Iranian Journal of Science and Technology, Transactions of Civil Engineering (2021) 45:335–357
https://doi.org/10.1007/s40996-020-00499-2

RESEARCH PAPER

A Practical Method for Rapid Assessment of Reliability in Deep Excavation Projects

Arefeh Arabaninezhad[1] · Ali Fakher[1]

Received: 5 October 2019 / Accepted: 1 October 2020 / Published online: 12 November 2020

Abstract

Many reliability analysis methods require complicated mathematical process or access to comprehensive datasets. Such shortcomings limit their application to solve the geotechnical problems. It is of advantage to develop simpler reliability analysis methods that can be employed in the design of geotechnical structures. The current study suggests a simple framework for quick reliability analysis of deep excavation projects in urban areas, which is a common geotechnical problem. To investigate the feasibility of the presented method, five case studies were considered. It is worth mentioning that the horizontal displacement at the crest of excavation was set to be the main system response. For verification purposes, the results were compared to the random set, point estimate and Monte Carlo methods results, which are also used for reliability analysis of geotechnical problems. All case studies were recognized as projects of high importance and monitored during the excavation process. The field observations confirmed that the estimated probabilities of excessive deformations were reasonable for all cases. Comparing the modeling results and field measurements suggests the feasibility of the presented method for evaluating the reliability of deep urban excavations and estimating the horizontal displacement at the crest of excavations.

Keywords Deep excavation · Monte Carlo simulation · Random set finite element method · Reliability analysis · Point estimate method · Uncertainty

1 Introduction

1.1 Review of Conventional Reliability Analysis Methods in Geotechnical Engineering

The uncertainty caused by soil properties poses a major challenge in geotechnical problems. Soil is variable, and its variability is not necessarily considered in the design procedure. Addressing uncertainty does not increase the level of safety, but allows a more rational design while the designer can calibrate the decisions based on the desired or required performance level of a structure (Uzielli et al. 2006). The outcome of reliability analysis methods could be used to assess the reliability of the design and system performance. Common deterministic design approaches assign only one constant value to each input variable and one constant threshold value to control the design. Hence, they are not the most appropriate methods to be applied to most geotechnical problems, where the uncertainty in soil properties is noticeable. The professional engineers need the reliability methods to assign different values to input variables in order to evaluate the effect of uncertainty on the system performance.

Various probabilistic and non-probabilistic reliability analysis methods have been proposed to reduce the likelihood of uncertainty in geotechnical structures (e.g., Kendall 1974; Christian et al. 1994; Peschl 2004; Kaymaz 2005; Low 2005; Kyung Park et al. 2007; Muhanna et al. 2007; Nadim 2007; Pula and Bauer 2007; Schweiger and Thurner 2007; Zhang et al. 2009; Suchomel and Mašı 2010; Beer et al. 2013; Goswami et al. 2016). Because of the complicated and time-consuming mathematical process and lack of thorough information required for input parameters, the proposed methods are not widely used by professional engineers in

Electronic supplementary material The online version of this article (https://doi.org/10.1007/s40996-020-00499-2) contains supplementary material, which is available to authorized users.

✉ Ali Fakher
afakher@ut.ac.ir

Arefeh Arabaninezhad
arefeh.arabani@ut.ac.ir

[1] School of Civil Engineering, University of Tehran, Tehran, Iran

real projects. This highlights the importance of developing more practical methods, which, in terms of mathematics, are simple and work well with limited available input data for estimating the system reliability.

In recent years, the Monte Carlo (MC) simulation method (Zhang et al. 2010; Jiang et al. 2015; Mahdiyar et al. 2017), random field (RF) method (Griffiths and Fenton 2004; Griffiths et al. 2009), point estimate (PE) method (Rosenblueth 1975; Harr 1989; Ahmadabadi and Poisel 2015) and random set (RS) method (Schweiger and Peschl 2005; Nasekhian and Schweiger 2011; Shen, and Abbas 2013; Ghazian Arabi and Fakher 2016; Arabaninezhad and Fakher 2016; Momeni et al. 2018) have been investigated by researchers for reliability analysis of geotechnical problems.

The MC simulation method is a powerful probabilistic technique, which is applicable to both linear and nonlinear problems. In the MC simulation, the best-fitted function (distribution) for each input variable is determined, and based on the estimated range for each input variable, one value is chosen randomly for each run, the model output is computed based on the chosen values, and the obtained results are recorded as the system response. This procedure is iterated several times (e.g., 1000 times) using a variety of values. The simulation process generates a range of values for model output. Finally, the reliability of the model can be determined using statistical analyses. The MC method requires a large number of simulations and takes a long time to provide a reliable distribution for the system response. Hence, except for simple problems, it seems to be impractical for geotechnical practices (Nadim 2007).

The RF method is a useful probabilistic-based reliability method which considers the spatial variability of soil parameters. In this method, the soil layer is divided into small elements and the relevant properties are defined for individual elements. Providing the required parameters for RF needs sufficient information about soil layers, which can rarely be obtained through common site investigation plans. This issue may reduce the popularity of the RF in geotechnical engineering practice.

The PE method is a probabilistic method, in which the probability distribution of input parameters is substituted by single values and their respective predefined weights. The simplicity and relatively low number of required analyses in PE makes it appropriate for reliability analysis. However, the available data in professional geotechnical problems are often insufficient for defining an accurate probability distribution function governing each soil variable, which could make the assumptions in the first step of the PE misleading.

The RS method is a non-probabilistic method that provides a general framework for dealing with set-based information and discrete probability distributions. The random set procedure is used to map the inputs onto the system response in terms of probability bounds. Because the required input data in RS are in the form of random sets and no assumption is made for the probability distribution of the soil variables, this method can deal with the system uncertainty when the exact input values are not available. Application of the RS method in geotechnical practice also has some disadvantages and limitations. In the RS method, at least two random sets with specific probability shares should be assigned to each soil variable. Selecting the required sets and assigning the corresponding probability share to each set are not common in practice. In geotechnical practice, an expert uses all available information to define only one preferred range for each geotechnical variable. In addition, compared to probabilistic methods, the RS results comprise lower and upper bounds, which could be confusing when applied in the design stage.

1.2 The Necessity to Develop a Practical Method for Reliability Analysis of Deep Urban Excavation

As a result of the extensive development of urban areas, deep excavation design has become an increasingly pursued topic in engineering analyses in recent years. In this study, the deep excavation projects are considered as a representative case for geotechnical problems. The authors applied the aforementioned reliability analysis methods to real deep excavation projects in urban areas (Ghazian Arabi and Fakher 2016; Arabaninezhad and Fakher 2016; Momeni et al. 2018). However, due to the shortcomings of these conventional methods in engineering practice, we developed a simple and rapid method called expert selected set (ESS) for reliability analysis of deep excavations.

Many researchers have investigated the distribution functions governing the geotechnical variables, but these functions are not generally considered in engineering practice. Overall, the availability of information in engineering projects is not sufficient for obtaining the governing distribution functions of geotechnical parameters. The statistical parameters (e.g., mean and standard deviation) are usually estimated by statistical inference from sampled observational data, and a point estimator is used to approximate the ‘true’ parameter. Thus, the distribution is exposed to some levels of uncertainty (Zhang et al. 2010).

Since professional engineers do not have access to sufficient information, the reliability analysis methods must be examined using the available data in real projects in order to demonstrate their applicability. In the real deep excavation projects, the site investigation data are not sufficient to define the distribution function of input parameters. On the other side, the large number of iterations needed for MC simulation looks to be impractical to perform. Therefore, a simple and quick method inspired by the concepts used in MC and RS methods is introduced in this study.

In order to evaluate the applicability of the ESS, it was tested on five real monitored deep excavation projects supported by nail-anchor systems. All of the cases have been constructed during the last 10 years. For verification purposes, the results of the ESS were compared to reliability analysis results obtained through RS, PE and MC methods. It is worth mentioning that the applicability of the suggested method should be examined for other structures. Because of the catastrophic effects of deep excavation-induced ground movements on the nearby structures, the horizontal displacement at the crest of excavation is considered as the major system response.

2 The Proposed Method for Reliability Analysis of Deep Excavations

The numerical modeling based on the finite element method (FEM) results in more than one system response without changing the model. In the current study, reliability analysis was performed using the ESS method in combination with the FEM. In the proposed ESS method, unlike the RS, only one set is defined for each input variable, and no probability share is required to be assigned to the sets. Also, unlike MC, no distribution is fitted to input variables and only the lower and upper bounds of input sets are considered as random values. Hence, the number of required FEM runs would be 2^N (N is the number of basic variables).The procedure for reliability analysis in the ESS method is summarized below:

- *Step 1* Define the geometry of the problem and select the appropriate properties and constitutive models for the soil layers and the support system. In this regard, a finite element model is prepared as the main file.
- *Step 2* Provide only one expected range for each input geotechnical variable. Expert judgment on the available information is the main source for selecting the expected range for the variables. However, the statistical knowledge and the suggested coefficients of variation for various geotechnical properties could help suggest more reliable ranges for geotechnical variables. Only one set is selected in the ESS method, so it is similar to engineering practice which also requires one range.
- *Step 3* The number of required finite element runs is 2^N (N is the number of basic variables). Sensitivity analysis could be performed to determine the most influential variables to reduce N and the computational effort.
- *Step 4* A matrix of different combinations of input variables is provided. Subsequently, each combination is keyed into a finite element model and the desired system response (here the horizontal displacement at the excavation top point) is recorded.
- *Step 5* The mean value and standard deviation of the recorded model outputs obtained in step 4 are calculated. Also, using statistical softwares such as EasyFit (Schittkowski 2008), the best distribution function is fitted on the recorded values of system response. The reliability analysis results are illustrated in the form of probability distribution function (PDF) and cumulative distribution function (CDF) curves.
- *Step 6* A threshold value is defined for the main system response in order to determine the probability of unsatisfactory performance of the system. In this study, an acceptable value is defined for the horizontal displacement at the crest of excavation as a target value. The probability of higher displacement values, which indicates the probability of unsatisfactory performance of the system, is determined.

As mentioned earlier, in order to evaluate the applicability of the ESS, it was tested on five real monitored deep excavation projects. For verification purposes, the results of the ESS method were compared to reliability analysis results obtained by RS, PE and MC methods and also the field measurements and observations. The steps of implementing the suggested ESS method are described in greater detail for case study 1, and the procedure for other case studies is described briefly.

