.B: 41996-13776, Rasht, Guilan, Iran

<sup>2</sup> Rocscience Incorporations, 54 St., Patrick St, 6137704857, Toronto, ON, Canada

calculation of the FS value. A geosynthetic-reinforced slope subjected to static or seismic loadings may fail by one of the three principal mechanisms including external, internal and transition failure modes. External failure happens when slip surface circumvents the reinforced zone, while internal failure occurs when slip surface intersects all the reinforcement layers. Transition mechanism, however, indicates the failure state when slip surface does not cross all the reinforcement layers while intercepting some of them (Javankhoshdel and Bathurst, 2017).

Since three decades ago, there have been a number of outstanding research studies to evaluate the satisfactory performance as well as the failure mechanisms of geosynthetic-reinforced slopes and walls under both static and seismic loadings (Kitch, 1994; Zheng et al., 2006; Basha and Basudhar, 2010; Kitch et al., 2011; Li et al., 2014; Khosravizadeh et al., 2016; Ferreira et al., 2016; Song et al., 2018; Farshidfar et al., 2020; Aroni Hesari et al., 2021; Agarwal et al., 2021; Manna et al., 2021; Basbug et al., 2021; Fathipour et al., 2021; Mirmoazen et al., 2021a, 2021b).

Ling et al. (1997) proposed seismic design procedures for reinforced slopes based on a pseudo-static analysis considering horizontal acceleration and a permanent displacement. Concerning different failure modes, internal and external stability analyses were conducted to determine the strength and length requirements for the reinforcement layers. The length required to resist against direct sliding has shown to increase as the seismic acceleration increased which, in turn, might become impractical at moderate design acceleration levels.

Ling and Leshchinsky (1998) examined the stability and permanent deformations of a geosynthetic-reinforced slope, under the influence of combined horizontal and vertical accelerations. The results showed that the vertical acceleration in the downward direction caused an increased tensile force mobilized in the reinforcement, while an upward acceleration required a larger geosynthetic length to resist against sliding.

Nouri et al., (2006) studied the effect of horizontal and vertical accelerations on the performance of reinforced slopes and walls using the well-established pseudo-static approach and considering horizontal slices method (HSM). It was shown that the effect of a horizontal acceleration would mainly depend on the strength properties of the back-fill soil. In addition, for the low values of horizontal seismic acceleration, the effect of vertical acceleration was shown to be insignificant.

Javankhoshdel and Bathurst (2017) carried out deterministic and probabilistic analyses on reinforced slopes using LEM with the circular slip surface for purely frictional and cohesive-frictional soils. The influence of soil properties and slope geometry on the failure mechanism, reinforcement length and tensile strength requirements was studied.

It was shown that for a particular FS value, there is a reinforcement minimum length and tensile strength, ensuring that the external failure occurs. In addition, it was observed that the internal failure is prone to occur with the same FS value when the reinforcement length exceeded requirement threshold limit.

Dastpak et al., (2021) adopted both deterministic and stochastic LEM approaches to study the stability of a reinforced slope under static loading. Assuming a non-circular slip surface, design charts were prepared and the required geosynthetic length and tensile strength were obtained for different deterministic values of factor of safety corresponding to the external, transitional and internal failure mechanisms.

It is evident from all the above studies that the behavior of reinforced slopes subjected to static and seismic loadings has been fairly well recognized using LEM. However, there have been no previous studies, throughout the literature, for the effect of seismic accelerations on the length and tensile strength requirements of geosynthetic layers, particularly on the change in failure mechanism for a certain value of FS. Therefore, this study taps into the well-established pseudo-static theory by means of the limit equilibrium-based program *Slide 2* aiming to provide further insight into the stability of reinforced slopes under seismic loading. Accordingly, the effects of vertical and horizontal acceleration coefficients on the length and tensile strength requirements of reinforcement layers are analyzed for two different failure mechanisms (i.e. internal and external). Following a comprehensive parametric study, the influences of soil properties and slope geometry as well as the acceleration components on the required geosynthetic properties are thoroughly elaborated.

## 2 Limit Equilibrium Methods for Stability Analyses

The Swedish slip circle or ordinary method of slices developed by Fellenius (1936) had been the most common limit equilibrium method used for slope stability purposes. This method simplifies the equilibrium equations by assuming that the inter-slice forces, both horizontal and vertical, counteract each other, thus leading to lower estimations of FS.

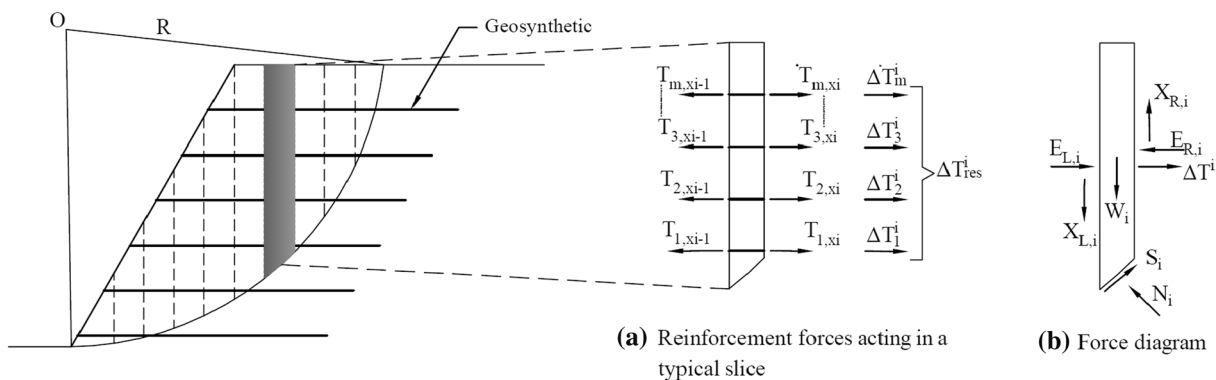
Later, Bishop (1955) refined the Fellenius approach by accounting for the inter-slice normal forces, thus enhancing the accuracy of the obtained FS. However, Bishop's simplified method does not satisfy all equilibrium equations as horizontal force equilibrium is not included. Janbu (1954) improved the above methods for non-circular failure surfaces using rigorous and simplified techniques. In the Janbu's rigorous method, only the vertical and horizontal equilibrium equations are satisfied and the moment

equilibrium is not guaranteed. In addition, a correction factor is applied in order to indirectly compensate for the inter-slice forces.

Spencer (1967) also proposed a limit equilibrium method satisfying all the equilibrium equations, including moment as well as horizontal and vertical forces, which could be ideally adopted for both circular and non-circular failure surfaces. In this method, both normal and shear inter-slice forces are considered to be constant. In a fairly similar approach, Morgenstern and Price (1965) introduced an alternative method for the circular and non-circular failure surfaces accounting for all the equilibrium equations known as GLE/Morgenstern–Price method. Quite similar to Spencer, GLE/Morgenstern–Price method considers the inter-slice normal forces while maintaining the moment equilibrium and also the horizontal and vertical forces equilibrium. Unlike the Spencer method, GLE/Morgenstern–Price approach defines particular equations for the inter-slice forces in the forms of constant, half-sine, clipped-sine, trapezoidal, etc. Figure 1 compares the assumptions and equilibrium equations satisfied in different limit equilibrium methodologies commonly utilized to assess the overall stability of earth slopes.

### 3 Verification Analyses

In the current study, the limit equilibrium-based program *Slide 2* (Rocscience Inc., 2018) is utilized to examine the performance of a reinforced soil slope subjected to pseudo-static seismic loading. Prior to performing the parametric analysis, static stability analysis of a typical reinforced slope is conducted and the results are compared with those presented in Javankhoshdel and Bathurst (2017). The reinforced slope used for the validation analysis is 5 m high supporting a purely frictional material with the drained internal friction angle and the unit weight equal to 30° and 20 kN/m<sup>3</sup>, respectively. The slope is reinforced using four layers of geosynthetic placed at equal vertical distances of  $s_v = 1.0$  m. Bishop’s simplified method is adopted while considering a circular failure surface as similarly used by Javankhoshdel and Bathurst (2017). The results of validation analyses are compared in Fig. 2 where the geosynthetic tensile strength ( $T$ ) is plotted against the required length ( $L$ ) for different slope inclinations ( $\beta$ ). As seen in Fig. 2, the adopted model and the corresponding limit equilibrium method seem to be able to successfully regenerate the results obtained by Javankhoshdel and Bathurst (2017) for a wide range of

