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RESEARCH PAPER

Confidence Intervals for Common Signal-to-Noise Ratio of Several Log-Normal Distributions

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Abstract

Log-normal distribution is used widely in application fields such as economics and finance. This paper considers confidence interval estimates for common signal-to-noise ratio of log-normal distributions based on generalized confidence interval (GCI), adjusted method of variance estimates recovery, and computational approaches. A simulation study is conducted to compare the performance of these confidence intervals. A Monte Carlo simulation is applied to report coverage probability and average length of the confidence intervals. Based on the simulation study, for $k = 3$, the GCI can be used. For $k = 6$, the results of GCI approach perform similarly to the results of computational approach. For $k = 10$, the computational approach can be considered as an alternative to estimate the confidence interval. A numerical example based on real data is presented to illustrate the proposed approaches.

Keywords Average length · Coverage probability · Log-normal distribution · Monte Carlo simulation · Signal-to-noise ratio

1 Introduction

The log-normal distribution, which is related to the normal distribution, has special characteristics which have led to its use as a model in many applications, particularly for analyzing biological, bioequivalence, medical, and environmental data. For instance, the log-normal distribution has been used to compare several drug formulations in bioequivalence studies (Hannig et al. 2006; Schaarschmidt 2013) and has been used to estimate rainfall frequency in climate sciences and hydrology (Cho et al. 2004; Ritzema 1994).

The coefficient of variation (CV) is the ratio of the standard deviation to the mean that is a unit-free measure of variability relative to the mean. It has been used to analyze physical, biological, clinical, and medical data. Various applications of the CV can be found in Miller and Karson (1977), Doornbos and Dijkstra (1983), Tsim et al. (1991), Vangel (1996), Gupta and Ma (1996), Fung and Tsang (1998), Wong and Wu (2002), Tian (2005), Mahmoudvand and Hassani (2009), Niwitpong (2013), Ng (2014), Thangjai et al. (2016), Nam and Kwon (2017), and Hasan and Krishnamoorthy (2017). For example, in climate sciences and hydrology, the CV index has been used as a statistical measure of variability in the ratio of plant water demand to precipitation and to identify regions with highly variable climates as potentially vulnerable to periodic water stress and scarcity.

The signal-to-noise ratio (SNR), the reciprocal of the CV, is the ratio of the mean to the standard deviation of a single variable. It is an important measure in many applications, such as the signal strength relative to the background noise in analog and digital communications, to explain the magnitude of the mean of a process compared to variation in quality control, and for regression problems in econometrics, is the ratio of the full variance and partial variance of an explanatory variable. In the environment, measurements can be disturbed by noise (Motchenbacher and Connelly 1993), not only electronic noise but also external events such as wind, vibrations, variation in

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temperature, etc. In other words, the SNR helps to reduce the noise by controlling the environment. The SNR has been used to measure the relationship between excess return and the risk of financial assets in finance for portfolio selection models (Holgersson et al. 2012; Soliman et al. 2012). For convenient inference, it is to set up a model for the data and estimate a single parameter that determines the SNR of entire population.

The problem when estimating the SNR has received considerable attention in the literature. Sharma and Krishna (1994) studied an asymptotic sampling distribution of the inverse coefficient of variation, while George and Kibria (2011) presented an interval estimator for the SNR of a Poisson distribution and later on improved the confidence intervals for the SNR by inverting the confidence intervals for the CV (George and Kibria 2012). Moreover, Albatineh et al. (2014) developed the asymptotic sampling distribution of the SNR and then, in another study, introduced a novel approach to construct the confidence interval for the SNR based on an asymptotic sampling distribution (Albatineh et al. 2017). Recently, Thangjai and Niwitpong (2019) presented the interval estimation for a single SNR and the difference between the SNRs of two log-normal distributions.

Under many circumstances, independent samples are collected from different log-normal distributions with a common SNR, which can be used to test the equality of two or more SNRs (Gupta 2006), and indeed, the problem of making statistical inference (estimation and hypothesis testing: estimation is more accurate than hypothesis testing) from the common SNR of log-normal distributions is of interest in this study. We approach this by combining the summary statistics of the samples to estimate the common SNR. In addition, interval estimation can provide information on more than a point estimation. In the literature, Niwitpong (2018) proposed confidence intervals for the single SNR of a normal distribution, the difference between the SNRs of two normal distributions, and the common SNR of k normal distributions. In this paper, we extend the idea to construct the confidence interval for a common SNR of k log-normal distributions. The concept of the generalized confidence interval (GCI) introduced by Weerahandi (1993) is used to estimate the interval estimator and has been widely applied to estimate the confidence intervals for common parameters (Krishnamoorthy and Lu 2003; Tian 2005; Tian and Wu 2007; Thangjai et al. 2018). The concept of the adjusted MOVER (method of variance estimates recovery) approach based on MOVER, as introduced by Thangjai et al. (2018), is also applied. The computational approach is computed based on simulation and numerical computations using the maximum likelihood estimate (Pal et al. 2007). The GCI, adjusted MOVER, and computational approaches are used to construct confidence intervals for the common SNR of log-normal distributions in this study.

The organization of this paper is as follows. In Sect. 2, the proposed confidence intervals for common SNR of log-normal distributions are constructed. In Sect. 3, simulation technique and results are presented. In Sect. 4, a numerical example is presented. In Sect. 5, some concluding remarks are presented.

