



# Semi-Analytical Approach for Estimating the Viscoelastic Settlement of a Footing Resting on a Slope

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## Abstract

The safety of a foundation can be viewed from two different perspectives, bearing capacity, and the settlement. There are many articles focused on the bearing capacity of a foundation built near the slope. However, investigating the settlement of the foundation rest near the slope is very limited. In order to increase the safety of the structure, besides the elastic settlement, the study of time-dependent behavior of footing is of great importance in geotechnical engineering. In this research, a semi-analytical solution has been proposed to obtain the viscoelastic settlement of a footing adjacent to a slope. Based on the developed Airy stress function, distributed stress within the slope due to foundation load was computed analytically and then displacement has been acquired by using the finite difference method. The outcome of the proposed method was compared with COMSOL finite element software and good agreement between those was observed.

**Keywords** Creep · Foundation settlement · Time-dependent settlement · Differential settlement · Elastic settlement

## 1 Introduction

Recently, investigating the safety of the foundation built near the slope has drawn more attention from researchers. Two critical aspects must be met during the design of a foundation, namely, ultimate bearing capacity as well as the relative settlement of the footing that should not trespass the allowable settlement (Meyerhof 1951).

For the half-space and horizontal ground surface cases, many published works have focused on the settlement of the foundation (Choobbasti et al. 2010; Díaz et al. 2018; Egorov and Nichiporovich 1961; Fisher 1957; Gazetas et al. 1985; Gunerathne et al. 2018; Naseri and Hosseininia 2015).

Also, many researchers studied the long term settlement of foundation resting on soil or rock mass. In this regard, Taylor and Merchant (1940) by considering the soil mass as a Kelvin body investigated the creep settlement of a foundation (Taylor and Merchant 1940). Booker and Small (1986) used the transformation of governing equation and studied the creep settlement of foundation resting on a horizontal soil layer (Booker and Small 1986). In order to study the creep settlement of the embankment, Justo and Durand (2000) employed the Merchant model (Justo and Durand 2000). Xie et al. (2008) considered the burger model and studied the creep settlement of the foundation applied to time depending loading (Xie et al. 2008). Zou et al. (2018) employed a semi-analytical approach to study the consolidation of clay by considering the clay as an elastic-viscoplastic material (Zou et al. 2018). Chen and Ai (2020) have presented a numerical solution in order to study the viscoelastic behavior of transversely isotropic multilayered porous rock foundation (Chen and Ai 2020).

From the mentioned literature, it can be concluded that the settlement of foundation resting on half-space has been widely investigated by many researchers. Nevertheless, despite many published studies that focus on the assessment of the ultimate bearing capacity of a foundation near the slope (Georgiadis 2010; Haghgoeui et al. 2020a, b; Kusakabe et al. 1981; Leshchinsky 2015; Meyerhof 1957; Narita

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and Yamaguchi 1990; Zhou et al. 2018), to the knowledge of authors, published research on the settlement of the foundation placed near the slope is limited. Shallow foundation sometimes needed to be constructed on or adjacent to the slope (Graham et al. 1988; Kusakabe et al. 1981; Meyerhof 1957; Ni et al. 2016; Saran et al. 1989; Shields et al. 1990; Zhou et al. 2018), and prediction of the settlement is crucial in the design of shallow foundations (Bungenstab and Bicalho 2016; Haghgouei et al. 2021). Also, Geomaterial's deformation is a time-dependent phenomenon and demonstrates rheological behavior (Chen and Ai 2020; Cong and Hu 2017; Deng et al. 2000; Malan 1999; Omar et al. 2011; Yao et al. 2018; Zeng and Kou 1992; Zhou and Cheng 2015). Therefore, investigation of time-dependent foundation settlement rest near the slope to increase the safety of the structures is amply clear.