3 Implementing the Proposed Method for Real Deep Excavation Projects

3.1 Selected Projects

Tall buildings are common in northern Tehran. In order to supply sufficient space for parking, multi-story basements are constructed for these buildings; thus, major deep excavation projects are performed to construct the basements. The routine depth of a deep excavation is in the range of 20 to 40 m. The soil layers generally consist of fill materials near the ground surface (1.5 to 3 m in depth), with clayey gravel and clayey sand being most frequently observed in order. In order to consider the mentioned specifications of general deep excavations in Tehran, five excavation walls from three important monitored projects were selected as summarized in Table 1. The excavation areas of the selected projects were large, and the inclination of the ground surface causes different excavation depths at different parts of the same project.

Figure 1 shows the excavations location and neighboring facilities. Project (I) is shown in Fig. 1a, where two walls were selected for study because of the differences in their field observation and surcharges. The monitoring records revealed several small cracks on the ground surface near wall 2. Project (II), shown in Fig. 1b, was launched in 2012. As construction proceeded, several cracks were observed around the northern part of the excavation, causing anxiety to the residents of nearby buildings; thus, excavation was suspended for a period

Table 1 General specifications of selected projects

Project	Excavation depth (m)	Fill material depth (m)	Support system	Main type of soil layers	Number of investigated walls
(I) South Atlas Plaza, Commercial center	23 and 25	3	Nail-anchor combination	Clayey gravel and clayey sand	2
(II) Shiraz Street, Golestan Administrative-commercial building	34 and 36.5	1.5	Nail-anchor combination	Clayey gravel and stiff clay	2
(III) North Atlas, Hotel	36.5	1.5	Nail-anchor combination	Clayey gravel and clayey sand	1

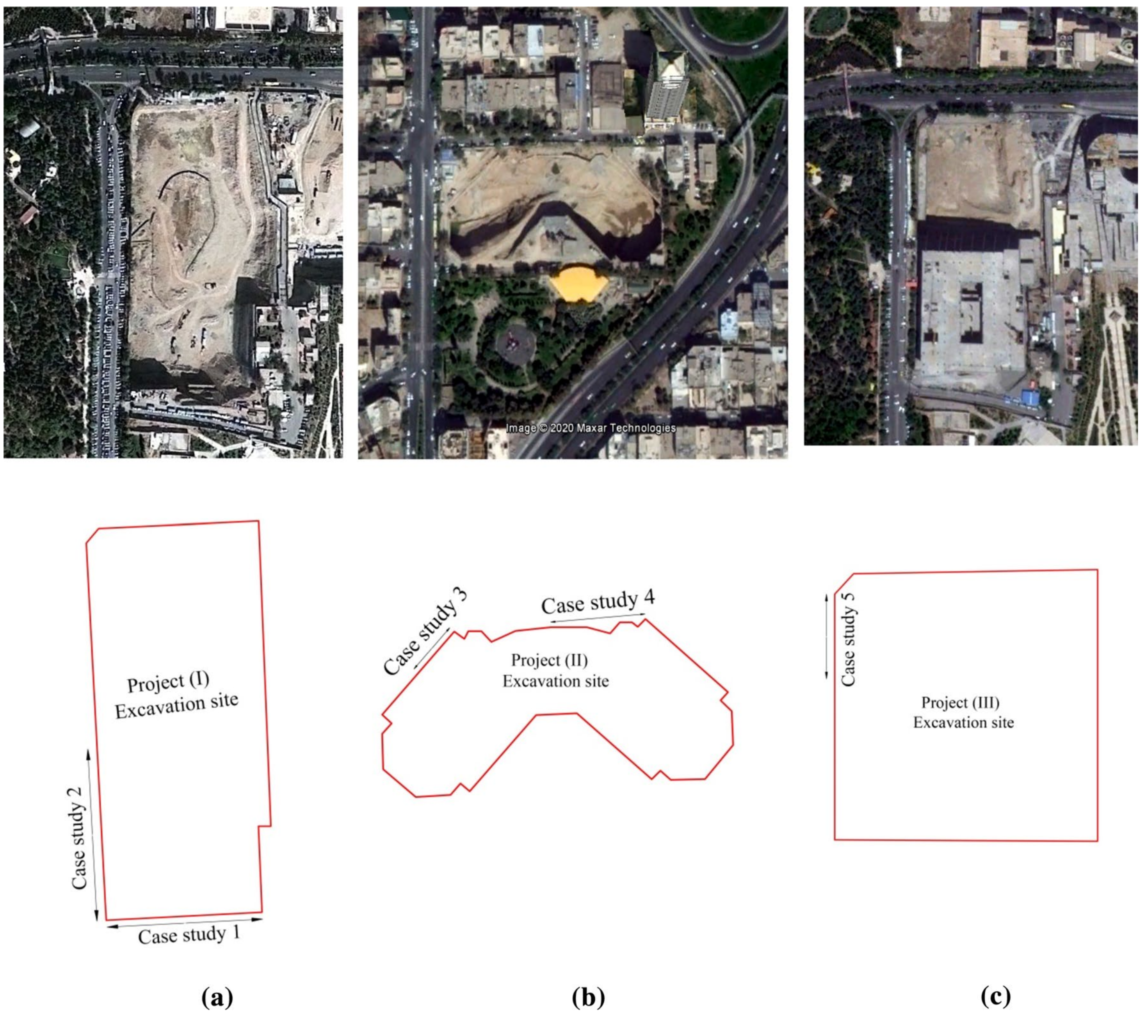

Fig. 1 Aerial view of intended deep excavation projects locations. **a** Case studies 1 and 2. **b** Case studies 3 and 4 **c** Case study 5.

in order to revise the stabilization plan. The soil profile that appeared during excavation indicated that the primary geotechnical investigations had not been consistent with actual soil conditions. Several residential buildings were located adjacent to the street on the north side of the project. Monitoring reports indicated that the majority of horizontal displacement occurred in the northwestern part of the excavation; hence, reliability analysis was performed for walls 3 and 4 which were located in this region. Project (III) was located in the northern half of the deep excavation project site of project (I) as shown in Fig. 1c. This project was carried out to construct a hotel. During excavation, a building was also being built in the southern half of the excavation project.

The excavation support system implemented for all of the walls was a nail-anchor combination. Support system plans were designed with the aid of deterministic methods by geotechnical engineers. The support system was considered to be equal in all finite element analyses performed for reliability analysis of each intended wall. Hence, for the sake of brevity the details of support systems are not explained in detail.

3.2 The Acceptable Value for Horizontal Displacement at the Crest of Excavation

Considering an acceptable (threshold) value for horizontal displacement of the excavation top point, one can estimate the probability of unsatisfactory performance of system. The acceptable displacement value depends on national codes and engineering judgment (Momeni et al. 2017).

Depending on project constraints, requirements with respect to controlling the wall and ground movements will vary. Estimates of wall and ground movements are typically made using semi-empirical relationships developed from past performance data. According to federal highway administration manual (Sabatini et al. 1999), the maximum horizontal deformation (δ_{max}) for anchored walls constructed in sands and stiff clays average approximately 0.2%H with a maximum of approximately 0.5%H, where H is the height of the wall. Navy design manual DM 7.2 (1982) suggested that walls in sand and silt might displace laterally up to 0.2%H. This value for stiff and soft clay was recommended to be 0.5%H and 0.2%H, respectively. PSCG (2000), based on the importance of utilities adjacent to excavation, sets some criteria for excavation protection levels in Shanghai, China. According to these criteria, δ_{max} should be less than 0.3%H in the case that important infrastructure or facilities exist within a distance of 1–2H from the excavation. If no important infrastructure and facilities exist within a distance of 2H from the excavation, then δ_{max} should not exceed 0.7%H.

Considering the literature, the acceptable values of horizontal displacements for the intended deep excavation walls are presented in Table 2.

3.3 Acceptable Probability of Excessive Deformation

Excessive movements can occur without a failure mechanism occurring (Marr and Hawkes 2010). In other words, sometimes an excessive amount of excavation-induced deformation damages the surrounding buildings, while the deep excavation is not collapsed (Momeni et al. 2017). Hence, investigating the serviceability failure of the excavations is of advantage. It is worth mentioning that the probability of excessive deformation for a deep excavation is different from the probability of ultimate failure or collapse. In order to decide whether the determined values for aforementioned probabilities are acceptable, a target value should be considered for each one. The acceptable range for the probability of failure reported in many researches is from 10^{-6} to 10^{-4} (Smith 1986; Santa Marina et al. 1992; HSE 2001). The acceptable probability of excessive deformation (APED) is certainly higher than these values because of the catastrophic consequences of deep excavation collapse compared to the excessive deformation which might cause serviceability failure. In this study, the value of 0.10 is considered for APED as proposed by Momeni et al. (2017).

3.4 Applying ESS Method for Case Study 1

The excavation, with an area of 32,000 m^2, was located adjacent to the Haghani metro station as shown in Fig. 1a. Numerical modeling was done using finite element software PLAXIS 2D. To minimize the boundary effects, a relatively large working area of 90 m width and 55 m depth was used and the geometry was simulated using a plane strain two dimensional model, which includes the soil mass surrounding the excavation. The excavation depth was set to be 23 m. At the bottom of the model, a fixed boundary condition was set. However, the vertical side boundaries were only fixed horizontally and they were set to be free in vertical

Table 2 The acceptable value for horizontal displacement of the deep excavations top point

Case study no.	Excavation depth (m)	Neighboring situation	Acceptable δ_{max}/H (%)	Threshold value for horizontal displacement at the crest of excavation (mm)
1	23	No important facility and building	0.5	100
2	25		0.5	100
3	36.5	Several residential buildings	0.2	65
4	34		0.2	65
5	36.5	No important infrastructure and facilities within a distance of 2H from the excavation	0.5	150

directions. The soil was modeled using 15-noded elements. The soil layers were defined almost in accordance with the soil profile. The ground water level was determined based on the site observation and is shown in Fig. 2a. As mentioned earlier, the excavation support was a nail-anchor combination. The supporting elements were defined in 12 levels with average vertical and horizontal distances of 2 m and 1.5 m, respectively.

Stage construction was utilized for the analysis. In the first stage, initial stresses were generated. The ground water level was defined in the second stage. The third stage dealt with excavation to the depth of 2 m, and in the next stage, the supporting elements in the first level below the excavation surface were activated. The activation process was repeated until all excavation and supporting elements were activated. Figure 2a shows a cross section of the model. The Mohr–Coulomb constitutive model was used to define the soil behavior. For each soil variable, the lower and upper bounds of the input range were selected according to expert judgment as shown in Table 3. As mentioned earlier, the statistical knowledge for various geotechnical properties could help suggest more reliable ranges. This issue is more discussed in Sect. 6.

Tables 4 and 5 show the main input parameters and material properties used in case study 1, which are obtained from site investigation data and laboratory tests.