2 Confidence Intervals for Common SNR

Let $X = (X_1, X_2, \ldots, X_n)$ be an independently and identically distributed (i.i.d.) random sample of size n from normal distributions with mean μ and variance σ^2. The SNR is μ/σ. Let $Y = (Y_1, Y_2, \ldots, Y_n)$ be i.i.d. random sample of size n from log-normal distributions with parameter μ_Y and σ_Y^2. The mean and variance of the random variable Y are $E(Y) = \exp(\mu + \sigma^2/2)$ and $\mathrm{Var}(Y) = (\exp(\sigma^2) - 1) \cdot (\exp(2\mu + \sigma^2))$, respectively. Hence, the random variable Y has SNR defined as

$$\theta = \frac{E(Y)}{\sqrt{\mathrm{Var}(Y)}} = \frac{\exp(\mu + \sigma^2/2)}{\sqrt{(\exp(\sigma^2) - 1) \cdot (\exp(2\mu + \sigma^2))}} = \frac{1}{\sqrt{\exp(\sigma^2) - 1}}. \quad (1)$$

Let $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1)$ be maximum likelihood estimators of μ and σ^2, respectively. The maximum likelihood estimator of θ is given by

$$\hat{\theta} = \frac{1}{\sqrt{\exp(S^2) - 1}}. \quad (2)$$

According to Thangjai et al. (2016) and Thangjai and Niwitpong (2019), the variance of $\hat{\theta}$ is given by

$$\mathrm{Var}\left(\hat{\theta}\right) = \frac{\sigma^4 \cdot \exp(2\sigma^2)}{2(n-1) \cdot (\exp(\sigma^2) - 1)^3}. \quad (3)$$

Consider k independent log-normal distributions with a common SNR θ. Let $Y_i = (Y_{i1}, Y_{i2}, \ldots, Y_{in_i})$ be a random sample of size n from i-th log-normal distributions as follows: $X_{ij} = \log(Y_{ij}) \sim N(\mu_i, \sigma_i^2)$. From the i-th sample, the maximum likelihood estimator of θ_i is given by

$$\hat{\theta}_i = \frac{1}{\sqrt{\exp(S_i^2) - 1}}. \quad (4)$$

The variance of $\hat{\theta}_i$ is

$$\mathrm{Var}\left(\hat{\theta}_i\right) = \frac{\sigma_i^4 \cdot \exp\left(2\sigma_i^2\right)}{2(n_i - 1) \cdot (\exp(\sigma_i^2) - 1)^3}. \quad (5)$$

According to Graybill and Deal (1959), the estimator of the common SNR θ, the weighted average of SNR $\hat{\theta}_i$ based on k individual samples, is defined by

$$\hat{\theta} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\hat{\mathrm{Var}}\left(\hat{\theta}_i\right)} \Bigg/ \sum_{i=1}^{k} \frac{1}{\hat{\mathrm{Var}}\left(\hat{\theta}_i\right)}, \tag{6}$$

where $\hat{\theta}_i$ is defined as in Eq. (4) and $\hat{\mathrm{Var}}\left(\hat{\theta}_i\right)$ denotes the estimator of $\mathrm{Var}\left(\hat{\theta}_i\right)$ which is defined in Eq. (5) with σ_i replaced by s_i.

In this section, interval estimators for common SNR are developed using the GCI, adjusted MOVER, and computational approaches.

2.1 GCI

The GCI uses generalized pivotal quantity (GPQ) to construct the confidence interval. For more details about the GPQ, see Weerahandi (1993). Let $X = (X_1, X_2, \ldots, X_n)$ be a random variable with probability density function $f(X|\theta, \delta)$, where θ is parameter of interest and δ is nuisance parameter. Let $x = (x_1, x_2, \ldots, x_n)$ be an observed value of X. The random quantity $R(X, x, \theta, \delta)$ is called be GPQ if the following two conditions are satisfied:

1. $R(X, x, \theta, \delta)$ has a probability distribution that is free of unknown parameters.
2. The observed value of $R(X, x, \theta, \delta)$, $X = x$, does not depend on nuisance parameters.

Let $R(\alpha/2)$ and $R(1-\alpha/2)$ be the $100(\alpha/2)$-th and the $100(1-\alpha/2)$-th percentiles of $R(X, x, \theta, \delta)$. The $100(1-\alpha)\%$ two-sided GCI for the parameter of interest is defined as $[R(\alpha/2), R(1-\alpha/2)]$.

For the i-th sample, let $\bar{x}_i$ and s_i^2 be the observed values of sample mean $\bar{X}_i$ and sample variance S_i^2, respectively. The sample variance has a Chi-squared distribution with $n_i - 1$ degree of freedom defined as

$$S_i^2 \sim \frac{\sigma_i^2 \chi^2_{n_i-1}}{(n_i - 1)}. \tag{7}$$

From Eq. (7), the population variance can be obtained as follows:

$$\sigma_i^2 \sim \frac{(n_i - 1) S_i^2}{\chi^2_{n_i-1}}. \tag{8}$$

The GPQ for population variance σ_i^2 is defined as

$$R_{\sigma_i^2} = \frac{(n_i - 1) s_i^2}{\chi^2_{n_i-1}}. \tag{9}$$

The GPQ for the SNR θ based on the i-th sample can be written as

$$R_{\theta_i} = \frac{1}{\sqrt{\exp\left(R_{\sigma_i^2}\right) - 1}}, \tag{10}$$

where $R_{\sigma_i^2}$ is defined as in Eq. (9).