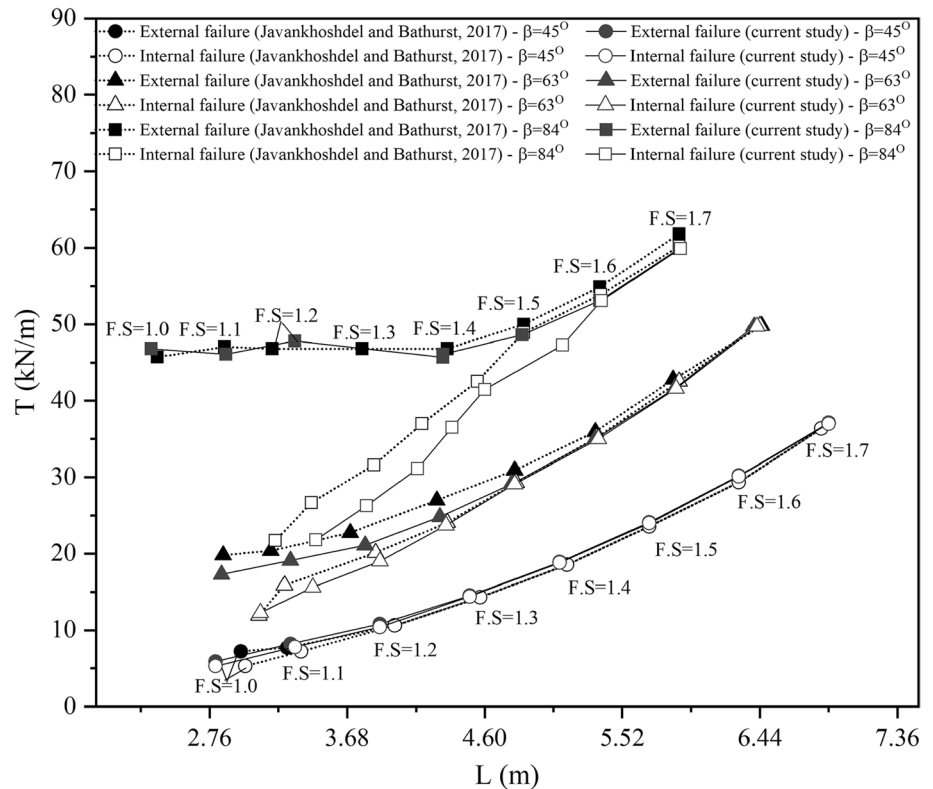


Method of analysis	Equilibrium satisfied			Inter-slice forces	
	Moment	Vertical force	Horizontal force	Normal	Shear
Bishop simplified	Yes	Yes	No	Yes	No
Janbu simplified	No	Yes	Yes	Yes	No
Spencer	Yes	Yes	Yes	Yes	Yes
GLE/Morgenstern-Price	Yes	Yes	Yes	Yes	Yes

$\Delta T_j^i$  = The reinforcement acting force on the  $i^{th}$  slice and the  $j^{th}$  reinforcement layer  
 $T_{j,xi}$  and  $T_{j,xi-1}$  = tensile forces, respectively, on the right and left sides of the  $i^{th}$  slice and the  $j^{th}$  reinforcement layer  
 $\Delta T^i$  = The resultant tensile reinforcement force on the  $i^{th}$  slice  
 $N_i$  = Normal force at the base of slice  
 $S_i$  = Shear force at the base of slice  
 $E_{R,i}$  and  $E_{L,i}$  = Inter-slice normal forces  
 $X_{R,i}$  and  $X_{L,i}$  = Inter-slice shear forces  
 $W_i$  = Weight of the  $i^{th}$  slice

Fig. 1 Comparison between the assumptions adopted in different limit equilibrium methods used for slope stability analysis (Modified after Fredlund et al., 2019 and Djefal and Belkacemi, 2020)

**Fig. 2** Variations of geosynthetic tensile strength vs. length to generate external and internal failure modes under static loading ( $H = 5$  m,  $n = 4$ ,  $\gamma = 20$  kN/m<sup>3</sup>,  $\varphi = 30^\circ$ ,  $c = 0$ )



slope inclinations. The required length and tensile strength of the geosynthetic layers as well as the probable failure mechanism for a certain value of FS are fairly well compared with the original model. For instance, when the slope angle is  $45^\circ$ , the tensile strength and length requirements of geosynthetic layers, to reach the same FS, nearly coincide with those reported by Javankhoshdel and Bathurst (2017). Moreover, the change in failure mechanism from internal to external, particularly for  $FS < 1.5$ , is clearly observed by increasing the geosynthetic tensile strength, confirming the satisfactory performance of the present model.

It is also worth noting that as the slope inclination increases, the transition zone becomes wider, but the overall patterns of external and internal failure mechanisms turn to be flatter and steeper, respectively. When the external failure mechanism governs, increasing the slope angle for a certain FS value results in an increased geosynthetic tensile strength demand, while geosynthetic minimum length requirement decreases. In contrast, for the internal failure mechanism, a steeper slope bears increased geosynthetic minimum length requirement, while the minimum tensile strength demand to hold internal failure mechanism decreases.

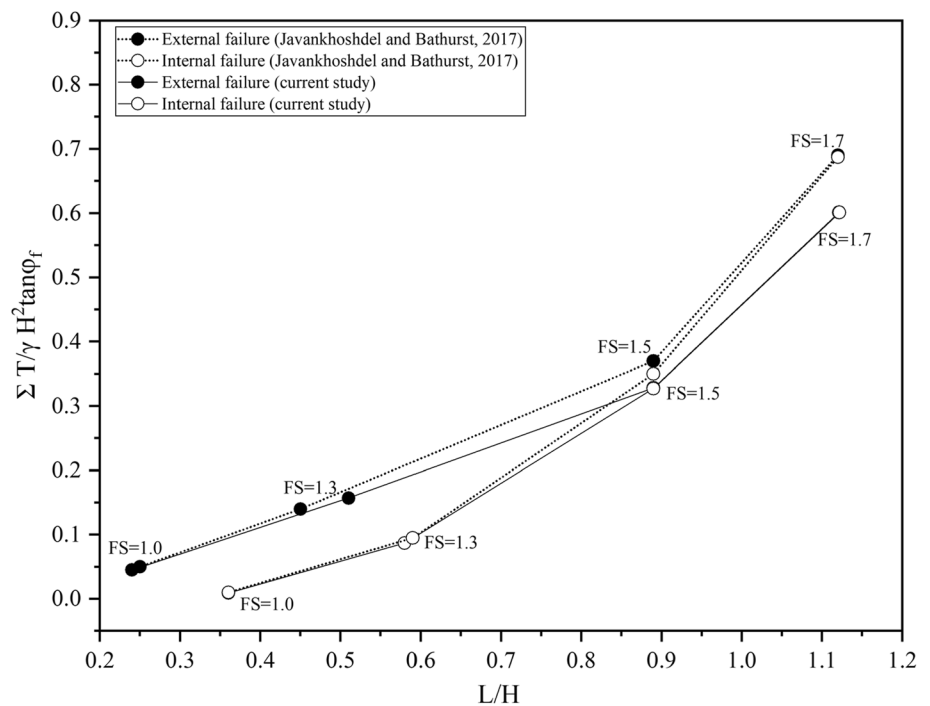
Figure 3 illustrates the variations of the summation of normalized required tensile strength of all geosynthetic layers against the normalized length of reinforcements. In these analyses, the slope supports a cohesive-frictional fill [ $c/(\gamma H \tan \varphi) = 0.04$ ] and the slope inclination is  $45^\circ$  while

the values of FS range from 1.0 to 1.7. It should be noted that the parameter  $\varphi_f$  is the factored soil internal friction angle defined as  $\varphi_f = \tan^{-1}(\frac{\tan \varphi}{F.S.})$ . From Fig. 3, it is clearly observed that for given values of normalized geosynthetic tensile strength and length, the present model could precisely capture the values of FS reported by Javankhoshdel and Bathurst (2017) for a particular failure mechanism.

Further verification analysis is performed, but herein subjected to the seismic accelerations. Both Bishop Simplified and Spencer methods with circular failure surface are utilized to perform the LEM analyses. In this contribution, results obtained from Bishop Simplified and Spencer methods are compared with those rendered by RS2 finite element code (Rocscience Inc., 2019) so as to show the reliability and validity of the corresponding limit equilibrium approaches.

For the sake of simplicity, the internal friction angle and the unit weight of soil were modified to  $30^\circ$  and  $18$  kN/m<sup>3</sup>, respectively. In addition, it is assumed that the slope is  $5$  m high reinforced by eight geosynthetic layers placed at equal vertical distances of  $s_v = 0.6$  m. Moreover, the hypothetical reinforced slope has an inclination angle equal to  $\beta = 60^\circ$  and is subjected to the vertical and horizontal acceleration coefficients of  $0.1$  and  $0.2$ , respectively. According to the results of finite element modeling (FEM), the minimum required tensile strength and length of geosynthetic layers to render

**Fig. 3** Variations of geosynthetic normalized tensile strength vs. normalized length to generate external and internal failure modes under static loading condition ( $H = 5$  m,  $n = 4$ ,  $\gamma = 20$  kN/m<sup>3</sup>,  $\beta = 45^\circ$ ,  $\varphi = 30^\circ$ ,  $c/(\gamma H \tan \varphi) = 0.04$ )



FS = 1.2 for external failure mechanism are  $T = 21$  kN/m and  $L = 4.8$  m, respectively, fairly close to those acquired by Spencer method, i.e.  $T = 25.3$  kN/m and  $L = 6.1$  m and also Bishop simplified method, i.e.  $T = 23.6$  kN/m and  $L = 6.25$  m, as illustrated in Fig. 4.