In the case of calculation of time-dependent displacements caused by footing load, usually viscoelastic, elastic-viscoplastic or elastoplastic-viscoplastic models have been used by researchers (Christie 1964; Justo and Durand 2000; Kaliakin and Dafalias 1990; Yin and Graham 1994). However, viscoelastic models can evaluate the time-dependent behavior more satisfactory when the stress level in geomaterial is low (Zhu et al. 2012). In the case of higher loading, where the geomaterials exhibit plastic deformation, the assessment of the bearing capacity of the foundation adjacent to a slope is more prominent than the evaluation of the settlement of the foundation, especially in the absence of supporting systems such as the retaining wall, due to slope failure.

In this research, a semi-analytical solution, that is able to consider all the effective parameters that play a significant role in the viscoelastic settlement of the foundation, will be proposed to obtain both elastic and visco-elastic settlement of a footing adjacent to a slope.

## 2 Detail of the Proposed Semi-Analytical Solution

### 2.1 Evaluating the Stress State Within the Slope

Assessment of the stress distribution within a slope is of great importance to evaluate the creep displacement. Based on the theory of elasticity, the stress distribution in a body can be found by employing the Airy stress function. Equation (1) is known as Bi-harmonic equation and satisfies the equilibrium and compatibility relations. In this equation  $\phi$  denotes the Airy stress function. By solving the Eq. (1) and finding the Airy stress function, one can find the stress components by using the derivatives of Airy stress function as presented in Eq. (2).

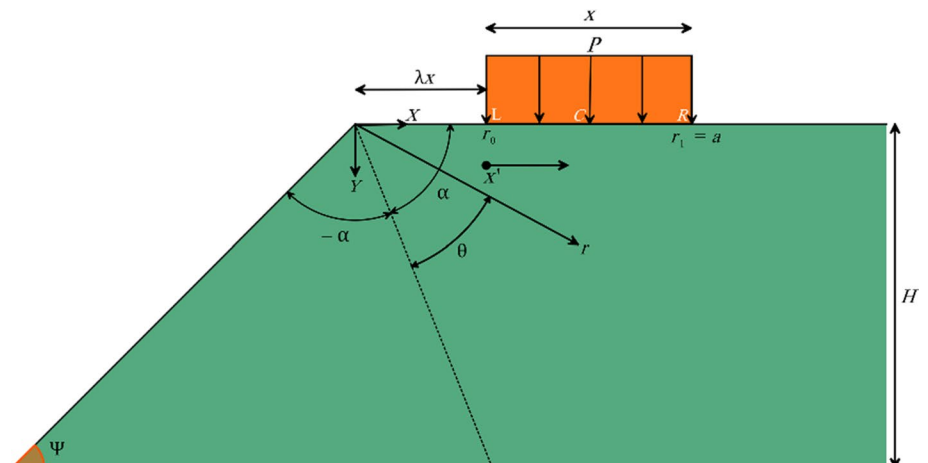
$$\nabla^4 \phi = \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \phi = 0 \quad (1)$$

$$\begin{aligned} \sigma_\theta &= \frac{\partial^2 \phi(r, \theta)}{\partial r^2} & \sigma_r &= \frac{1}{r} \frac{\partial \phi(r, \theta)}{\partial r} \\ &+ \frac{1}{r^2} \frac{\partial^2 \phi(r, \theta)}{\partial \theta^2} & \tau_{r\theta} &= -\frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \phi(r, \theta)}{\partial \theta} \right) \end{aligned} \quad (2)$$

The authors, suggested an analytical solution to calculate the stress distribution due to the loads of footing, within a slope with high accuracy (Haghgouei et al. 2020a, b) by transforming the Airy stress function into the Mellin transformed domain. The definition of Mellin transformation and its inversion is presented in Eq. (3).

$$\begin{aligned} f(z) &= \int_0^\infty t^{z-1} f(t) dt \\ f(t) &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} t^{-z} f(z) dz \end{aligned} \quad (3)$$

**Fig. 1** Schematic representation of the foundation resting near the slope



The proposed transformed Airy stress function is also presented in Eq. (4).