The GPQ for the common SNR θ is a weighted average of the GPQ R_{θ_i} based on k individual sample as

$$R_\theta = \sum_{i=1}^{k} \frac{R_{\theta_i}}{R_{\mathrm{Var}(\hat{\theta}_i)}} \Bigg/ \sum_{i=1}^{k} \frac{1}{R_{\mathrm{Var}(\hat{\theta}_i)}}, \tag{11}$$

where

$$R_{\mathrm{Var}(\hat{\theta}_i)} = \frac{R^2_{\sigma_i^2} \cdot \exp\left(2R_{\sigma_i^2}\right)}{2(n_i - 1) \cdot \left(\exp\left(R_{\sigma_i^2}\right) - 1\right)^3}. \tag{12}$$

In other words, $R_{\mathrm{Var}(\hat{\theta}_i)}$ is $\mathrm{Var}\left(\hat{\theta}_i\right)$ with σ_i^2 replaced by s_i^2.

It is easy to verify that R_θ is GPQ for θ. Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common SNR of several log-normal distributions based on the GCI approach is given by

$$\mathrm{CI}_{\mathrm{GCI}} = [L_{\mathrm{GCI}}, U_{\mathrm{GCI}}] = [R_\theta(\alpha/2), R_\theta(1-\alpha/2)], \tag{13}$$

where $R_\theta(\alpha/2)$ and $R_\theta(1-\alpha/2)$ denote the $100(\alpha/2)$-th and $100(1-\alpha/2)$-th percentiles of R_θ, respectively.

The following algorithm is useful in constructing the GCI for θ.

Algorithm 1.

For a given $\bar{x}_i$ and s_i^2 based on the i-th sample, where $i = 1, 2, \ldots, k$
For $g = 1$ to m
Generate $\chi^2_{n_i-1}$ from Chi-squared distribution with $n_i - 1$ degrees of freedom
Compute $R_{\sigma_i^2}$ from Eq. (9)
Compute R_{θ_i} from Eq. (10)
Compute $R_{\mathrm{Var}(\hat{\theta}_i)}$ from Eq. (12)
Compute R_θ from Eq. (11)
End g loop
Compute $R_\theta(\alpha/2)$ and $R_\theta(1-\alpha/2)$

2.2 Adjusted MOVER Confidence Interval

The MOVER approach discussed by Donner and Zou (2012) involves the sum of two parameters. Let θ_1 and θ_2 be the parameters of interest. Also, let $\hat{\theta}_1$ and $\hat{\theta}_2$ be the estimators of θ_1 and θ_2, respectively. The lower and upper limits of confidence interval for sum of two parameters $\theta_1 + \theta_2$ are defined as L_{12} and U_{12}, respectively. Here, the central limit theorem and assumption of independence between the point estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ are used to construct interval estimator. The lower limit L_{12} is obtained by

$$L_{12} = \hat{\theta}_1 + \hat{\theta}_2 - z_{\alpha/2}\sqrt{\mathrm{Var}\left(\hat{\theta}_1\right) + \mathrm{Var}\left(\hat{\theta}_2\right)}, \tag{14}$$

where $z_{\alpha/2}$ is the $100(\alpha/2)$-th percentile of the standard normal distribution.

Now, confidence limits for θ_i are separated as $[l_i, u_i]$, where $i = 1, 2$. Therefore, $[l_1, u_1]$ contains the parameter values for θ_1 and $[l_2, u_2]$ contains the parameter values for θ_2. Since the lower limit L_{12} must be closer to $l_1 + l_2$ than to $\hat{\theta}_1 + \hat{\theta}_2$. For $i = 1, 2$, the variance estimates for $\hat{\theta}_i$ at $\theta_i = l_i$ based on the central limit theorem can be written as

$$\hat{\mathrm{Var}}\left(\hat{\theta}_{l_i}\right) = \frac{\left(\hat{\theta}_i - l_i\right)^2}{z^2_{\alpha/2}}. \tag{15}$$

Substituting back into Eq. (14) yields

$$L_{12} = \hat{\theta}_1 + \hat{\theta}_2 - \sqrt{\left(\hat{\theta}_1 - l_1\right)^2 + \left(\hat{\theta}_2 - l_2\right)^2}. \tag{16}$$

And similarly, the upper limit U_{12} must be closer to $u_1 + u_2$ than to $\hat{\theta}_1 + \hat{\theta}_2$. For $i = 1, 2$, the variance estimates for $\hat{\theta}_i$ at $\theta_i = u_i$ based on the central limit theorem can be written as

$$\hat{\mathrm{Var}}\left(\hat{\theta}_{u_i}\right) = \frac{\left(u_i - \hat{\theta}_i\right)^2}{z^2_{\alpha/2}}. \tag{17}$$

Hence, the upper limit U_{12} is obtained by

$$U_{12} = \hat{\theta}_1 + \hat{\theta}_2 + \sqrt{\left(u_1 - \hat{\theta}_1\right)^2 + \left(u_2 - \hat{\theta}_2\right)^2}. \tag{18}$$

From Eqs. (15) and (17), the variance estimates for $\hat{\theta}_i$ at $\theta_i = l_i$ and $\theta_i = u_i$ are the average variance between these two variances given by

$$\begin{aligned}\hat{\mathrm{Var}}\left(\hat{\theta}_i\right) &= \frac{\hat{\mathrm{Var}}\left(\hat{\theta}_{l_i}\right) + \hat{\mathrm{Var}}\left(\hat{\theta}_{u_i}\right)}{2} \\ &= \frac{1}{2}\left(\frac{\left(\hat{\theta}_i - l_i\right)^2}{z^2_{\alpha/2}} + \frac{\left(u_i - \hat{\theta}_i\right)^2}{z^2_{\alpha/2}}\right).\end{aligned} \tag{19}$$

As documented by Graybill and Deal (1959), the common SNR θ is weighted average of the SNR $\hat{\theta}_i$ based on k individual samples. The common SNR is obtained by

$$\hat{\theta} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\hat{\mathrm{Var}}\left(\hat{\theta}_i\right)} \Bigg/ \sum_{i=1}^{k} \frac{1}{\hat{\mathrm{Var}}\left(\hat{\theta}_i\right)}, \tag{20}$$

where $\hat{\theta}_i$ is defined as in Eq. (4) and $\hat{\mathrm{Var}}\left(\hat{\theta}_i\right)$ is defined as in Eq. (19).

