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RESEARCH PAPER

Predicting Discharge Coefficient of Rectangular Broad-Crested Gabion Weir Using M5 Tree Model

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Abstract Currently made alternative structures from loose stones, gabion weirs are preferred with respect to solid concrete weirs formerly used in the past. By being more stable and flexible, gabion weirs have great advantage over their rigid (impervious) counterparts. The aim of this study was to investigate the overflow and through flow in rectangular broad-crested gabion weirs in order to evaluate the discharge coefficient C_d. Eight physical models of broad-crested gabion weirs with four different porosities were made. The results of the experiments revealed that C_d tended to be 20% less in submerged flow than in free flow. In addition, the average values of C_d in both free and submerged flow were 0.66 and 0.53, respectively. Though increasing the gabion porosity led to an increase in C_d, this amount became less and less with higher discharge values. M5 tree model as a sub-technique of data mining used to model C_d values is capable of constructing tree-based piecewise linear equations for continuous datasets. The results showed that M5 tree model presents 12 linear equations for both free and submerged flows with R and RMSE of 0.95 and 0.036, respectively.

Keywords Broad-crested weir · Discharge coefficient · Gabion · M5 tree model · Porosity

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Abbreviations

B	Weir width
d_{50}	Mean stone size used in gabion construction
Fr	Froude number $= \frac{q}{\sqrt{g}H_1^{1.5}}$
G	Acceleration due to gravity
H_1	Water depth in upstream of the weir measured from weir crest
H_2	Water depth in downstream of the weir measured from weir crest
L	Weir length
n	Porosity of gabion materials
P	Weir height
Q	Discharge
q	Discharge per unit width
Re	Reynolds number
S_r	Submergence ration $= H_2/H_1$
ρ	Fluid density
μ	Dynamic viscosity of the fluid (water)

1 Introduction

In general the conventional weirs typically consist of an impermeable body made from concrete; its primary function is raising the water level and thus regulating the flow efficiently. However, the very nature of its construction acts as a barrier, preventing the longitudinal movement of aquatic flow along with disrupting the transportation of physical and chemical substances in water, eventually leading to an unsavory impact on the water environment. Gabion weirs as a building material have numerous advantages over the more conventional ones, to name a few: the movement of each individual stone comprising the weir is not the issue of concern. Additionally, the design of gabion weirs can be adapted to particular situations such as flash flood mitigation (Mohamed 2010).

Broad-crested weirs are typical hydraulic structures offering the flow control and measurements at different flow depths. The flow characteristics in broad-crested weirs having different cross-sectional throats have been of great interest to many investigators (Hager and Schwalt 1994; Gogos et al. 2006; Salmasi et al. 2011, 2013). Recent studies are mainly focused on hydraulic behavior, flow conditions and the discharge coefficient in various weir types (Gonzalez and Chanson 2007). A set of laboratory experiments was conducted to investigate the effect of upstream corner rounding in broad-crested weirs (Ramamurthy et al. 1998). Similarly, experiments were conducted to investigate the flow characteristics of broad-crested weirs with a sharp upstream corner (Hager and Schwalt 1994). Based on the research study of Sarker and Rhodes (2004), the rectangular broad-crested weir with measurements of free-surface profile over a laboratory scale was performed and compared with numerical simulation. Results indicated that for a given value of flow rate (or discharge), the upstream water depth was suitably predicted compared with the rapidly varied flow over the crest. A stationary wave profile in the supercritical flow downstream was also observed. Gonzalez and Chanson (2007) conducted experiments for a nearly full-scaled broad-crested weir. A comprehensive analysis of velocity/pressure measurements was implemented for various configurations. Study of energy dissipation over stepped gabion spillways with low heights was carried out by Salmasi et al. (2012). Results showed that with higher discharges, energy dissipation was greater in previous (gabion) spillways than those with impermeable horizontal or vertical faces. The results of the experiment on gabion weir by Mohammad (2010) were compared with those of solid weirs having the same dimension and showed a large deviation when the solid weirs equation was applied to gabion weirs (permeable weirs). Kells (1993, 1994) studied spatially varied flow over the rock-filled embankments for two flow conditions. The first condition was for partial overtopping of the embankment, while the latter involved a complete overtopping. Michioku et al. (2005) examined the hydrodynamics of a rubble-mound weir both theoretically and experimentally. In another study, Michioku et al. (2007) investigated the flow field around rubble-mound weirs and groins experimentally.

