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ORIGINAL ARTICLE

Nonlinear visco-elastic and accelerating creep model for coal under conventional triaxial compression

Sheng-Qi Yang · Peng Xu · Tao Xu

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Abstract On basis of triaxial creep experimental results of coal with various deviatoric stress levels, a new nonlinear visco-elastic and accelerating creep model is put forward. First, Burgers visco-elastic triaxial creep model is deduced and used to identify the creep parameters by the discrimination criterion of the square of correlation coefficient (R^2) and sum of least error square (Q). In Burgers visco-elastic triaxial creep model, the visco-elastic modulus G_2 of coal decreases but the visco coefficient η_1 and η_2 all increase with increasing time, which has distinct time scale effect. The Burgers visco-elastic triaxial creep model fails in predicting long-term deformation characteristics. Then by introducing a time index, a nonlinear visco-elastic triaxial creep model is put forward, which can predict better for long-term deformation behavior of coal. R^2 identified by nonlinear visco-elastic triaxial creep model is higher, but Q identified by nonlinear visco-elastic triaxial creep model is smaller 1 order than that by Burgers triaxial creep model. However, Burgers and nonlinear visco-elastic triaxial creep model is very difficult to describe the accelerating creep stage of coal. Therefore in the end, a new nonlinear accelerating creep model is put forward in order to describe the complete creep curve of coal. The nonlinear accelerating creep model has a better agreement with the experimental data.

Keywords Coal · Triaxial compression · Nonlinear · Visco-elastic triaxial creep model · Accelerating creep model

1 Introduction

Coal is a kind of special rock material containing many minor microscopic fissures. In the past, a lot of short-term strength, deformation and acoustic emission (AE) behaviors of coal have been experimentally investigated (Medhurst and Brown 1998; Feng et al. 2010; Li et al. 2011; Xie et al. 2012). However, very fewer test results have been reported regarding creep behaviors of coal under uniaxial and conventional triaxial compression. Cao et al. (2007) carried out the experiment on outburst-hazardous coal under uniaxial compression creep to study the AE characteristics of coal at different creep stages. But the creep time of coal was too short, only 4 h. Wang et al. (2010) used self-creep testing apparatus to carry out triaxial creep tests on outburst coal specimens from Songzao coal mine, which found that the attenuation creep property

S.-Q. Yang (✉) · P. Xu
State Key Laboratory for Geomechanics and Deep Underground Engineering, School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou 221116, People's Republic of China
e-mail: yangsqi@hotmail.com

T. Xu
Center for Rock Instability and Seismicity Research, Northeastern University, Shenyang 110819, People's Republic of China

was shown in the conditions of less than its long-term strength load, which non-attenuation creep property for the greater conditions. Yin et al. (2011) carried out the triaxial creep experiment of coal containing gas under unloading condition. The results showed that unloading could accelerate the failure and the macrofissure induced by unloading could increase the gas flow velocity significantly. Xu et al. (2009) analyzed the influence of creep on permeability of gas-bearing coal by means of experimental equipments of coal seepage. The experimental results showed that with different temperatures and effective stress coupling, the influence of creep on coal permeability was also different. The above creep test results of coal build a better foundation for the construction of creep models of coal.

In recent years, a few progresses on triaxial creep model of coal have been reported. Based on triaxial creep test results of out burst prone coal, by introducing a visco-elasto-plastic body which could describe the deformation property of non-Newtonian fluids, Wang et al. (2010) put forward the visco-elasto-plastic creep model of out burst prone coal combined with a Poyting–Thomson model. On the basis of the triaxial creep experimental data of coal containing gas, Yin et al. (2011) modified Chaboche viscoplasticity constitutive relation, which could describe the short term creep failure of coal containing gas. However, the previous triaxial creep models do not take into account time scale effect and verify the predicted effect of triaxial creep model with various differential stress levels, therefore which can not be used to predict long-term creep deformation of coal. Up to now, the theories of triaxial creep model of coal materials have not been very mature. Nowadays, triaxial creep model of coal is still one of the most difficult problems in the rock rheological mechanics study.

Therefore in this research, to better understand triaxial creep behavior and predict long-term deformation behavior of deep coal tunnel, triaxial creep tests were conducted for coal located in Xingan mine of China with various axial differential stress levels. Based on visco-elastic triaxial creep test results of coal, the variance of creep parameters with time were analyzed in detail. The paper focuses on modeling of triaxial creep deformation by putting forward nonlinear visco-elastic and accelerating creep model for coal under conventional triaxial compression, and evaluating the accuracy and reasonability of identified triaxial creep parameters.

2 Triaxial creep test result of coal

The coal located in the deep coal tunnel (at a depth of about 700 m) of Xingan mine was chosen for the test object. Coal blocks were collected from in-site engineering and then were further machined in the laboratory. We made cylindrical coal specimen with the length of 100 mm and the diameter of 50 mm according to ASTM requirement (1996) for creep testing. The coal specimens have no obvious fractures and cracks, but exist some minor microscopic fissures in accordance with the scanning electric microscopic (SEM) results (Fig. 1). At the same time, coal intrinsically has anisotropy due to its cleat structure; therefore, we machined the coal specimens along the same direction from the same block in order to avoid the influence of anisotropy on the creep experimental results of coal.

We carried out the triaxial creep tests for coal specimen on a rock servo-controlled triaxial rheology testing system. The testing system is made up of a loading system, constant-stability pressure equipment, a hydraulic pressure transferring system, pressure chamber equipment, a water pressure system and an automatic data collection system. This testing system can be used to perform hydrostatic pressure tests, conventional triaxial compression tests under drained or undrained conditions, triaxial seepage tests, triaxial creep tests and chemical corrosion tests.

