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ORIGINAL ARTICLE

Thermal variations of water in the Nam Song stream/Mekong river: I. A mathematical model

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Abstract River water temperature affects nearly every physical property related to water quality management. Therefore, anthropogenic change of water temperature is often undesirable. The objectives of this study were to develop an analytical model describing the thermal regime of a river–dam system. The model will be tested in a second manuscript using data from the Nam Song Basin in East Asia. The model incorporates three stages: (a) short time water heating process, (b) longer time water heating and (c) water cooling. Unified and continuous presentation of the above three solutions describes the complete thermal cycle. It showed that the longer the travel time from a dam (the Nam Song dam in this study) to the measuring point (slow flow rate), the larger the amount of energy absorbed and the higher its associated water temperature. According to the model the temperature of a water volume increases at a rate of ∼1.2 °C/km downstream when water flow rate is 1 km/h. When water flows at a rate of 4 km/h, temperature increases at a rate of ∼0.25 °C/km downstream. The model turns out to be a simple case of the kinematic wave theory, which is much easier to solve than large-scale computer simulation. For successful test of the model the specific parameters of the Nam Song River were used. Naturally, these parameters are changed from one river to another. Thus, to use the model for prediction of thermal regime in other rivers, their local specific parameters should be taken.

Introduction

Water temperature has always been considered one of the most important factors determining the geographic distribution of fish and other aquatic organisms (Bartholow 1989; Chen et al. 1998; Bashar and Stefan 1993). Most aquatic organisms are cold blooded, so their body temperatures are the same as the temperature of the surrounding water. Human activity affect stream-water temperature due to impoundments, industrial use, irrigation, and global warming (LeBlanc et al. 1997). Water temperature affects nearly every physical property related to water quality management, and unfortunately change of water temperature is often undesirable.

Water withdrawal for irrigation purposes often reduces stream flow, when flows are already at minimum levels. Slow water flow creates problems with daily temperature fluctuation. Thus, one of our objectives was to develop an analytical model that can help to evaluate the relationships between water flow velocity in a river and its temperature. Oxygen is a major water quality parameter that needs to be controlled. It is strongly affected by temperature as warm water contains less oxygen than cold water. At standard barometric pressure (101.3 kPa = 1 Atmosphere) and fresh water oxygen depletion is about 0.23 ppm/1 °C (Kondo 1995) starting from oxygen saturation of 14.6 ppm at 0 °C and decreasing to 7.5 ppm at 30 °C. The height of

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Table 1 Steady-state conditions for Fig. 4 under hot climate $\overline{T_a} = 40\ °C$

Parameter Units	ke/h MJ m^{-3}h^{-1}	K h^{-1}	X m	V mh^{-1}	X/V h	ρC_p MJ m^{-3} °C^{-1}	a °C	$T_w(t_0)$ °C	SS T_w °C
4A	0.2	0.07	2000	1000	2	4.186	2	24	∼26.0
4A	0.2	0.07	4000	1000	4	4.186	2	24	∼29.0
4A	0.2	0.07	6000	1000	6	4.186	2	24	∼31.0
4B	0.2	0.07	4000	1000	4	4.186	2	24	∼29.0
4B	0.2	0.07	4000	2000	2	4.186	2	24	∼26.0
4B	0.2	0.07	4000	4000	1	4.186	2	24	∼25.0

Calculated steady-state water temperature (SSTw) for 4A = changing distance from the source and constant flow rate (1 km h^{-1}). 4B = constant distance (4 km) and changing water flow rates

the dam is about 200 m ASL which is associated with a barometric pressure of 99 kPa. Under these conditions at temperature of 30 °C oxygen solubility is reduced to less than 7 ppm. Solubility of oxygen in saline water, decreases with increasing salinity, but in the Nam Song Dam that collects rain water salinity effect can be neglected. As the temperature rises, animals use oxygen at a faster rate—the metabolic rate doubles with each 10 °C increase (Nemerow 1985). Contrarily, with increased temperature, green water plants and algae grow faster and release more oxygen into the water, but their decomposition consumes more oxygen. Thus, there are several processes within a stream that are affected by its temperature and provide the rationale for this study.

Various models have been developed for calculating water temperature. They were classified into statistic regression models and physically based models.

Regression models have been developed to quantify and predict stream temperature at various time scales (Mohseni et al. 1998; Caissie et al. 2001).