According to Krishnamoorthy and Oral (2017), the lower and upper limits of the confidence interval for the common SNR θ are given by

$$L_{\mathrm{AM}} = \hat{\theta} - \sqrt{\sum_{i=1}^{k} \frac{\left(\hat{\theta}_i - l_i\right)^2}{\left(\hat{\mathrm{Var}}\left(\hat{\theta}_{l_i}\right)\right)^2} \Bigg/ \sum_{i=1}^{k} \frac{1}{\left(\hat{\mathrm{Var}}\left(\hat{\theta}_{l_i}\right)\right)^2}} \tag{21}$$

and

$$U_{\mathrm{AM}} = \hat{\theta} + \sqrt{\sum_{i=1}^{k} \frac{\left(u_i - \hat{\theta}_i\right)^2}{\left(\hat{\mathrm{Var}}\left(\hat{\theta}_{u_i}\right)\right)^2} \Bigg/ \sum_{i=1}^{k} \frac{1}{\left(\hat{\mathrm{Var}}\left(\hat{\theta}_{u_i}\right)\right)^2}}, \tag{22}$$

where $\hat{\theta}$ is defined as in Eq. (20).

According to Casella and Berger (2002), it is well known that the confidence interval for the SNR is given by

$$[l_i, u_i] = \left[\hat{\theta}_i - t_{1-\alpha/2}\sqrt{\mathrm{Var}\left(\hat{\theta}_i\right)}, \hat{\theta}_i + t_{1-\alpha/2}\sqrt{\mathrm{Var}\left(\hat{\theta}_i\right)}\right], \tag{23}$$

where $t_{1-\alpha/2}$ is the $100(1-\alpha/2)$-th percentile of a Student's t distribution, $\hat{\theta}_i$ is defined as in Eq. (4) with S_i replaced by s_i, and $\mathrm{Var}\left(\hat{\theta}_i\right)$ is defined as in Eq. (5) with σ_i replaced by s_i.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common SNR of several log-normal distributions based on the adjusted MOVER approach is given by

$$\mathrm{CI}_{\mathrm{AM}} = [L_{\mathrm{AM}}, U_{\mathrm{AM}}], \tag{24}$$

where L_{AM} is defined as in Eq. (21), U_{AM} is defined as in Eq. (22), and l_i and u_i are defined as in Eq. (23).

2.3 Computational Confidence Interval

The computational approach introduced by Pal et al. (2007) uses the maximum likelihood estimates. To apply our proposed approach, the common SNR based on maximum likelihood estimator is obtained by

$$\hat{\theta}_{ML} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\hat{\mathrm{Var}}(\hat{\theta}_i)} \Bigg/ \sum_{i=1}^{k} \frac{1}{\hat{\mathrm{Var}}(\hat{\theta}_i)}, \tag{25}$$

where $\hat{\theta}_i$ is defined as in Eq. (4) and $\hat{\mathrm{Var}}(\hat{\theta}_i)$ is defined as in Eq. (5) with σ_i replaced by s_i.

The restricted maximum likelihood estimates (RMLEs) of parameters are used to obtain the computational approach. The maximum likelihood estimates of μ_i, σ_i^2, and θ under $\theta_1 = \theta_2 = \ldots = \theta_k = \theta$ provide the RMLEs of these parameters. The RMLE of μ_i is defined as $\hat{\mu}_{i(\mathrm{RML})} = \bar{X}_i$. The RMLE of σ_i^2 is defined as $\hat{\sigma}^2_{i(\mathrm{RML})} = S_i^2$. And the RMLE of θ is defined as $\hat{\theta}_{i(\mathrm{RML})} = 1/\sqrt{\exp\left(S^2_{i(\mathrm{RML})}\right) - 1}$.

Let $X_{i(\mathrm{RML})} = \left(X_{i1(\mathrm{RML})}, X_{i2(\mathrm{RML})}, \ldots, X_{in_i(\mathrm{RML})}\right)$ be artificial sample of size n_i from normal distributions with mean $\hat{\mu}_{i(\mathrm{RML})}$ and variance $\hat{\sigma}^2_{i(\mathrm{RML})}$. For i-th artificial sample, let $\bar{X}_{i(\mathrm{RML})}$ and $S^2_{i(\mathrm{RML})}$ be the mean and variance of the log-transformed sample from a log-normal distribution. Let $\bar{x}_{i(\mathrm{RML})}$ and $s^2_{i(\mathrm{RML})}$ be the observed values of $\bar{X}_{i(\mathrm{RML})}$ and $S^2_{i(\mathrm{RML})}$, respectively. The common SNR based on k individual samples is obtained by

$$\hat{\theta}_{\mathrm{RML}} = \sum_{i=1}^{k} \frac{\hat{\theta}_{i(\mathrm{RML})}}{\hat{\mathrm{Var}}(\hat{\theta}_{i(\mathrm{RML})})} \bigg/ \sum_{i=1}^{k} \frac{1}{\hat{\mathrm{Var}}(\hat{\theta}_{i(\mathrm{RML})})}, \tag{26}$$