The triaxial creep tests were conducted in the laboratory with constant temperature and humidity to avoid the influence of the disturbance of surrounding environment on the experimental results. During the triaxial compression tests, the confining pressure was firstly increased to the pre-confirmed value at a rate of 0.5 MPa/s in order to ensure coal specimen was in a uniform hydrostatic stress. Then the axial differential stress was applied on the rock specimens at a constant rate of 0.127 MPa/s. In accordance with short-term triaxial strength of coal, the axial differential stress level of triaxial creep test can be determined. The confining pressure is set to 30 MPa, and the corresponding short-term triaxial strength of coal is about 82.43 MPa, denoting with the axial differential stress. Therefore two axial differential stress levels, i.e. 73

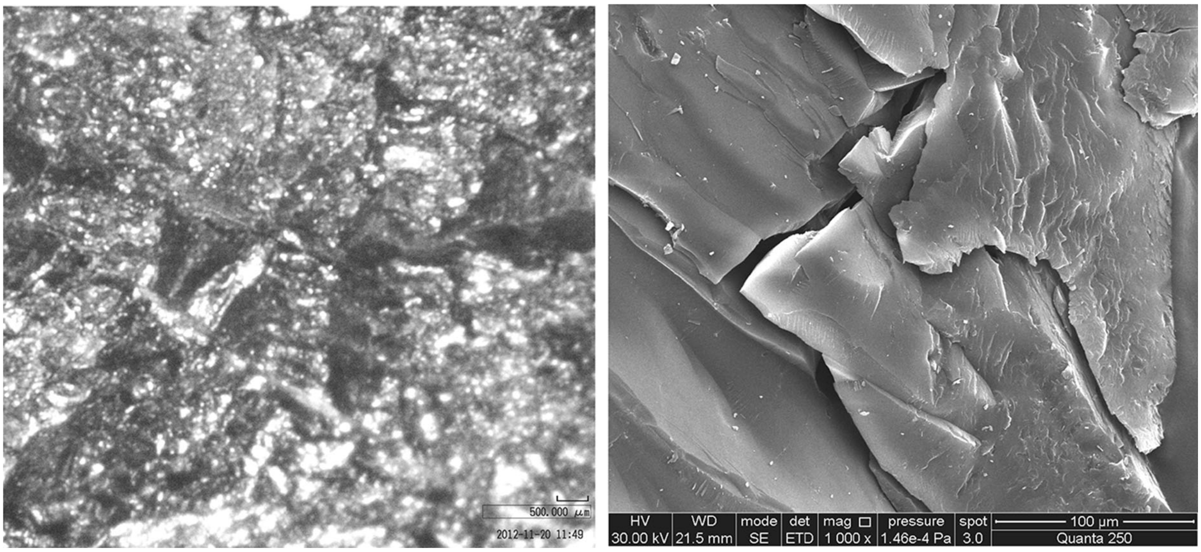


Fig. 1 Microscopic observation of tested coal specimen in the present specimen

and 75 MPa, were used to investigate the triaxial creep behavior of coal. When testing, at first, a constant confining pressure (30 MPa) was loaded on coal specimen, and then applied the corresponding axial differential stress level. The relation between axial strain and time were obtained at varying axial differential stress levels.

Figure 2a shows obtained typical visco-elastic triaxial creep test results of coal at constant axial differential stress level. In terms of Fig. 2a, instant deformation occurs in the coal at constant confining pressure. With the increase of time, the axial strain of coal specimen also increases step by step, but axial creep rate decreases gradually. After the time increases to 50 h (about 2 days), the coal specimen transforms from primary creep stage to steady-state creep stage, and the axial deformation increases linearly with time by a constant steady-state creep rate.

It should be noted that the coal specimen does not occur the creep failure at the axial differential stress of 73 MPa and only two creep stages (i.e. primary creep stage and steady-state creep stage) occurs. However, when the axial differential stress level is increased to 75 MPa, the creep failure occurs as illustrated in Fig. 2b, which shows a different creep behavior compared with that at the axial differential stress of 73 MPa.

From Fig. 2b, it can be seen that when the confining pressure equals to 30 MPa and the axial differential stress reaches 75 MPa, the axial deformation of coal specimen undergoes three stages of primary creep (AB), steady-state creep (BC) and accelerating creep (CD) and corresponds to primary creep rate, steady-state creep rate and accelerating creep rate, respectively in the axial direction. At primary creep stage, axial creep strain rate decreases to a constant value quickly with the increase of time. At steady-state creep stage, axial creep strain rate keeps basically a constant with the increase of time. At accelerating creep stage, axial creep strain rate increases sharply with the increase of time and the creep failure occurs in coal specimen.

3 Burgers visco-elastic triaxial creep model and time scale effect

Based on triaxial creep curves of coal in Fig. 2a, it can be seen that the coal has distinct visco-elastic behavior with a steady-state creep rate higher than zero. To describe visco-elastic triaxial creep behavior of coal, two basic components, i.e. an elastic component (Hookean body) and a visco-elastic component (Newton body), are often connected in series or parallel, which can form all kinds of visco-elastic creep models, such as Maxwell model (Hookean body and Newton body are connected in series), Kelvin model (Hooker body and Newton body are connected in parallel). In this paper, the Burgers visco-elastic model