Sensitivity analysis was performed to identify the variables with the highest influence on system response, and to reduce the number of required finite element runs. The method provided by the US Environmental Protection Agency (1999) was used in this study.

Three main coefficients, including the sensitivity ratio (Eq. 1), sensitivity score (Eq. 2) and relative sensitivity (Eq. 3), were calculated for each input variable according to the system response.

The sensitivity ratio is defined as the ratio of the relative change in the model output f(x) to a relative change in a parameter x:

$$\eta_S = \left| \frac{\left[\frac{f(x_{L/U}) - f(x)}{f(x)}\right]}{\left[\frac{x_{L/U} - x}{x}\right]} \right| \tag{1}$$

where $f(x_{L/U})$: The system response obtained by assigning the lower or upper bound of the selected set to each input variable, while keeping all other parameters fixed to the reference value.

f(x): The system response obtained by assigning the reference values to all input variables. $x_{L/U}$: The lower or upper bound of the selected set for each input variable. x: The reference value for each input variable.

After calculating the sensitivity ratio for the lower and upper bounds of each input variable, the sensitivity score is obtained using Eq. (2). The sensitivity score is calculated in order to make the sensitivity ratio dimensionless.

$$\eta_{SS} = (\eta_{SxL} + \eta_{SxU}) \cdot \frac{(x_L - x_U)}{x} \tag{2}$$

Finally, the relative sensitivity α for each variable is obtained by dividing the value of sensitivity score for each variable to the sum of sensitivity scores of all variables through Eq. (3). A threshold value is considered to determine the most influential input variables.

Table 3 Selected set for input soil variables (case study 1)

Variable no.	Parameter		Ref value	Selected set	
				Lower bound	Upper bound
1	c (kN/m^2)[a]	Fill material	9.64	6.75	12.53
2		Clayey gravel (dry)	85	59.50	110.50
3		Clayey gravel (saturated)	75	52.50	97.50
4		Clayey sand	55	38.50	71.50
5	φ[b]	Fill material	31	28.21	33.79
6		Clayey gravel (dry)	40.36	36.73	43.99
7		Clayey gravel (saturated)	33	30.03	35.97
8		Clayey sand	31.5	28.67	34.34
9	E (MN/m^2)[c]	Fill material:	35	24.50	45.50
10		Clayey gravel (dry)	95	66.50	123.50
11		Clayey gravel (saturated)	106.79	74.75	138.82
12		Clayey sand	71.79	50.25	93.32

[a]Cohesion

[b]Friction angle

[c]Elastic modulus

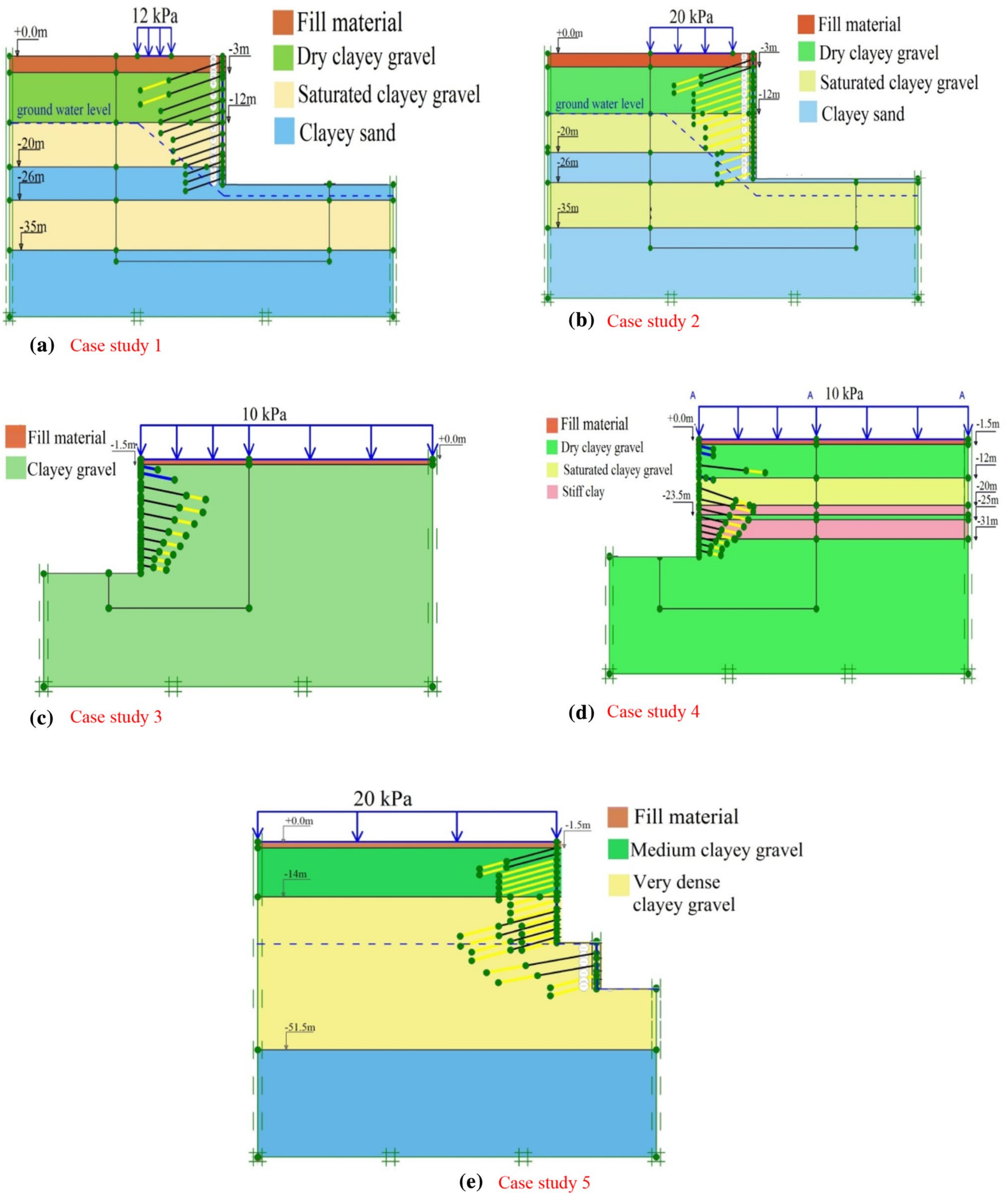

Fig. 2 Cross section of the model for investigated case studies

Table 4 Parameters for structural elements in numerical model (case study 1)

Reinforcement level	Depth below the top of excavation	Structural element	Behavior	EA (kN/m)	EI (kNm^2/m)	Tensile capacity (kN/m)	Horizontal spacing (m)
1 and 2	1 to 3.5	R32 self-drilling anchor	Elasto-plastic	9.82E+04	–	288	1.5
3 to 8	6 to 16	R32 self-drilling nail		9.76E+04	–	168	2
9	17.5	Nail φ 32		1.61E+05	–	202.7	2
10	19	Nail φ 32		1.61E+05	–	202.7	1.5
11 and 12	20.5 and 22	Nail φ 28		1.23E+05	–	155.2	1.5
		Shotcrete wall (concrete facing)	Elastic	2.82E+06	3384		

Table 5 Soil parameters in main numerical model (case study 1)

Material	Behavior	Unit weight (kN/m^2)		Cohesion (kN/m^2)	Internal Friction angle (°)	Elastic modulus (MN/m^2)	Dilation angle (°)
		Unsaturated	Saturated				
Fill material	Drained	17.4	19	9.64	31	3.5E+04	4
Clayey gravel (dry)	Drained	19	21	85	40.36	9.5E+04	12
Clayey gravel (saturated)	Drained	19	20.1	75	33	1.068E+05	5
Clayey sand	Drained	19	20.1	55	31.5	7.18E+04	3

Table 6 Detail of sensitivity analysis for case study 1

Variable no.	Parameter		Sensitivity ratio (η_S) considering lower bound	Sensitivity ratio (η_S) considering upper bound	Sensitivity score η_{SS}	relative sensitivity α (%)
1	c (kN/m^2)	Fill material	1.648	0.120	2.121	23.20
2		Clayey gravel (dry)	0.007	0.001	0.010	0.11
3		Clayey gravel (saturated)	0.264	0.129	0.471	5.16
4		Clayey sand	0.586	0.300	1.064	11.64
5	φ	Fill material	1.468	0.298	0.636	6.96
6		Clayey gravel (dry)	0.003	0.008	0.004	0.04
7		Clayey gravel (saturated)	0.196	0.032	0.082	0.90
8		Clayey sand	1.476	1.374	1.026	11.22
9	E (MN/m^2)	Fill material	0.413	0.205	0.742	8.11
10		Clayey gravel (dry)	0.218	0.021	0.287	3.13
11		Clayey gravel (saturated)	1.121	0.651	2.127	23.26
12		Clayey sand	0.283	0.195	0.574	6.28

$$\alpha(x_i) = \frac{\eta_{SS,i}}{\sum_{i=1}^{n} \eta_{SS,i}} \tag{3}$$

The relative sensitivities for 12 variables in case study 1 were calculated and are shown in Table 6.

As depicted in Fig. 3, based on the calculated values for relative sensitivity and a threshold value of 10% [as recommended in literature (Shen and Abbas 2013)] the following four variables were selected as the most influential variables: cohesion of fill material, the cohesion and friction angle of clayey sand and the elastic modulus of the saturated clayey gravel layers.

The general procedure for constructing all combinations of input soil variables inspired by Tonon et al. (2000) is

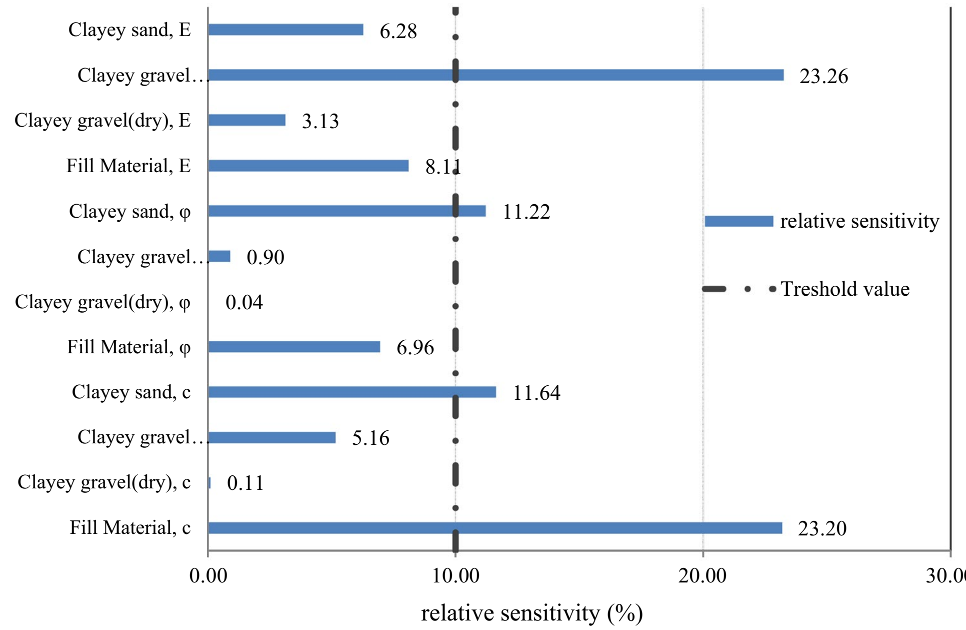

Fig. 3 Relative sensitivity of input variables for case study 1

as follows using the lower and upper bounds of the suggested ranges.

