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ORIGINAL ARTICLE

Study on variations of peak strength of a sandstone during cyclic loading

A. Taheri · A. Royle · Z. Yang · Y. Zhao

Abstract An experimental investigation was carried out on Hawkesbury sandstone samples to study the peak strength variations during triaxial monotonic and cyclic compressive testing. Axial load, confining pressures and axial and lateral deformations were measured precisely from start of loading until post-peak state. Cyclic tests were carried out at different stress levels and unloading amplitudes. All the tests were performed at 4 MPa confining pressure. A methodology to conduct cyclic loading testing was devised including the development of an innovative peak strength prediction method. The results confirmed that beginning of cyclic loading at higher stress will result in failure after fewer cycles. Moreover, it was found that if the specimen experience more cyclic loading the failure mode will be more brittle. The mechanical properties of the rock were altered by systematic cyclic loading; dependent upon the applied stress level during cyclic loading the rock may become weaker or stronger as a result of cyclic loading. It was identified that a critical maximum normalized deviator stress (i.e. deviator stress at the beginning of unloading normalized by rock peak strength) exists between 93.7 and 94.1 % which defines the limit between peak strength hardening and weakening during cyclic loading. If the cyclic loading deviator stress is lower than this critical boundary, the peak strength increases up to 11 %. This increase in peak strength is directly proportional to the normalized deviator stress in unloading; the amount of increase in peak strength decreases with a decrease in cyclic loading deviator stress.

Keywords Hawkesbury sandstone · Cyclic loading · Peak strength

1 Introduction

Rock mass in underground and surface mines is exposed to systematic cyclic loading during drilling, mechanical excavation and truck haulage vibration or due to mine seismology. The mechanical properties of the rock are altered by cyclic loading and consequently rock may fail at stress levels lower than the monotonic failure stress. Such mechanisms could reduce the stability of the haul road and excavation boundaries and lead to failure which poses a significant hazard. In order to manage the risk, it is required to predict the effect of cyclic loading on mechanical properties of rocks.

Bieniawski ([1967]) defined the stages of brittle fracture of rock under triaxial compression and it was determined that cyclic testing should commence just prior to the onset of the unstable crack propagation stage to ensure fatigue failure of a specimen in a timely

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manner. This occurs between 70 and 85 % of the peak strength where the stress–strain relation becomes non-linear. Liu et al. (2012) and Xiao et al. (2010) defined damage as the creation and propagation of micro-cracks in a specimen which is considered to be continuous on a larger scale. Wasantha et al. (2015) looked at effect of joint geometry on fracturing behavior of rock joints. They observed a strong correlation between the fracture progression behaviours of the specimens with different failure mechanisms. Ge et al. (2003) examined the three stages of fatigue behaviour; volumetric compaction, volumetric dilation with strain hardening behaviour, and volumetric dilation with strain softening behaviour. It was found that the threshold for fatigue failure occurred at the transition point between volumetric compaction and volumetric dilation. Attewel and Farmer (1973) proposed that in cyclic loading failure occurs when the strain energy stored in the specimen exceeds a critical energy release level. They mentioned that just prior to this point, the rock have experienced a deformation state in which the incremental strains increase in proportion to the number of loading cycles. Bagde and Petros (2005) concluded that the dynamic energy sustained by the sample at a given testing condition could be treated as a rock characteristic indicating that this could influence the failure mechanism. Several studies performed by Haimson and Kim (1972), Eberhardt (1998) and Bagde and Petros (2005) confirmed that progressive micro-fracturing is the main cause of fatigue failure.

To date, a number of models have been established to predict damage of a rock as a result of compressive cyclic loading (Eberhardt et al. 1999; Chen et al. 2006; Xiao et al. 2010; Taheri and Tani 2013). However, the effect of various parameters during cyclic loading including unloading amplitude, stress level and confining pressure have not been studied together to predict peak strength during cyclic loading.

A significant problem that was noted with a majority of the studies was that the peak strength of the sample during monotonic loading was not accurately predicted, meaning that the effects of heterogeneity and variability among specimens were not taken into account (Eberhardt et al. 1999; Chen et al. 2006; Xiao et al. 2010; Koseki et al. 2003; Wang et al. 2013; Yang et al. 2015). In a study performed by Bastian et al. (2014) on Hawkesbury sandstone, the variability between the samples obtained from a single

rock block was significant. Therefore, due to a lack of consistent and a representative amount of data it was not possible to develop a reliable model for predicting the damage and deformability attained by a rock under cyclic loading. This is the problem other ran into as well when performing cyclic loading on rock (e.g. Eberhardt et al. 1999; Zhengyu and Haihong 1990). Figure 1 shows the results of a study performed by Eberhardt et al. (1999) to quantify damage due to cyclic loading on rock. As can be seen in this figure, the unloading boundary is changed three times throughout the test, mainly because due to underestimation of rock monotonic peak strength, the selected unloading boundaries were not sufficient to fail the sample.