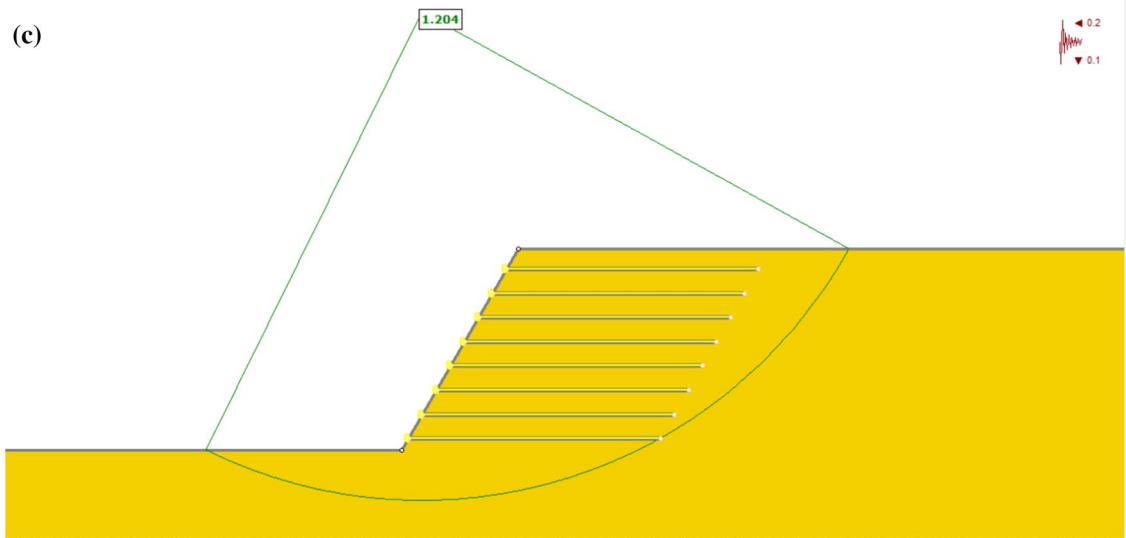
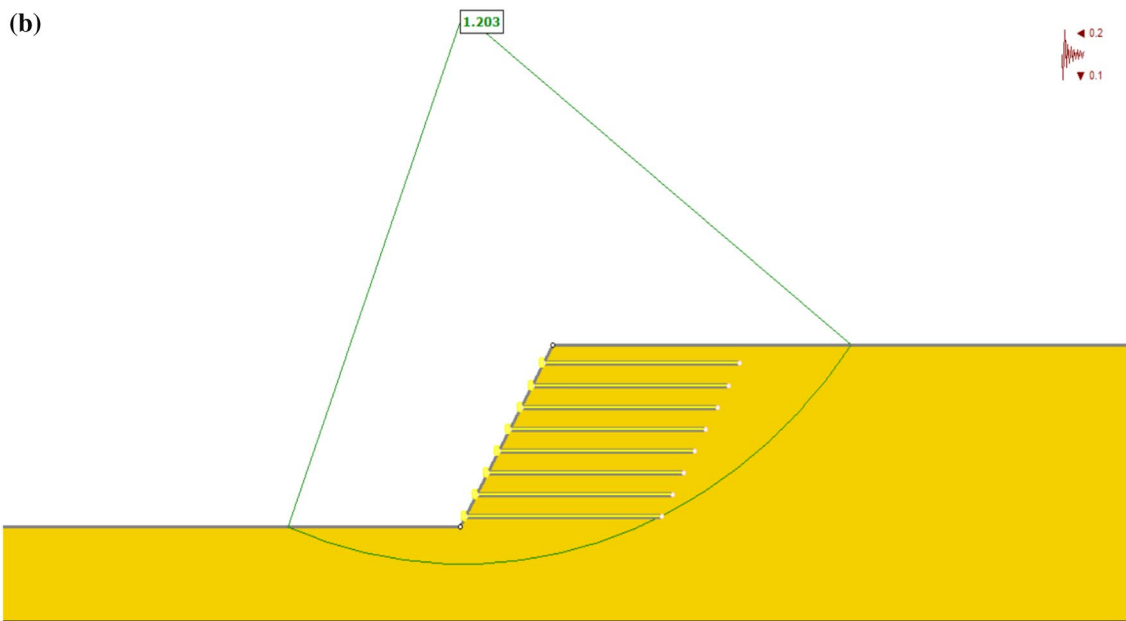
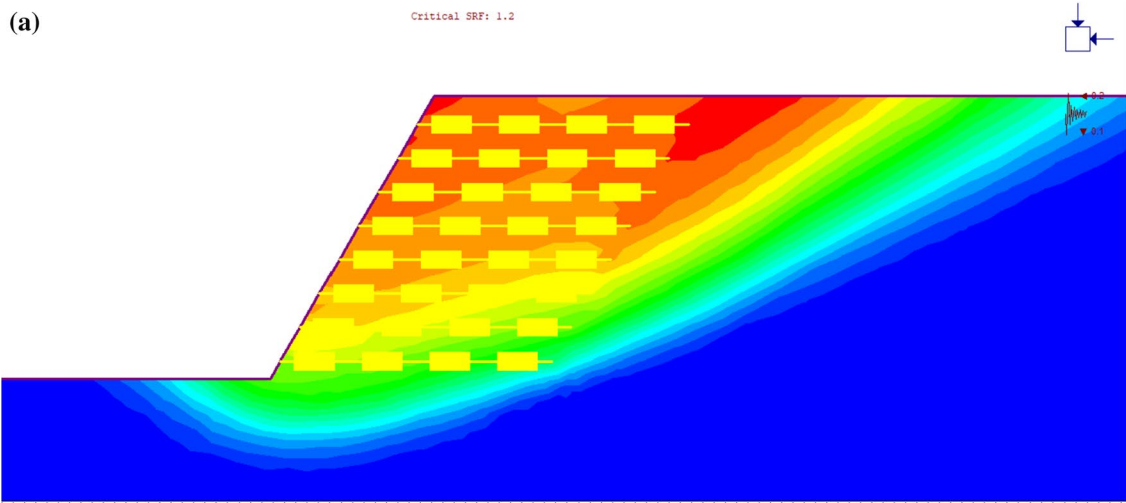
#### 4 Comparison of Limit Equilibrium Methods

In order to assess the effect of different limit equilibrium methods on the required geosynthetic tensile strength and length, stability analysis of a reinforced slope is performed using four limit equilibrium approaches mentioned earlier. The analyses are performed under both static and seismic conditions and the results are then compared. Figure 5a shows the plots of required geosynthetic tensile strength against its length to generate internal and external failure mechanisms under static loading condition. It is observed that the values of FS calculated by Spencer and GLE/Morgenstern–Price for both internal and external failure mechanisms coincide as all the equilibrium equations are totally satisfied in both methods. Nevertheless, when the internal failure mechanism governs, the Janbu simplified method overestimates the geosynthetic length requirements compared to others. However, when the failure mechanism becomes of the external type, the Janbu simplified method leads to a lower geosynthetic tensile strength demand for all FS values. The notable deviation between the results of Janbu simplified method and others can be presumably ascribed to the very fact that this method is devoid of any

moment calculations in slope stability analysis, thus leading forces mobilized in the reinforcements to solely participate in satisfying the equilibrium of horizontal forces. Therefore, overlooking the potential moment resistance contribution of geosynthetic layers in Janbu simplified method gives rise to the longer reinforcement layers to be required so as to guarantee the occurrence of internal mode of failure. It is noteworthy that the outcomes for Bishop simplified approach are in general agreement with the Spencer and GLE/Morgenstern–Price’s.

Figure 5b compares the effect of implementation of different LEMs on required geosynthetic length and tensile strength when the slope is subjected to the seismic acceleration coefficients of  $k_h = 0.2$  and  $k_v = 0.1$ . When failure mechanism is external, the required tensile strength calculated by Janbu simplified and then Bishop simplified methods are lower than those computed by Spencer and GLE/Morgenstern–Price methods. In contrast, when the internal failure occurs, for a particular FS, the Janbu simplified method overestimates the geosynthetic length demand among others. This can be attributed predominantly to the lack of moment equilibrium satisfaction in the corresponding approach as compared to the other methods. Indeed, this simplification would fail to consider the resisting moment offered by geosynthetic reinforcement layers, thus giving rise to a substantial increase in the required length of reinforcement to maintain a particular level of safety factor.