Within the last decade, several studies reported the use of M5 model tree, the tree-based regression approach for water resource applications. Pal and Deswal (2009) used M5 model tree to simulate daily reference evapotranspiration using climatic data of Davis station and found it performed well in comparison to empirical relations. Londhe and Dixit (2011) applied M5 model tree to forecast the stream flow one day in advance at two stations, one in Narmada river basin and the other in Krishna river basin in India. Pal et al. (2012) applied M5 model tree for pier scour prediction using field dataset. Comparison of results with four predictive equations suggests an improved performance by M5 model tree in predicting the pier scour depth with dimensioned data and found it performing equally well to a back-propagation neural network. Ditthakit and Chinnarasri (2012) presented the development of new pan coefficient (K_p) equations for Class A pan along with Colorado sunken pan under green and dry fetch conditions by using M5 model tree. Sattari et al. (2013) compared the capabilities of M5 model tree and support vector machine (SVM) in predicting daily stream flows in Sohu River, located within the municipal borders of Ankara, Turkey. Also, Sattari et al. (2013) applied M5 tree model and artificial neural networks for modeling monthly reference evapotranspiration (ET_0) in Ankara.

The aim of this study is to determine the effect of various hydraulic and geometric parameters of rectangular broad-crested gabion weirs on discharge coefficient (C_d) by experimental tests and modeling the C_d using M5 tree model. In literature review, the authors did not find application of M5 model in prediction of C_d. This paper also considers both free- and submerged-flow conditions with eight tested physical models. These models differ in porosity, heights and lengths. Analysis of variance (ANOVA) was carried out to determine the importance of each dimensionless parameter in this study too.

2 Materials and Methods

2.1 Experimental Setup

Figure 1 represents the typical flow characteristics above the broad-crested weir. As shown in Fig. 1, H_1 is maximum water level above the weir (m), and P and L are weir height and crest length (m), respectively. In Fig. 1, the discharge above the weir is evaluated from the following expression:

$$Q = \frac{2}{3} C_d B \sqrt{2g} H_1^{1.5} \quad (1)$$

where Q is discharge ($m^3\ s^{-1}$); C_d is discharge coefficient; B is weir width (m); g is acceleration due to gravity ($m\ s^{-2}$); and H_1 is water depth over the weir crest (m). Usually Eq. (1) can be used in both free and submerged-flow conditions; hence, clearly in submerged-flow condition, C_d will act as a function of H_2 (downstream water level) too.

Experimental measurements indicate that the C_d in solid weir is a function of the weir height (P), length of weir crest (L), unit discharge ($q = Q/B$), upstream corner shape

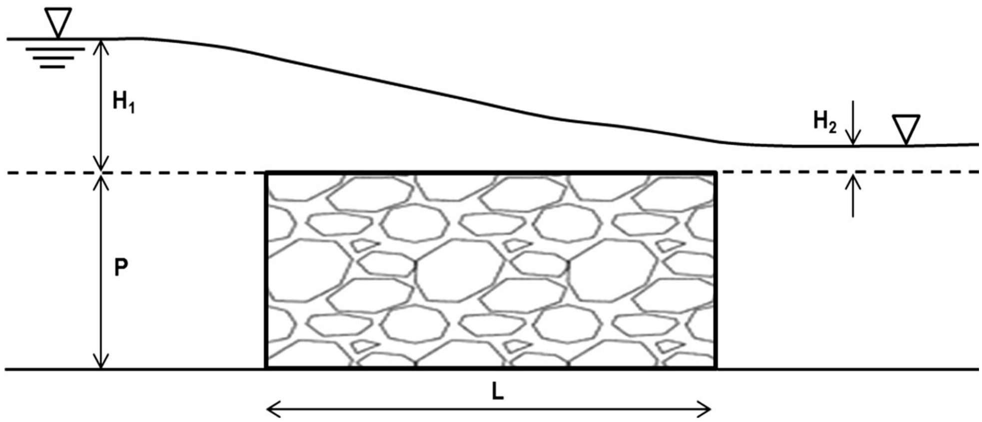

Fig. 1 Flow characteristics through a broad-crested gabion weir

and upstream/downstream water depth (H_1, H_2). As the nature of gabion weir construction, C_d will be a function of stone porosity too.