Let $x \in X$ be a vector of set value variables in which $x = (c_f, c_{sc}, \varphi_{sc}, E_{GC})$ and a random relation is defined on the Cartesian product $c_f \times c_{sc} \times \varphi_{sc} \times E_{GC}$. Since each basic variable was presented in terms of interval and there were lower and upper bounds, 2^4 combinations of basic variables were formed. Table 7 includes all of these possible combinations and their respective finite element model output.

The best fitted distribution function for the recorded values of system responses, shown in column 7 of Table 7, was determined using EasyFit and the reliability analysis results were depicted in the form of PDF and CDF curves as shown in Fig. 4. Table 8 shows the mean, standard deviation and the best distribution function fitted on the recorded values of system responses.

In order to check the consistency of the ESS method to reality, the reliability analysis result was checked against field measurements as graphically shown in Fig. 4. It was observed that the range of system response covered the value of the field measurement (19 mm).

The results of reliability analysis for other case studies are presented in the following sections.

4 Verification of the ESS Method

For verification purposes, the ESS-based results are compared with (I) field measurements and observations and (II) other reliability analysis results which were obtained using other theoretical reliability analysis methods (MC, PE and RS).

4.1 Verification Procedure Considering Case Study 1

In order to provide verification of the suggested method, the RS as a non-probabilistic method, the PE as a probabilistic method and the MC as a widely spread reliability analysis technique are utilized. For the sake of brevity, the details of reliability analysis procedure using the RS and PE and MC methods are not presented here, but their procedures are briefly discussed for case study 1.

4.1.1 Reliability Analysis of Case Study 1, Random Set Method Implementation

In order to implement the RS method, for each soil variable, according to the geotechnical reports and engineering judgment, two ranges with a weight of 0.5 for each were suggested. In order to consider the spatial variations in soil parameters, the primary values of the variables were modified slightly using a variance reduction technique. In this study, the method proposed by Schweiger and Peschl (2005) was applied. The modified upper and lower bounds of the suggested ranges, and the reference values for each soil variable, are summarized in Tables 9.

A random relation was defined on the Cartesian product $x = (c_f, c_{sc}, \varphi_{sc}, E_{GC})$. 2^4 combinations were generated considering two datasets defined for each of the most influential variables. Then, the lower and upper bounds of information sources were assigned to each variable. Total of 256 combinations of input data were formed in order to perform the

Table 7 Input values for $x = (c_f, c_{sc}, \varphi_{sc}, E_{GC})$ for deterministic finite element runs (case study 1)

Deterministic run number	Possible combinations	Fill material c (kPa)	SC-CL c (kPa)	SC-CL φ	GC-Sat E (MPa)	Model output for horizontal displacement of the excavation top point (mm)
1	LLLL[a]	6.75	38.5	28.67	74.75	43.13
2	ULLL	12.53	38.5	28.67	74.75	64.4
3	LULL	6.75	71.5	28.67	74.75	33.98
4	LLUL	6.75	38.5	34.34	74.75	37.12
5	LLLU[b]	6.75	38.5	28.67	138.83	20.17
6	UULL	12.53	71.5	28.67	74.75	55.6
7	ULUL	12.53	38.5	34.34	74.75	58.54
8	ULLU	12.53	38.5	28.67	138.83	40.86
9	LUUL	6.75	71.5	34.34	74.75	30.02
10	LULU	6.75	71.5	28.67	138.83	12.6
11	LLUU	6.75	38.5	34.34	138.83	15.35
12	UUUL	12.53	71.5	34.34	74.75	51.97
13	UULU	12.53	71.5	28.67	138.83	33.75
14	ULUU	12.53	38.5	34.34	138.83	36.28
15	LUUU	6.75	71.5	34.34	138.83	10.1
16	UUUU	12.53	71.5	34.34	138.83	31.34
Assigning the reference values to input variables		9.64	55	31.5	106.8	41.32

[a]Lower bound of set

[b]Upper bound set

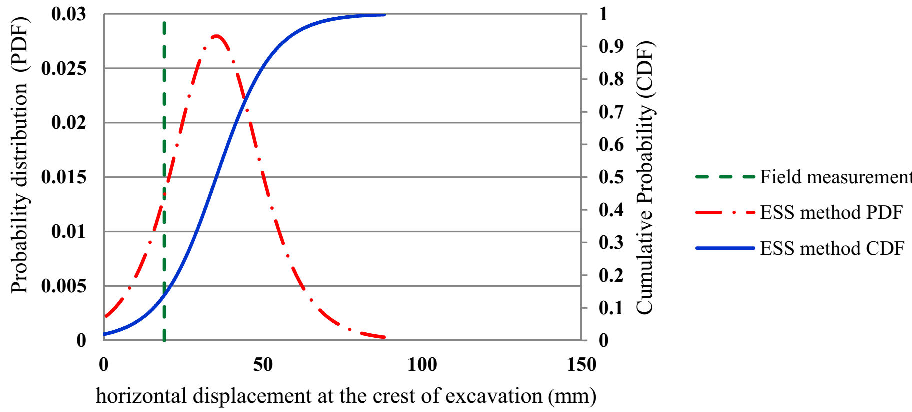

Fig. 4 Reliability analysis results compared to the field measurement for case study 1

Table 8 Statistical specification of reliability analysis results (case study 1)

System response	Mean (μ)	SD (σ)	Best fitted distribution
Horizontal displacement at the crest of excavation (mm)	35.95	16.31	Log-logistic

Table 9 The ranges of random sets and reference values for soil variables considering spatial variation (case study 1)

Layer		c (kN/m^2)		φ°		E (MN/m^2)	
		Range of sets	Ref	Range of sets	Ref	Range of sets	Ref
Fill Material	Set 1	6.93–13.21	9.65	29.29–33.29	31	26.43–46.43	35
	Set 2	6.07–11.79		28.71–32.71		23.57–43.57	
Clayey gravel(dry)	Set 1	72.86–102.86	85	38.57–43.29	40.36	83.57–103.57	95
	Set 2	67.14–97.14		37.43–42.71		86.43–106.43	
Clayey gravel (Saturated)	Set 1	62.86–92.86	75	31.29–35.29	33	92.86–126.43	106.79
	Set 2	57.14–87.14		30.71–34.71		87.14–123.57	
Clayey sand	Set 1	46.43–66.43	55	29.93–33.93	31	66.43–86.43	71.79
	Set 2	43.57–63.57		29.07–33.07		57.14–83.57	

required finite element analyses. Assuming that the input variables were stochastically independent (Tonon et al. 2000), the joint probability for the system response focal element was equal to 0.0625 (the product of the probability assignment 0.5 for each input focal element).

Finally, the reliability analysis results obtained by random set method were represented in the form of probability box (upper and lower bounds) for the horizontal displacement at the crest of excavation. A probability box (p-box) is a pair of cumulative probability distribution functions that represents the imprecise probability distribution of a random variable (Nasekhian and Schweiger 2011).

4.1.2 Reliability Analysis of Case Study 1, Applying Point Estimate Method

To implement the PE method, an approach suggested by Zhou and Nowak (1988) was utilized. The $2n^2+1$ (n is the number of basic variables) integration rule was employed, which is considered as an optimum compromise between accuracy and computational effort (Thurner 2000). The concept of PE method could be found in the literature (e.g., Rosenblueth (1975), Lind (1983), Harr (1989), Hong (1998), Zhou and Nowak (1988)) and is not discussed here in detail. The approach considered in this study is describes as follows.

A distribution function was assigned to each soil variable at the first step. According to the available data shown in Table 9, a uniform distribution was constructed whose left and right extreme values were the medians of left and right random set bounds, respectively. Then, typical distributions were fitted and an appropriate distribution was selected for further analysis. For instance, considering the cohesion of fill material, as shown in Fig. 5, the normal distribution was selected since it covers the whole range of random set values, and it is a commonly used distribution for cohesion of soil layers.

The governing distributions for other effective input variables were selected in a similar way and are summarized in Table 10.

According to the integration rule applied in $2n^2+1$ method, 33 combinations of input variables were generated

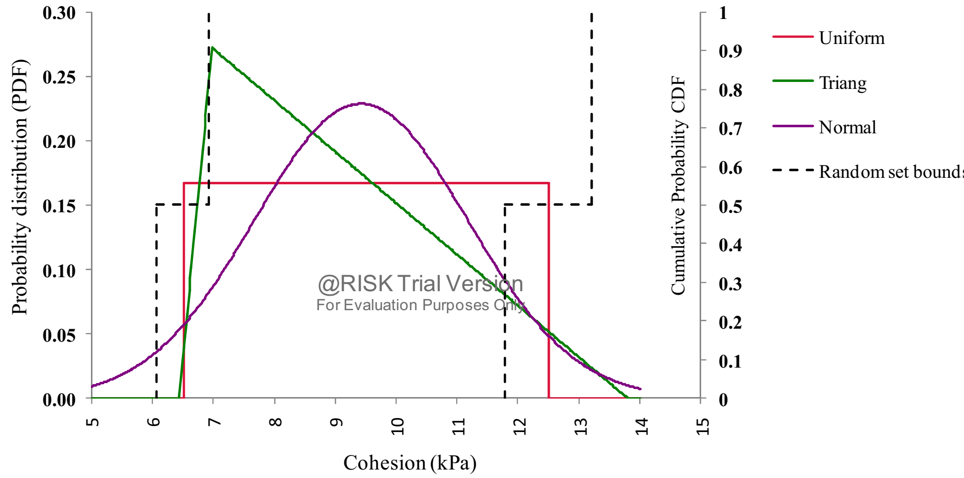

Fig. 5 Assumed distributions for cohesion of fill material

Table 10 The governing distribution for effective input variables in case study 1

Variable	Distribution function	Mean	Standard dev
Fill material, c(kPa)	Normal	9.42	1.75
Clayey sand, c(kPa)	Normal	55.24	5.85
Clayey sand, φ	Normal	31.44	1.18
Clayey gravel (Saturated), E(MPa)	Normal	107.52	10

and the relevant model outputs were recorded. Then, the mean and standard deviation of the results were calculated. In the last step, using @Risk software (2018), the best distribution function was fitted on the recorded values in order to illustrate the PE method reliability analysis result in the form of CDF curve.