$$\Phi(z, \theta) = \frac{F(z)}{2z(z+1)} \left[ \frac{z \sin(z\alpha) \cos((z+2)\theta) - (z+2) \sin(z+2)\alpha \cos(z\theta)}{(z+1) \sin(2\alpha) + \sin 2(z+1)\alpha} + \frac{(z+2) \cos(z+2)\alpha \sin(z\theta) - z \cos(z\alpha) \sin((z+2)\theta)}{(z+1) \sin(2\alpha) - \sin 2(z+1)\alpha} \right] \tag{4}$$

By considering Eqs. (2), (3) and (4), the stress component can be defined as Eq. (5) (Tranter 1951).

$$\begin{aligned} \sigma_\theta &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Phi(z, \theta) z(z+1) r^{-z-2} dz \\ \sigma_r &= \frac{-1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left[ z\Phi(z, \theta) - \frac{d^2\Phi(z, \theta)}{d\theta^2} \right] r^{-z-2} dz \\ \tau_{r\theta} &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{d\Phi(z, \theta)}{d\theta} (z+1) r^{-z-2} dz \end{aligned} \tag{5}$$

In order to investigate the effect of footing load’s placement on the settlement of the foundation, the footing load should have the flexibility to be considered at any place on the slope with a certain distance from the crest of the slop. To implement this configuration in the proposed method a step function,  $s$ , can be considered in the loading function. This loading function may be illustrated by Eq. (7) as follows,

$$\begin{aligned} F(z) &= \int_0^1 \left[ \sum_{j=0}^1 s(r-r_j)(-1)^j \right] r^{z+1} dr \\ &= \frac{1}{z+2} \sum_{j=0}^1 r_j^{z+2} (-1)^j \end{aligned} \tag{7}$$

In this Equation  $r_0$  and  $r_1$  are equal to  $\lambda x$  and  $a$ , respectively. By solving Eq. (5) based on Eq. (4) and doing some algebra, the transformed stress components are calculated as Eq. (8).

$$\begin{aligned} \sigma_\theta &= \frac{-1}{4\pi i} \int_{c-i\infty}^{c+i\infty} \left( \frac{(\lambda x)^{z+2} - a^{z+2}}{r^{z+2}} \right) \left[ \frac{-\cos((z+2)\alpha)\sin(z\theta) + \frac{z}{z+2} \cos(z\alpha)\sin((z+2)\theta)}{(z+1)\sin(2\alpha) - \sin(2(z+1)\alpha)} + \frac{-\frac{z}{z+2} \sin(z\alpha) \cos((z+2)\theta) + \sin((z+2)\alpha) \cos(z\theta)}{(z+1)\sin(2\alpha) + \sin(2(z+1)\alpha)} \right] dz \\ \sigma_r &= \frac{-1}{4\pi i} \int_{c-i\infty}^{c+i\infty} \left( \frac{(\lambda x)^{z+2} - a^{z+2}}{r^{z+2}} \right) \left[ \frac{-\frac{z+4}{z+2} \cos(z\alpha) \sin((z+2)\theta) + \cos((z+2)\alpha)\sin(z\theta)}{(z+1)\sin(2\alpha) - \sin(2(z+1)\alpha)} + \frac{\frac{z+4}{z+2} \sin(z\alpha) \cos((z+2)\theta) - \sin((z+2)\alpha)\cos(z\theta)}{(z+1)\sin(2\alpha) + \sin(2(z+1)\alpha)} \right] dz \\ \tau_{r\theta} &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left( \frac{(\lambda x)^{z+2} - a^{z+2}}{r^{z+2}} \right) \left[ \frac{\cos(z\alpha) \cos((z+2)\theta) - \cos((z+2)\alpha) \cos(z\theta)}{(z+1)\sin(2\alpha) - \sin(2(z+1)\alpha)} + \frac{\sin(z\alpha) \sin((z+2)\theta) - \sin((z+2)\alpha) \sin(z\theta)}{(z+1)\sin(2\alpha) + \sin(2(z+1)\alpha)} \right] dz \end{aligned} \tag{8}$$