where $\hat{\theta}_{i(\mathrm{RML})} = 1/\sqrt{\exp\left(S^2_{i(\mathrm{RML})}\right) - 1}$ with $S_{i(\mathrm{RML})}$ replaced by $s_{i(\mathrm{RML})}$ and $\hat{\mathrm{Var}}(\hat{\theta}_{i(\mathrm{RML})})$ is defined as in Eq. (5) with σ_i replaced by $s_{i(\mathrm{RML})}$.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common SNR of several log-normal distributions based on the computational approach is given by

$$\mathrm{CI}_{\mathrm{CA}} = [L_{\mathrm{CA}}, U_{\mathrm{CA}}] = [\hat{\theta}_{\mathrm{RML}}(\alpha/2), \hat{\theta}_{\mathrm{RML}}(1-\alpha/2)], \tag{27}$$

where $\hat{\theta}_{\mathrm{RML}}(\alpha/2)$ and $\hat{\theta}_{\mathrm{RML}}(1-\alpha/2)$ denote the $100(\alpha/2)$-th and $100(1-\alpha/2)$-th percentiles of $\hat{\theta}_{\mathrm{RML}}$, respectively.

The following algorithm is useful in constructing computational confidence interval for θ.

Algorithm 2.

For a given $\bar{x}_i$ and s_i^2 based on the i-th sample, where $i = 1, 2, \ldots, k$, and θ
Compute $\hat{\mu}_{i(\mathrm{RML})} = \bar{X}_i$ and $\hat{\sigma}^2_{i(\mathrm{RML})} = S_i^2$
For $g = 1$ to m
Generate $x_{ij(\mathrm{RML})}$ from $N\left(\hat{\mu}_{i(\mathrm{RML})}, \hat{\sigma}^2_{i(\mathrm{RML})}\right)$
Compute $\bar{x}_{i(\mathrm{RML})}$ and $s^2_{i(\mathrm{RML})}$
Compute $\hat{\theta}_{\mathrm{RML}}$ from Eq. (26)
End g loop
Compute $\hat{\theta}_{\mathrm{RML}}(\alpha/2)$ and $\hat{\theta}_{\mathrm{RML}}(1-\alpha/2)$ from Eq. (27)

3 Simulation Studies

A simulation study was performed to evaluate the coverage probabilities and average lengths of the three confidence intervals using the R statistical program. The SNR of log-normal distribution does not depend on parameter μ but it depends on parameter σ only. Although Fung and Tsang (1998), Tian (2005), and Ng (2014) suggested that the CV rarely exceeds 0.50 ($\sigma = 0.4724$ and SNR $= 1.9999$) for most medical and biological studies, this research followed George and Kibria (2012); in the simulation study, the sample cases were $k = 3$, 6, and 10. Clearly, the value of SNR in Eq. (1) depends only on the shape parameter σ. For simulation purposes, set the population means $\mu_1 = \mu_2 = \ldots = \mu_k = \mu = 1$ (arbitrary), and in order to get SNR $\theta = 10$, 3.33, 2, 1, we must set $\sigma = 0.0998$, 0.2938, 0.4724, 0.8326, respectively. The population standard deviations $\sigma_1, \sigma_2, \ldots, \sigma_k$ and the sample sizes $n_1, n_2, \ldots, n_k$ were given in the following tables. For each set of parameters, 5,000 random samples were generated. For the GCI and computational approaches, $1000 R_\theta$'s and $1000\hat{\theta}_{\mathrm{RML}}$'s were obtained for each of the random samples. For $100(1-\alpha)\%$ confidence interval, the coverage probability of the confidence interval would equal to $c \pm z_{\alpha/2}\sqrt{\frac{c(1-c)}{M}}$, where c is the nominal confidence level and M is a number of simulation runs. Therefore, the 95% confidence interval would have the coverage probability in a range of between [0.9440,0.9560], with the shortest average length is preferable.

The following algorithm is used to evaluate the coverage probabilities and average lengths of three confidence intervals:

Algorithm 3.

For a given $(n_1, n_2, \ldots, n_k)$, $(\mu_1, \mu_2, \ldots, \mu_k)$, $(\sigma_1, \sigma_2, \ldots, \sigma_k)$, and θ
For $h = 1$ to M
Generate x_{ij} from $N(\mu_i, \sigma_i^2)$, where $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n_i$
Calculate $\bar{x}_i$ and s_i^2
Construct the confidence interval based on the GCI approach $[L_{\mathrm{GCI}(h)}, U_{\mathrm{GCI}(h)}]$
Construct the confidence interval based on the adjusted MOVER approach $[L_{\mathrm{AM}(h)}, U_{\mathrm{AM}(h)}]$
Construct the confidence interval based on the computational approach $[L_{\mathrm{CA}(h)}, U_{\mathrm{CA}(h)}]$
Record whether or not all the values of θ fall in their corresponding confidence intervals
Compute the length of interval $U_{(h)} - L_{(h)}$
End h loop
Compute the coverage probability and average length for each confidence interval

The results in terms of the coverage probabilities and average lengths for $k = 3$, 6, and 10 sample cases are given in Tables 1, 2, and 3, respectively. The results of $k = 3$ are presented in Table 1, in which it can be seen that the GCI approach was preferable in terms of the coverage probability, whereas the coverage probabilities of the computational approach were close to the nominal confidence level of 0.95 when the sample sizes were large. Moreover, the coverage probabilities of the adjusted MOVER approach were less than the nominal confidence level of 0.95. From