Figure 6 compares the magnitude of FS obtained from different methods versus geosynthetic length while its tensile strength is  $T = 18$  kN/m under both static and seismic



**Fig. 4** Comparison between FEM and LEM to generate external failure mechanism for FS = 1.2: **a** FEM by RS2; **b** LEM by Spencer method; **c** LEM by Bishop Simplified method ( $H = 5$  m,  $s_v = 0.6$  m,  $\gamma = 18$  kN/m<sup>3</sup>,  $\beta = 60^\circ$ ,  $\varphi = 30^\circ$ ,  $k_h = 0.2$ ,  $k_v = 0.1$ )

loading conditions. According to the results presented in Fig. 6a, where the variations of FS under static loading condition are plotted, it is clearly observed that using Bishop simplified method leads to the lowest value of FS among others. However, using other three methods, for which both the vertical and horizontal forces equilibrium equations are satisfied, the computed values of FS are in good agreement. It is also worth noting that, irrespective of the methodology used, the static FS values remain unchanged for the internal failure mechanism, with the increase of the geosynthetic length.

However, as illustrated in Fig. 6b, when the slope is subjected to the seismic loading with a horizontal and vertical acceleration coefficients of  $k_h = 0.2$  and  $k_v = 0.1$ , the values of FS computed by GLE/Morgenstern–Price approach and, much pronounced, the Spencer method fluctuate by increasing the geosynthetic length in the stable zone. This observation could be attributed primarily to the very fact that the horizontal earthquake force component plays a dual role by both increasing the motive and the resistant forces, as schematically demonstrated in Fig. 7. In other words, the net contribution of the motive and resistant forces increase would act as intermittent change of failure mechanism from internal to transient and vice versa as illustrated in Fig. 6b.

The fluctuating behavior of GLE/Morgenstern–Price and Spencer methods, when incorporating horizontal earthquake loading, can come into scrutiny by depicting the active and passive zones, as schematically illustrated in Fig. 7. It is evident that the horizontal earthquake loading increases the motive force by dragging the sliding body outward. On the other hand, the pullout capacity, in the passive loading zone, is delegated to resist the abovementioned outward movement. However, the horizontal earthquake loading exerts outward loading component to the passive zone as well. This will, in turn, brings about cancelled pullout force, which acts at the interface between the reinforcement layers and the surrounding passive soil. Therefore, for the methods in which the inter-slice horizontal forces are taken into account (i.e. GLE/Morgenstern–Price and Spencer methods), any variations in the length of geosynthetic in the passive zone alter the pullout capacity of the reinforcement; thus, the values of FS obtained by these methodologies do not follow a consistent pattern, as seen in Fig. 6b. This will, as such, give rise the critical failure surface to bypass the reinforcement layers in order to yield the minimum factor of safety; hence, leading the transitional failure mechanism to govern on some occasions. The illustrations on the inset diagrams in Fig. 6b are showing this effect efficiently. These observations can

also be experienced by a direct shear test experiment, where both its halves are moving, expressed herein as "moving direct shear phenomenon". In a conventional direct shear test on the soil-geosynthetic interface, one of the half boxes is stationary while the other is moving horizontally so as to yield the pull-out failure at the interface. However, if both boxes are considered to move simultaneously in the same direction, the pull-out capacity of the geosynthetic reinforcement would increase. Similar analogue could be recognized for the active and passive wedges in the reinforced earth slopes (Fig. 7). According to the so-called moving direct shear phenomenon, the pseudo-static acceleration increases the driving and resisting forces in the active and passive zones, respectively. Therefore, the resultant of these forces may change with the increase in the reinforcement length, resulting in the observed fluctuations in the safety factors obtained.

## 5 Parametric Study

### 5.1 General Outlines and Adopted Parameters

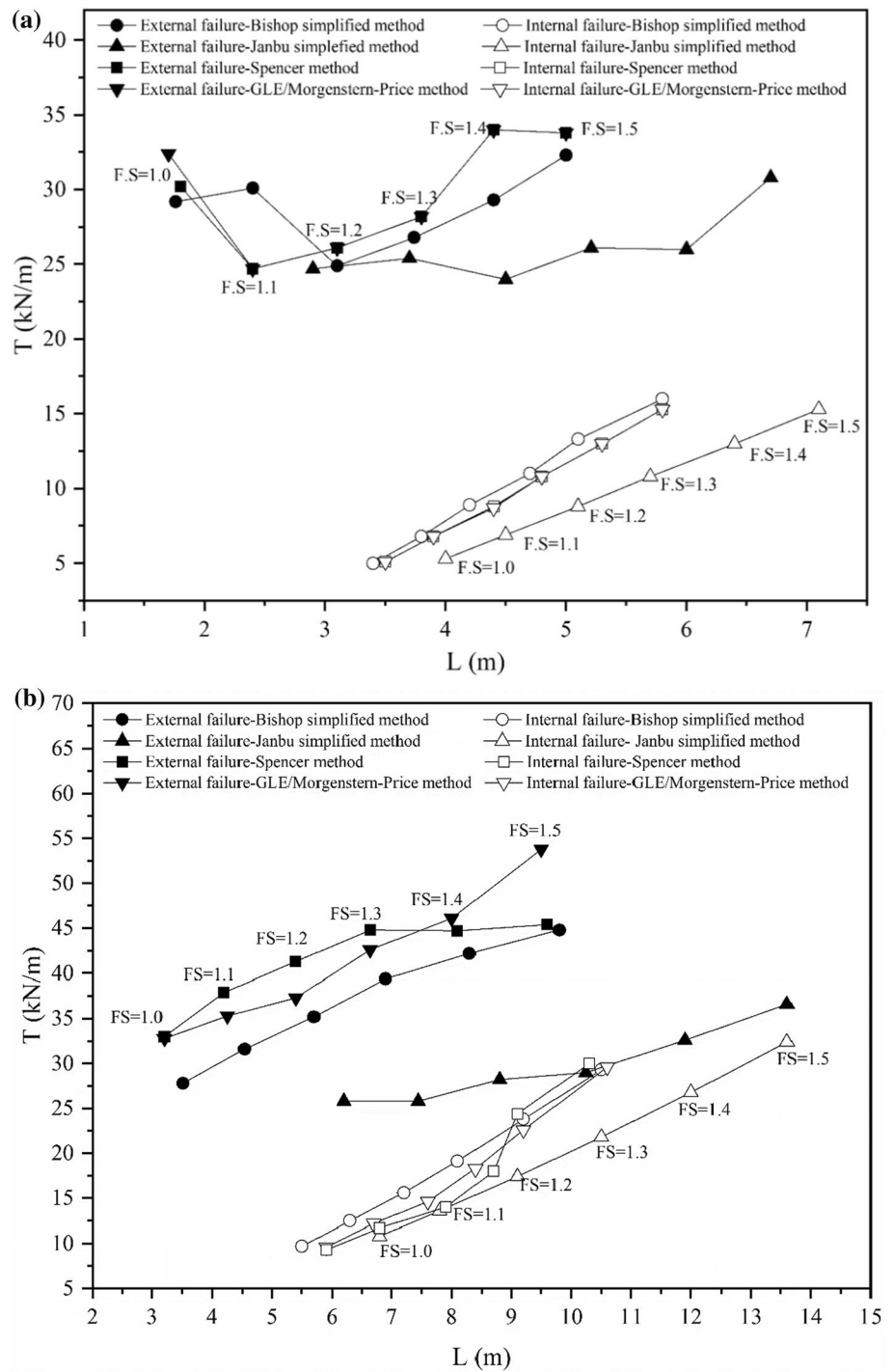
A set of parametric studies is conducted herein to analyze the stability of a typical reinforced earth slope under seismic loading using the pseudo-static approach. Well-established Bishop's simplified method is used and design charts are prepared where the variations of the geosynthetic reinforcement tensile strength demand are plotted against the required reinforcement length to generate external and internal failure mechanisms. During the 1923's Kanto earthquake in Japan, Mononobe (1924) showed that a combination of vertical and horizontal accelerations would cause significant damages on the earth structures. In order to address specific concerns raised in such cases, the combined effect of vertical and horizontal accelerations on the stability of the reinforced slopes is considered in this study.

For all the slope models in the parametric study, the fill soil is considered to be purely frictional material with a constant unit weight of  $\gamma = 18$  kN/m<sup>3</sup>. In addition, the lowermost reinforcement layer is placed 0.3 m above the slope base. The other reinforcement layers are uniformly distributed along the slope height with an equal vertical distance of  $s_v = 0.6$  m, while the number of reinforcement layers undergoes changes with the slope height. Table 1 summarizes the magnitudes of the variables adopted in the parametric analyses.