For the case of foundation resting on a slope, based on Fig. 1, the boundary conditions for the slope and its upper foundation can be represented by Eq. (6) as follows,

$$\begin{aligned} \sigma_\theta &= f(r) = P, \quad \theta = \alpha, \quad r_0 \leq r \leq r_1 \\ \sigma_\theta &= 0, \quad \theta = -\alpha \\ \tau_{r\theta} &= 0, \quad \theta = \pm\alpha \end{aligned} \tag{6}$$

Equation (8) is a meromorphic function that has a simple pole in the  $z = -1$  imaginary path. Thus, the stress component in the real domain can be obtained by taking integration on the mentioned vertical imaginary path and subtracting the answer from the residue of the function multiplied by  $\pi i$ . The stress components in the real domain are defined by Eq. (9).

$$\begin{aligned}
(\sigma_\theta - \sigma_r) &= \frac{\lambda x}{r\pi} \left[ \int_0^\infty (g_1 + g_2) \left[ \text{Sin} \left( y \text{Log} \left( \frac{\lambda x}{r} \right) \right) \right] dy \right] \\
&\quad - \frac{a}{r\pi} \left[ \int_0^\infty (g_1 + g_2) \left[ \text{Sin} \left( y \text{Log} \left( \frac{a}{r} \right) \right) \right] dy \right] + \left[ (\text{Residu}) \frac{a - \lambda x}{r\pi} \right] \\
(\sigma_\theta + \sigma_r) &= \frac{\lambda x}{r\pi} \left[ \int_0^\infty \frac{(g_5 y + g_3) + (g_6 y + g_4)}{1 + y^2} \text{Cos} \left( y \text{Log} \left( \frac{\lambda x}{r} \right) \right) dy \right. \\
&\quad \left. + \int_0^\infty \frac{(g_3 y - g_5) + (g_4 y - g_6)}{1 + y^2} \text{Sin} \left( y \text{Log} \left( \frac{\lambda x}{r} \right) \right) dy \right] \\
&\quad - \frac{a}{r\pi} \left[ \int_0^\infty \frac{(g_5 y + g_3) + (g_6 y + g_4)}{1 + y^2} \text{Cos} \left( y \text{Log} \left( \frac{a}{r} \right) \right) dy \right] \\
&\quad + \int_0^\infty \frac{(g_3 y - g_5) + (g_4 y - g_6)}{1 + y^2} \text{Sin} \left( y \text{Log} \left( \frac{a}{r} \right) \right) dy - \left[ (\text{Residu}) \frac{a - \lambda x}{r\pi} \right] \\
\tau_{r\theta} &= \frac{\lambda x}{2r\pi} \int_0^\infty (g_7 - g_8) \text{Cos} \left( y \text{Log} \left( \frac{\lambda x}{r} \right) \right) dy \\
&\quad - \frac{a}{2r\pi} \int_0^\infty (g_7 - g_8) \text{Cos} \left( y \text{Log} \left( \frac{a}{r} \right) \right) dy
\end{aligned} \tag{9}$$

The functions  $g_1$  to  $g_8$  are presented in Appendix. Also, in the proposed Airy stress function, the role of gravity has been neglected. However, one of the useful principles in the linear elastic method is to use the superposition scheme, such that in order to consider the effect of gravity stress, the outcome of gravity stress distribution suggested by Goodman and Brown (Goodman and Brown 1963) has been added to the outcome of the proposed method. Therefore, the final

## 2.2 Computing the Integrals

Equation (9) cannot be solved explicitly and it should be computed numerically. Here, the Filon (1930) integration method was used to estimate the stress component within the slope. The Filon approach will divide the integrals into  $2n$  subdivision and the length size of each part is equal to  $l$ . Equation (11) may represents the Filon numerical integration method.

$$\begin{aligned}
\int_{x_0}^{x_{2n}} f(x) \text{Cos}(tx) dx &= l \left[ \Omega(th) [f_{2n} \text{Sin}(tx_{2n}) - f_0 \text{Sin}(tx_0)] + \chi(t) c_{2n} + \omega(t) c_{2n-1} \right] \\
\int_{x_0}^{x_{2n}} f(x) \text{Sin}(tx) dx &= l \left[ \Omega(th) [-f_{2n} \text{Cos}(tx_{2n}) + f_{x_0} \text{Cos}(tx_0)] + \chi(t) S_{2n} + \omega(t) S_{2n-1} \right]
\end{aligned} \tag{11}$$

result is able to include the gravity effect. Stress components due to gravity load are presented in Eq. (10).