Analytical deterministic models attempt to simulate the process that affect stream temperature such as channel geometry, conduction, radiation, advection, and dispersion (Chen et al. 1998). Although the models could calculate very accurate the actual fluctuations of temperatures in a river water, they required many detailed input data.

The analytical model which is used here is based on the law of energy conservation that resulted in unique dependence of the thermal regime on the water flow velocity, a distance downstream and temperature differences between the surface of the water and the atmosphere. The time-dependent solution is useful for describing heating and cooling processes. This occurs when the rate of energy gain starts at sunrise and loss start in the afternoon. Data of thermal variations are often collected at a single point along rivers during several days or even longer period. The data are helpful to formulate the dependence of water temperature on time and position and to study the physical characteristics of variations in temperature. The model turn out to be a simple case of the kinematic wave theory which is much easier to

solve than large-scale computer simulation provided that its parameters are determined experimentally.

The objectives of this study were to develop and test a physical model to predict the thermal dynamic processes in a river and later to compare it with actual data of the Nam Song River. The Nam Song has a diversion dam to divert water from the Nam Song Basin to the Nam Ngum reservoir. The water release discharge from the Nam Song Dam decreased from 10.0 m^3 s^{-1} to a minimum of only 2.0 m^3 s^{-1}); The average flow rate during the study period was 1800 m h^{-1} (Yoshida et al. 2003a, b, 2004, 2005; Roel and Sean 2001).

Theory

General approach

Various models have been developed for calculating water temperature. They were classified according to their time scales (Deas and Lowney 2000), including annual monthly daily and hourly models. Another classification was based on the mathematical approach including statistic regression models (Bogan et al. 2003) and physically based models on the thermal regime of the water (Yoshida et al. 2004, 2005).

Regression models have been developed to quantify and predict stream temperature at various time scales (Mohseni et al. 1998; Caissie et al. 2001).

Physical process models simulated the underlying processes that affect stream temperature such as channel geometry, conduction, radiation, advection, and dispersion (Chen et al. 1998; Rutherford et al. 1997). Although the models could calculate very accurately the actual fluctuations of temperature in river water, they required many detailed input data.

From the collected data we found that modeling the diurnal cycles of water temperature can be simplified if one assumes that the water flow velocity and air temperature are constant averages, independent of time and position

along the river channel. This simplification results in a thermal formulation that can be solved analytically, thus providing additional insight into the problem of thermal behavior of river water flow. The analytical model that is used here, based on the law of energy conservation, resulted in unique dependence of the thermal regime on the water flow velocity, distance downstream, and temperature difference between the surface of the water and the atmosphere. The time-dependent solution is useful for describing heating and cooling processes. This occurs when the rate of energy gain starts at sunrise and loss starts in the afternoon. The model incorporates the mechanisms of the thermal variation process as described by an energy balance function and energy flow mechanisms. Data of thermal variations are often collected at a single point along rivers during several days or longer periods. The data are helpful to formulate the dependence of the water temperature on time and position and to study the physical characteristics of variations in temperature flow. According to the results, the model turns out to be a simple case of hourly based kinematic wave theory. This was much easier to solve than a large-scale computer simulation, provided that its parameters can be determined experimentally.
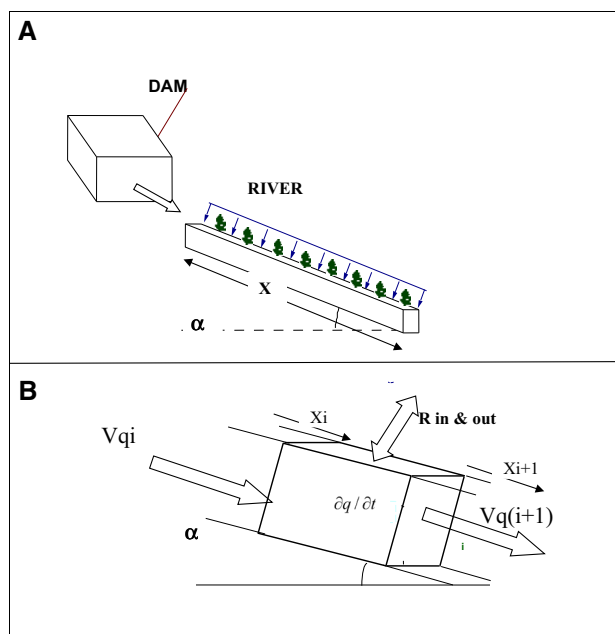
Description of the heat flow model

A typical river system (Fig. 1a) is represented here by a channel X m in length, the volume of which is homogeneous at temperature $T(t)$ (°C), and into which a given quantity of thermal energy is entering (or leaving) from a controlling dam and at a constant discharge V (mh^{-1}). A second source of energy comes from the surrounding atmosphere through radiation.