### 5.2 Influence of Fill Internal Friction Angle

Figure 8 shows the effect of internal friction angle of the slope fill material on the geosynthetic required length and tensile strength to generate internal and external

**Fig. 5** Comparison of different LEMs performance in generating internal and external failure mechanisms: **a** static loading; **b** seismic loading with  $k_h = 0.2$ ,  $k_v = 0.1$  ( $H = 7.5$  m,  $s_v = 0.6$  m,  $\gamma = 18$  kN/m<sup>3</sup>,  $\beta = 60^\circ$ ,  $\varphi = 35^\circ$ )



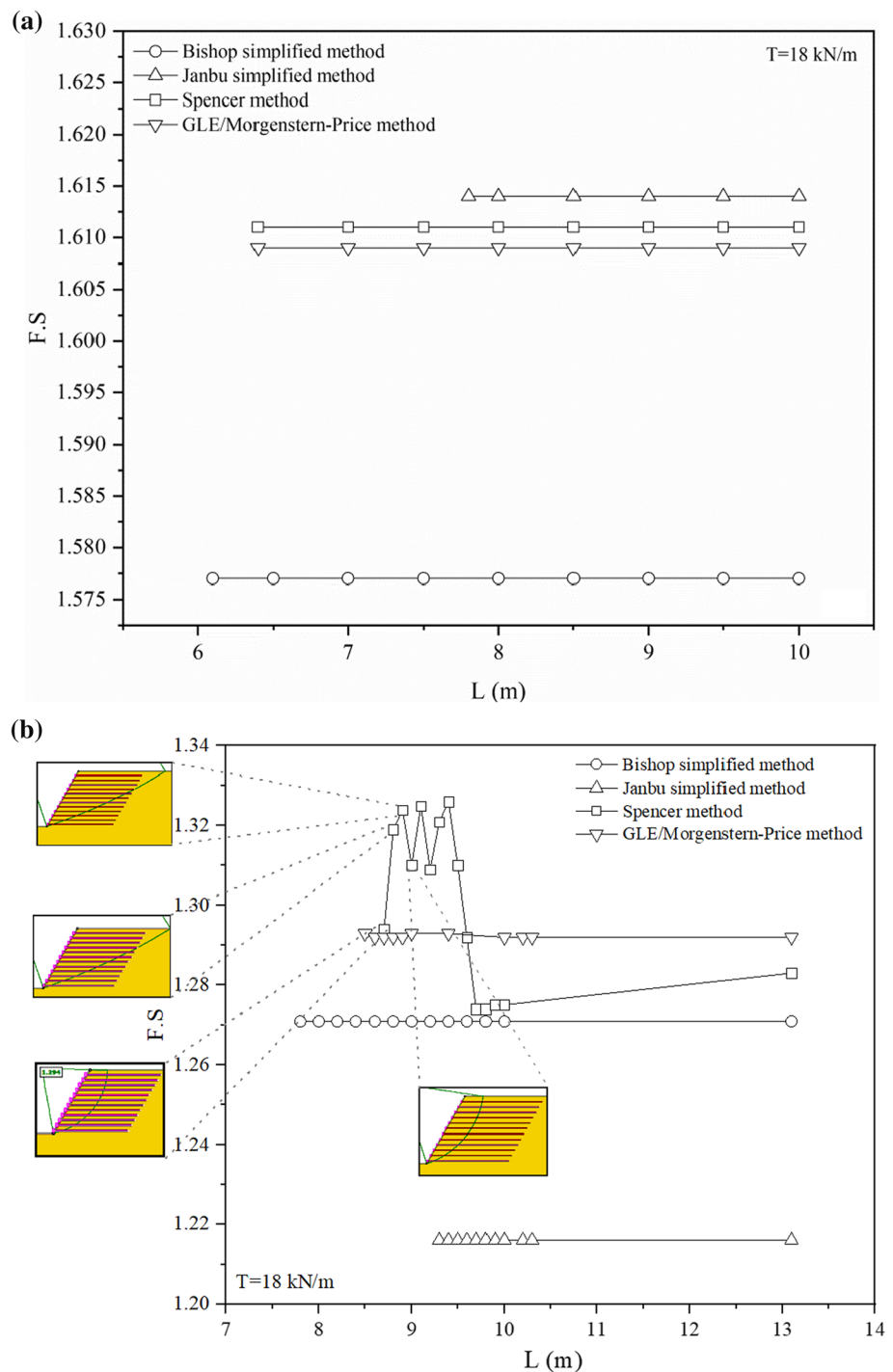
failure mechanisms. In these analyses, the reference slope is assumed to be 7.5 m high, having an inclination angle of  $\beta = 60^\circ$ , while subjected to static loading and seismic acceleration coefficients of  $k_h = 0.2$  and  $k_v = 0.1$ .

What can be simply concluded is that, for any internal friction angle of the fill material, when the studied slope is subjected to the seismic accelerations, the required length and tensile strength of the reinforcement layers to produce

a specific factor of safety significantly increase compared to the static loading. For instance, when the fill internal friction angle is  $\varphi = 40^\circ$  and the external failure mechanism governs, the required length and tensile strength of geosynthetic layers to produce a factor of safety  $FS = 1.5$  increases dramatically from  $L = 3.5$  m and  $T = 30$  kN/m for the static loading to about  $L = 6$  m and  $T = 42$  kN/m for the pseudo-static seismic loading condition, respectively. In other words, in



**Fig. 6** Comparison of different LEMs for prediction of FS for internal failure mechanism under: **a** static loading; **b** seismic loading,  $k_h = 0.2$ ,  $k_v = 0.1$  ( $H = 7.5$  m,  $s_v = 0.6$  m,  $\gamma = 18$  kN/m<sup>3</sup>,  $\beta = 60^\circ$ ,  $\varphi = 35^\circ$ )



order to reach a specific seismic FS value, similar to its corresponding value at static loading condition, the required length and tensile strength of the reinforcement layers have to be adequately increased to compensate for the effect of the horizontal acceleration on increasing the driving force and moment caused by the sliding mass in the active zone.

Concerning the effect of the internal friction angle of the slope fill, it is observed that, irrespective of the

failure mechanism and loading conditions, increasing the internal friction angle noticeably reduces the demand for reinforcement length and tensile strength. For example, when the reference slope is subjected to seismic loading, adopting a target safety factor of FS = 1.3 against the internal failure mechanism will require reinforcement layers with the length and tensile strength being  $L = 8$  m and  $T = 18$  kN/m, respectively. However, increasing the internal

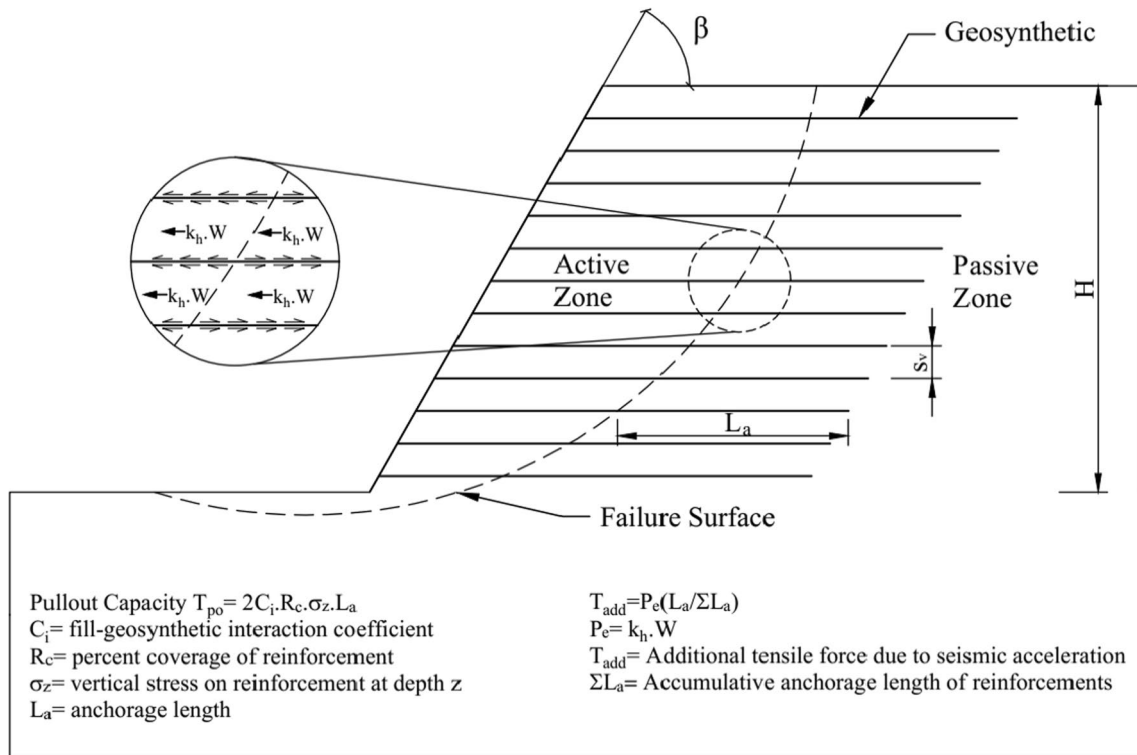


Fig. 7 Influence of pseudo-static force on geosynthetic pullout capacity

friction angle from  $\varphi = 35^\circ$  to  $45^\circ$ , remarkably diminishes the required length and tensile strength of reinforcement layers to  $L = 4.5$  m and  $T = 8$  kN/m, respectively. It is evident that for a typical reinforced slope under seismic loading, the required length and tensile strength for the reinforcement layers reduce to approximately half as fill internal friction angle improves from  $35^\circ$  to  $45^\circ$ .