4.1.3 Reliability Analysis of Case Study 1, Monte Carlo Simulation Implementation

The distribution functions as defined in the first step of PE method were assigned to each effective input variable. The @Risk software (2018) was utilized to perform MC simulation. Once the database was prepared, a multiple linear regression (MLR) based on the given sets of data was performed in order to relate the input variables to the model output. This MLR equation was used to generate 10^6 simulations. Finally, the MC-based reliability analysis results were obtained in the form of CDF curve.

4.1.4 Comparison and Discussion for Case Study 1

For verification purposes, the reliability analysis results from different theoretical methods and the relevant most likely values ranges were compared with the field measurement and the threshold value for the horizontal displacement at the crest of excavation. The most likely range is usually estimated by technical judgment, but in this study, the statistical concepts were also used to define the most likely values ranges.

For the RS method, the range of response values with a cumulative probability of 0.5 for the lower and upper bounds of reliability analysis result were considered as the most likely values zone. For other implemented methods, the following concepts were used to define the most likely values zone.

In probability theory, it is known that if a distribution is unimodal, the interval of $\{\mu - 1.8\sigma < x < \mu + 1.8\,\sigma\}$ represents the 86% confidence level independent of what distribution the system response has. Also, according to the *3σ* rule, the probability of x falling away from mean value μ by more than *3σ* is at most 5%. Pukelsheim (1994) based on Vysochanskij-Petunin inequality implies that: where X is a real random variable with unimodal distribution, mean μ, variance σ^2 and radius $r > 0$, then for any $r > 1.63\sigma$:

$$\Pr(|X - \mu \geq r|) \leq \frac{4}{9}\left(\frac{\sigma^2}{r^2}\right) \tag{4}$$

The aforementioned equation is proposed for distributions that are unimodal and have finite variance, whether they are non-normal or skewed distributions. These concepts elucidate the most likely and warning zones for the system response obtained by ESS as well as PE and MC. Hence, for these methods, the interval of $\{\mu - 1.8\sigma < x < \mu + 1.8\sigma\}$ was considered as the most likely values zone. The reliability analysis results consist of three zones: (a) most likely, (b) warning and (c) unlikely. These are quantified by the intervals: $\{\mu - 1.8\sigma < x < \mu + 1.8\sigma\}$, $\{\mu - 3\sigma < x < \mu - 1.8\sigma \;\&\; \mu + 1.8\sigma < x < \mu + 3\sigma$ and $\{\mu - 3.0\sigma < x \;\&\; x < \mu - 3.0\sigma\}$, respectively. The CDF curves and the range of most likely

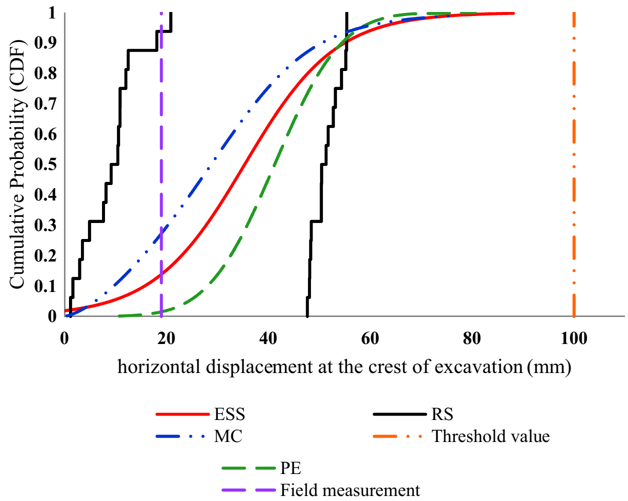

Fig. 6 Results of the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 1

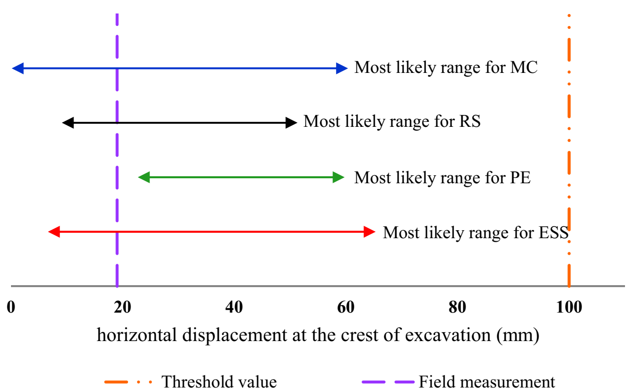

Fig. 7 The range of most likely values obtained by the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 1

values for the horizontal displacement at the crest of excavation obtained by different reliability analysis methods are shown in Figs. 6 and 7, respectively.

According to Figs. 6 and 7:

- A good agreement was found between the results of the ESS and other implemented methods.
- The field measurement value was within the most likely zones estimated by the RS, the MC and the ESS method.
- For PE, the field measurement value fell within the warning zone.
- The CDF curves for horizontal displacement of the excavation top point obtained through all of the aforementioned methods were considerably less than the threshold value (100 mm according to Table 2). Hence, it can be concluded that the implemented support system for case study 1 was designed conservatively.

The summary of the results obtained by different reliability analysis methods is presented in Table 11.

Unsatisfactory system performance occurs when the horizontal displacement at the crest of excavation exceeds the threshold value. The calculated probabilities of unsatisfactory system performances obtained by different methods are presented in Table 11. All of these values were less than the acceptable probability of excessive deformation (APED = 0.1). This conclusion is in agreement with the field observations where no crack was found around the deep excavation.

4.2 Comparison and discussion for case study 2

Figure 1a displays the location of case study 2, and Fig. 2b shows a cross section of the model. The soil profile at the location of wall 2 was the same as that investigated in case study 1. Table 3 shows the properties of the soil variables.

Table 11 Comparative table on the reliability analysis results (Case study 1)

Reliability analysis method	Field observation		Zone within field measurement fell	The range of most likely values for the horizontal displacement (mm)	Threshold value for horizontal displacement at the crest of excavation (mm)	Probability of unsatisfactory performance
	Observed cracks	Measured horizontal displacement (mm)				
RS	None	19	Most likely	(9.1–51.33)	100	Lb: 0 Ub: 0
PE			Warning	(22.71–59.76)		0
ESS			Most likely	(6.59–65.31)		0
MC			Most likely	(0.08–60.37)		0

Table 12 Statistical specification of reliability analysis results (case study 2)

System response	Mean (μ)	SD (σ)	Best fitted distribution
Horizontal displacement at the crest of excavation (mm)	75.04	21.08	Log-logistic

Sensitivity analysis was used to select the following five variables as the most influential ones: cohesion, friction angle and elastic modulus of clayey sand, cohesion and elastic modulus of saturated clayey gravel layers. A total of 2^5 combinations were generated to perform the required finite element runs. The horizontal displacements at the crest of excavation related to all possible combinations were recorded. Table 12 shows the mean, standard deviation and the best distribution function fitted on the recorded values of system response.

The CDF curves and the range of most likely values for the horizontal displacement at the crest of excavation, obtained by different reliability analysis methods, are shown in Figs. 8 and 9, respectively.

According to Figs. 8 and 9:

- A good agreement was found between the results of the ESS and other implemented methods.
- The field measurement value fell within the most likely zones estimated by all reliability analysis methods.
- Unlike the PE, the RS and the MC methods, the threshold value was within the most likely zone estimated by the ESS method.

The summary of the results obtained by different reliability analysis methods is presented in Table 13.

According to the last column of Table 13, the calculated probabilities of unsatisfactory system performance (considering the ESS method and the upper bound of the RS method) were larger than the acceptable probability of 0.10. These values were in line with the small cracks observed on the ground surface near the excavation.

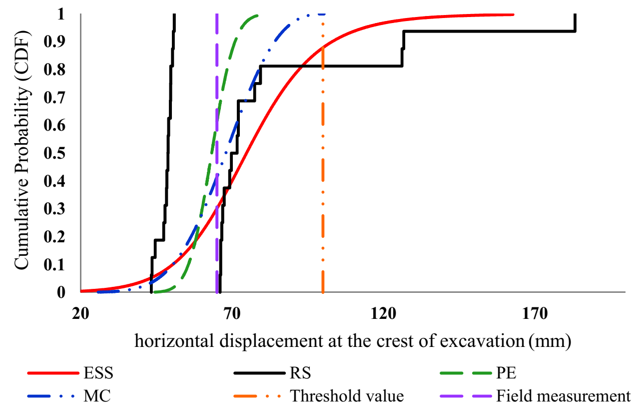

Fig. 8 Results of the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 2

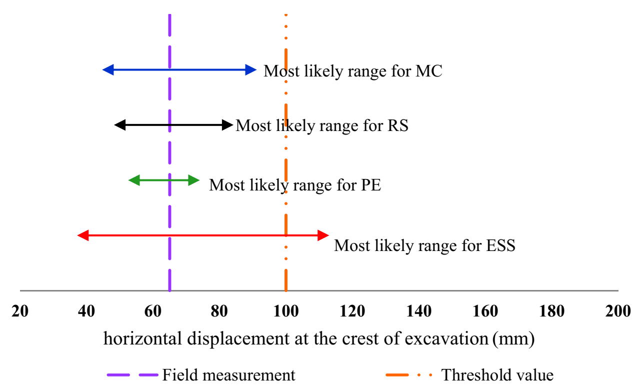

Fig. 9 The range of most likely values obtained by the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 2

Table 13 Comparative table on the reliability analysis results (Case study 2)

Reliability analysis method	Field observation			The range of most likely values for the horizontal displacement (mm)	Threshold value for horizontal displacement at the crest of excavation (mm)	Probability of unsatisfactory performance
	Observed cracks	Measured horizontal displacement (mm)	Zone within field measurement fell			
RS	Small	65	Most likely	(48.22–84.21)	100	Lb: 0 Ub: 0.2
PE			Most likely	(52.48–74.06)		0
ESS			Most likely	(37.1–112.98)		0.14
MC			Most likely	(44.64–91.22)		0

Table 14 Selected set for input soil variables (case study 3)

Variable no.	Parameter		Mean value	Selected set	
				Lower bound	Upper bound
1	c (kN/m^2)	Fill material	9.81	6.87	12.75
2		Clayey gravel	65	45.50	84.50
3	φ°	Fill material	29	26.39	31.61
4		Clayey gravel	35	31.85	38.15
5	E_{50}^{ref} (MN/m^2)[a]	Fill material	13	9.10	16.90
6		Clayey gravel	53.11	37.18	69.04

[a]Secant stiffness in standard drained triaxial test

Table 15 Statistical specification of reliability analysis results (case study 3)

System response	Mean (μ)	SD (σ)	Best fitted distribution
Horizontal displacement at the crest of excavation (mm)	119.37	49.17	Normal

4.3 Comparison and Discussion for Case Study 3

The excavation site, in an area of 8500 m^2, was adjacent to Vanak Park, Tehran. Figure 1b shows the project location, and Fig. 2c shows a cross section of the model. The hardening soil (HS) constitutive model was used to describe the soil behavior. The lower and upper bounds for each input soil variable were selected according to the expert judgment as shown in Table 14. The stiffness parameters of the HS model were obtained using famous correlations, and one of them is shown in the table of parameters.