Experiments on gabion broad-crested weir test runs were conducted in the Hydraulic Laboratory of Water Engineering Department of Tabriz University. The test runs were installed in a flume with 0.25 m width, 12 m length and 0.6 m height. Table 1 contains geometrical characteristics of the physical models corresponding to gabion rectangular broad-crested weirs. In Table 1, model type refers to weir height (P) and gabion porosity. For example, G15.41 refers to weir height ($P = 15$ cm) and porosity ($n = 41\%$). In addition, d_{50} refers to mean diameter of stones filled in the gabion basket. Figure 2 illustrates the gabion broad-crested weir, after installation in laboratory flume. To measure the porosity of gabions, at first total volume of gabion basket was determined with its dimension. Then based on the relation $n = V_{\text{Void}}/V_{\text{Total}}$, void volume was calculated by entering gabion basket into a water container and measuring the replaced water volume in it.

In the present study, both free-flow and submerged-flow conditions were tested. The flow condition (either free or submerged) was monitored by discharge rate at the entrance and at the end of the laboratory flume using a gate. Discharge was measured by a calibrated sharp triangle weir (of 53° angle) installed at the downstream of the flume. Discharge water was supplied by a pump (maximum value 50 l/s). Discharge ranged from 7 to 50 l/s with the accuracy level of ±0.9 l/s. Upstream/downstream water levels were measured using a point gauge within ±0.1 mm accuracy. All measurements were made along the centerline of the flume.

Table 1 Geometrical characteristic of physical models of gabion broad-crested weirs

Model type	P (cm)	L (cm)	n (%)	d_{50} (cm)
G15.39	15	15	39	0.5
G15.41	15	15	41	1
G15.45	15	15	45	1.55
G15.50	15	15	50	3
G30.39	30	30	39	0.5
G30.41	30	30	41	1
G30.45	30	30	45	1.55
G30.50	30	30	50	3

Total set of 195 test runs was carried out in the gabion broad-crested weirs (95 tests for free flow and 100 tests for submerged flow) with four different porosities ($n = 39$, 41, 45 and 50%), two different weir heights and weir length (P, $L = 15$, 30 cm) as well as varying discharge rates. It should be mentioned that the total number of experiments was 250 including both the solid (traditional) broad-crested weir and gabion broad-crested weirs, but in this study, only gabion broad-crested weir data were used for estimating the discharge coefficient. The selection of gabion broad-crested weir dimensions was based on laboratory flume facilities.

In applying the M5 tree model technique, 80% of all data points were separated for training and 20% for testing the validation of the proposed model.

2.2 Theory

Basically, the discharge coefficient (C_d) depends on hydraulic and geometric variables expressed as:

$$f(q, H_1, H_2, P, L, d_{50}, n, \rho, g, \mu) = 0 \quad (2)$$

where q is discharge per unit width; H_1 and H_2 are water depths in upstream and downstream of the gabion weir, respectively; P is weir height; L is length of the weir; d_{50} is mean stone size used in gabion construction; n is porosity of gabion materials; ρ is fluid density; g is gravity of acceleration; and μ is dynamic viscosity of the fluid (water in this study).

Flow rate per width (q) was calculated by ($q = Q/B$), where Q is total discharge and B is the weir width equal to 25 cm in this experimental study. Using the Buckingham Π theorem, C_d can be expressed as:

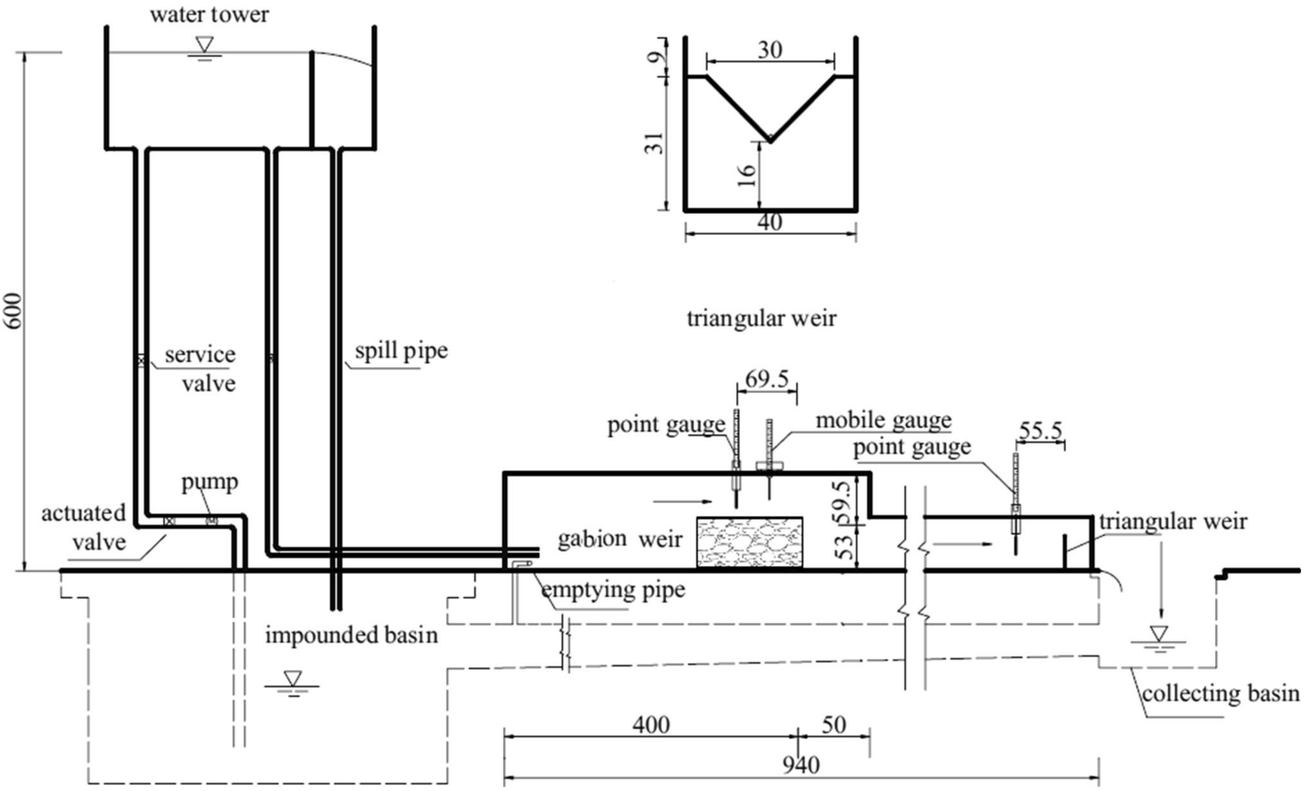

Fig. 2 Experimental setup (dimensions in centimeters)

$$\frac{q}{\sqrt{g}H_1^{1.5}} = f_1\left(\frac{\rho q}{\mu}, \frac{H_1}{L}, \frac{d_{50}}{P}, n, \frac{H_2}{H_1}\right) \quad (3)$$

Equation (3) is rewritten as Eq. (4):

$$C_\mathrm{d} = f_1(Re, H_1/L, d_{50}/P, n, S_\mathrm{r}) \quad (4)$$

where $Re = \rho q/\mu$ is Reynolds number used by Mohammad (Mohamed 2010). The fifth parameter on the right side of Eq. (4) is only relevant at the submerged-flow condition in weir and is known as submergence ratio. In free-flow condition, if $H_2 = 0$, S_r will be zero.

In the following section, the dimensionless groups in Eq. (4) will be correlated to give explicit equations for computing the C_d over the gabion weir at the free- and submerged-flow regimes.

The M5 model tree algorithm is the novel data mining technique which is used as a classifier of decisions trees family. An M5 model tree creates the form of a decision tree with linear regression functions instead of terminal class values at its leaves. Model trees generalize the concepts of regression trees, which have constant values at their leaves (Witten and Frank 2005). Model trees have several advantages, making them a suitable simple regression method for performance analysis. Thus, they are analogous to piecewise linear functions. M5 model tree is a binary decision tree having linear regression function at the terminal (leaf) nodes, which can predict continuous numerical attributes (Quinlan 1992). Tree-based models are constructed by a divide-and-conquer method. A model tree generation requires two different stages. The first stage involves using a splitting criterion to create a decision tree. The splitting criterion for the M5 model tree algorithm is based on treating the standard deviation of the class values that reach a node as a measure of the error at that node and calculates the expected reduction in this error as a result of testing each attribute at that node. The formula to compute the standard deviation reduction (SDR) is:

$$\mathrm{SDR} = \mathrm{sd}(T) - \sum \frac{|T_i|}{|T|}\mathrm{sd}(T_i) \quad (5)$$

where T represents a set of examples that reaches the node; T_i represents the subset of examples that have the ith outcome of the potential set; and sd represents the standard deviation. Due to the splitting process, the data in child nodes have less standard deviation compared to parent node and thus are more pure. After examining all the possible splits, M5 tends to choose the one that maximizes the expected error reduction. This division often produces a large tree-like structure which may cause overfitting. To solve this problem, the tree must be pruned back, for example, by replacing a sub-tree with a leaf. Thus, second stage in the design of model tree involves pruning the overgrown tree and replacing the sub-trees with linear regression functions. Pruning process occurs if the estimated error for the linear regression functions at the root of a sub-tree is smaller or equal to the expected error for the sub-tree. The technique of generating the model tree splits the parameter space into areas (subspaces) and builds in each of them a linear regression model. For further details of M5 model tree, readers are referred to Quinlan (1992).