To solve the above mentioned problem, a method to predict rock peak strength, q_f, without failing the specimen is required to allow results to be confidently attributed to cyclic loading processes, rather than estimating rock strength from previous tested specimens, assuming that all the samples are completely identical. A successful cyclic loading test requires the rock specimen to fail after a sufficient number of
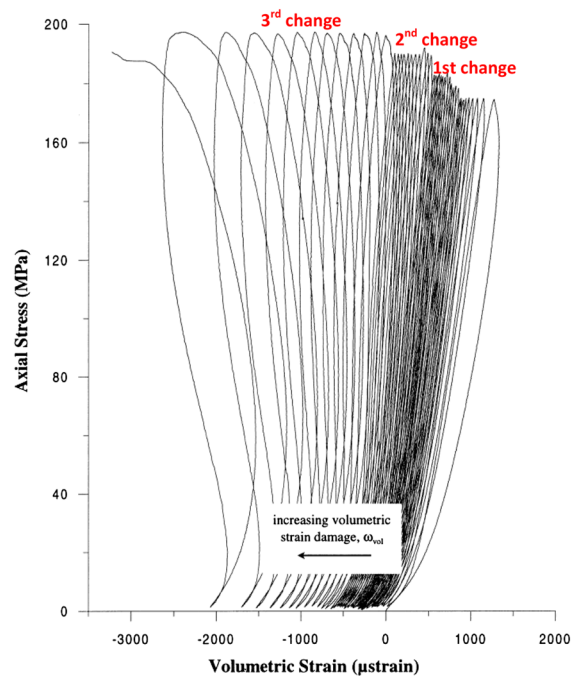

Fig. 1 Change of unloading stress boundary during cyclic loading test in axial stress–volumetric strain relations (adopted from Eberhardt et al. 1999)

cycles without any change in unloading stress level and loading/unloading rate. During a cyclic loading test, when the predicted q_f value was much lower than the real peak strength, the designed cyclic loading axial stress level, q_{un}, would not be adequate to cause sufficient damage to break the rock. As the damage caused by every cycle, if any, was small, the number of cycles needed before the rock failed would be large hence the rock would not fail unless the q_{un} or loading rate value was modified. Adjustments to q_{un} or loading rate during a cyclic loading violate the principle of control variables during the test and render the results unreliable for damage model development. In addition, if the predicted q_f value was much higher than the real peak strength, the sample may experience a premature failure in the first cycle of loading. Hence, to prevent these situations, in this study the main challenge was to develop a method to accurately predict q_f of each rock sample and therefore obtain reliable test results. The results were then used to evaluate the effect of cyclic loading on peak and residual strength of sandstone.

2 Methodology

A total of 5 monotonic tests at loading rate of 0.05 mm/min were undertaken to study the rock behaviour during monotonic loading and to develop a method to predict monotonic peak strength, q_f during cyclic loading tests. In addition 15 displacement controlled cyclic triaxial compression tests under confining pressure of 4 MPa with loading/unloading rates of 5/10 mm/min were completed on Hawkesbury sandstone specimens to study the effect of cyclic loading on peak strength. The two variables that were altered for each test were:

1. The maximum deviator stress applied to the specimen at each cycle, q_{un}.
2. Loading amplitude, q_b, which will be set at 50, 75 and 100 % of q_{un}.

The Hawkesbury sandstone specimens were prepared according to International Society for Rock Mechanics (ISRM) standards. The specimens had dimensions of 100 mm in height and 42 mm in diameter. Two lateral and two axial strain gauges were fixed to the surface of the specimens in order to measure the true strain experienced by the specimen during loading.

Based on previous research, the Hawkesbury sandstone has an average strength around 40 MPa, therefore, a closed-loop servo-controlled testing machine with a loading capacity of 250 kN was used for testing. The machine had a loading rate capability in the range of 0.001–10 mm/s which was set at 5 mm/min to ensure the confining pressure remained constant throughout the test. A high pressure Hoek cell and hydraulic pressure system were used to apply and control the confining pressure. The machine has a Linear Variable Differential Transducer (LVDT) to measure axial displacements.