The dependence of the water temperature on time and distance from the dam can be derived from considerations of energy flow in this system and the equation of continuity. In each volume element (Fig. 1b) of length dX, the change of energy storage in the river is through thermal emission and heat flux to and from the surface of the water. By conservation of energy, the rate of thermal change in the river is equal to the difference of the rate of entry of the water across the plane at Xi and $Xi + $ dX.

Approximating the behavior of this system by a one-dimensional continuity equation with a sink/source term, we can write:

$$\frac{\partial q}{\partial t} + \frac{\partial (V_q(x))}{\partial X} = \Delta L(t,x), \tag{1}$$

where t is time (h), q is the total quantity of energy within the given volume of the river segment (referring to energy density—MJ m^{-3}), V_q is the amount of energy entering across the plane at Xi and $Xi + $ dX (MJ m^{-2} h^{-1}), V is the flow velocity (m h^{-1}), ΔL (x, t) is the sum of long wave

Fig. 1 a Schematic description of the model. b A sub-unit of the river model. qi and $q(i + 1)$ are the energy fluxes into and out of a river unit. V is a constant water flow velocity, $q(x, t)$ is the total energy storage in that unit. It depends on time (t) and location along the river (x). R is the input and output of radiation on the unit

heat flux and the sensible rate of energy gain or loss from the water surface to the atmosphere and vice versa (MJ m^{-3} h^{-1}). The rate depends on the temperature at time and position along the river.

Using

$$q = \rho C_p \times \Delta T_w = 4.186 \times \Delta T_w \tag{2}$$

where q is the energy storage as above (MJm^{-3}), ρ is the water density (1000 kg m^{-3}), Cp is the heat capacity of water (= 4.186 kJ (kg °C)$^{-1}$= 4.186 MJ (1000 kg °C)$^{-1}$), and ΔTw is the change in water temperature (°C). Introducing Eq. (2) into Eq. (1), we obtain:

$$\rho C_p \frac{\partial T_w}{\partial t} + \rho C_p V \frac{\partial T_w}{\partial x} = \Delta L(x,t) \tag{3}$$

Equation (3) is a reduced form of the continuity equation (Benyahya et al. 2010) for the following assumptions:

1. The contribution of dispersion relative to energy flow with the stream flow is negligible, that is, $D\rho C_p \partial^2 T / \partial X^2 = 0$
   because the relatively high velocity of flow and the low longitudinal temperature gradient along the modeled stream, indicating that the system was advection dominant. Therefore, D_L was set equal to zero in the model.

2. The volumetric water flux density along the river is constant, namely: $\rho C_p T \partial V / \partial X = 0$;

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Based on observations made by Yoshida et al. (2004) the above assumptions are experimentally satisfied.

Note that Eq. (3) is a kinematic wave equation in terms of volumetric heat flux units (MJ m^{-3} h^{-1}). The relationships between ΔL and T will be clarified hereafter.

The heating process

For the solution of Eq. (3) we used a well-known technique of parameters transformation (Zarmi et al. 1983) with the transformation:

$$\zeta = x - v(t - t_0), \tag{4}$$

where t_0 is an arbitrary reference time that we choose to be the starting time of the heating process. Water temperature is now described as a function of t and ζ rather than t and X. Note that ζ is constant for the volume element which at $t = t_0$ starts at $X = \zeta$ and is followed as it moves downstream.

Equation (3) now becomes:

$$\frac{\partial T(t, \xi)}{\partial t} = \Delta L^*(t) \tag{5}$$

ΔL^* is the net rate of energy input after conversion to thermal units (°C h^{-1}). The net thermal inflow ΔL^* is independent of position since neither the net radiation nor the air temperature depends on X. According to Eq. (5), the change in water temperature is only a function of time. It reflects the change in temperature as felt by an observer moving along the X direction through the time dependence of $\square \Delta L^*$. For such an observer, moving at the flow velocity of the river (V), a change in time implies a change in position (X) as well.