3 Results and Discussion

The values of C_d for all of the eight models (distinguished with free and submerged-flow conditions) are plotted as a function of *Re*. The results are shown in Fig. 3.

Based on Fig. 3, in free-flow condition, C_d is higher than in the submerged-flow condition. With increase in *Re*, value of C_d decreases slightly; thus, C_d is not significantly dependent on *Re*. The scatters of data points illustrate the various value of gabion porosity (n) versus various weir geometries (P and L).

Figure 4 shows the values of C_d versus H_1/L in free-flow condition with different values of stone size per weir height (d_{50}/P) along with stone porosity (n). According to Fig. 4, the gabion weir with high porosity ($n = 0.5$) contributes to higher values of C_d. Gabion with $n = 0.39$ and has the lowest C_d as illustrated. With increase in H_1/L, values of C_d in all models converge to 0.66. This shows that value of C_d is fixed with increase in H_1/L greater that about 0.9 ($H_1/L > 0.9$).

According to Fig. 4, C_d increases with H_1/L in most cases except for $d_{50}/P = 0.11$ and $n = 0.45$. This increase is slight, but it needs more experimental tests in future studies. Average value for C_d in free flow is 0.66.

The values of C_d versus H_1/L for submerged-flow condition are shown in Fig. 5. As mentioned earlier, the gabion weir with high porosity ($n = 0.5$) has higher values of C_d. Gabion with $n = 0.39$ has the lowest C_d, and average value of C_d in submerged flow is equal to 0.53 which is approximately 20% lower than C_d in free-flow.

One of the aims of this study is to explore the potential of the M5 model tree for the modeling of discharge coefficient (C_d) in both free and submerged-flow conditions using laboratory datasets. Several models were created utilizing various input combinations, whereas modeled C_d