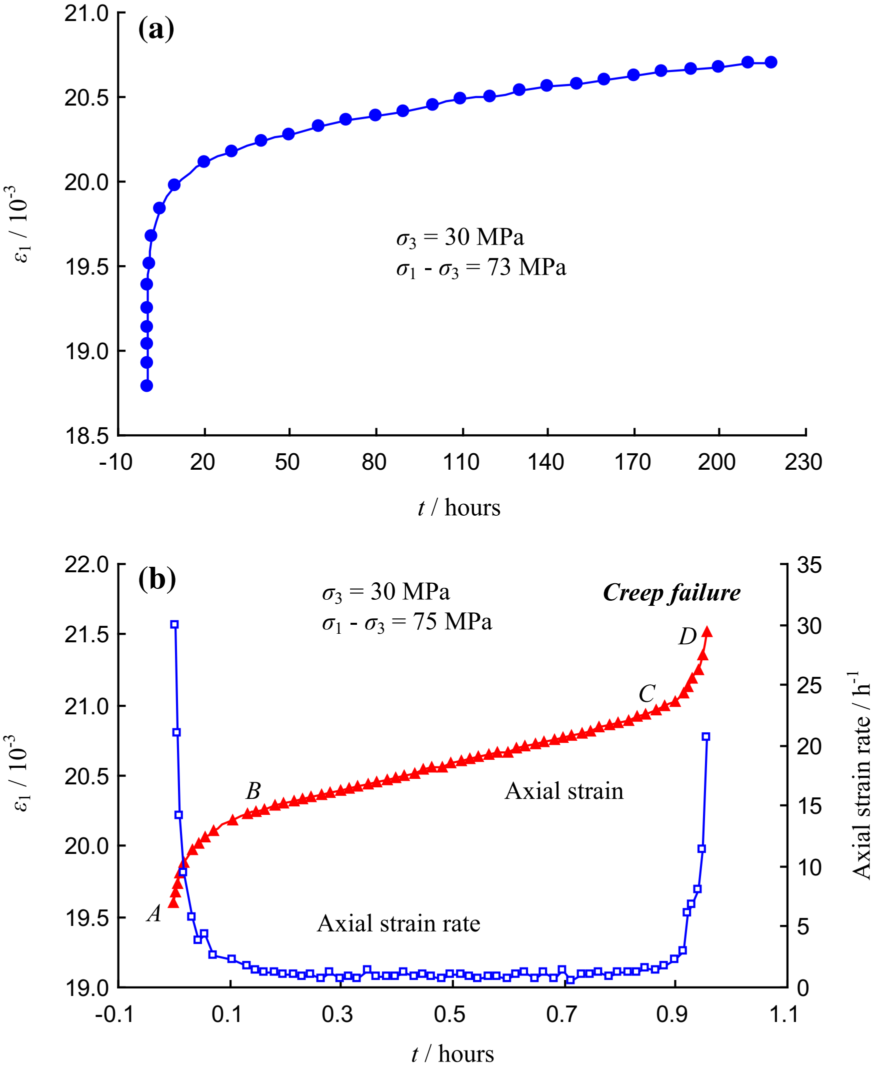


Fig. 2 Typical visco-elastic and accelerating triaxial creep test curves of coal specimens at different deviatoric stresses

is chosen to study the visco-elastic triaxial creep behavior of coal. The creep equation of Burgers model under one-dimensional stress state (i.e. uniaxial compression state) is expressed as

$$\varepsilon_1 = \frac{\sigma}{E_1} + \frac{\sigma}{E_2}\left[1 - \exp\left(-\frac{E_2}{\eta_1}t\right)\right] + \frac{\sigma}{\eta_2}t \tag{1}$$

where E_1 is the elastic modulus; E_2 is the visco-elastic modulus; η_1 and η_2 are all the visco coefficient.

However, engineering rock mass is usually located in a triaxial compressive stress state. Therefore, we will deduce further the creep equation of rock under three-dimensional stress state based on the above mentioned creep equation under one-dimension stress state (Bonini et al. 2009).

When rock is in a three-dimension stress state, its internal stress tensor may be decomposed into a spherical stress tensor σ_m and a differential stress tensor S_{ij},

$$\begin{cases} \sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3}\sigma_{kk} \\ S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_m = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk} \end{cases} \tag{2}$$

where, δ_{ij} is Kronecker function. Thus, in accordance with Eq. (3), the following equation holds

$$\sigma_{ij} = S_{ij} + \delta_{ij}\sigma_m \tag{3}$$

The spherical tensor σ_m can only change the volume of object, but not its shape. On the other hand, the differential stress tensor S_{ij} can only change the shape of object, but not its volume. Therefore, strain tensor can also be decomposed into a spherical strain tensor ε_m and a differential strain tensor e_{ij},

$$\begin{cases} \varepsilon_m = \frac{1}{3}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \frac{1}{3}\varepsilon_{kk} \\ e_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_m = \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk} \end{cases} \quad (4)$$

Therefore, the following equation can also be obtained.

$$\varepsilon_{ij} = e_{ij} + \delta_{ij}\varepsilon_m \quad (5)$$

Suppose the shear modulus of rock is G, and bulk modulus of rock is K, the following relation relates these two properties

$$G = \frac{E}{2(1+\mu)} \quad (6)$$

$$K = \frac{E}{3(1-2\mu)} \quad (7)$$

where, E and μ are, respectively, the elastic modulus and Poisson's ratio of the rock material.

For the Hookean Body under a three-dimension stress state, we have

$$\sigma_m = 3K\varepsilon_m \quad (8)$$

$$S_{ij} = 2Ge_{ij} \quad (9)$$

For a visco-elastic body under three-dimension stress state, its creep equation can be derived in accordance with the creep equation under uniaxial stress state, the constant differential stress of rock, written as

$$\begin{cases} \varepsilon_m = \frac{1}{3K}\sigma_m \\ e_{ij} = \frac{S_{ij}}{2G_1} + \frac{S_{ij}}{2G_2}\left[1 - \exp\left(-\frac{G_2}{\eta_1}t\right)\right] + \frac{S_{ij}}{2\eta_2}t \end{cases} \quad (10)$$

Therefore, the creep equation of rock under conventional triaxial compression stress condition ($\sigma_2 = \sigma_3$) can be written as

$$\varepsilon_1 = \frac{\sigma_1 + 2\sigma_3}{9K} + \frac{\sigma_1 - \sigma_3}{3G_1} + \frac{\sigma_1 - \sigma_3}{3G_2}\left[1 - \exp\left(-\frac{G_2}{\eta_1}t\right)\right] + \frac{\sigma_1 - \sigma_3}{3\eta_2}t \quad (11)$$

In accordance with triaxial creep test curves, the square of correlation coefficient (R^2) and sum of least error square (Q) are simultaneously chosen as discrimination criterion to evaluate the creep model parameters obtained by nonlinear least square method (NLSM). It needs to be noted that R^2 more tends to 1.0 and Q more approaches to zero, which means that the theoretical model agrees better with the experimental data. Therefore for a given creep model and experimental data, we always can obtain a maximum R^2 and a minimum Q.