With ζ the transformed variable, the general solution of Eq. (5) becomes:

$$T(t, \zeta) = \int_{t_0}^{t} \Delta L^*(t') \mathrm{d}t' + Z(t_0, \zeta), \tag{6}$$

where $Z(t_0, \zeta)$ is a constant of integration depending explicitly on the reference time t_0 and ζ but not on time t.

In order to express ΔL in terms of temperature, we first analyze the thermal balance of the water in the river.

Microclimatological approximation of the time-dependent heating process

The net rate of exchange of heat across the air–water interface is the sum of rates of heat exchange processes. A thorough analysis of this process was presented long ago by Edinger et al. (1968). They concluded that the net rate of heat exchange processes can be evaluated in terms of thermal exchange coefficients, the air–water temperature, and its variations with time. In our research, the thermal exchange between water and air was first studied through analytical analysis of observations of the Nam Ngum River energy balance in the heating stage (Yoshida et al. 2004).

We assume that for homogenous thermal distribution in the water profile the effective energy for heating is composed from the sum of four sources: (a) long wave radiation to the water ($L\downarrow$); (b) long wave radiation from the water ($L\uparrow$); (c) the sensible heat flux into the system (H) and (d) a residual short wave input (A). The term A may include the contribution of changes of heat storage and evaporation. Thus, after Brutsaert (1982) total energy input (ΔL) is:

$$\Delta L = H + L \downarrow - L \uparrow + A \tag{7}$$

In Eq. (7), ΔL is the available energy for water heating and its meaning is now clarified. It is convenient to employ the widely used unit system, which in this case means that all units are in Wm^{-2} but later we will change it to MJ m^{-2} h^{-1}. We employed the following sign convention: positive sign for ΔL is towards the water surface and negative sign is for heat leaving the water surface. Thus A, for example, is negative when potential evaporation is the dominant factor (Zhou et al. 2006) and positive when rain or high humidity is the dominant factor of the residual terms.

The relationship between sensible heat flux and temperature is given by:

$$H = \frac{\rho_a C_{pa}(T_a - T_w)}{r_a} = \beta(T_a - T_w), \tag{8}$$

where ρ_a is air density (1.2 kg m^3), C_{pa} is the specific heat of air (1010 J kg^{-1} K^{-1}), r_a is an exchange coefficient for sensible heat flux, which is represented here as an aerodynamic resistance (sm^{-1}), T_w and T_a are water surface and air temperatures (K).

Long wave radiation input/output is related to temperature through the Stephan–Boltzmann law (Brutsaert 1982):

$$\Delta L_w = L_a - L_w = \varepsilon\sigma(T_a^4 - T_w^4), \tag{9}$$

where ΔL_w is net long wave radiation from (or to) the water surface, σ is the Stefan–Boltzmann constant (5.67×10^{-8} J s^{-1} m^{-2} K^{-4} or on hourly basis: K$^{-4} = 2.04 \times 10^{-4}$ J h^{-1} m^{-2} K^{-4}), and ε is the emissivity (taken to be an average of 0.95).

An approximation to Eq. (9) can be obtained using average air temperature ($\overline{T}_a$) and the derivative of L with respect to T:

$$\Delta L_w = 4\varepsilon\sigma\overline{T}_a^3\Delta T = 4\varepsilon\sigma\overline{T}_a^3(\overline{T} - T_w) = \gamma(\overline{T}_a - T_w) \tag{10}$$

Thus, the sum of Eqs. (8) and (10), with ke $= \gamma + \beta$ represents the available energy for heating during day time hours:

$$H + \Delta L_\mathrm{w} + A = \mathrm{ke}[\overline{T_\mathrm{a}} - T_\mathrm{w}(t)] + A \qquad (11)$$

Note that ke is an energy term coefficient of proportionality. In that case ke becomes an integrating coefficient that includes the sum of the missing energy balance coefficients.