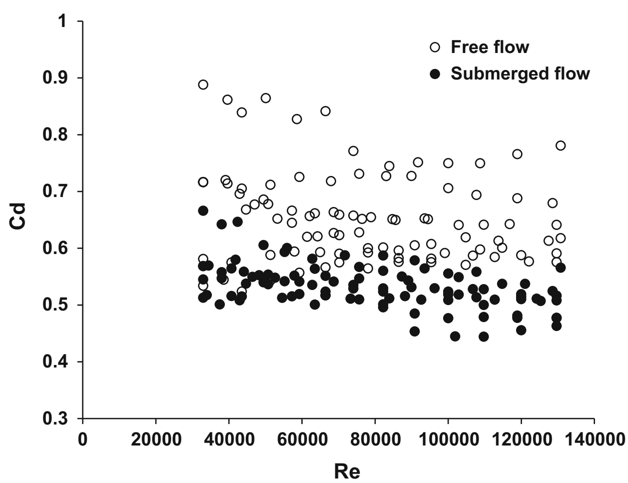

Fig. 3 C_d versus *Re* for free and submerged-flow conditions

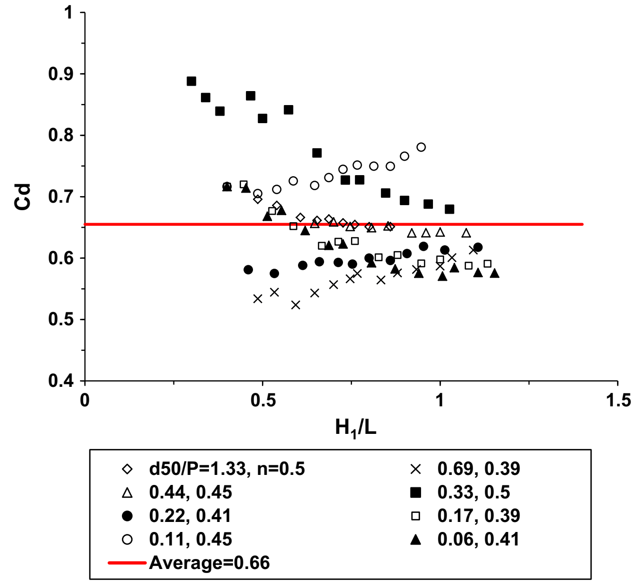

Fig. 4 C_d versus H_1/L for varying d_{50}/P and n values in free-flow condition

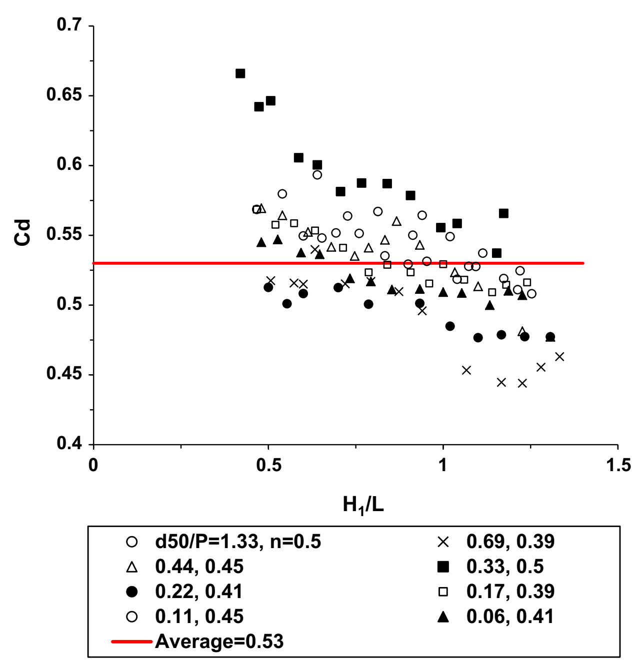

Fig. 5 C_d versus H_1/L for varying d_{50}/P and n values in submerged-flow condition

Table 2 M5 tree models output values for both free and submerged-flow condition to predict C_d

No.	Inputs	R	RMSE	Number of linear models from M5
1	$\boldsymbol{H_1/L, n, d_{50}/P, S_r}$	**0.95**	**0.036**	**12**
2	$H_1/L, Re, n, d_{50}/P, S_r$	0.94	0.039	14
3	$H_1/L, Re, n, S_r$	0.89	0.049	12
4	$H_1/L, n, S_r$	0.87	0.051	6
5	$H_1/L, Re, S_r$	0.85	0.057	12
6	Re, n, Sr	0.85	0.055	4
7	$H_1/L, Re, d_{50}/P, S_r$	0.84	0.058	16
8	$Re, d_{50}/P, S_r$	0.84	0.058	25
9	$H_1/L, S_r$	0.72	0.070	2
10	$H_1/L, d_{50}/P, S_r$	0.68	0.074	20