Sensitivity analysis indicated that the following three variables were the most influential ones: cohesion, friction angle and stiffness of the clayey gravel layer. The model outputs related to all 2^3 possible combinations of input variables were recorded. Table 15 shows the mean, standard deviation and the best distribution function fitted on the recorded values of system response.

Inaccurate geotechnical site investigation reports, used during the deterministic design stage, caused the numerically calculated horizontal displacement to be less than the threshold value. This inaccuracy led to the proposing an inappropriate support system and the appearance of large cracks around the project location during excavation. The CDF curves and the range of most likely values for the horizontal displacement at the crest of excavation obtained by different reliability analysis methods are shown in Figs. 10 and 11, respectively.

According to Figs. 10 and 11:

- A good agreement was found between the results of all implemented methods.
- The only method for which the most likely values range covered the field measurement was the ESS method. Hence, for case study 3, the ESS could be suggested as

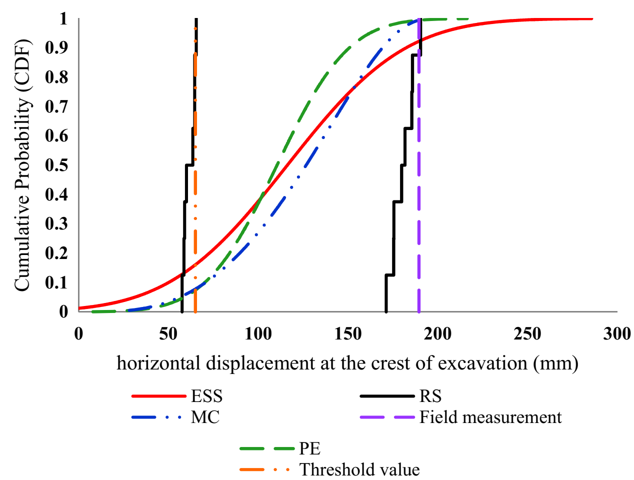

Fig. 10 Results of the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 3

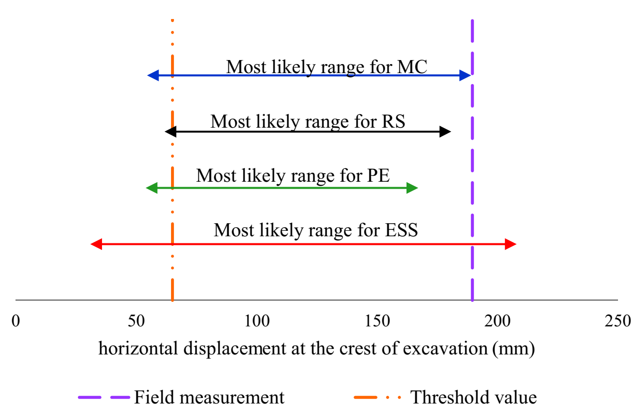

Fig. 11 The range of most likely values obtained by the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 3

the most appropriate method to predict the actual horizontal displacement at the crest of excavation.

The summary of the results obtained by different reliability analysis methods is presented in Table 16.

According to the last column of Table 16, the calculated probabilities of unsatisfactory system performance, estimated by the ESS method, were in good agreement with other applied methods. All of the calculated values were more than the acceptable probability of 0.10. This conclusion was in line with the large cracks observed on the ground surface near the excavation.

4.4 Comparison and Discussion for Case Study 4

Figure 1b displays the location of case study 2, and Fig. 2d shows a cross section of the system model. The HS constitutive model was used to describe the soil behavior. For each soil variable, the lower and upper bounds of the input soil variables were selected according to expert judgment as shown in Table 17.

Sensitivity analysis indicated that the following four variables were the most influential ones: cohesion of dry clayey gravel, stiffness of the dry/saturated clayey gravel, friction angle of the dry/saturated clayey gravel and stiffness of the stiff clay layers. The system responses related to all 2^4 possible combinations of input variables were recorded. Table 18 shows the mean, standard deviation and the best distribution function fitted on the recorded values of model output.

It should be noted that, similar to case study 3, inaccurate geotechnical site investigation led to proposing improper support system and the observation of large cracks around the deep excavation project.

Table 16 Comparative table on the reliability analyses results (Case study 3)

Reliability analysis method	Field observation			The range of most likely values for the horizontal displacement (mm)	Threshold value for horizontal displacement at the crest of excavation (mm)	Probability of unsatisfactory performance
	Observed cracks	Measured horizontal displacement (mm)	Zonewithin field measurement fell			
RS	Large	189.5	Warning	(61.7–180.73)	65	Lb: 0.2 Ub: 1
PE			Warning	(53.87–167.01)		0.93
ESS			Most likely	(30.87–207.87)		0.93
MC			Warning	(54.41–189.01)		0.85

Table 17 Selected set for input soil variables (case study 4)

Variable no.	Parameter		Mean value	Selected set	
				Lower bound	Upper bound
1	c (kN/m^2)	Fill material	9.81	6.867	12.75
2		Clayey gravel (dry)	53.09	37.16	69.02
3		Clayey gravel (saturated)	25	17.5	32.5
4		Stiff clay	94.04	65.83	122.25
5	φ°	Fill material	28.99	26.38	31.60
6		Clayey gravel (dry)	35	31.85	38.15
7		Clayey gravel (saturated)	35	31.85	38.15
8		Stiff clay	11	10.01	11.99
9	E_{50}^{ref}(MN/m^2)	Fill material	13	9.1	16.9
10		Clayey gravel (dry)	53.09	37.16	69.017
11		Clayey gravel (saturated)	53.09	37.16	69.02
12		Stiff clay	26.54	18.58	34.50

Table 18 Statistical specification of reliability analysis results (case study 4)

System response	Mean (μ)	SD (σ)	Best fitted distribution
Horizontal displacement at the crest of excavation (mm)	201.63	58.34	Gen.Extreme Value

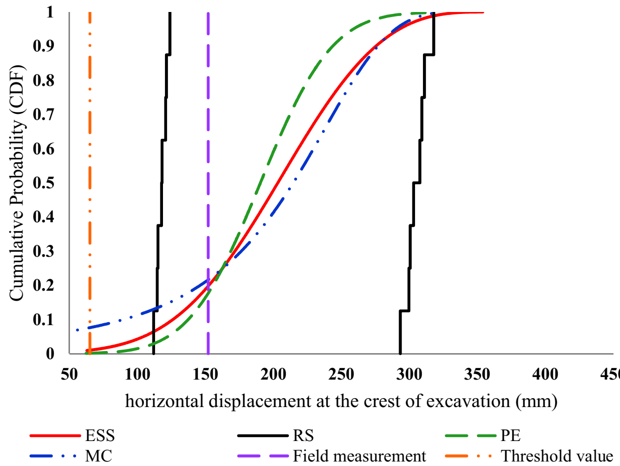

Fig. 12 Results of the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 4

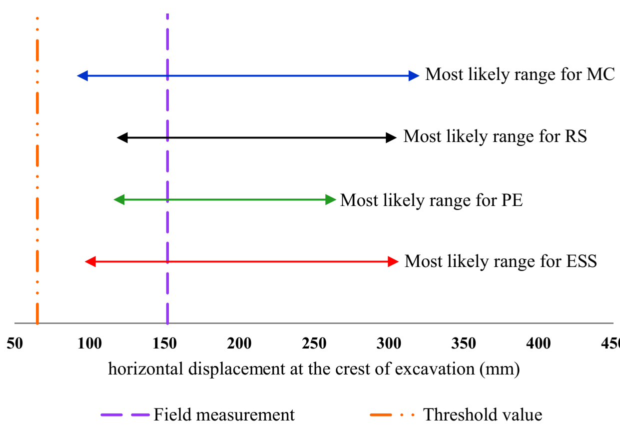

Fig. 13 The range of most likely values obtained by the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 4

The CDF curves and the range of most likely values for the horizontal displacement of deep excavation top point obtained by different reliability analysis methods are shown in Figs. 12 and 13, respectively.

According to Figs. 12 and 13:

- A good agreement was found between the results of all implemented methods.
- The field measurement value fell within the most likely zones estimated by all methods.

The summary of the results obtained by different reliability analysis methods is presented in Table 19.

According to the last column of Table 19, the calculated probabilities of unsatisfactory system performance, estimated by all of the methods, were more than the acceptable probability of 0.10. This conclusion was in line with the large cracks observed on the ground surface near the excavation.

4.5 Comparison and Discussion for Case Study 5

The excavation site, in an area of 16,000 m^2, was located in the northern half of the deep excavation project site (I), as illustrated in Fig. 1c. Figure 2e represents a cross section of the system model. The HS constitutive model was used to describe the soil behavior. The lower and upper bounds of the input sets were selected according to expert judgment as shown in Table 20.

Based on the sensitivity analysis results, the following four variables were selected as the most influential ones: cohesion, friction angle and stiffness of the saturated clayey gravel and stiffness of the dry clayey gravel layers. The model outputs related to all 2^4 possible combinations of input variables were recorded. Table 21 shows the mean, standard deviation and the best distribution function fitted on the recorded values of system response.