$$\begin{aligned}
\sigma_{xx} &= \frac{\rho g}{\alpha - \tan(\alpha)} [Y(\alpha - \sin(\alpha) \cos(\alpha)) \\
&\quad - 2 \sin(\alpha) \cos^2(\alpha) (X \sin(\alpha) - Y \cos(\alpha)) \log \left( \frac{\sin(\alpha - \beta)}{\sin(\alpha)} \right) \\
&\quad - \beta \sin(\alpha) [Y \sin(\alpha) (1 + 2 \cos^2(\alpha)) + X \cos(\alpha) (1 - 2 \sin^2(\alpha))] \\
\sigma_{yy} &= \rho g y - \frac{\rho g}{\alpha - \tan(\alpha)} [-Y \sin(\alpha) \cos(\alpha) \\
&\quad + 2 \sin^3(\alpha) (X \sin(\alpha) - Y \cos(\alpha)) \log \left( \frac{\sin(\alpha - \beta)}{\sin(\alpha)} \right) \\
&\quad + \beta \sin(\alpha) [Y \sin(\alpha) (1 - 2 \cos^2(\alpha)) + X \cos(\alpha) (1 + 2 \sin^2(\alpha))] \\
\sigma_{xy} &= \frac{-\rho g}{\alpha - \tan(\alpha)} [Y \sin^2(\alpha) \\
&\quad + 2 \cos(\alpha) \sin^2(\alpha) (X \sin(\alpha) - Y \cos(\alpha)) \log \left( \frac{\sin(\alpha - \beta)}{\sin(\alpha)} \right) \\
&\quad + \beta \sin(\alpha) (1 - 2 \sin^2(\alpha)) (X \sin(\alpha) - Y \cos(\alpha))]
\end{aligned} \tag{10}$$

where,

$$\begin{aligned}
c_{2n} &= 2 \sum_{i=0}^n f_{2i} \text{Cos}(tx_{2i}) - [f_{2n} \text{Cos}(tx_{2n}) + f_0 \text{Cos}(tx_0)] \\
c_{2n-1} &= \sum_{i=1}^n f_{2i-1} \text{Cos}(tx_{2i-1}) \\
S_{2n} &= \left[ 2 \sum_{i=0}^n f_{2i} \text{Sin}(tx_{2i}) \right] - [f_{2n} \text{Sin}(tx_{2n}) + f_0 \text{Sin}(tx_0)] \\
S_{2n-1} &= \sum_{i=1}^n f_{2i-1} \text{Sin}(tx_{2i-1})
\end{aligned}$$

**Table 1** Visco-elastic parameters of studied cases

Case	$G_M$ (Pa)	$G_k$ (Pa)	$\eta_M$ (Pa.s)	$\eta_k$ (Pa.s)	Ref
1	7.6e6	1.8e5	1.08e16	1.08e15	Paraskevopoulou et al. (2018); Pellet (2010)
2	1.2e7	7.5e8	1.0e15	9.7e21	Paraskevopoulou (2016); Paraskevopoulou et al. (2018)

$$\Omega(tl) = \frac{1}{tl} + \frac{\text{Sin}(2tl)}{2(tl)^2} - \frac{2\text{Sin}^2(tl)}{(tl)^3}$$

$$\chi(tl) = 2 \left[ \frac{1 + \text{Cos}^2(tl)}{(tl)^2} - \frac{\text{Sin}(2tl)}{(tl)^3} \right]$$

$$\omega(th) = 4 \left[ \frac{\text{Sin}(tl)}{(tl)^3} - \frac{\text{Cos}(tl)}{(tl)^2} \right]$$