Table 1 Coverage probabilities (CP) and average lengths (AL) of 95% two-sided confidence intervals for the common SNR of several log-normal distributions: 3 sample cases

$n(3)$*	$\sigma(3)$*	CI_{GCI}		CI_{AM}		CI_{CA}	
		CP	AL	CP	AL	CP	AL
10(3)	0.10,0.29,0.47	**0.9550**	2.3030	0.9262	2.0781	0.9312	2.5368
	0.29,0.47,0.83	**0.9550**	1.3864	0.9384	1.2463	0.9268	1.4664
10,20,30	0.10,0.29,0.47	**0.9564**	1.1999	0.9310	1.0887	0.9450	1.2505
	0.29,0.47,0.83	0.9490	0.7326	0.9252	0.6699	0.9438	0.7619
30(3)	0.10,0.29,0.47	**0.9516**	1.2582	0.8852	1.0128	0.9438	1.2958
	0.29,0.47,0.83	0.9468	0.7444	0.8930	0.6098	0.9382	0.7613
50(3)	0.10,0.29,0.47	0.9428	0.9643	0.8760	0.7592	0.9436	0.9808
	0.29,0.47,0.83	0.9476	0.5695	0.8886	0.4579	0.9436	0.5782
30,50,100	0.10,0.29,0.47	**0.9508**	0.6545	0.9184	0.5778	0.9492	0.6642
	0.29,0.47,0.83	**0.9500**	0.4003	0.9158	0.3550	0.9498	0.4065
50,100,200	0.10,0.29,0.47	**0.9506**	0.4581	0.9148	0.4038	0.9494	0.4619
	0.29,0.47,0.83	0.9482	0.2812	0.9156	0.2485	**0.9514**	0.2834
100(3)	0.10,0.29,0.47	0.9426	0.6779	0.8650	0.5261	0.9438	0.6836
	0.29,0.47,0.83	0.9478	0.4001	0.8824	0.3169	0.9480	0.4030
200(3)	0.10,0.29,0.47	**0.9512**	0.4785	0.8688	0.3683	**0.9500**	0.4802
	0.29,0.47,0.83	0.9452	0.2822	0.8702	0.2218	0.9404	0.2832

*$n(3) = (n_1, n_2, n_3)$ and $\sigma(3) = (\sigma_1, \sigma_2, \sigma_3)$

Bold values indicate that the coverage probability is greater than or close to the nominal confidence level of 0.95

Table 2 Coverage probabilities (CP) and average lengths (AL) of 95% two-sided confidence intervals for the common SNR of several log-normal distributions: 6 sample cases

$n(6)$*	$\sigma(6)$*	CI_{GCI}		CI_{AM}		CI_{CA}	
		CP	AL	CP	AL	CP	AL
10(6)	0.10(2),0.29(2),0.47(2)	0.9320	1.7398	0.9262	1.4416	**0.9592**	1.7472
	0.10,0.29(2),0.47(2),0.83	0.9356	1.3430	0.9034	1.0533	0.9468	1.3300
10(2),20(2),30(2)	0.10(2),0.29(2),0.47(2)	0.9360	0.8754	0.9132	0.7644	**0.9538**	0.8767
	0.10,0.29(2),0.47(2),0.83	0.9446	0.6960	0.8912	0.5674	**0.9526**	0.7082
30(6)	0.10(2),0.29(2),0.47(2)	0.9458	0.9176	0.8812	0.7119	**0.9510**	0.9131
	0.10,0.29(2),0.47(2),0.83	0.9428	0.7042	0.8646	0.5199	0.9492	0.7079
50(6)	0.10(2),0.29(2),0.47(2)	0.9464	0.6967	0.8760	0.5363	0.9458	0.6940
	0.10,0.29(2),0.47(2),0.83	**0.9508**	0.5377	0.8600	0.3917	**0.9538**	0.5392
30(2),50(2),100(2)	0.10(2),0.29(2),0.47(2)	0.9446	0.4684	0.9116	0.4077	0.9496	0.4685
	0.10,0.29(2),0.47(2),0.83	0.9488	0.3796	0.8840	0.3022	**0.9532**	0.3825
50(2),100(2),200(2)	0.10(2),0.29(2),0.47(2)	0.9434	0.3260	0.9086	0.2851	0.9440	0.3261
	0.10,0.29(2),0.47(2),0.83	**0.9500**	0.2667	0.8850	0.2116	**0.9504**	0.2677
100(6)	0.10(2),0.29(2),0.47(2)	0.9480	0.4853	0.8730	0.3716	0.9472	0.4839
	0.10,0.29(2),0.47(2),0.83	0.9478	0.3762	0.8420	0.2711	0.9488	0.3775
200(6)	0.10(2),0.29(2),0.47(2)	**0.9510**	0.3402	0.8680	0.2601	**0.9512**	0.3394
	0.10,0.29(2),0.47(2),0.83	0.9452	0.2651	0.8310	0.1899	0.9456	0.2653

*$n(k) = (n_1, n_2, \ldots, n_k)$ and $\sigma(k) = (\sigma_1, \sigma_2, \ldots, \sigma_k)$

Bold values indicate that the coverage probability is greater than or close to the nominal confidence level of 0.95

Table 2, for $k = 6$, the coverage probabilities of the GCI approach were less than the nominal confidence level of 0.95 as the sample sizes were small and became closer to the nominal confidence level of 0.95 as the sample sizes were large. The coverage probabilities of the adjusted MOVER approach were less than the nominal confidence