Replacing the energy term (ke) with thermal term k by dividing both sides by $\rho C_\mathrm{p}/r_\mathrm{s}$ and determining:

$$\Delta T_\mathrm{w} = [T_\mathrm{w}(t) - \overline{T_\mathrm{a}}], \qquad (12)$$

Eq. (11) is reorganized to obtain:

$$\frac{\mathrm{d}\Delta T}{\mathrm{d}t} = -k\Delta T_\mathrm{w}(t) + a, \qquad (13)$$

where k is the cooling rate parameter, often called the thermal time constant, that has dimensions of t^{-1} and a is the thermal expression of the residual short wave radiation (a method to determine them will be discussed in the next paper of this analysis).

Integration of Eq. (13) with the conditions that at $t_0 = 0$, $\Delta T(\mathrm{t_0}) = \Delta T_0$, provides an explicit expression for the rising of water temperature with time:

$$T_\mathrm{w} = \overline{T_\mathrm{a}} + \frac{a}{k} + \left(\Delta T_\mathrm{w0} - \frac{a}{k}\right)e^{-kt}. \qquad (14)$$

Equation (14) shows that the rate of heat gain or loss is proportional to the difference $\Delta T_0 = T_\mathrm{w0} - T_\mathrm{a}$. Heat is gained by the water when $T_\mathrm{w} < \overline{T_\mathrm{a}}$ and is lost when $T_\mathrm{w} > \overline{T_\mathrm{a}}$. The initial condition requires that at $t = 0$ when the heating process starts: $\Delta T_0 = T\mathrm{w}_0 - \overline{T_\mathrm{a}}$, which is the largest difference between air and water temperature.

An example of the change of water temperature $[T_\mathrm{w}(t)]$ with time as given by Eq. (14) is displayed in Fig. 2.

The figure demonstrates the heating properties of the system. In cold weather it started with a low temperature $T\mathrm{w}_0 = 5$ °C at $t_0 = 0$. The upper limit of river water after a long time approached 17 °C, some two degrees above the onset value of $\overline{T_\mathrm{a}} = 15$ °C The higher water temperature resulted from the positive input of short wave radiation "a/k". Under cold and humid conditions, the effect of evaporative cooling is small compared to the energy input, and hence the water temperature can be above the air temperature. In hot weather at $t_0 = 0$ minimum temperature was set at 10 °C and it asymptotically approached 28 °C ($=\overline{T_\mathrm{a}} - a/k$), two degrees below the onset temperature (30 °C). In this case hot weather was simulated not only by its high $\overline{T_\mathrm{a}}$, but also by the effect of evaporative cooling that kept the water temperature below $\overline{T_\mathrm{a}}$.
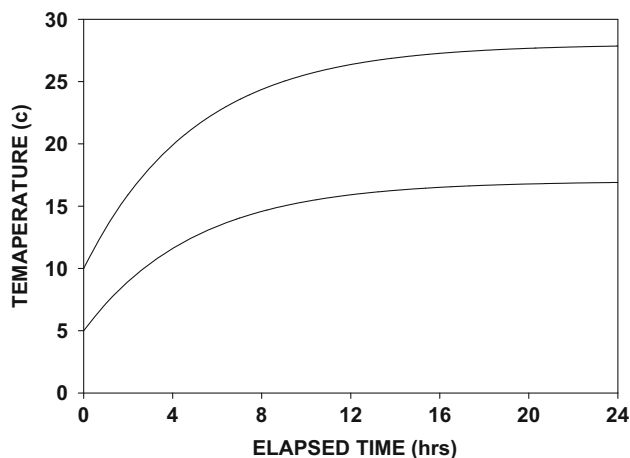
Fig. 2 The heating process of water in a river as a function of time for two climatic conditions: cold climate with $\overline{T_\mathrm{a}} = 15$ °C and $T_\mathrm{w0} = 5$ °C and hot climate $\overline{T_\mathrm{a}} = 30$ °C and $T_\mathrm{w0} = 10$ °C ($k = 0.5/$h)

Equation (14) applies only for time-dependent heating, not for the effect of position along the river or for the cooling phase which is typical to afternoon conditions. These two aspects will be discussed in the following section.