### 2.3 Computing the Strain

The vertical strain induced by foundation load within the slope during the time can be represent by Eq. (12) as follows,

$$\begin{aligned} \epsilon &= \epsilon_{\text{dev}} + \epsilon_V \\ \epsilon_V &= \frac{I_1}{9K} \\ \epsilon_{\text{dev}} &= \frac{1}{2G^*(t)} \sigma_{\text{dev}}|_{t=0} + \int_0^t \frac{1}{2G^*(t-t')} \left( \frac{d}{dt'} \sigma_{\text{dev}}(t') \right) dt' \end{aligned} \tag{12}$$

where the subscript dev and  $V$  denotes deviatoric and volumetric strain, respectively.  $I_1$  represents the first invariant of the stress tensor,  $K$  is the Bulk modulus, and  $G^*(t)$  is the time-dependent modulus which can be defined for Maxwell, Kelvin and Burger model as Eqs. (13), (14), and (15), respectively. It should be noted that Burger’s model is the linear combination of Maxwell and Kelvin bodies.

$$\frac{1}{G_M^*(t)} = \frac{1}{G_M} + \frac{t}{\eta_M} \tag{13}$$

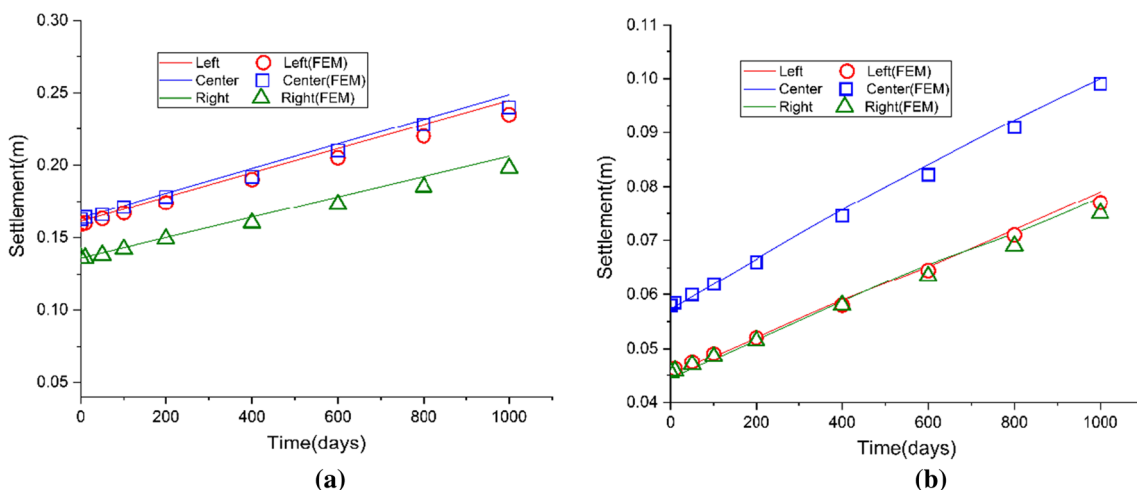
$$\frac{1}{G_K^*(t)} = \frac{1}{G_K} \left( 1 - e^{-\frac{G_K t}{\eta_K}} \right) \tag{14}$$

$$\frac{1}{G_B^*(t)} = \frac{1}{G_M^*(t)} + \frac{1}{G_K^*(t)} = \frac{1}{G_M} + \frac{t}{\eta_M} + \frac{1}{G_K} \left( 1 - e^{-\frac{G_K t}{\eta_K}} \right) \tag{15}$$