To measure peak strength, q_f, change of a sample subjected to cyclic loading, the peak strength under monotonic loading should be known prior to cyclic test. In the previous researches (e.g. Eberhardt et al. 1999; Gatelier et al. 2002; Bagde and Petros 2005; Yang et al. 2015) rock strength at monotonic loading was defined by conducting monotonic test(s) assuming that all the samples have similar strength. However, when testing Hawkesbury sandstone, as mentioned by Bastian et al. (2014), due to sample variability a wide range of monotonic strengths were measured at 4 MPa confining pressure, ranging between 61 and 82 MPa. Hence the prediction of monotonic peak strength, q_f, using the average monotonic tests results is not accurate for individual specimens. A model to predict the rock strength for each specimen before the cyclic loading is required.

In order to derive a method to predict the monotonic peak strength, q_f, with potential indicator points, results of monotonic tests were further investigated. Figure 2 shows axial stress–strain relations obtained
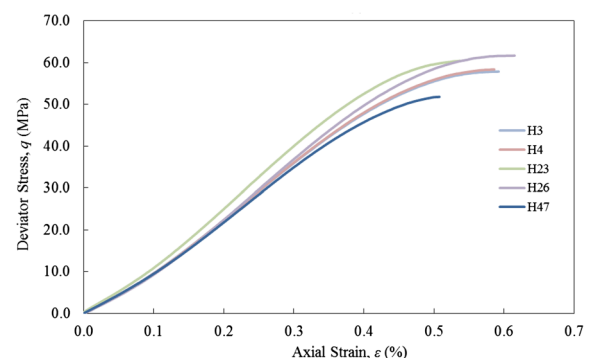

Fig. 2 Stress strain relations of monotonic triaxial tests

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by monotonic testing. As can be seen in this figure neither peak strength, nor stiffness of sample are identical. Peak strength of five specimens varies between 51.8 and 61.7 MPa at $\sigma_3 = 4$ MPa. It was intended to use stress–strain relations to obtain potential indicator point to predict monotonic peak strength, q_f, of each sample during cyclic loading without damaging it. The indicator point should be easily monitored during testing and it must be able to predict q_f for Hawkesbury sandstone samples tested at 4 MPa confining pressure, σ_3.

Analysing the results of monotonic tests, it was found that during monotonic loading the secant Young's modulus, E_{sec}, evaluated by the external LVDT, keeps increasing until the axial stress come to a point closer to the failure point, after which the E_{sec} starts decreasing. The results showed that this point is a reliable indicative point to predict peak strength without damaging the sample. As schematically shown in Fig. 3, the deviator stress value at this indicator point, q_{id}, always provides a constant value for the ratio of q_{id}/q_f. As demonstrated in Table 1, the possibility of applying an offset to the secant Young's modulus was investigated in order to reduce the variability of q_{id}/q_f. The purpose of the offset also was to smooth out small scale fluctuations in the data as shown in Eq. 1. As shown schematically in Fig. 4 and then for a monotonic test in Fig. 5, as the offset

increases, the deviator stress at $E_{sec(max)}$ with off-set also increases. Figure 6 schematically shows Secant Young's modulus, E_{sec}, values calculated from axial stress–strain results, by applying an offset. As shown in this figure after the indicator point, q_{id}, material shows large plastic deformation, stiffness reduces significantly and therefore, E_{sec} starts decreasing.

Table 1 shows that at 0.4 % or higher value of offset, the ratio of q_{id}/q_f is always constant for sandstone samples having different peak strength and stiffness values. The maximum E_{sec} with 0.5 % offset is the most accurate indicator for q_f prediction, however, in order to achieve a trade-off between accuracy and potential damage to the specimen, 0.4 % offset was employed in this study using Eq. 2. The results obtained during cyclic loading didn't show any noticeable damage on sample due to pre-monotonic loading.

$$E_{sec}\ with\ offset = \frac{Deviator\ stress}{strain + offset} \quad (1)$$

$$q_{f\ Prediction} = \frac{q_{id}}{96\%} \quad (2)$$

After defining the indicator point, a methodology was designed such that a pre-monotonic load was applied to every sample prior to the commencement of cyclic loading. As the specimen must be intact when cyclic loading commences, the pre-monotonic load will be removed when E_{sec} peaks with 0.4 % offset and rock peak strength predicted using Eq. 2. With post-test analysing the results after completing the cyclic loading test, the accurate peak strength can be estimated by knowing the exact axial stress where secant Young's modulus is the maximum value ($E_{sec} = max$).