Table 19 Comparative table on the reliability analyses results (Case study 4)

Reliability analysis method	Field observation		Zone within field measurement fell	The range of most likely values for the horizontal displacement (mm)	Threshold value for horizontal displacement at the crest of excavation (mm)	Probability of unsatisfactory performance
	Observed cracks	Measured horizontal displacement (mm)				
RS	Large	152	Most likely	(117.92–305.32)	65	Lb: 1 Ub: 1
PE			Most likely	(115.83–264.95)		1
ESS			Most likely	(96.62–306.64)		1
MC			Most likely	(91.3–320.48)		0.97

Table 20 Selected set for input soil variables (case study 5)

Variable no.	Parameter		Mean value	Selected set	
				Lower bound	Upper bound
1	c (kN/m^2)	Fill material	6.5	4.55	8.45
2		Clayey gravel (dry)	58.44	40.91	75.97
3		Clayey gravel (saturated)	51.555	36.09	67.02
4	φ°	Fill material	33	30.03	35.97
5		Clayey gravel (dry)	35	31.85	38.15
6		Clayey gravel (saturated)	35	31.85	38.15
7	E_{50}^{ref}(MN/m^2)	Fill material	12.5	8.75	16.25
8		Clayey gravel (dry)	20	14.00	26.00
9		Clayey gravel (saturated)	78.445	54.91	101.98

Table 21 Statistical specification of reliability analysis results (case study 5)

System response	Mean (μ)	SD (σ)	Best fitted distribution
Horizontal displacement at the crest of excavation (mm)	80.49	24	Nakagami

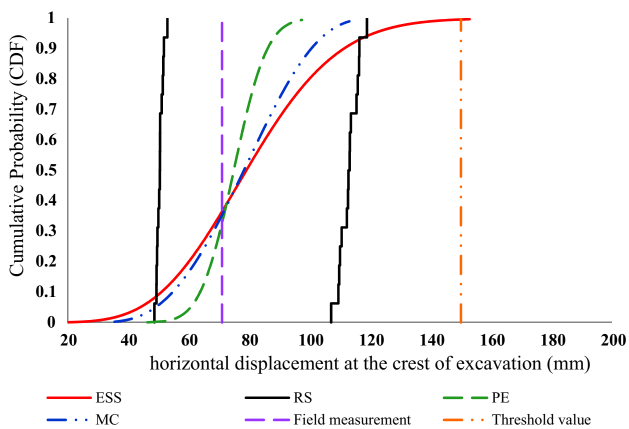

Fig. 14 Results of the ESS method compared to the RS, PE, MC overlaid with the field measurement and threshold values for case study 5

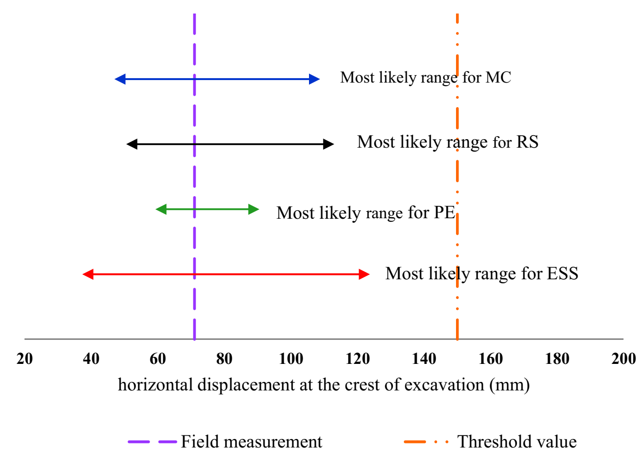

Fig. 15 The range of most likely values obtained by the ESS method compared to RS, PE, MC overlaid with the field measurement and threshold values for case study 5

The reliability analysis results are graphically illustrated in Fig. 14. This figure also shows the related field measurement and threshold values.

According to Figs. 14 and 15:

- A good agreement was found between the results of ESS and other implemented methods.
- The field measurement value was within the most likely zones estimated by all methods with approximately the same probability of occurrence.

The summary of the results obtained by different reliability analysis methods is presented in Table 22.

According to the last column of Table 22, the calculated probabilities of unsatisfactory system performance, estimated by all of the methods were equal to zero. This conclusion was in line with the monitoring reports which indicated that no issue was observed near the deep excavation project.

4.6 Overall Comparison and Discussion

Table 23 summarizes the reliability analysis results for all case studies obtained through four implemented reliability analysis methods.

Comparing the concepts and the results of different reliability analysis methods, the following issues were found to approve the applicability of the ESS method for real deep excavation projects:

Table 22 Comparative table on the reliability analyses results (Case study 5)

Reliability analysis method	Field observation		Zone within field measurement fell	The range of most likely values for the horizontal displacement (mm)	Threshold value for horizontal displacement at the crest of excavation (mm)	Probability of unsatisfactory performance
	Observed cracks	Measured horizontal displacement (mm)				
RS	None	152	Most likely	(50.46–113)	65	Lb: 0 Ub: 0
PE			Most likely	(59.24–90.52)		0
ESS			Most likely	(37.27–123.71)		0
MC			Most likely	(46.93–108.74)		0

Table 23 Summary of results comparing four methods

Case no.	Reliability analysis method	Number of FE runs	Field observation		Zone of results where the field measurement values fall	Mean value for reliability analysis results displacement (mm)	Threshold value for horizontal displacement at the crest of excavation (mm)	Probability of unsatisfactory performance
			Observed cracks	Measured horizontal displacement (mm)				
1	RS	256	None	19	Most likely	Lb:5.89 Ub:52.43	100	Lb: 0 Ub: 0
	PE	25			Warning	41.23		0
	ESS	16			Most likely	35.95		0
	MC	300			Most likely	29.73		0
2	RS	256	Small	65	Most likely	Lb:47.89 Ub:78.22	100	Lb: 0 Ub: 0.2
	PE	25			Most likely	63.27		0
	ESS	32			Most likely	75.04		0.14
	MC	300			Most likely	67.93		0
3	RS	64	Large	189.5	Warning	Lb:63.18 Ub:180.74	65	Lb: 0.2 Ub: 1
	PE	19			Warning	110.44		0.93
	ESS	8			Most likely	119.37		0.85
	MC	180			Warning	122.67		0.93
4	RS	64	Large	152	Most likely	Lb:117.92 Ub:305.32	65	Lb: 1 Ub: 1
	PE	19			Most likely	190.39		1
	ESS	16			Most likely	201.63		1
	MC	180			Most likely	205.89		0.97
5	RS	256	None	71	Most likely	Lb:49.46 Ub:113	150	Lb: 0 Ub: 0
	PE	25			Most likely	74.88		0
	ESS	16			Most likely	80.49		0
	MC	300			Most likely	77.84		0

- For all studied cases, the field measurement values fell within the range of most likely values estimated by the ESS method.
- The probabilities of excessive deformation obtained through the ESS method were in line with the observed cracks around the deep excavation projects for all cases.

- The number of required FEM runs for the ESS method was noticeably smaller than that of the RS and the MC methods.
- Considering column 7 of Table 23, the mean values obtained by the ESS method were close to the values obtained by the MC and PE methods and between the values of upper and lower bounds of the RS results. Hence, the reliability analysis results obtained through the ESS were in good agreement with other implemented theoretical methods.
- Unlike the ESS method, in order to implement the PE and MC, the best fitted distribution for each input soil variable has to be determined. In most of real deep excavation projects, the site investigation data are not thorough enough for this necessity. Hence, it is considered as a drawback for either MC or PE implementation. It also suggests the efficiency of ESS in reliability analysis of deep excavation projects.
- In geotechnical practice, the experts use all available data to define only one preferred range for each geotechnical variable. Also, in the ESS method, unlike the RS, only one set is defined for each input variable and no probability share is required to be assigned to the sets.

5 Selection of the Appropriate Range for Geotechnical Variables

In the ESS method, only one expected range was defined for each soil variable based on expert judgment. However, it should be noted that:

> The expert defines sets based on the available geotechnical data, experience and the engineering judgment.
> The expert could use the statistical knowledge in combination with the engineering judgment to select more appropriate lower and upper bounds for input variables.
> The mean value for each input variable was calculated equal to average of the proposed set by the expert.

No special distribution function was assumed to define input variables. although the available literature about common governing distribution functions and the values of coefficients of variations for different soil properties can be used to help the expert to suggest more proper ranges.

The normal and lognormal functions are the most common probability distribution functions assigned to geotechnical properties (Nadim 2007). The empirical rule in statistics states that for a normal distribution, 68% of data will fall within the first standard deviation, 95% within the first two standard deviations and 99.7% within the first three standard deviations of the distribution average. When a mean value (μ) is considered for a soil variable, the suggested values for the coefficient of variation (COV) can be used to calculate the standard deviation (σ) as follows:

$$\mathrm{COV} = \left(\frac{\sigma}{\mu}\right) \tag{5}$$

Table 24 summarizes the recommended COVs for different soil parameters.

It is of advantage to investigate how the statistical knowledge could be applied to select more reliable sets. Three different ranges [($\mu - 0.68\sigma$), ($\mu + 0.68\sigma$)], [($\mu - \sigma$), ($\mu + \sigma$)] and [(μ—2σ), ($\mu + 2\sigma$)] were assigned as the input sets for soil variables. The effect of considering each range on the results of the ESS method was investigated for all case studies. Finally, it was concluded that which of the above mentioned ranges would lead to more reliable estimates of the system response. The COVs for each soil variable were considered according to the standard values shown in Table 24. The summary of ESS-based reliability analysis results is represented in Table 25.

Table 25 shows that:

- For all case studies, the ranges of most likely values for system response, considering [($\mu - 2\sigma$), ($\mu + 2\sigma$)] as the input set, were quite broad compared to the results obtained through other selected sets.
- For all case studies, when [($\mu - \sigma$), ($\mu + \sigma$)] was considered as the input set, the field measurements fell within the range of most likely values obtained.

Table 24 Recommended ranges for COV

Soil property	Reported COV (%)	Standard COV (%)	Source
Cohesion (undrained clays)	20–50	30	Singh (1971) and Lumb (1974)
Cohesion (undrained sands)	25–30	30	Lumb (1974)
Friction angle (various soil types)	9	9	Lumb (1966)
Elastic modulus	2–42	30	Kennedy (1978) and Otte (1978)

Table 25 Summary of the ESS results considering different input ranges

Case no	Selected set	Percentage of statistical confidence to fall the data within selected set (%)	System response (horizontal displacement on top of excavation) Range of most likely values (mm)	Field measurement (mm)	Zone within field measurement value falls
1	$\mu \pm 0.68\sigma$	50	22.54–56	19	Warning
	$\mu \pm \sigma$	68	6.58–56.31		Most likely
	$\mu \pm 2\sigma$	95	− 17.67–123.16		Most likely
2	$\mu \pm 0.68\sigma$	50	45.91–90.34	65	Most likely
	$\mu \pm \sigma$	68	37.1–112.98		Most likely
	$\mu \pm 2\sigma$	95	0.45–241.87		Most likely
3	$\mu \pm 0.68\sigma$	50	53.57–166.13	189.5	Warning
	$\mu \pm \sigma$	68	30.87–207.87		Most likely
	$\mu \pm 2\sigma$	95	− 95.75–497.57		Most likely
4	$\mu \pm 0.68\sigma$	50	120.24–254.6	152	Most likely
	$\mu \pm \sigma$	68	96.62–306.64		Most likely
	$\mu \pm 2\sigma$	95	− 29.62–607.8		Most likely
5	$\mu \pm 0.68\sigma$	50	48.52–109.32	71	Most likely
	$\mu \pm \sigma$	68	37.27– 123.71		Most likely
	$\mu \pm 2\sigma$	95	− 26.65–274.67		Most likely

- For case studies 1 and 3, the field measurements fell within the ranges of warning values obtained when [($\mu - 0.68\sigma$), ($\mu + 0.68\sigma$)] was selected as the input set. This means that the calculated values of horizontal displacement at the crest of excavation were overestimated for case 1 and underestimated for case 3 compared to the field measurements.