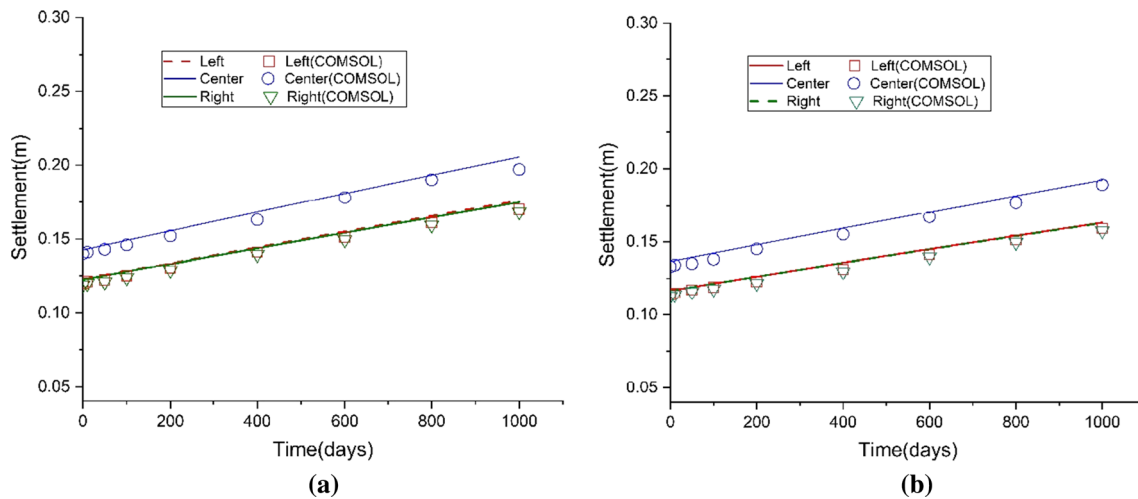
In Eqs. (13) to (15),  $G$  and  $\eta$  are the shear modulus and viscosity, respectively. In the current study, the two-dimensional plane strain condition was assumed, and therefore, the first invariant of stress can be calculated as  $I_1 = (1 + \nu)(\sigma_x + \sigma_y)$ . Since this paper aims to examine the creep settlement of the foundation, it will be assumed that the stress state remains constant during the time.

### 2.4 Computing the Settlement

In order to compute the settlement of the foundation based on the distributed strain in the slope, the FDM was employed as presented in Eq. (16). The domain was meshed such that



**Fig. 2** The settlement of the left, center and right side of a footing placed at **a**70-degree slope with  $H/x=10$  and  $\lambda=0$  and **b** 60-degree slope with  $H/x=5$  and  $\lambda=1$  by considering the Burgers model



**Fig. 3** The settlement of the left, center and right side of a footing placed at **a** 70-degree slope with  $H/x=10$  and  $\lambda=2$  and **b** 30-degree slope with  $H/x=10$  and  $\lambda=0$  by considering the case 1 material for Burgers model

at each grid point, the vertical displacement was calculated through Eq. (16).

$$e = \frac{\partial V}{\partial y} = \frac{V(y+h) - V(y)}{h} \quad (16)$$

In Eq. (16),  $h$  is the size of the mesh, and  $V$  represents the vertical displacement. In order to obtain accurate answer from the Eq. (16), the proper mesh size should be considered. Based on the suggestion by (Ghosh and Sharma 2010) an analysis has been conduct to find the displacement by considering different mesh size and results show that beyond the mesh size smaller than 0.06 m, no noteworthy difference in the outcomes can be detected, and therefore, a mesh size of 0.06 m is considered as the optimal mesh size in the present study.

### 3 Results and Discussion

In order to validate the proposed method, two different slope configurations were considered and the outcome of the semi-analytical method was compared with the COMSOL FEM software. In the first model, a 70-degree slope with  $H/x=10$  and  $\lambda=0$  was assumed. In the second case, a 60-degree slope with  $H/x=5$  and  $\lambda=1$  was modeled. For both cases, the Burger model was assigned for slope material. Moreover, the poison ratio of 0.3 and density equal to 20 KN/m<sup>3</sup> was implemented into the model. The properties of the Burgers model are presented in Table 1.

The settlement of the left, right and center of the foundation (as defined in Fig. 1) were recorded during the time. As can be seen in Fig. 2, there is a good agreement between the proposed method's outcome and numerical simulation.