Figure 7 shows the proposed cyclic loading test procedure that was used in this study. Pre-monotonic loading was applied to every rock specimen until the indicator point was reached. The q_f value for each individual rock sample was then calculated based on the stress at indicator point, q_{id}. The q_{id} was different for every sample however the ratio of q_{id}/q_f was the same for all samples. As q_{un} is always smaller than q_f, premature failure was avoided. The X value given in $q_{un} = X\% \times q_f$ was also implemented confidently. Therefore the likelihood of a time consuming test due to an insufficient q_{un} value was also reduced and large
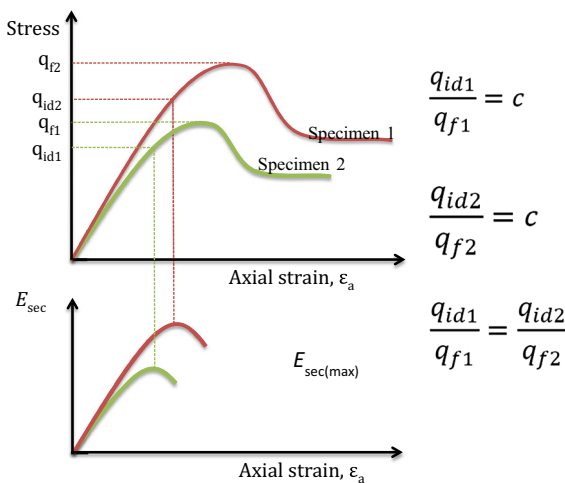

Fig. 3 Use of $E_{sec} = max$ as an indicator point to predict peak strength

Table 1 Monotonic test results use to develop an indicator point to predict monotonic peak strength using stress–strain relations in cyclic loading tests

Specimen	Monotonic peak strength, q_f (MPa)	q_{in}*/q_f at 0 % offset	q_{in}/q_f at 0.1 % offset	q_{in}/q_f at 0.2 % offset	q_{in}/q_f at 0.3 % offset	q_{in}/q_f at 0.4 % offset	q_{in}/q_f at 0.5 % offset
H3	57.884	0.81	0.88	0.91	0.93	0.95	0.97
H4	58.398	0.81	0.87	0.89	0.92	0.95	0.97
H23	60.457	0.84	0.91	0.93	0.97	0.97	0.98
H26	61.713	0.85	0.91	0.94	0.95	0.96	0.97
H47	51.830	0.88	0.91	0.95	0.97	0.97	0.98
	Average:	0.84	0.90	0.92	0.95	0.96	0.97

* q_{in} is equal to the deviator stress where secant Young's modulus peaks (E_{sec} = max)
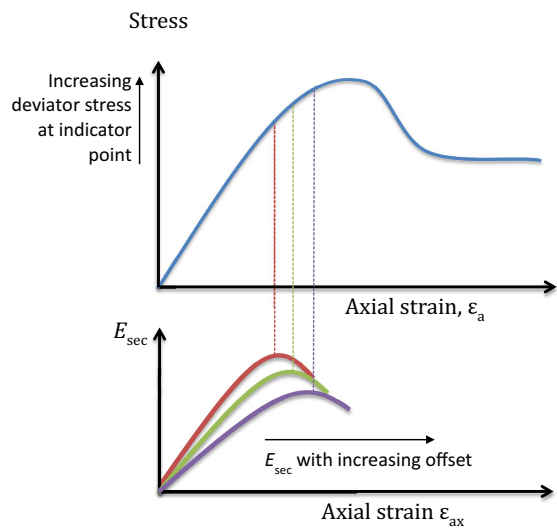

Fig. 4 Effect of increasing the offset applied to E_{sec} calculations on q_{id}

cycle numbers could be attributed to rock properties rather than inaccurate testing conditions. If the rock specimen did not fail after a large number of cycles, a monotonic load was applied until failure in order to examine the effect of long cyclic loading on the q_f value.

3 Analysis of results

Figure 8 shows the deviator stress versus axial strain graph for a specimen failed as a result of cyclic loading. For Test HS33, cyclic loading was initiated at
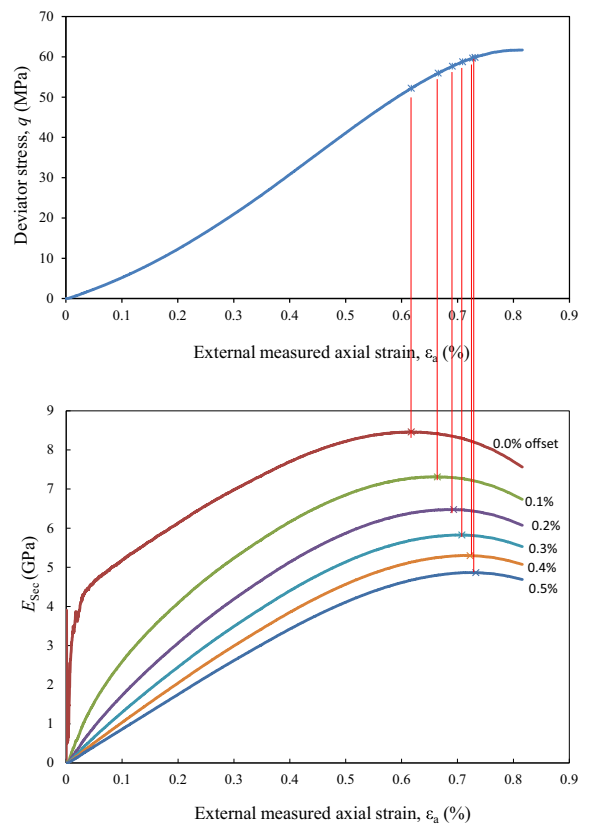