It was concluded that considering the input sets with 95% of statistical confidence interval is not recommended, because it would result in a vast and inefficient range for system response. On the other side, considering the ranges with 50% of statistical confidence interval may cause underestimation of the values calculated for displacements and subsequently an inappropriate design for the support system. The reliability analysis results were in best consistency with the field measurement values, when [($\mu - \sigma$), ($\mu + \sigma$)] was utilized. These findings can be used to help experts with little knowledge in statistics to suggest more reliable input sets for basic soil variables.

6 Conclusions

A practical method called the expert selected set (ESS) method was proposed for rapid reliability analysis of deep excavation projects. The feasibility of the ESS in professional practice was evaluated for five monitored deep excavation case studies. In addition, the results of the ESS method were compared to the reliability analysis results obtained from three other theoretical methods (PE, MC and RS) along with the field measurements and observations. The study main conclusions are as below:

a) The ESS method is a simple and practical technique for reliability analysis of deep excavations in urban areas and could be easily implemented by experts in geotechnical projects.
b) The selected sets for input soil variables have significant effect on the reliability analysis results. Simple statistical knowledge could help the experts suggest more reliable ranges. According to the findings in Sect. 5, considering the [($\mu - \sigma$), ($\mu + \sigma$)] range as the input set results in an appropriate estimate for the horizontal displacement at the crest of excavation.
c) The monitoring reports confirmed the ESS-based results. For all cases, the field measurement values fell within the most-likely values zones. Also, in the cases with cracks observed around the project, the threshold values of horizontal displacements at the crest of excavations were within or are less than the most-likely values zones. The ESS method can be applied in the design stage to have a reliable estimate of the displacements that would occur in reality.
d) A good agreement is found between the results of the ESS and other implemented reliability analysis methods (RS, PE and MC), while the ESS was found to be more practical than other methods.

References

Ahmadabadi M, Poisel R (2015) Assessment of the application of point estimate methods in the probabilistic stability analysis of slopes. Comput Geotech 69:540–550

Arabaninezhad A, Fakher A (2016) Application of non-probabilistic random set method for reliability analysis of deep excavation. In: 4th international reliability engineering conference, Sahand University of Technology, Tabriz, Iran, 2016

Beer M, Zhang Y, Quek ST, Phoon KK (2013) Reliability analysis with scarce information: comparing alternative approaches in a geotechnical engineering context. Structural Safety. 41(Supplement C):1–10

Christian JT, Ladd CC, Baecher GB (1994) Reliability applied to slope stability analysis. Geotech Eng 120(12):2180–2207

Ghazian Arabi M, Fakher A (2016) Reliability analysis in Iran-Zamin excavation by application of random set finite element method. In: 4th international reliability engineering conference, Sahand University of Technology, Tabriz, Iran, 2016

Goswami S, Ghosh S, Chakraborty S (2016) Reliability analysis of structures by iterative improved response surface method. Structural Safety. 60(Supplement C):56–66

Griffiths DV, Fenton GA (2004) Probabilistic slope stability analysis by finite elements. J Geotech Geoenviron Eng 130(5):507–518

Griffiths DV, Huang J, Fenton GA (2009) Influence of spatial variability on slope reliability using 2-D random fields. J Geotech Geoenviron Eng 135(10):1367–1378

Harr ME (1989) Probabilistic estimates for multivariate analyses. Appl Math Model 13(5):313–318

Health and Safety Executive (2001) Reducing risks: protecting people

Hong HP (1998) An efficient point estimate method for probabilistic analysis. Reliab Eng Syst Saf 59:261–268

Jiang S-H, Li D-Q, Cao Z-J, Zhou C-B, Phoon K-K (2015) Efficient system reliability analysis of slope stability in spatially variable soils using Monte Carlo simulation. J Geotech Geoenviron Eng 141(2):04014096

Kaymaz I (2005) Application of kriging method to structural reliability problems. Struct Saf 27(2):133–151

Kendall DG (1974) foundations of a theory of random sets. In: Harding EF, Kendall DG (eds) Stochastic Geometry. Wiley, New York

Kennedy TW (1978) Practical use of the indirect tensile test for the characterisation of pavement materials. In: Australian Road Research Board conference proc, vol 9, no 3

Kyung Park J, Tanner Blackburn J, Gardoni P (2007) Reliability assessment of excavation systems considering both stability and serviceability performance. Georisk Assess Manag Risk Eng Syst Geohazards. 1(3):123–41

Lind NC (1983) Modelling uncertainty in discrete dynamical systems. Appl Math Model 7:146–152

Low BK (2005) Reliability-based design applied to retaining walls. Geotechnique 55(1):63–75

Lumb P (1966) The variability of natural soils. Can Geotech J 3(2):74–97

Lumb P (1974) Application of statistics in soil mechanics. In: Lee IK (ed) Soil mechanics: new horizons. Newnes-Butterworth, London, pp 44–112

Mahdiyar A, Hasanipanah M, Armaghani DJ, Gordan B, Abdullah A, Arab H et al (2017) A Monte Carlo technique in safety assessment of slope under seismic condition. Eng Comput 33(4):807–817

Marr WA, Hawkes M (2010) Displacement-based design for deep excavations. Earth Retent Conf 3:82–100

Momeni E, Poor Moosavian S, Fakher A (2017) Acceptable probability of excessive deformation for deep urban excavations. In: Proceeding of the 70th Canadian geotechnical conference, Ottawa, Ontario, Canada

Momeni E, Poor Moosavian SM, Mahdiyar A, Fakher A (2018) Evaluating random set technique for reliability analysis of deep urban excavation using Monte Carlo simulation. Comput Geotech 100:203–215

Muhanna RL, Zhang H, Mullen RL (2007) Interval finite elements as a basis for generalized models of uncertainty in engineering mechanics. Reliable Comput 13(2):173–194

Nadim F (2007) Tools and strategies for dealing with uncertainty in geotechnics. In: Griffiths DV, Fenton GA (eds) Probabilistic methods in geotechnical engineering. Springer, Vienna, pp 71–95

Nasekhian A, Schweiger HF (2011) Random set finite element method application to tunnelling. Int J Reliab Saf 5(3–4):299–319

Navy, U. S. 1982. Foundations and earth structures, NAVFAC design manual. DM-7.2.

Otte EA (2018) Structural design procedure for cement treated layers in pavements, Civil Engineering, University of Pretoria, Dissertation, 1978. palisade. @RISK, Risk Analysis and Simulation. 2018 [cited 2018; https://www.palisade.com/risk/default.asp]

Peschl GM(2004) Reliability analyses in geotechnics with the random set finite element method. Institute for Soil Mechanics and Foundation Engineering, Graz University of Technology, Dissertation

PSCG (2000) Specification for Excavation in Shanghai Metro Construction. Professional Standards Compilation Group, Shanghai

Pukelsheim F (1994) The three sigma rule. Am Stat 48(2):88–91

Pula W, Bauer J (2007) Application of the response surface method. In: Griffiths DV, Fenton GA (eds) Probabilistic methods in geotechnical engineering. Springer, Vienna, pp 147–168

Rosenblueth E (1975) Point estimates for probability moments. Proc Natl Acad Sci USA 72(10):3812–3814

Sabatini PJ, Pass DG, Bachus RC (1999) Geotechnical Engineering Circular No. 4, Ground anchors and anchored systems, FHWA Publication No. FHWA-IF-99-015, Technical Manual, pp 281.

Santamarina J, Altschaeffl A, Chameau J (1992) Reliability of slopes: incorporating qualitative information (abridgment). Transport Res Rec; 19921343

Schittkowski K (2008) EASY-FIT user guide. Department of Mathematics, University of Bayreuth, Germany

Schweiger HF, Peschl GM (2005) Reliability analysis in geotechnics with the random set finite element method. Comput Geotech 32(6):422–435

Schweiger HF, Thurner R (2007) Basic concepts and applications of point estimate methods in geotechnical engineering. In: Griffiths DV, Fenton GA (eds) Probabilistic methods in geotechnical engineering. Springer, Vienna, pp 97–112

Shen H, Abbas SM (2013) Rock slope reliability analysis based on distinct element method and random set theory. Int J Rock Mech Min Sci 61:15–22

Singh A (1971) How reliable is the factor of safety in foundation engineering?. In: First international conference on applications of statistics and probability in soil and structural engineering, Hong Kong, Hong Kong University Press, pp 389–424

Smith GN. A suggested method of reliability analysis for earth retaining structures; 1986.

Suchomel R, Mašın D (2010) Comparison of different probabilistic methods for predicting stability of a slope in spatially variable c–φ soil. Comput Geotech 37(1):132–40

Thurner R (2001) Probabilistische Untersuchungen in der Geotechnik mittels deterministischer Finite Elemente-Methode. In: Riedmüller G, Schubert W, Semprich S (eds) Gruppe Geotechnik Graz, Vol.11. 2001, Graz: Technische Universität Graz, Gruppe Geotechnik Graz, Institut für Bodenmechnaik und Grundbau

Tonon F, Bernardini A, Mammino A (2000) Determination of parameters range in rock engineering by means of Random Set Theory. Reliab Eng Syst Saf 70(3):241–261

USEPA T (1999) Total risk integrated methodology status report. US Environmental Protection Agency Office of Air Quality Planning and Standards: Research Triangle Park

Uzielli M, Lacasse S, Nadim F, Phoon K-K (2006) Soil variability analysis for geotechnical practice. In: 2nd International workshop on characterisation and engineering properties of natural soils, vol 3, pp 1653–752

Zhang H, Mullen RL, Muhanna RL (2010) Interval Monte Carlo methods for structural reliability. Struct Saf 32(3):183–190

Zhang J, Zhang LM, Tang WH (2009) Bayesian framework for characterizing geotechnical model uncertainty. J Geotech Geoenviron Eng 135(7):932–940

Zhou J, Nowak AS (1988) Integration formulas to evaluate functions of random variables. Struct Saf 5(4):267–284