The creep parameters in Eq. (11) for Burgers model can be identified by the following method. First, the bulk modulus K and shear modulus G_1 can be obtained by ε_0, where ε_0 is instant axial strain for each axial differential stress level. Take the axial differential stress level $\sigma_1 - \sigma_3 = 73$ MPa as an example, ε_0 is equal to 18.782×10^{-3}. Thus, we can obtain the following equation,

$$\varepsilon_0 = \frac{\sigma_1 + 2\sigma_3}{9K} + \frac{\sigma_1 - \sigma_3}{3G_1} = 18.782 \times 10^{-3} \quad (12)$$

In accordance with Eq. (12) and Eqs. (6–7), we can obtain the bulk modulus K and shear modulus G_1 of coal are respectively 4.131 and 1.690 GPa. After confirming the parameters K and G_1, other three creep parameters G_2, η_1 and η_2 can be obtained by direct iterative method using NLSM. According to m groups of experimental data (t_i, ε_i) (i = 1,…, m), a group of initial approximate values (G_2^0, η_1^0 and η_2^0) is firstly given, and then many iterations are carried out on basis of initial approximate values until the creep parameters satisfies required precision by R^2 and Q as discrimination criterion. When R^2 and Q, respectively, reach a maximum and minimum value, the corresponding creep parameter values are that we need.

By adopting the above method, the creep model parameters with eight various time scales (i.e. $t = 20$, 50, 80, 110, 140, 170, 200 and 218 h) are identified to analyze the influence of long-and-short of creep test time on Burgers creep model parameters. Table 1 lists the Burgers visco-elastic triaxial creep model parameters of coal with different time scales. A maximum R^2 and a minimum Q are also obtained. Notice, K and G_1 are constant for a given axial differential stress level, which are independent on time scale. However, the creep parameters G_2, η_1 and η_2 change with the increase of time scale, as shown Fig. 3. From Fig. 3,

Table 1 Burgers visco-elastic creep model parameters of coal with axial differential stress level of 73 MPa ($\sigma_3 = 30$ MPa)

Time scale (h)	K (GPa)	G_1 (GPa)	G_2 (GPa)	η_1 (GPa h)	η_2 (GPa h)	R^2	Q
20	4.131	1.690	29.24	6.98	882.15	0.974	6.38×10^{-6}
50	4.131	1.690	25.22	8.71	1967.29	0.970	9.50×10^{-6}
80	4.131	1.690	23.35	9.86	2945.65	0.970	1.065×10^{-5}
110	4.131	1.690	22.14	10.79	3862.63	0.970	1.114×10^{-5}
140	4.131	1.690	21.28	11.59	4696.98	0.970	1.126×10^{-5}
170	4.131	1.690	20.61	12.31	5474.32	0.970	1.130×10^{-5}
200	4.131	1.690	20.05	13.00	6211.93	0.970	1.133×10^{-5}
218	4.131	1.690	19.73	13.45	6700.88	0.970	1.142×10^{-5}
Average	4.131	1.690	22.70	10.84	4092.73	0.971	1.037×10^{-5}

we can see that with the increase in time, the visco-elastic shear modulus G_2 of coal decrease step by step and the variance tends to slow down. But it can be seen that the visco coefficient η_1 of coal increases step by step and the variance tends to slow down. However, it can be seen that the visco coefficient η_2 of coal increase with time, which has almost a linear behavior.

From the above analysis, we conclude that Burgers visco-elastic triaxial creep model parameters have obvious time scale effect, i.e. identified visco-elastic triaxial creep model parameters of coal are distinctly different with various time scales. Therefore, the Burgers visco-elastic triaxial creep parameters identified by short-term creep test results are very difficult