Fig. 5 Deviator stress calculated in a monotonic triaxial test (Test H26) when E_{sec} = max with different offsets

65 MPa and requiring 68 cycles until failure. The stress–strain results in this figure are similar to those obtained by Xiao et al. (2009). The hysteretic loops are developed according to law of loose-dense-loose. Figure 9 shows the relations for Test HS43 which did
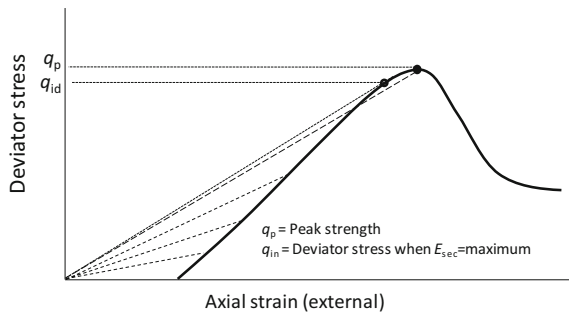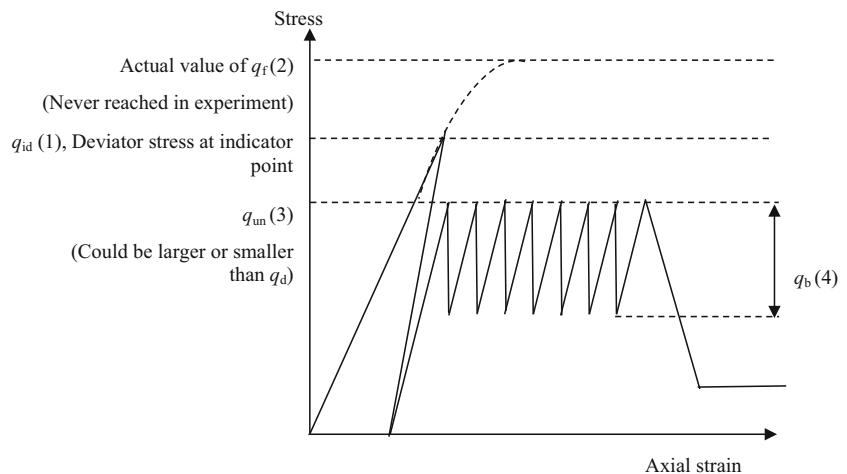
Fig. 6 Secant Young's modulus variations during axial loading

not fail during cyclic loading. In this test after 2255 loading/unloading a post-monotonic loading was applied to fail the specimen. As can be seen in this figure, the sample does not experience much axial plastic deformation after a large number of cycles and comparing with Fig. 8, the amount of permanent lateral and volumetric strain in the specimen during cyclic loading is very small. As discussed by Singh (1989), if the applied stress level in cyclic loading is considerably lower than the monotonic peak strength, rock won't fail during cyclic loading and it can even become stronger. The amount of permanent deformation during cyclic loading in peak or valley is usually considered as the amount of damage that the sample is experienced (e.g. Eberhardt et al. 1999). It will be shown in Sect. 5 that the sample for Test HS43 shows an increase in peak strength during a monotonic loading after a cyclic loading. Obviously this sample wasn't damaged during cyclic loading and therefore,

the amount of permanent strain measured in this test during cyclic loading is very low.

It can be obtained that the deviator stress at failure for the specimen that did not fail after cyclic test is much higher than the upper limit of cyclic loading. An accurate post-test analysis was competed after the test by using the precise maximum E_{sec} obtained from the data file. Table 2 shows the results of cyclic loading tests carried out in this study.