From Fig. 2a, one can readily see that the settlement of the right side of the footing is less than the left side, and therefore, the differential settlement will occur. By decreasing the slope angle or increasing the normalized distance of the footing from the crest of the slope, the vertical stress distribution is tend to be more symmetric (Haghgoei et al. 2020a, b) under the footing. Consequently, the settlement of the two sides of the foundation will be almost found equal. Figure 3 also depicts settlement resulted from another alternatives of the slope angle as well as the normalized footing distance. As can be seen in Fig. 3a, increasing the normalized footing distance from the crest of the slope leads to uniform settlement of the two edges of the foundations. The same behavior can be observed by decreasing the slope angle in Fig. 3b. Since the main aim of this research work is to propose a new solution and the sensitivity analysis of the parameters is out of the scope of this work, the authors will not delve further into the effect of slope characteristics on the settlement.

### 4 Conclusion

Due to the significant role of foundation settlement on the safety of the structures as well as the limited research work on the evaluation of time-dependent settlement, an attempt has been made to evaluate the visco-elastic settlement of the foundation placed near the slope. In this regard, a semi-analytical solution has been proposed and the time-dependent settlement of the footing, based on the four-element Burgers model, has been investigated. The outcome of the FEM and the proposed model demonstrates a very good agreement. Although numerical modeling is able to solve complex problems under various situations, due to time-consuming procedure it is not a cost-effective option in the initial stage

of the design, and therefore, the proposed method can be considered as a suitable alternative for evaluating the viscoelastic settlement. Also, results manifested that a foundation built near a steep slope is prone to a differential settlement. By decreasing the slope angle or increasing the normalized footing distance from the crest of the slope, a uniform settlement of the two side edges will occur.

## Appendix

$$g_1 = \frac{-\sin(\alpha - \theta) \cos h(\alpha + \theta)y + \sin(\alpha + \theta) \cos h(\alpha - \theta)y}{(y \sin 2\alpha - \sin h(2\alpha y))}$$

$$g_2 = \frac{\sin(\alpha + \theta) \cos h(\alpha - \theta)y + \sin(\alpha - \theta) \cos h(\alpha + \theta)y}{(y \sin 2\alpha + \sin h(2\alpha y))}$$

$$g_3 = \frac{-\cos(\alpha - \theta) \sin h(\alpha + \theta)y + \cos(\alpha + \theta) \sin h(\alpha - \theta)y}{(y \sin 2\alpha - \sin h(2\alpha y))}$$

$$g_4 = \frac{\cos(\alpha + \theta) \sin h(\alpha - \theta)y + \cos(\alpha - \theta) \sin h(\alpha + \theta)y}{(y \sin 2\alpha + \sin h(2\alpha y))}$$

$$g_5 = \frac{-\sin(\alpha - \theta) \cosh(\alpha + \theta)y + \sin(\alpha + \theta) \cosh(\alpha - \theta)y}{(y \sin 2\alpha - \sinh(2\alpha y))}$$

$$g_6 = \frac{\sin(\alpha + \theta) \cos h(\alpha - \theta)y + \sin(\alpha - \theta) \cos h(\alpha + \theta)y}{(y \sin 2\alpha + \sin h(2\alpha y))}$$

$$g_7 = \frac{\sin(\alpha - \theta) \sin h(\alpha + \theta)y + \sin(\alpha + \theta) \sin h(\alpha - \theta)y}{y \sin(2\alpha) - \sin h(2\alpha y)}$$

$$g_8 = \frac{\sin(\alpha - \theta) \sinh(\alpha + \theta)y - \sin(\alpha + \theta) \sinh(\alpha - \theta)y}{y \sin(2\alpha) + \sinh(2\alpha y)}$$

$$\text{Residue} = \left[ \frac{\pi \sin \alpha \cos \theta}{\sin 2\alpha - 2\alpha} + \frac{\pi \sin \alpha \cos \theta}{\sin 2\alpha + 2\alpha} \right]$$

**Author contributions** All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by HH, MHK, ARK and MA. The first draft of the manuscript was written by HH and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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## Declarations

**Conflict of interest** The authors have no affiliation with any organization with a direct or indirect financial interest in the subject matter discussed in the manuscript.

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