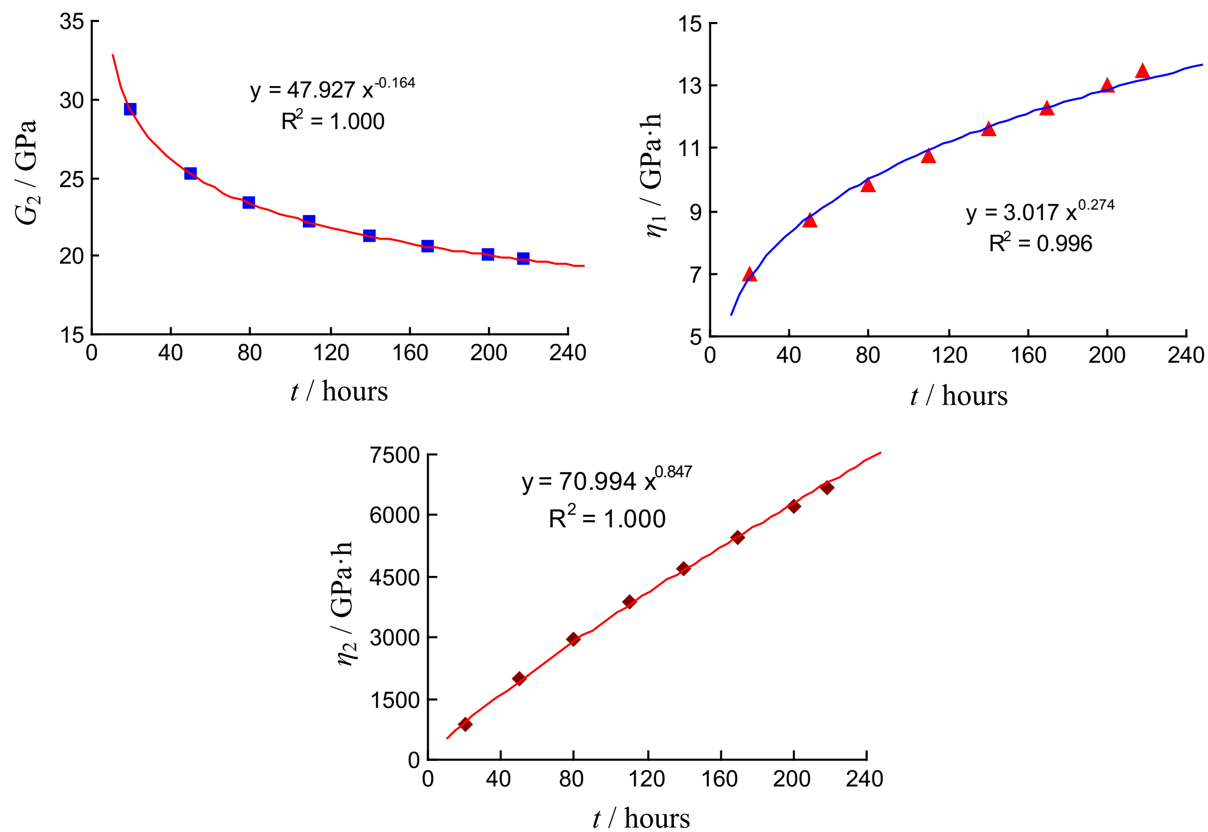


Fig. 3 Influence of time on creep parameters in Burgers visco-elastic triaxial creep model

to predict accurately long-term creep deformation, which can be further illustrated by the following description.

The triaxial creep experimental data in 80 and 170 h are used to inverse the creep parameters on the basis of Burgers triaxial creep model, while the triaxial creep experimental data in 80–218 and 170–218 h are used to verify the predicted effect of Burgers triaxial creep model. Figure 4a shows the comparison between Burgers triaxial creep model curves on basis of inversing parameters with the experimental data within time scale of 80 h and long-term visco-elastic triaxial creep deformation (80–218 h). Figure 4b depicts the comparison between Burgers triaxial creep model curves on basis of inversing parameters with the experimental data within time scale of 170 h and long-term visco-elastic triaxial creep deformation (170–218 h). From Fig. 4a, we can see that Burgers triaxial creep model is not very good for predicting visco-elastic creep behavior in 80–218 h, and the predicted creep deformation is distinctly higher than the experimental value. However by comparing Fig. 4a with Fig. 4b, we can conclude that Burgers triaxial creep model curve on basis of inversing parameters within the larger time scale is in a better agreement with the experimental result.

4 Nonlinear visco-elastic triaxial creep model

From Fig. 4, it can be seen that Burgers visco-elastic triaxial creep model (Eq. 11) with five parameters is very difficult to describe accurately visco-elastic triaxial creep characteristics of coal. The main reason for this is that four components in Burgers visco-elastic creep model are all linear, which can not make a reasonable reflection for nonlinear visco-elastic creep behavior of coal. Taking into account that the Burgers visco-elastic triaxial creep model is not very good for predicting long-term deformation characteristics of coal specimen (Fig. 4), therefore here a modified nonlinear visco-elastic triaxial creep model is constructed by introducing a time index to better predict the long-term deformation behavior of coal than that by linear visco-elastic triaxial creep model. The nonlinear visco-elastic triaxial creep model can be shown as follows.

$$\varepsilon_1 = \frac{\sigma_1 + 2\sigma_3}{9K} + \frac{\sigma_1 - \sigma_3}{3G_1} + \frac{\sigma_1 - \sigma_3}{3G_2}\left[1 - \exp\left(-\frac{G_2}{A}\left(\frac{t}{t_0}\right)^n\right)\right] \tag{13}$$

where t_0 is a reference time, which is set to 1.0 (the units of t_0 are the same as those to t), n is time index and A is a creep parameter.

The five parameters in the above nonlinear triaxial creep model (Eq. 13) can be identified by the same method as presented in Burgers triaxial creep model. Table 2 lists nonlinear triaxial creep model parameters of coal with different time scales. From Table 2, it can be seen that the nonlinear triaxial creep model parameters are not distinctly dependent on time scale, which is different from that for Burgers triaxial creep model. The above analysis shows that nonlinear triaxial creep model parameters identified by short-term creep test results can be used to predict accurately long-term visco-elastic triaxial deformation. In order to understand this

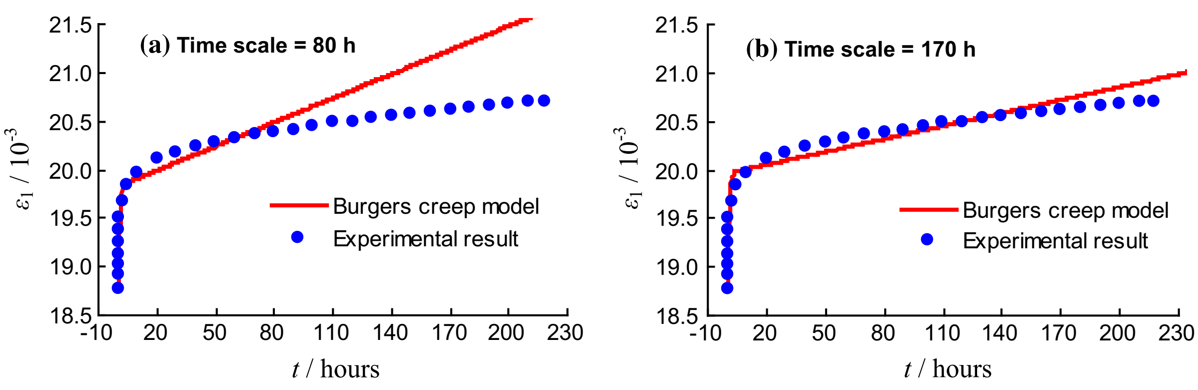


Fig. 4 Comparison between Burgers triaxial creep model curves on basis of inversing parameters with the experimental data within time scale of 80, 170 h and long-term visco-elastic triaxial creep deformation