4 Rock damage

The normalized deviator stresses for cyclic loading, q_{un}/q_f, and the normalized residual stresses, q_r/q_f, were investigated. In so doing, for specimens with unloading amplitude of 100 %, q_{un}/q_f versus damage increment, i, at failure and q_r/q_f versus axial strain at failure were plotted and shown in Fig. 10. To provide a consistent comparison, the results for the test at 75 and 50 % of unloading amplitude are not included in this figure. Further tests should be conducted in the future to let the authors analyze the results of those tests as well. As shown in Fig. 10a, a higher normalized stress at unloading will result in failure after fewer cycles. In addition, Fig. 10b shows that if the specimen experience more axial strain before failure due to an increase of fatigue life, the failure is more brittle. In other words, during cyclic loading, the difference between peak strength and residual strength increases with an increase in axial strain at failure. This is mainly because by an increase in fatigue life, specimen is

Fig. 7 Proposed cyclic loading test procedure; (*1*) the indicator point was found during the first single loading; (*2*) the q_f value was predicted based on the relationship between the q_{id} and q_f; (*3*) q_{un} was designed based on $q_{un} = X \% \times q_f$; and (*4*) q_b was defined based on q_{un}
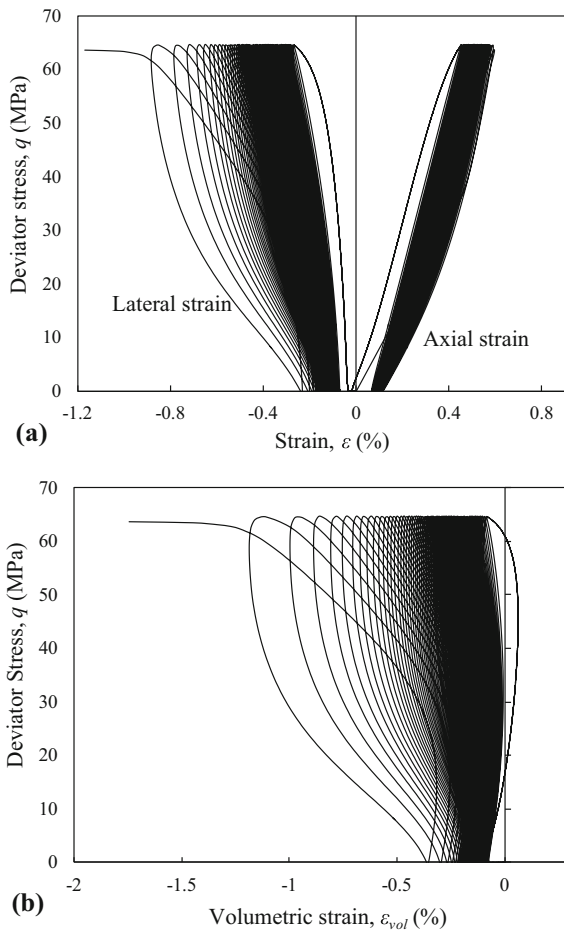
Fig. 8 An example of cyclic loading test results where the sample failed during cyclic loading (Test HS33); **a** deviator stress–axial and lateral strain relations; **b** deviator stress–volumetric strain relations
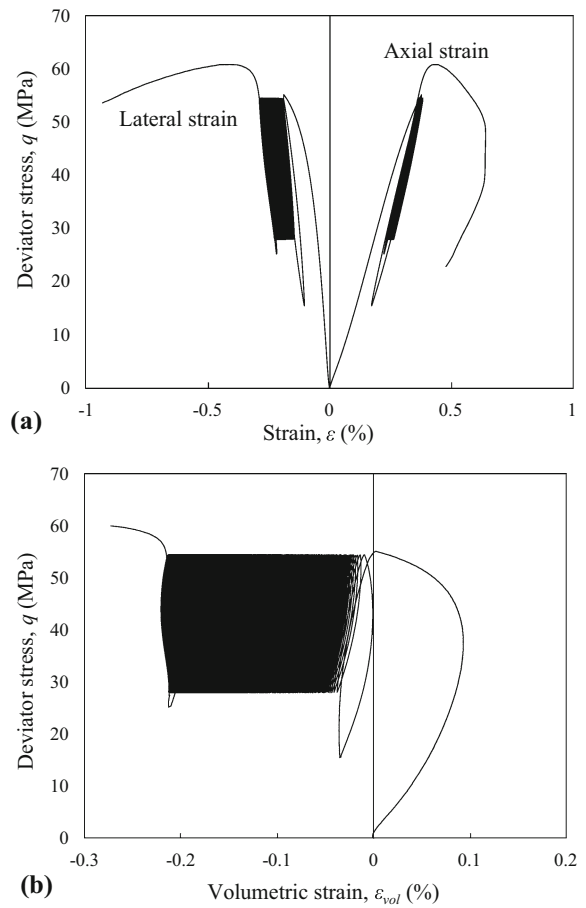

Fig. 9 An example of cyclic loading test results where the sample didn't fail during cyclic loading (Test HS43); **a** deviator stress–axial and lateral strain relations; **b** deviator stress–volumetric strain relations

subjected to more damage until failure point. Therefore, failure behavior of a sample which experienced more damage before failure is more eruptive.