The influence of internal friction angle on the required reinforcement properties is, however, less pronounced when the reinforced slope is subjected to the static loading compared to seismic loading, confirming the critical role of the strength properties of slope fill material in its seismic performance.

Table 1 Variables considered in the parametric analyses

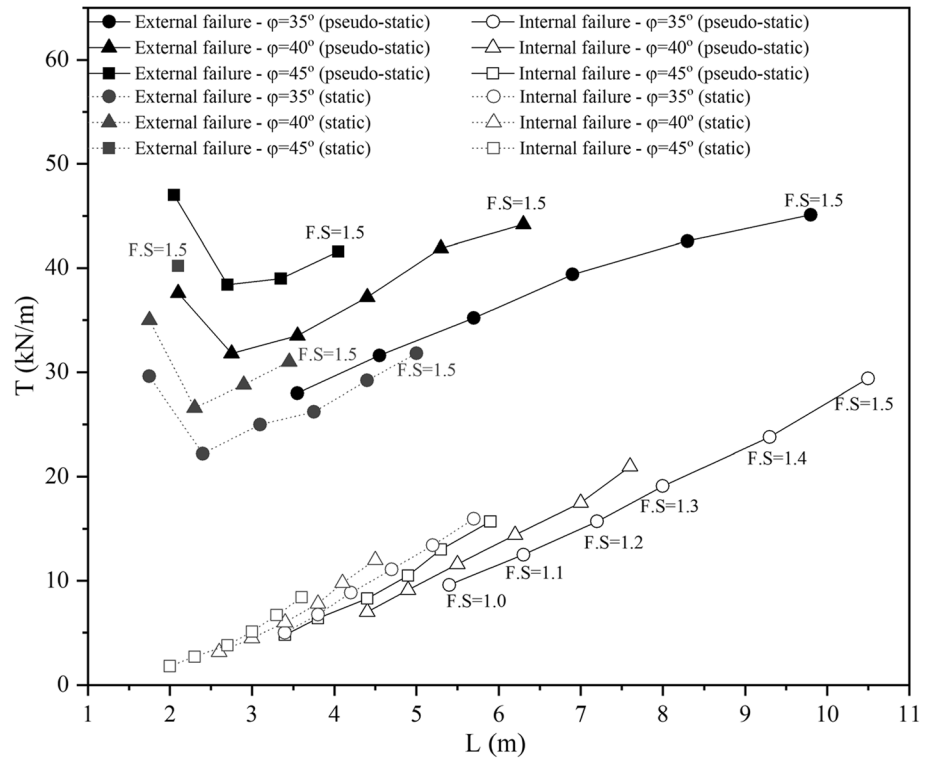
Parameter	Values*
Internal friction angle of soil, $\varphi$ ( $^\circ$ )	35, <b>40</b> , 45
Slope angle, $\beta$ ( $^\circ$ )	45, <b>60</b> , 75
Slope height, $H$ (m)	5.0, <b>7.5</b> , 10
Horizontal seismic acceleration coefficient, $k_h$ (-)	0.05, 0.1, 0.15, <b>0.2</b> , 0.25, 0.3
Vertical to horizontal acceleration ratio, $k_v/k_h$ (-)	0, $\pm$ <b>0.5</b> , $\pm$ 1.0

\*Bold values are the reference model parameters

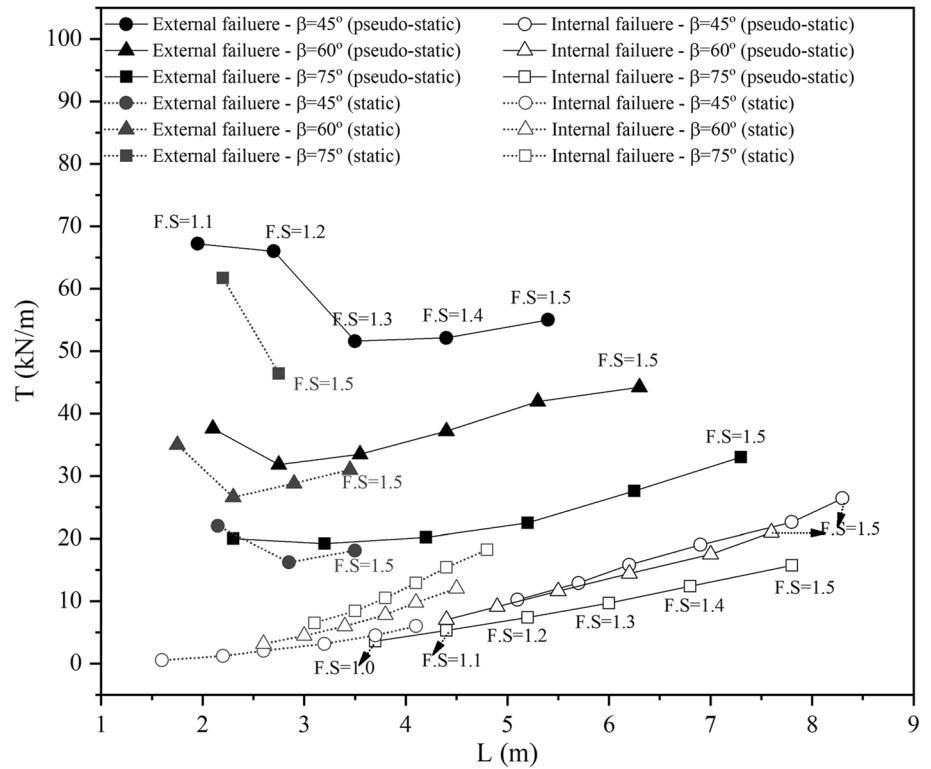
### 5.3 Influence of Slope Inclination

The effect of slope angle on minimum reinforcement layers requirements to generate internal and external failure mechanisms for both static and seismic loading conditions is illustrated in Fig. 9. In these analyses, a 7.5 m-high slope is assumed to support a fill material with  $\varphi = 40^\circ$ , which is the reference internal friction angle. According to the results, regardless of the loading conditions and triggered failure mechanism, an increase in the slope angle from  $\beta = 45^\circ$  to  $75^\circ$ , brings about a longer geosynthetic reinforcement requirement to maintain a certain safety level. It is eminent from Fig. 9 that the minimum reinforcement length requirement to attain FS = 1.5 against seismic external failure increases from  $L = 5.5$  m to around  $L = 7.5$  m, when the slope steepens from  $\beta = 45^\circ$  to  $75^\circ$ . In other words, the steeper the slope is, the longer geosynthetic reinforcement layers would be required to withstand a possible external failure mechanism under seismic loading condition. Nevertheless, this observation is less highlighted when the slope is under static loading. To be more specific, the minimum required length for the geosynthetic reinforcement layers due to the slope angle increasing from  $\beta = 45^\circ$  to  $75^\circ$ , increases from  $L = 2.8$  m to 3.5 m for gravitational force only; hence, bearing witness to the insignificant contribution of slope angle to the

**Fig. 8** Influence of internal friction angle on the required length and tensile strength of geosynthetic layers ( $H=7.5$  m,  $s_v=0.6$  m,  $\gamma=18$  kN/m<sup>3</sup>,  $\beta=60^\circ$ ,  $k_h=0.2, k_v=0.1$ )



**Fig. 9** Influence of slope angle on the length and tensile strength requirements of geosynthetic layers ( $H=7.5$  m,  $s_v=0.6$  m,  $\gamma=18$  kN/m<sup>3</sup>,  $\phi=40^\circ$ ,  $k_h=0.2, k_v=0.1$ ). ++ Same as the above ++



geosynthetic layers length demand under static loading compared to the seismic loading condition.

It is also observed that, the effect of slope angle on geosynthetic strength and length requirements would be less pronounced when the failure mechanism turns from external to internal mode. For instance, for a slope subjected to seismic loading, to maintain a FS value of 1.5 against the internal failure, the change in slope angle from  $\beta = 45^\circ$  to  $75^\circ$  does not pose notable effect on the geosynthetic length and strength requirements, as opposed to the external failure mechanism elucidated earlier. This observation could be ascribed to the very fact that the safety factor against internal failure mechanism, passing all the reinforcement layers in the active zone, is mainly influenced by the tensile strength of the reinforcement layers, with the facing inclination of the sliding mass playing less important role. On the other side of the spectrum, when the failure surface circumvents all the reinforcement layers, it is the slope angle which directly affects the weight of the sliding mass, i.e. the motive force in other words. Therefore, longer geosynthetic layers are ideally needed for steeper slopes to preserve similar FS values against external failure mechanism.