Table 2 Nonlinear visco-elastic creep model parameters of coal with axial differential stress level of 73 MPa ($\sigma_3 = 30$ MPa)

Time scale (h)	K (GPa)	G_1 (GPa)	G_2 (GPa)	A (GPa)	n	R^2	Q
20	4.131	1.690	17.74	22.31	0.431	0.995	1.11×10^{-6}
50	4.131	1.690	15.73	24.49	0.386	0.996	1.32×10^{-6}
80	4.131	1.690	14.46	25.85	0.357	0.996	1.55×10^{-6}
110	4.131	1.690	13.41	26.91	0.334	0.995	1.77×10^{-6}
140	4.131	1.690	12.39	27.84	0.314	0.995	2.00×10^{-6}
170	4.131	1.690	11.43	28.65	0.298	0.994	2.15×10^{-6}
200	4.131	1.690	10.54	29.35	0.284	0.994	2.24×10^{-6}
218	4.131	1.690	10.05	29.71	0.277	0.994	2.23×10^{-6}
Average	4.131	1.690	13.22	26.89	0.340	0.995	1.80×10^{-6}

point more deeply, Fig. 5a, b, respectively, show the comparison between nonlinear triaxial creep model curves on the basis of inversing parameters with the experimental data within time scale of 80, 170 h and long-term visco-elastic triaxial creep deformation (80–218 h) and (170–218 h). From Fig. 5a, we can see that the predicted creep deformation by nonlinear triaxial creep model within time scale of 80 h is a little lower than the experimental value, while when time scale reaches 170 h, the nonlinear triaxial creep model is much better for predicting visco-elastic creep behavior.

In order to further illustrate the dominance of the nonlinear triaxial creep model by comparing with Burgers triaxial creep model, Fig. 6 shows the comparison of R^2 and Q identified by nonlinear and Burgers triaxial creep model with various time scales, respectively. From Fig. 6, it can be seen that R^2 for nonlinear triaxial creep model is higher than that for Burgers triaxial creep model. However, Q for

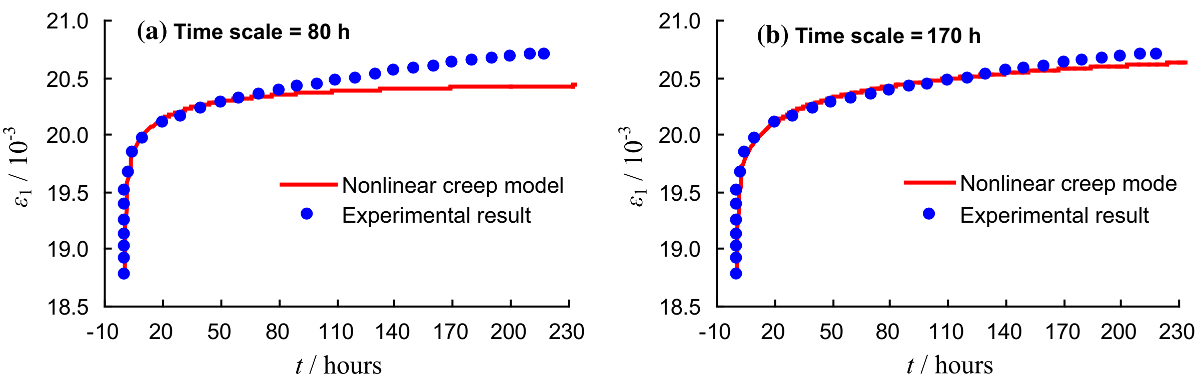
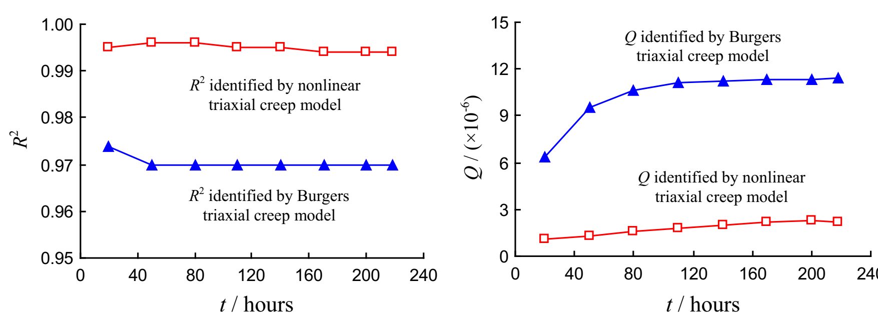


Fig. 5 Comparison between nonlinear triaxial creep model curves on basis of inversing parameters with the experimental data within time scale of 80, 170 h and long-term visco-elastic triaxial creep deformation

Fig. 6 Comparison of R^2 and Q identified by nonlinear and Burgers triaxial creep model, respectively



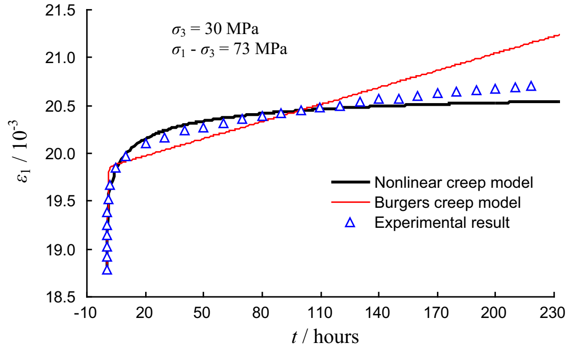


Fig. 7 Comparison between nonlinear, Burgers triaxial creep model (the creep parameters are the average value with various time scale) and experimental result

nonlinear triaxial creep model is smaller by one order than that obtained by Burgers triaxial creep model. Moreover, there are five parameters in the nonlinear triaxial creep model, which is the same as those in Burgers triaxial creep model. Besides, nonlinear triaxial creep model can predict very well for long-term triaxial deformation tendency of coal, therefore, nonlinear triaxial creep model of coal is better than Burgers triaxial creep model.