Heating process (sunrise) as a function of time and position along the river

Basic storage of energy in water is reflected by its temperature and follows the daily sunrise and sunset. The fluctuations of energy during the heating and cooling cycles and the basic energy storage in the water means that at any time reference initial conditions cannot be specified. However, an important feature of Eq. (14) is that it is totally insensitive to the choice of the time reference $t_0 = 0$. Thus, this uncertainty in time origin can be avoided by taking the initial conditions at: $(t - t_0) = 0$ at which $T_\mathrm{w0}(t_0) \leq T_\mathrm{w}(t)$. This means that at the selected time $t - t_0 = 0$, $T_\mathrm{w0}(t_0)$ is the minimal storage of thermal energy. The stored thermal energy increases, and consequently T_w in Eq. (14) also increases. The selected initial conditions also require that the constant of integration at $(t = t_0, X) \equiv T_\mathrm{w0}$ everywhere along the river channel. In order to have some insight into Eq. (6) and obtain a solution, we multiplied both sides of Eq. (14) by the energy constant ke and divided it by the depth of the river "h" obtaining an expression "Q" for the energy density inflow through the water surface (units MJ m^{-3} h^{-1} °C^{-1}).

$$Q(t) = \frac{\mathrm{ke}}{h} T_\mathrm{w}(t) = \frac{\mathrm{ke}}{h}\left[\overline{T_\mathrm{a}} + \frac{a}{k} + \left(\Delta T_\mathrm{w0} - \frac{a}{k}\right)e^{-kt}\right] \qquad (15)$$

Thus, $Q(t)$ is the total energy input into the water of the river (MJ m^{-3}) at any time t. It is the energy used to

increase the temperature of the water above a given initially low temperature at sunrise.

With these conditions, combining Eqs. (15) and (6) and integrating, the solution to Eq. (6) can be taken from Zarmi et al. (1983) to yield two continuous solutions (Eqs. 16a, 16b), which satisfy the boundary and the initial conditions:

$$Q(t) = \frac{ke}{h}\left(\overline{T}_a - \frac{a}{k}\right)\left[(t - t0) - \frac{1 - e^{-k(t-t0)}}{k}\right]$$
$$+ Q_0; \ t \leq (t_0 + X/V) \quad (16a)$$

$$Q(t) = \frac{ke}{h}\left(\overline{T}_a - \frac{a}{k}\right)\left[x/v - \frac{e^{k(x/v)} - 1}{k}\left(e^{-k(t-t0)}\right)\right]$$
$$+ Q_0; \ (t_0 + X/V) \leq t \quad (16b)$$

Equation (16a) satisfies the conditions that at $t = t_0$ at any position X, $Q(t) = Q_0$ ($= \rho C_p T_{w0}$), which is the minimal amount of energy storage in the river water. Beyond $t = t_0$, $Q(t)$ increases with time. Both equations are identical at $t = X/V$ (Eq. 16b ≡ 16a), which satisfies the condition of continuity of the heating–cooling process. Equation (16b) also shows that at $t \gg t_0$, the amount of energy in the water is asymptotically approaching a constant (Eq. 17) representing maximal water temperature.

$$Q(t) \approx \frac{ke}{h}\left(\overline{T}_a - \frac{a}{k}\right)x/v + Q_0. \quad (17)$$

This constant is also the starting point for the cooling process

Cooling process (sunset) as a function of time and position along the river

The cooling process results in a decreasing difference between maximum and initial (minimum) water temperature or the equivalent stored energy in the water. This decreased difference between the two temperature extremes can be written as:

$$T_w(t) - T_{w0} = \Delta T_w$$
$$= \frac{ke}{\rho C_p h}\int_{tr}^{T} -\Delta T_w m \, dt = \frac{ke}{\rho C_p h}\Delta T_w(t - t_r)$$
$$+ Z(t_r, \zeta), \quad (18)$$

where ΔT_{wm} is the difference between maximum and initial (minimum) water temperature, t_r marks the end of the heating process and the start of the cooling process, when the water temperature is at its daily maximum, and T marks the end of the thermal cycle (heating and cooling process).