Table 3 Coverage probabilities (CP) and average lengths (AL) of 95% two-sided confidence intervals for the common SNR of several log-normal distributions: 10 sample cases

$n(10)$*	$\sigma(10)$*	CI_{GCI}		CI_{AM}		CI_{CA}	
		CP	AL	CP	AL	CP	AL
10(10)	0.10(2),0.29(3),0.47(3),0.83(2)	0.8984	1.1265	0.9014	0.7921	**0.9576**	1.0197
	0.10(5),0.83(5)	0.9218	0.9287	**0.9510**	0.6731	**0.9672**	0.7795
10(3),20(4),30(3)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9308	0.5350	0.8836	0.4269	**0.9540**	0.5325
	0.10(5),0.83(5)	0.9394	0.4125	0.9468	0.3637	**0.9600**	0.3944
30(10)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9314	0.5431	0.8556	0.3930	0.9496	0.5352
	0.10(5),0.83(5)	0.9358	0.4021	0.9160	0.3327	**0.9528**	0.3899
50(10)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9408	0.4104	0.8568	0.2960	**0.9510**	0.4075
	0.10(5),0.83(5)	0.9448	0.2999	0.9152	0.2511	**0.9586**	0.2952
(0(3),50(4),100(3)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9450	0.2858	0.8810	0.2284	**0.9502**	0.2863
	0.10(5),0.83(5)	0.9460	0.2153	0.9386	0.1964	**0.9544**	0.2125
50(3),100(4),200(3)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9470	0.1999	0.8768	0.1597	**0.9514**	0.1999
	0.10(5),0.83(5)	0.9488	0.1489	0.9352	0.1373	**0.9524**	0.1481
100(10)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9420	0.2861	0.8362	0.2053	0.9490	0.2850
	0.10(5),0.83(5)	0.9456	0.2068	0.9068	0.1740	**0.9542**	0.2051
200(10)	0.10(2),0.29(3),0.47(3),0.83(2)	0.9490	0.2005	0.8384	0.1438	0.9470	0.2002
	0.10(5),0.83(5)	0.9460	0.1445	0.9026	0.1219	0.9472	0.1439

*$n(k) = (n_1, n_2, \ldots, n_k)$ and $\sigma(k) = (\sigma_1, \sigma_2, \ldots, \sigma_k)$

Bold values indicate that the coverage probability is greater than or close to the nominal confidence level of 0.95

level of 0.95 for all sample sizes. The coverage probabilities of the computational approach were close to the nominal confidence level of 0.95 for all sample sizes. The GCI approach performed as well as the computational approach in terms of the coverage probability when the sample sizes were large; otherwise, the average lengths of the computational approach were little wider than those of the GCI approach. From Table 3, for $k = 10$, the computational approach provided much better interval estimates than the other approaches in terms of the coverage probability. The coverage probabilities of the GCI and adjusted MOVER approaches were less than the nominal confidence level of 0.95.

For the sake of saving space, we do not show the simulation results for $k = 20$ and 30 as the results are similarly to the results for $k = 10$ sample cases when the coverage probability of the GCI approach is far below the nominal confidence level of 0.95 and thus the GCI approach is not recommended for $k = 10$, 20 and 30 cases.

4 Empirical Application

The proposed approaches are applied to a real daily rainfall data in this section. The data set is given by Thangjai et al. (2019) on the daily rainfall data on 17 July 2018. The data are divided into three regions: central, eastern, and southern regions. For central region, the summary statistics are $n_1 = 22$, $\bar{x}_1 = 1.1642$, $\bar{y}_1 = 8.8273$, $s_{X_1} = 1.6073$, and $s_{Y_1} = 15.8404$. For eastern region, the summary statistics are $n_2 = 15$, $\bar{x}_2 = 2.5592$, $\bar{y}_2 = 44.6467$, $s_{X_2} = 1.8549$, and $s_{Y_2} = 71.9642$. For southern region, the summary statistics are $n_3 = 28$, $\bar{x}_3 = 3.0854$, $\bar{y}_3 = 26.9321$, $s_{X_3} = 0.6373$, and $s_{Y_3} = 19.5774$. Thangjai et al. (2019) indicated that three datasets come from log-normal distributions. The common SNR of log-normal distributions was 0.3761. The 95% two-sided confidence intervals for the common SNR of log-normal distributions were constructed using the GCI, adjusted MOVER, computational approaches. The 95% GCI is [0.0301, 0.5691] with a length of interval of 0.5390. The 95% adjusted MOVER confidence interval is [0.2137, 0.5561] with a length of interval of 0.3424. The 95% computational confidence interval is [0.1032, 0.6327] with a length of interval of 0.5295. The results indicate that all of the confidence intervals contain the true common SNRs, but the length of adjusted MOVER confidence interval was shorter than the lengths of the GCI and computational confidence interval. Therefore, these results confirm our simulation study in the previous section in term of length for $k = 3$. In simulation, the adjusted MOVER confidence interval is the shortest average lengths, but the coverage probabilities are less than the nominal confidence level of 0.95. Furthermore, the coverage probability and length in this example are computed by using only one sample, whereas the coverage probability and average length in the simulation are computed by using 5,000 random samples.

Therefore, the adjusted MOVER confidence interval is not recommended to construct the confidence intervals for common SNR.