Figure 7 illustrates the comparison between nonlinear triaxial creep model and experimental result, and the creep parameters are the average value with various time scales, as listed in Table 2. At the same time, the comparison between Burgers triaxial creep model using the average creep parameters in Table 1 and experimental results is also plotted in Fig. 7. From Fig. 7, it can be seen clearly that nonlinear triaxial creep model using the average values as creep parameters has a good agreement with experimental result. However, Burgers triaxial creep model using the average values as creep parameters is not in a good agreement with experimental result, which is due to that Burgers triaxial creep model parameters has a certain time scale effect.

5 Nonlinear accelerating creep model

From Fig. 7, it can be seen that Burgers and nonlinear visco-elastic triaxial creep model can only describe the primary creep stage and steady-state creep stage, but can not describe the accelerating creep stage. Therefore, it is necessary to put forward a new nonlinear accelerating creep model in order to describe the complete creep curve of coal as shown in Fig. 2b.

Up to now, there are mainly two methods to construct non-linear accelerating creep models of rock. One method is to replace conventional linear rheological components such as Hooker body, Newton body and Plastic body with nonlinear components (Bahar et al. 1995; Yang et al. 2007). The other method is to adopt new theory (such as fracture and damage theory etc.) to construct creep model of rock (Grgic et al. 2003; Enrico and Tsutomu 2001; Pellet et al. 2005; Fabre and Pellet 2006; Sterpi and Gioda 2009). The proposed rheological models by the above two methods can all describe the accelerating creep stage of rock material. In this research, the first method is adopted to construct a new nonlinear accelerating creep model of coal.

Heap et al. (2009) depicted an evolution process of spatial AE hypocenters during the complete triaxial creep test of Darley Dale sandstone. A total of 1877 AE hypocenters were located during the experiment, of which 592 occurred during the primary creep phase, 702 during secondary creep and 583 during tertiary creep. The experimental result from Heap et al. (2009) showed that: (1) diffuse and distributed during primary creep, (2) distributed with some suggestion of localization or ''mixed mode'' during secondary creep and (3) increasingly localized along the eventual fault plane during tertiary creep. Therefore, the main reason that rock material produces nonlinear creep deformation can be described as follows. Due to the long-term action of a constant axial differential stress level, microscopic crack in the rock will undergo a process of propagation and fracture with the increase of time. So the nonlinear creep deformation of rock material is actually a function of time. By assuming that the nonlinear creep deformation of rock is the Weibull distribution function of time, a new nonlinear rheological component (Fig. 8a) is put forward, which can describe accurately the accelerating creep deformation of rock material. Figure 8b shows the relation between the creep deformation at a constant stress level and time, and the creep equation under one-dimensional stress state is

$$\varepsilon_1 = \frac{\sigma}{E}\left[1 - \exp\left(-\left(\frac{H(t-t_2)}{t_c - t_2}\right)^{\gamma}\right)\right] \tag{14}$$

where E is elastic modulus, t_2 is the time from steady-state creep stage to accelerating creep stage; t_c is the time of creep failure; γ is time index. In Eq. (14), H can be expressed as follows.

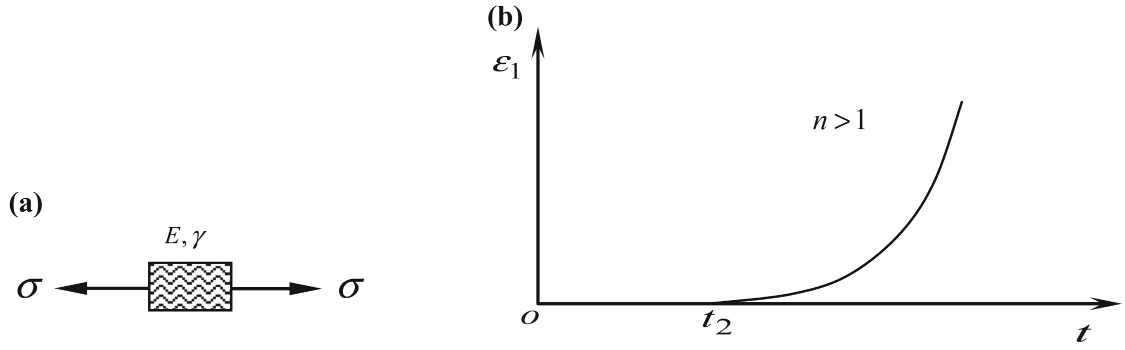


Fig. 8 Nonlinear rheological component and its creep curve. **a** Nonlinear rheological component, **b** Creep curve of nonlinear rheological component

$$H(t-t_2)=\begin{cases}0, & t\le t_2\\ t-t_2, & t>t_2\end{cases} \tag{15}$$

In accordance with Eq. (14) and Fig. 8, it can be seen that with the increase of time, the axial strain and axial strain rate of rock take on a nonlinear increase; therefore, the nonlinear rheological component can reflect the behavior of nonlinear accelerating creep of rock. If the nonlinear rheological component and Burgers visco-elastic model are connected in series, a new nonlinear accelerating rheological model (Fig. 9) can be obtained.