#### 5.4 Influence of Slope Height

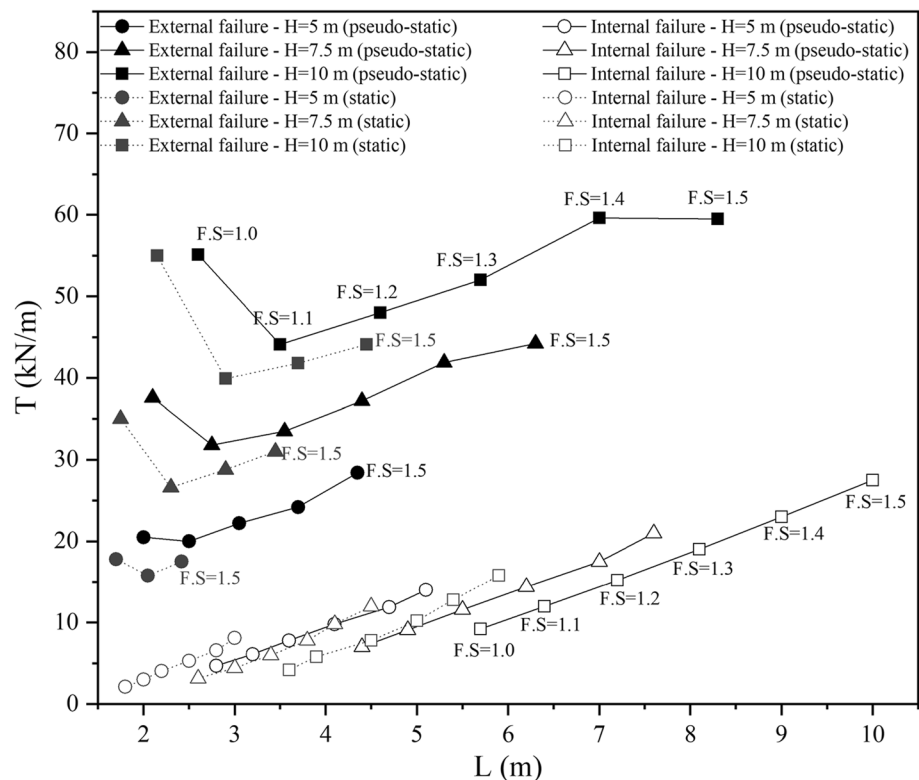
Figure 10 represents the effect of slope height on geosynthetic minimum length and tensile strength requirements for internal and external failure mechanisms under both static

and seismic loadings. It should be noted that, in all analyses, the reference model is inclined at an angle of  $\beta = 60^\circ$  and also bears an internal friction angle of  $\varphi = 40^\circ$ . In addition, the seismic acceleration coefficients are  $k_h = 0.2$  and  $k_v = 0.1$ , while the slope height varies within the range of the values presented in Table 1.

Generally, for any failure mechanism and seismic loading conditions, an increase in the slope height causes the reinforcement layers' length and tensile strength requirements to substantially increase in order to retain a certain FS value. As clearly observed, to yield FS = 1.5 against internal failure under seismic loading, the demand for reinforcement length and strength critically increases from  $L = 5.2$  m and  $T = 14$  kN/m to about  $L = 10$  m and  $T = 25$  kN/m, respectively, when the slope becomes heightened from  $H = 5$  m to  $H = 10$  m. As expected, the total weight of the sliding mass would dramatically increase by augmenting the slope height leading the driving force and moment caused by the pseudo-static load to substantially increase. Therefore, a set of longer and stronger geosynthetic reinforcements must be placed to reach the same FS value. A similar trend of variation could be vividly observed when the same slope is subjected to static loading only; but the influence of slope height on required geosynthetic properties of the static case seems to be less significant compared to the seismic loading.

It is also noted that, to produce FS = 1.5 against external failure under seismic loading, the required length and strength of geosynthetic reinforcement layers critically

**Fig. 10** Influence of slope height on the length and tensile strength requirements of geosynthetic layers ( $s_v = 0.6$  m,  $\beta = 60^\circ$ ,  $\gamma = 18$  kN/m<sup>3</sup>,  $\varphi = 40^\circ$ ,  $k_h = 0.2$ ,  $k_v = 0.1$ ). ++ Same as the above ++



increase from  $L = 4.4$  m and  $T = 28$  kN/m to about  $L = 8.7$  m and  $T = 60$  kN/m, respectively, when the slope height increases from  $H = 5$  m to  $H = 10$  m. According to the results, it can be concluded that any increase in the slope height, under either static or seismic loadings, would have a great impact on geosynthetic layers tensile strength and length requirements so as to yield a certain safety level.

### 5.5 Influence of Pseudo-Static Seismic Accelerations

In order to examine the influence of seismic loading on the length and tensile strength requirements of the geosynthetic reinforcement layers, the reference model is subjected to the different magnitudes of horizontal and vertical acceleration coefficients according to Table 1. In all analyses, the reference model is 7.5 m high, inclined at  $\beta = 60^\circ$ , while supporting a fill material with the reference internal friction angle of  $\varphi = 40^\circ$ .

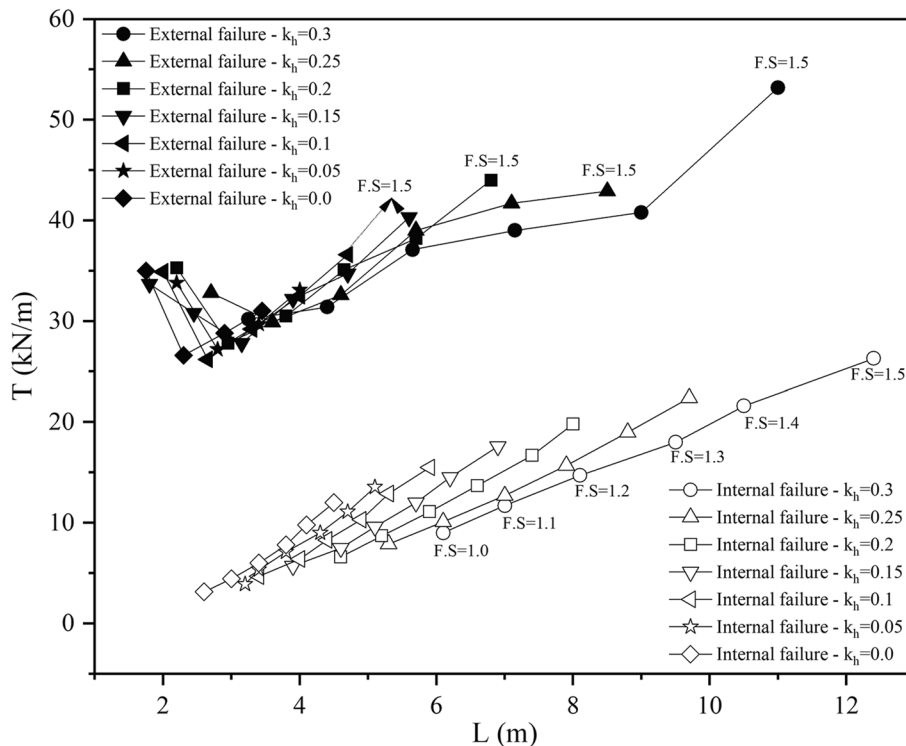
The effect of horizontal acceleration on the length and tensile strength requirements of geosynthetic layers is depicted in Fig. 11 for both internal and external modes of failure mechanism. According to the results, regardless of the failure mechanism, increasing the horizontal acceleration coefficient from  $k_h = 0$  (i.e. static loading condition) to 0.3, requires a longer and stronger reinforcement in order to imitate the same FS value. For instance, to obtain a factor of safety of 1.5 against external failure, the minimum geosynthetic length and tensile strength demands increase from  $L = 3.5$  m and  $T = 33$  kN/m to approximately  $L = 11.2$  m

and  $T = 52$  kN/m, respectively, when the horizontal acceleration coefficient augments from  $k_h = 0$  to 0.3. Indeed, the increased pseudo-static horizontal force is translated in form of the increased pullout load in the part of the reinforcement layers located in the active wedge. This, will as such, lead to longer reinforcement requirements in the passive zone to withstand the mobilized horizontal load, as schematically illustrated in Fig. 7. In other words, longer geosynthetic reinforcement layers are needed to mobilize sufficient resisting force against the augmented pseudo-static horizontal motive force due to increased horizontal seismic acceleration.