5 Discussion and Conclusions

Thangjai and Niwitpong (2019) proposed confidence intervals for the single SNR of a log-normal distribution and for the difference between SNRs of two log-normal distributions. In this paper, we extend the work of Thangjai and Niwitpong (2019) to construct confidence intervals for the common SNR of k log-normal distributions. The study was carried out to examine the performance of confidence intervals based on the GCI, adjusted MOVER, and computational approaches. The simulation study results indicate that the GCI approach was better than the other approaches for $k = 3$. Additionally, the computational approach can be used when the sample sizes were large. For $k = 6$ and $k = 10$, the computational approach was preferable for all sample sizes. The GCI approach was preferable when the sample sizes were large.

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

R code for outputs in Tables 1, 2, and 3

```
CI.SNRLN = function(M,m,k,n(k),mean,sqrt.var(k)){
ni = rep(0,k); s = rep(0,k); xbar = rep(0,k); mu.hat.RML = rep(0,k)
sigma.hat.RML = rep(0,k); Rtheta.GCI = rep(0,m)
theta.CA = rep(0,m); CP.GCI = rep(0,M); CP.AM = rep(0,M)
CP.CA = rep(0,M); Length.GCI = rep(0,M); Length.AM = rep(0,M)
Length.CA = rep(0,M); alpha = 0.05; z.alpha = qnorm(1-(alpha/2))
sqrt.var = c(sqrt.var(k)); theta(k) = 1/sqrt(exp(sqrt.var(k)^2)-1)
var.theta(k) = ((sqrt.var(k)^4)∗(exp(2∗(sqrt.var(k)^2))))/(2∗(n(k)-1)
∗(((exp(sqrt.var(k)^2))-1)^3))
theta.p = c(theta(k)); var.theta.p = c(var.theta(k))
theta = (sum(theta.p/var.theta.p))/(sum(1/var.theta.p))
for(i in 1:M){
x(k) = rnorm(n(k),mean,sqrt.var(k)); xbar.(k) = mean(x(k))
s.(k) = sd(x(k)); ni = c(n(k)); xbar = c(xbar.(k)); s = c(s.(k))
thetahat = 1/sqrt((exp(s^2))-1)
var.thetahat = ((s^4)∗(exp(2∗(s^2))))/(2∗(ni-1)∗(((exp(s^2))-1)^3))
frac1 = sum(thetahat/var.thetahat); frac2 = sum(1/var.thetahat)
thetahat.large = frac1/frac2
for(j in 1:m){
V = rchisq(k,ni-1); Rsig.sqrt = ((ni-1)∗(s^2))/V
Rvar = ((Rsig.sqrt^2)∗(exp(2∗(Rsig.sqrt))))/(2∗(ni-1)∗(((exp(Rsig.sqrt))-1)^3)); Rtheta = 1/sqrt((exp(Rsig.sqrt))-1)
Rtheta.GCI[j] = sum(Rtheta/Rvar)/sum(1/Rvar)}
L.CI1 = quantile(Rtheta.GCI,0.025,type=8)
U.CI1 = quantile(Rtheta.GCI,0.975,type=8)
CP.GCI[i] = ifelse(L.CI1<theta&&theta<U.CI1,1,0)
Length.GCI[i] = U.CI1-L.CI1
t.x = qt((1-alpha/2),(ni-1)); l1 = (1/sqrt((exp(s^2))-1))-(t.x∗sqrt(var.thetahat))
u1 = (1/sqrt((exp(s^2))-1))+(t.x∗sqrt(var.thetahat)); z = qnorm(alpha/2)
var.l = ((thetahat-l1)^2)/(z^2); var.u = ((u1-thetahat)^2)/(z^2)
var.t = (var.l+var.u)/2; thetahat.w = (sum(thetahat/(var.t)))/(sum(1/(var.t)))
L.CI2 = thetahat.w-(z.alpha∗sqrt(1/(sum(1/var.l))))
U.CI2 = thetahat.w+(z.alpha∗sqrt(1/(sum(1/var.u))))
CP.AM[i] = ifelse(L.CI2<theta&&theta<U.CI2,1,0)
Length.AM[i] = U.CI2-L.CI2
mu.hat.RML = xbar; mu.hat.RML(k) = mu.hat.RML[[k]]
sigma.hat.RML = s; sigma.hat.RML(k) = sigma.hat.RML[[k]]
for(j in 1:m){
x.RML(k) = rnorm(n(k),mu.hat.RML(k),sigma.hat.RML(k))
xbar.RML.(k) = mean(x.RML(k)); s.RML.(k) = sd(x.RML(k))
xbar.RML = c(xbar.RML.(k)); s.RML = c(s.RML.(k))
thetahat.RML = 1/sqrt((exp(s.RML^2))-1)
var.RML = ((s.RML^4)∗(exp(2∗(s.RML^2))))/(2∗(ni-1)∗(((exp(s.RML^2))-1)^3))
frac1.RML = sum(thetahat.RML/var.RML); frac2.RML = sum(1/var.RML)
theta.CA[j] = frac1.RML/frac2.RML }
L.CI3 = quantile(theta.CA,0.025,type=8)
U.CI3 = quantile(theta.CA,0.975,type=8)
CP.CA[i] = ifelse(L.CI3<theta&&theta<U.CI3,1,0)
Length.CA[i] = U.CI3-L.CI3}
```

cat("CP|GCI=", mean(CP.GCI),"Length|GCI=", mean (Length.GCI),"\ n")

cat("CP|AM=", mean(CP.AM),"Length|AM=", mean (Length.AM),"\ n")

cat("CP|CA=", mean(CP.CA),"Length|CA=", mean(-Length.CA),"\ n")}

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