Bold values refer to an alternative with maximum R and minimum RMSE

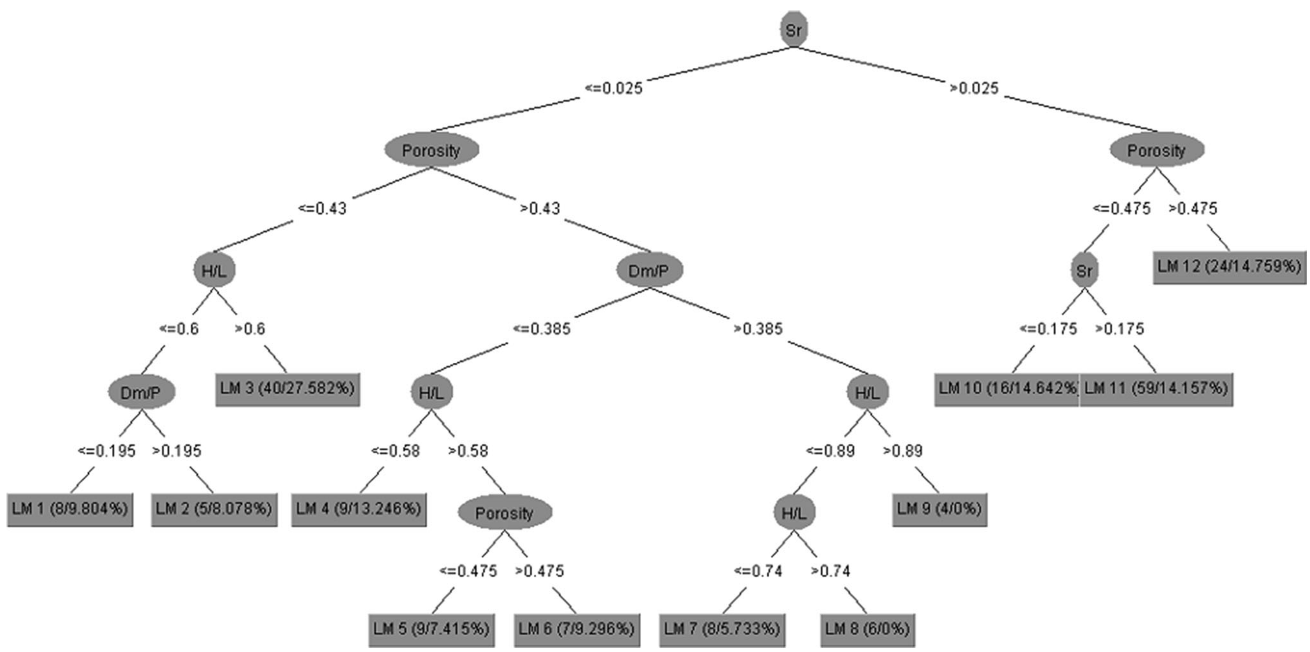

Fig. 6 Tree visualize for the best model (no. 1 in Table 2)

was considered as the output for all models. The performance evaluation process included two statistical terms, i.e., correlation coefficient (R) and the root mean square error (RMSE) and a number of generated linear equations for datasets by selected model. The selection of various dimensionless hydraulics parameters as M5 tree model inputs was determined by trial and error method using RMSE as the main evaluating criteria. A large number of trials were carried out using different combinations of parameters to obtain the optimal performance of M5 tree model. Table 2 contains the results for ten different selected models.

The results in Table 2 indicate that the model no. 1 including four dependent parameters, i.e., H_1/L, n, d_{50}/P, S_r, has the best modeling efficiency performance with $R = 0.95$ and RMSE $= 0.036$.

The total set of 12 linear equations is generated from M5 model tree for this choice. In such groups of studies the fewer number of linear equations is preferred. Even though the model no. 9 generated two linear equations, it is an undesirable scenario regarding the low values of R and RMSE. As the comparison between number one and number two models shows that R and RMSE values are roughly similar, it could be gathered that that elimination of Re has no significant effect on C_d prediction. Independency of C_d regarding the Re values was illustrated in Fig. 3, previously.

Figure 6 illustrates that the tree model generated the best scenario (no. 1). The major advantage of M5 model tree approach is the availability of simple linear relations in predicting C_d. Figure 6 explains the application of linear models for any kind of hydraulic parameters. For example, if S_r is greater than 0.025 and porosity is greater than 0.475,

Fig. 7 Linear models provided by M5 model tree approach (model no. 1)

LM num: 1 Cd = -0.1857 * H/L - 0.03 * Sr - 0.7856 * Porosity - 0.1401 * Dm/P + 1.0878

LM num: 2 Cd = -0.1563 * H/L - 0.03 * Sr - 0.7089 * Porosity - 0.1497 * Dm/P + 1.0327

LM num: 3 Cd = -0.0635 * H/L - 0.03 * Sr + 0.1485 * Porosity - 0.0705 * Dm/P + 0.6092

LM num: 4 Cd = -0.1568 * H/L - 0.03 * Sr + 1.3856 * Porosity - 0.059 * Dm/P + 0.2065

LM num: 5 Cd = -0.1147 * H/L - 0.03 * Sr + 0.8656 * Porosity - 0.059 * Dm/P + 0.4203

LM num: 6 Cd = -0.1412 * H/L - 0.03 * Sr + 0.858 * Porosity - 0.059 * Dm/P + 0.4431

LM num: 7 Cd = -0.1435 * H/L - 0.03 * Sr + 0.8988 * Porosity - 0.0649 * Dm/P + 0.3981

LM num: 8 Cd = -0.1341 * H/L - 0.03 * Sr + 0.8988 * Porosity - 0.0649 * Dm/P + 0.3912

LM num: 9 Cd = -0.1275 * H/L - 0.03 * Sr + 0.8988 * Porosity - 0.0649 * Dm/P + 0.3856

LM num: 10 Cd = -0.0752 * H/L + 0.0061 * Sr + 0.6207 * Porosity - 0.0197 * Dm/P + 0.3244