In order to have the steady-state temperature, $\frac{ke}{h}\left(\overline{T}_a - \frac{a}{k}\right)x/v$, at $t = t_r$ we must have

$$Z(t_r, \zeta) = \frac{ke}{\rho C_p h}\frac{\xi}{V} = \frac{ke}{\rho C_p h}\left(\overline{T}_a - \frac{a}{k}\right)\left\{\frac{X - V(t - t_r)}{V}\right\} \quad (19)$$
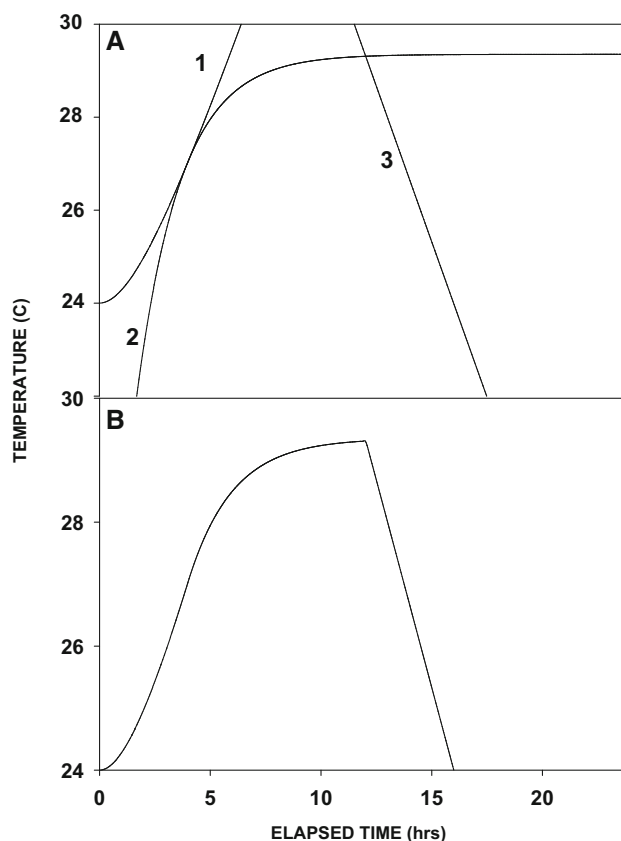
then

$$\Delta T_w(t, X) = \frac{ke}{\rho C_p h}\Delta T_w m(t - t_r) + \frac{ke}{\rho C_p h}\left(\overline{TK}_a - \frac{a}{k}\right)\frac{X}{V}$$
$$- \frac{ke}{\rho C_p h}\left(\overline{T}_a - \frac{a}{k}\right)(t - t_r). \quad (20)$$

We have measured data only at $L = 4000$ m, thus for $X = L$

$$\Delta T_w(t, L) = \frac{ke}{\rho C_p h}\left(\overline{T}_a - \frac{a}{k}\right)\frac{L}{V}$$
$$- \frac{ke}{\rho C_p h}\left[\left(\overline{T}_a\frac{a}{k}\right) + \Delta T_w m\right](t - t_r). \quad (21)$$

It follows from Eq. (21) that the thermal depletion is a linear function of time and its rate is given by the slope: $-\frac{ke}{\rho C_p h}\left(\left(\overline{T}_a - \frac{a}{k}\right) + \Delta T_w m\right)$.


Fig. 3 Theoretical presentation of the thermal cycle. **a** *Line 1* is the solution for a short time $t \leq t_0 + X/V$ (Eq. 16a), *line 2* is the solution for a long time $t \geq t_0 + X/V$ (Eq. 16b), and *line 3* is the cooling process Eq. 17 for the steady state at $t = t_r$ and Eq. (21) for the falling limb. **b** Unified and continuous presentation of the above three solutions to describe a complete thermal cycle

At $t = t_r$, Eq. (21), which describes the cooling process, coincides with Eq. (17), which describes the steady state of the heating process in Eq. (16b).

Now we have a consistent solution for the whole thermal cycle. The heating stage is given by Eqs. (16a, 16b) and the cooling stage by Eq. (21), such that the continuity of the process is maintained.

The combined solution is presented in Fig. 3.