The creep equation of nonlinear accelerating rheological model under one-dimensional stress state (i.e. uniaxial compression state) is

$$\varepsilon_1=\frac{\sigma}{E_1}+\frac{\sigma}{E_2}\left[1-\exp\left(-\frac{E_2}{\eta_1}t\right)\right]+\frac{\sigma}{\eta_2}t+\frac{\sigma}{E_3}\left[1-\exp\left(-\left(\frac{H(t-t_2)}{t_c-t_2}\right)^{\gamma}\right)\right] \tag{16}$$

In accordance with the same method as deducing Burgers visco-elastic triaxial creep model under three-dimensional stress state, we also can obtain the nonlinear accelerating creep equation of rock under conventional triaxial compression stress condition ($\sigma_2=\sigma_3$)

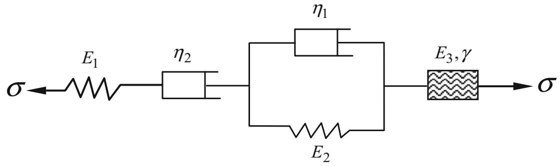


Fig. 9 Nonlinear accelerating rheological model

$$\varepsilon_1=\frac{\sigma_1+2\sigma_3}{9K}+\frac{\sigma_1-\sigma_3}{3G_1}+\frac{\sigma_1-\sigma_3}{3G_2}\left[1-\exp\left(-\frac{G_2}{\eta_1}t\right)\right]+\frac{\sigma_1-\sigma_3}{3\eta_2}t+\frac{\sigma_1-\sigma_3}{3G_3}\left[1-\exp\left(-\left(\frac{H(t-t_2)}{t_c-t_2}\right)^{\gamma}\right)\right] \tag{17}$$

The seven parameters in the above nonlinear accelerating creep model (Eq. 17) can be identified by the following method. First using the creep experimental data in time interval (0, t_2), the five parameters K, G_1, G_2, η_1 and η_2 can be confirmed as the same method presented in Burgers triaxial creep model.

After confirming the five parameters K, G_1, G_2, η_1 and η_2, when $t=t_c$ and $\varepsilon_1=\varepsilon_c$, the following equation can be obtained.

$$\varepsilon_c-\frac{\sigma_1+2\sigma_3}{9K}-\frac{\sigma_1-\sigma_3}{3G_1}-\frac{\sigma_1-\sigma_3}{3G_2}\left[1-\exp\left(-\frac{G_2}{\eta_1}t_c\right)\right]-\frac{\sigma_1-\sigma_3}{3\eta_2}t_c=\frac{\sigma_1-\sigma_3}{3G_3}\left(1-e^{-1}\right) \tag{18}$$

In this research, according to Fig. 2, we can confirm that t_2 equals to 0.8306 h and t_c equals to 0.9556 h. Combined with Eq. (18), we can confirm that $G_3=32.05$ GPa. And then by using a certain creep experimental data in time interval (t_2, t_c), we can obtain the creep parameter γ. Thus, we confirm seven parameters K, G_1, G_2, η_1, η_2, G_3 and γ in nonlinear accelerating creep model, which are listed in Table 3.

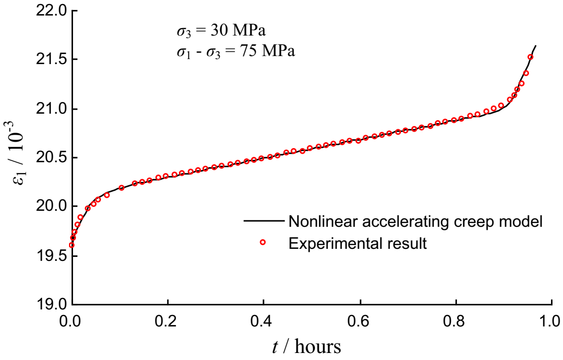


Fig. 10 Comparison between nonlinear accelerating creep model and experimental results of coal

Figure 10 shows the comparison between nonlinear accelerating creep model and experimental results of coal. From Fig. 10, it can be seen that the nonlinear accelerating creep model is in a good agreement with experimental result, which shows the rightness and reasonability of nonlinear accelerating creep model in this research.

6 Conclusions

(1) The tested coal shows good visco-elastic deformation behavior at differential stress of 73 MPa and confining pressure of 30 MPa. Based on obtained visco-elastic triaxial creep experimental results of coal, Burgers visco-elastic triaxial creep model are firstly used to identify the creep parameters by R^2 and Q. Burgers triaxial creep model parameters have a strong time scale effect, which shows that the visco-elastic shear modulus G_2 of rock decreases but the visco coefficient η_1 and η_2 increases with increasing time.

(2) Taking into account that Burgers triaxial creep model is not very good for predicting long-term deformation behavior of coal, a nonlinear visco-elastic triaxial creep model is constructed by introducing a time index, which can predict better for long-term deformation behavior of coal than that by Burgers triaxial creep model. R^2 identified by nonlinear triaxial creep model is higher but Q is smaller by one order than that by Burgers triaxial creep model. The predicted curve by nonlinear triaxial creep model agrees very well with experimental data, which shows the reasonability of nonlinear visco-elastic triaxial creep model.

(3) Burgers and nonlinear visco-elastic triaxial creep models can only describe the primary creep stage and steady-state creep stage, but can not describe the accelerating creep stage. Therefore in order to describe the complete creep curve of coal, a new nonlinear accelerating creep model is put forward, which is in a good agreement with experimental result.

Finally, it should be pointed out that this research only constructs the nonlinear visco-elastic and accelerating creep model for coal under conventional triaxial compression, which the reasonability of constructed nonlinear triaxial creep model is also testified. However, the creep mechanical of rock material is dependent to a lot of factors, such as the confining pressure, water pressure, temperature and humidity, etc. Therefore, in the future, we will carry out more triaxial creep tests for rock material and strengthen nonlinear triaxial creep model of rock material by taking into account more affected factors.

Table 3 Nonlinear accelerating creep model parameters of coal with axial differential stress level of 75 MPa (σ_3 = 30 MPa)

Specimen	K (GPa)	G_1 (GPa)	G_2 (GPa)	η_1 (GPa h)	η_2 (GPa h)	G_3 (GPa)	γ
Coal	4.054	1.658	49.11	1.357	26.01	32.05	5.171

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