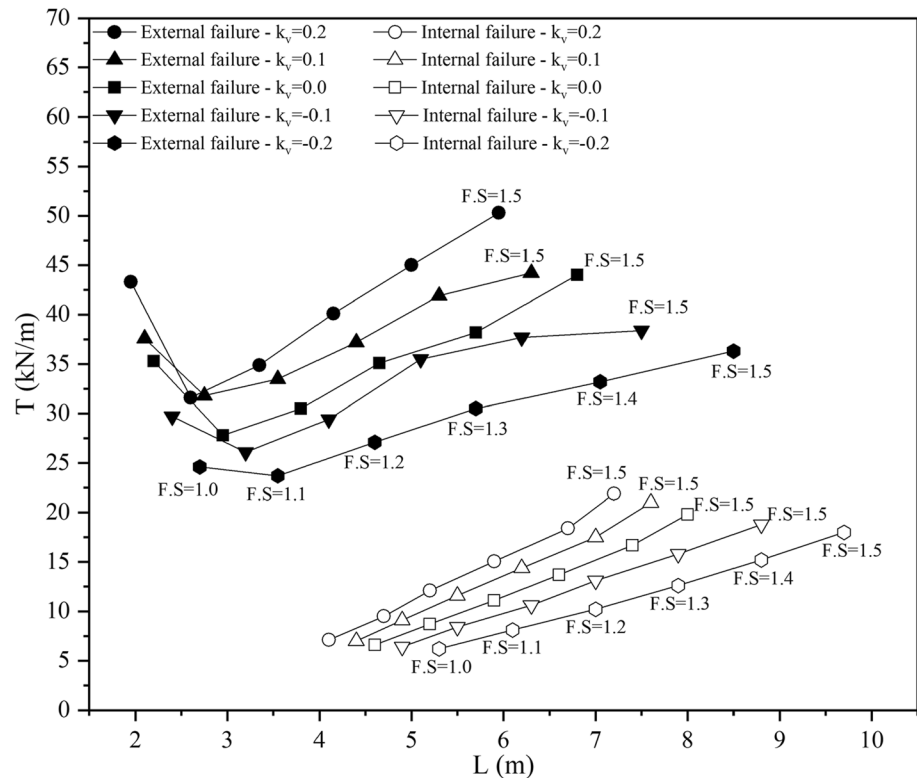
A direct comparison between different failure mechanisms reveals that for a constant magnitude of the horizontal acceleration coefficient  $k_h$ , a stronger geosynthetic reinforcement is needed to reach a particular FS value when the failure mechanism transforms from internal to external mode. For example, under horizontal acceleration coefficient of  $k_h = 0.3$  and FS = 1.5, the required tensile strength of geosynthetic increases from  $T = 24$  kN/m to roughly  $T = 53$  kN/m when the failure mechanism turns from internal to external mode. In fact, there is no resisting pullout capacity in the passive zone when the external failure occurs; thus, the geosynthetic tensile force has to be proportionally increased to resist the augmented driving force imposed by horizontal acceleration.

Figure 12 shows the effect of vertical acceleration coefficient on the geosynthetic length and tensile strength requirements for both the internal and external modes of failure mechanism. Similarly, the reference model is 7.5 m

**Fig. 11** Influence of horizontal acceleration coefficient on the length and tensile strength requirements of geosynthetic layers ( $H = 7.5$  m,  $s_v = 0.6$  m,  $\beta = 60^\circ$ ,  $\gamma = 18$  kN/m<sup>3</sup>,  $\varphi = 40^\circ$ ,  $k_v = 0$ )



**Fig. 12** Influence of the vertical acceleration coefficient on the length and tensile strength requirements of geosynthetic layers ( $H = 7.5$  m,  $s_v = 0.6$  m,  $\beta = 60^\circ$ ,  $\gamma = 18$  kN/m<sup>3</sup>,  $\varphi = 40^\circ$ ,  $k_h = 0.2$ )



high with the slope inclined at  $\beta = 60^\circ$  while the horizontal acceleration coefficient was kept constant at  $k_h = 0.2$ . Based on the results, irrespective of the failure mechanism considered, it is evident that the reversal of the direction of vertical acceleration from upward ( $k_v = 0.2$ ) to downward ( $k_v = -0.2$ ), gives rise the longer geosynthetic reinforcement layers to be deployed in order to maintain a particular FS value; however, the required strength seems to change to the contrary. For example, for the external failure mechanism, the minimum length and tensile strength requirements for the geosynthetic layers to render  $FS = 1.5$  alter from  $L = 8.5$  m and  $T = 35$  kN/m to about  $L = 6$  m and  $T = 50$  kN/m, respectively, when the vertical acceleration coefficient varies from  $k_v = -0.2$  to  $0.2$ . The rationale behind this observation could be sought in the effect of downward vertical acceleration on the total mass of the sliding zone. To shed more light, the negative  $k_v$  value basically magnifies the total weight of the sliding mass, which subsequently accentuates the driving force. Therefore, a longer geosynthetic has to be used to mobilize an adequate pullout capacity resisting the additional driving force across the failure slip caused by the increased sliding mass.

It is also observed that under similar conditions in terms of FS and  $k_v$  values, the required tensile strength substantially increases when the failure mechanism alters from internal to external. As clearly seen, for given values of  $FS = 1.5$  and  $k_v = 0.2$ , the required geosynthetic tensile

strength is critically increased from  $T = 17$  kN/m to  $T = 52$  kN/m when the failure mechanism changes from internal to external. As stated above, when the failure mechanism is external, there is no mobilized pullout force along the passive zone to resist the pseudo-static force; thus, the required geosynthetic tensile strength must be adequately increased to reach the same FS value as for the internal failure mechanism. This behavior, however, is less pronounced as the magnitude of the vertical acceleration turns from a positive (i.e. upward acceleration) to a negative value (i.e. downward acceleration). In other words, the influence of failure mechanism on the required geosynthetic tensile strength is less significant when the reinforced slope is subjected to the negative (i.e. downward) vertical acceleration.

## 6 Conclusions

In this study, the well-established pseudo-static approach was employed to investigate the stability of a typical reinforced earth slope under seismic loading. To this end, Bishop's simplified method was utilized with circular failure surface. Design charts were also prepared where the variations of the required tensile strength of the geosynthetic reinforcement layers were plotted against the reinforcement length demand for different failure mechanisms. The main findings of the current study are as follow:

- In general, the required length and tensile strength of the reinforcement layers to produce a particular FS substantially increased as the slope was subjected to the seismic loading compared to the static loading condition. It can be concluded that the length and tensile strength requirements have to be proportionally increased to resist the additional driving pseudo-static force caused by seismic accelerations.
- It was observed that, regardless of the failure mechanism and loading conditions, increasing the internal friction angle of the fill remarkably reduced the length and tensile strength demands for the reinforcement layers. The influence of internal friction angle on reinforcement demands, however, was less significant as the slope was subjected to the static loading condition compared to the seismic loading scenario. This lends support to the contention that the impact of the fill strength properties on the geosynthetic length and tensile strength requirements is more critical under seismic loading.
- Under seismic loading, slope inclination has shown to have a great impact on required tensile strength and length of the reinforcement layers. Regardless of the failure mechanism, an increase in the slope inclination necessitates longer geosynthetic reinforcement layers to be used in order to obtain a certain value of FS under seismic loading condition. This behavior; however, was insignificant when the slope was subjected to the static loading. The geosynthetic tensile strength and length requirements were also found to be less sensitive to the slope inclination when the failure mechanism turned from external to internal.
- An increase in the slope height caused the required length and tensile strength of reinforcement layers to substantially increase. Indeed, higher slope increased the magnitude of driving force and moment through enlarged sliding mass; thus, longer and stronger geosynthetic reinforcements were needed to maintain a specific FS value. In external failure mechanism, the change in geosynthetic tensile strength requirement was more obvious when the slope height altered from  $H = 5$  m to 10 m. Under static loading, a similar trend was observed, but the influence of slope height on required geosynthetic properties was not as considerable as in the case for the seismic loading.
- As expected, when the horizontal acceleration intensified from  $k_h = 0$  to 0.3, the required geosynthetic length and strength substantially increased. It was also observed that, unlike the internal failure mechanism, the effect of horizontal acceleration on geosynthetic length and strength requirements was quite remarkable when the failure mechanism was expected to be of external type. Moreover, a reversal of the vertical acceleration direction from upward to downward entails longer and weaker geo-

synthetic layers to be exploited in order to meet a certain safety level. The reason was due primarily to the effect of downward vertical acceleration on the total mass of the sliding zone imposing an additional driving moment.

- The design charts presented in this paper can be simply used for the preliminary estimation of the reinforcement length and tensile strength requirements when a typical reinforced slope is subjected to different combinations of vertical and horizontal seismic accelerations.

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**Data Availability** All data, models or codes that support the findings of this study are available from the corresponding author upon reasonable request.

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