Figure 3a contains separate presentations of each of the layers of the solution. It can be seen that the solution for short time (line 1) starts at $t = 0$ and increases exponentially shortly afterward, but it is diverted by the second solution (line 2). This line starts at a very, unrealistically, low temperature. As discussed above in Eqs. (16a, 16b), it crosses line 1 at $t = x/v$ and approaches a maximum that is crossed by the cooling solution (line 3) at $T_w(t) = T_w$ $(t = t_r) = \frac{ke}{\rho C_p h}\left(\overline{T_a} + \frac{a}{k}\right)$ $\frac{X}{V} + T_{w0}$. The full heating and cooling cycle is displayed in Fig. 3b as a continuous line that lasts (in this case) some 16 h.
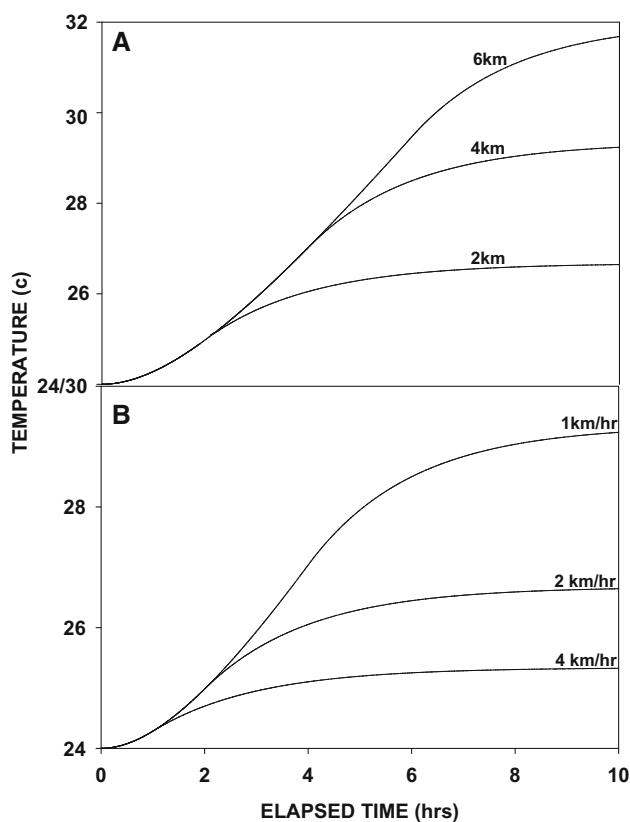
Fig. 4 a The effect of observation distance on the steady-state water temperature, and **b** the effect of water flow rate on the steady-state water temperature. Both **a** and **b** are presented as a function of elapsed time (the parameters selected for the steady-state solution are given in Table 1)

Concluding discussion

Because of a continued interest in water temperature as a critical parameter for aquatic health, temperature modeling remains an important issue in planning and management of any aquatic systems. The purpose of this water temperature model was to provide the major parameters for reservoirs and stream flow measurements.

Analytic simplification of the more complicated kinematic wave equation can be obtained when a constant river flow rate is assumed and average air temperature is used.

The model incorporates an analytical analysis of the energy exchange process as described by Eqs. (16a, 16b) and (19) and energy flow mechanisms. It is given for both steady-state and time-dependent cases. Our primary aim was to generalize the factors that are involved in the rate of thermal exchange between the atmosphere and water in a river in which stored energy is flowing downstream. An interesting aspect of system characteristics is demonstrated in Fig. 4.

In this figure, we examined the effect of combinations between the main river constants, $\overline{T_a}$, V and the distance along the channel length (X). The indicating river was the Nam Song River/Mekong and a comparison with actual data measured in this river is given in the next paper (Yoshida et al. this issue). At the gate of the dam $X = 0$, the temperature is T_{w0} (24 °C), and V is controlled by changing it from 1 to 4 km h^{-1} (Table 1)

The sensitivity of the water temperature to water flux density can be seen in Fig. 4b. For $V = 4$ km h^{-1} $T_w \sim 25$ °C (Fig. 4) and at the same distance from the dam ($L = 4$ km), when V is only 1 km h^{-1} $T_w \sim 29$ °C because the water element has a longer time to absorb heat. This means that increasing V at the dam can be used to compensate for problems of high temperatures downstream. Figure 4a demonstrates that the water temperature increases at a rate of ~ 1.2 °C/km when the stream flow rate is 1 km/h. When water flows at a rate of 4 km/h, temperature increases at a rate of ~ 0.25 °C/km downstream.

The test of the model has indicated that in terms of the volumetric water flux density (or the discharge) V is one of the most important variables that can be used to control the proper distribution of temperature downstream. For successful test of the model the specific parameters of the Nam Song River were used in the next paper (Yoshida et al. 2016 this issue). Naturally, these parameters are site specific and are changed from one river to another. Thus, to use the model for prediction of thermal regime in other rivers their local specific parameters should be taken, but the model developed here determines the variables that can

be approximate and help to obtain a realistic approximation of the thermal regime of many other streams.

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