5 Rock strength hardening

In this section the behaviour of the rock specimens that did not fail as a result of the applied cyclic loading are discussed. Previous studies mostly support that the strength of rock decreases as a result of cyclic loading due to progressive damage of rock (Bieniawski 1967; Eberhardt et al. 1999; Chen et al. 2006; Xiao et al. 2010; Koseki et al. 2003; Wang et al. 2013). According to Ray et al. (1998), the percentage decrease in

uniaxial compressive strength increases with the number of cycles applied. Along with this, Singh (1989) discusses, however, that in some cases the applied stress level in cyclic loading does not cause rock failure during a very large number of cycles. Instead, the rock can become "strain hardened" and in turn increases its uniaxial compressive strength. Burdine (1963) compared the static strength of Berea sandstone samples with and without 1000 pre-cyclic loading under the same confining pressure. The results revealed that the static failure values for the samples experienced pre-cyclic loading are relatively higher than fresh samples. Cyclic loading can have very significant rock strength hardening effect. An investigation carried out by Ma et al. (2013) showed that the strength of rock salt can increase up to 171 % after a

Table 2 Summary of test results including amount of rock strength increase

Test no.	Actual q_{un}/q_f (%)	Unloading amplitude (%)	Number of cycles	Failed during cyclic loading?	% of rock strength increase (%)
HS33	99.2	100	68	Y	–
HS41	98.8	100	99	Y	–
HS34	97.1	100	193	Y	–
HS39	97.0	75	648	Y	–
HS45	96.5	50	836	Y	–
HS37	96.4	100	230	Y	–
HS42	94.1	75	1344	Y	–
HS43	93.7	50	2255	N	4.0
HS36	92.7	100	1265	N	7.2
HS40	91.9	100	1365	N	11
HS35	86.6	100	3739	N	9.1
HS50	84.0	100	1300	N	5.3
HS47	81.5	100	1300	N	6.1
HS49	70.4	100	1300	N	3.0
HS48	61.3	100	1300	N	0.07

certain number of cyclic loading during triaxial compressive testing.

Of the 15 tests, seven of the specimens failed as a result of cyclic loading. The fatigue stress for those specimens was found to be smaller than the predicted q_f, which indicates a progressive damage mechanism due to cyclic loading. Eight of the specimens, however, did not fail after a large number of cycles. Therefore a monotonic load was applied until failure and post-peak state. It was found that in all eight tests which did not fail due to cyclic loading, the deviator stress at failure was higher than the predicted q_f value (see Table 2). This indicates that the strength of the rock specimens increased as a result of cyclic loading. In other words, if the stress level at the beginning of cyclic loading is not high enough to generate a failure in sandstone due to progressive cyclic loading damage, the rock strength, as a result of cyclic loading for a long period of time, may be increased.

The percentage of rock hardening is summarised in Table 2 in descending order of the actual q_{un}/q_f applied to the rock specimens. The seven tests with the highest normalised deviator stresses all failed as a result of cyclic loading with 94.1 % the lowest q_{un}/q_f resulting in failure (specimen HS42). In the remaining eight tests, the specimens did not fail as a result of cyclic loading. These specimens had a lower q_{un}/q_f

than those that failed. Based on these results it is hypothesised that there exists a value of q_{un}/q_f which lies between 93.7 and 94.1 % that indicates the critical boundary of rock strength hardening and weakening under cyclic loading. In addition, Table 2 shows that the rock peak strength can increase up to 11 % as a result of pre-cyclic loading.

Figure 11 shows that an increase in fatigue strength is directly proportional to the normalised deviator stress, q_{un}/q_f in unloading. Hence, after 1300 cycles of systematic cyclic loading with 100 % unloading amplitude in Hawkesbury sandstone specimens, the relationship between the rock strength increase percentage and normalised deviator stress at the upper limit of cyclic loading is given by the following Equation:

$$R = 0.2916 \times \frac{q_{un}}{q_f} - 0.1769 \qquad (3)$$

where R is the percentage of rock strength increase as a results of cyclic loading, q_{un} is the unloading amplitude and q_f is the initial peak strength of the rock. The application of Eq. 3 is limited by the fixed damage increment and unloading amplitude. Further study into the behaviour of different rock types under different unloading amplitudes, q_b, is required to define a generalized equation.
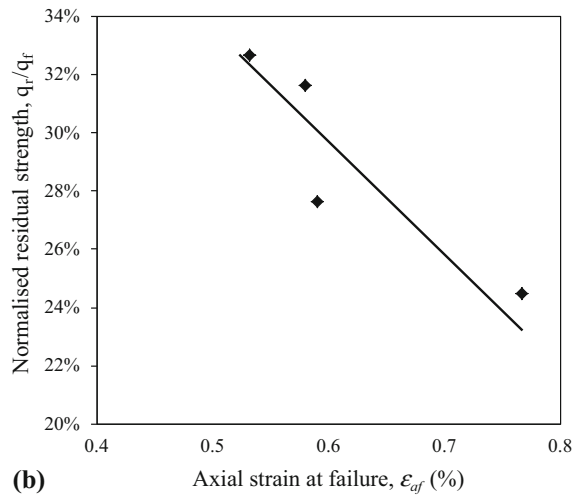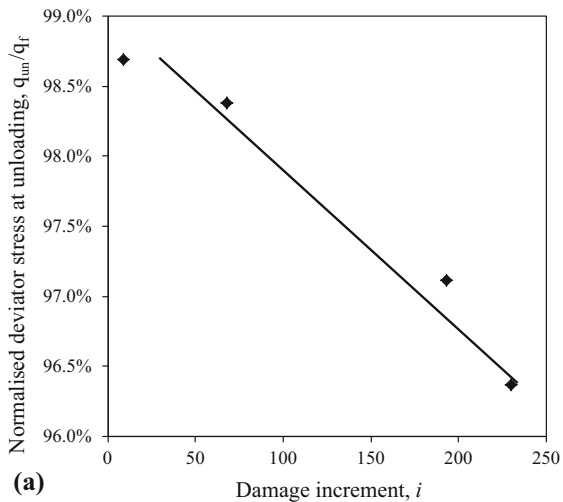
Fig. 11 Percentage increase in rock peak strength after cyclic loading versus normalised deviator stress at upper limit of cyclic loading

Fig. 10 a Normalized peak deviator stress at unloading versus damage increment at failure; b normalized residual deviator stress at unloading versus axial strain at failure

6 Conclusions

An experimental investigation was carried out using Hawkesbury sandstone to predict peak strength during triaxial cyclic compressive testing. A Methodology for successful cyclic loading testing was devised including the implementation of an innovative peak strength prediction method. The results give a strong indication that the peak strength prediction method was successful for all the tests undertaken in this study.
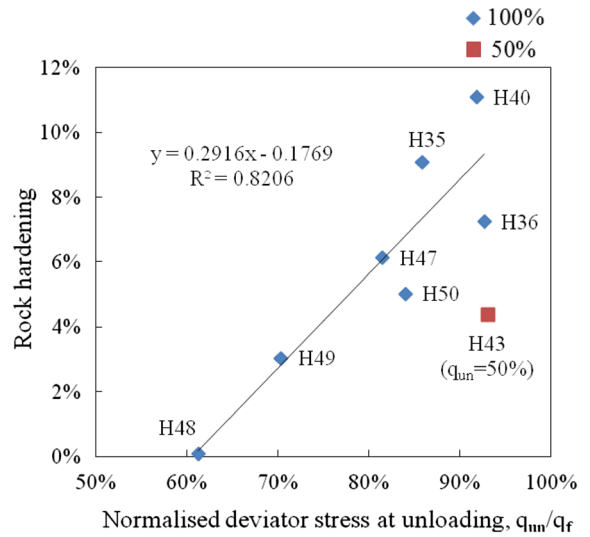
The normalized deviator stresses for cyclic loading, q_{un}/q_f, and the normalized residual stresses, q_r/q_f, were used to develop a preliminary damage model. The model confirmed that a higher normalized stress at unloading will result in failure after fewer cycles. Moreover, it was found that if the specimen experience more axial strain before the failure point, the failure will be more brittle, i.e. the difference between peak strength and residual strength increases.

This study confirmed that the peak strength of the sandstone is altered by cyclic loading. Dependent upon the stress level where the cyclic loading commences the rock may become weaker or stronger as a result of cyclic loading. It was found that, during cyclic loading fatigue failure happens if the stress level in cyclic loading is equal or higher that 94 % of rock peak strength. If the cyclic loading deviator stress is lower than this critical boundary, the peak strength obtained by post-monotonic loading may increase up to 11 %. This increase in peak strength is directly proportional to the normalised deviator stress in unloading; the amount of increase in peak strength decreases with a decrease in cyclic loading deviator stress.

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