LM num: 11 Cd = -0.057 * H/L + 0.0202 * Sr + 0.5068 * Porosity - 0.0148 * Dm/P + 0.3607

LM num: 12 Cd = -0.1101 * H/L - 0.0051 * Sr + 0.3587 * Porosity - 0.0287 * Dm/P + 0.5137

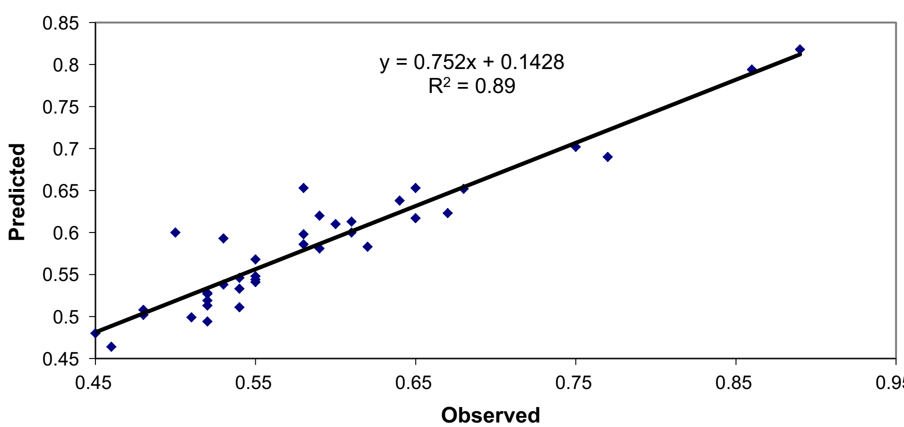

Fig. 8 Scatter plot by M5 model tree approach (model no. 1)

C_d value must be computed with LM num: 12; $C_d = -0.1101 * H/L - 0.0051 * S_r + 0.3587 *$ Porosity $- 0.0287 * D_m/P + 0.5137$.

Figure 7 presents 12 linear formula provided by M5 tree approach based on model no. 1 in Table 2.

Figure 8 illustrates the scatter plot between observed and predicted values of C_d using M5 model tree. As the plot suggests, most of the predicted values lie on the line of perfect agreement (model no. 1 with 12 linear functions). A high value of R (0.95) and smaller value of RMSE (0.036) with M5 model tree also confirm that this approach is useful in predicting values of C_d using this dataset.

In this research, analysis of variance (ANOVA) method was applied to determine importance of predictor variables (hydraulics and physical variables) in modeling of discharge coefficient (C_d). Generally, the ANOVA results

Table 3 Analysis of variance results

Source	*df*	Sum of squares	Mean squares	*F*	Pr > *F*
Model	5	1.3752	0.2750	437.5311	<0.0001
Error	189	0.1188	0.0006		
Corrected total	194	1.4940			

Table 4 ANOVA results for five predictor variables

Source	*df*	Sum of squares	Mean squares	*F*	Pr > *F*
H_1/L	1	0.4569	0.4569	726.7848	<0.0001
Re	1	0.3053	0.3053	485.7134	<0.0001
S_r	1	0.0328	0.0328	52.1969	<0.0001
n	1	0.0228	0.0228	36.2369	<0.0001
d_{50}/P	1	0.0086	0.0086	13.6921	0.0003

demonstrated that all predictor variables should be included in prediction model. The details of test results are presented in Tables 3 and 4.

The F value in Table 1 indicates that cumulative effect of predictor variables on C_d variable is significant ($P < 0.01$) and the detailed contribution of each of them is shown in following table (Table 4).

4 Conclusion

Weirs are hydraulic structures that are built for regulating, controlling and diverting water in the flow direction. Gabion structures are used extensively in water projects due to the ease of construction, permeability, accessibility and economic efficiency. Porous gabion structures are adaptable to the environment due to their material and performance and also are valuable from an ecologic viewpoint. In this study, eight physical models of gabion weirs were built for determining discharge coefficient (C_d) in gabion rectangular broad-crested weirs. The results of C_d were achieved by computing the data using M5 model. The assessment of the results by applying M5 clearly indicates that M5 is an efficient tool in predicting C_d in both free and submerge flow conditions. The excellent prediction performance of M5 yielded R to be 0.95 and RMSE to be 0.036. Thus, it should be emphasized that Reynolds number (Re) cannot be considered an important parameter in predicting C_d within this dataset. Results showed that the C_d of gabion weirs in free condition is 20% greater than that of the submerged condition.

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