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ORIGINAL PAPER

# Experimental and Numerical Modeling of Nano-clay Effect on Seepage Rate in Earth Dams

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**Abstract**

Earth dams control and store river water. Type of the earth dam is selected on the basis of available borrow sources. Where it is not possible to have access to suitable fine-grained sources for constructing an earth dam, then using homogeneous materials and an impermeable blanket is recommended. In this research, a mixture of sandy soil with 0.25, 0.5, 0.75, and 1 wt% of montmorillonite nano-clay was used to make the impermeable blanket. After initial tests of gradation, permeability, and optimum moisture content on the soils, experimental models of homogeneous earth dam with an impermeable blanket were constructed. The time needed to reach the steady-state phreatic line and seepage discharge in the models was compared in transient and unsaturated cases. Results showed that increasing the amount of nano-clay from 0.25 to 1.0%, decreased seepage discharge by 19, 67, 89, and 97%, respectively, compared to the control model. Then, a numerical model of the earth dam was prepared using SEEP/W software and was validated with the experimental results. Results of measured and modeled phreatic lines indicated that the numerical model is accurate enough. Results of sensitivity analysis for blanket thickness showed that seepage rate in 0.5% nano-clay model was $9.46\times10^{-6}$, $8.93\times10^{-6}$, and $8.01\times10^{-6}$ $m^3/s$ and in 1.0% nano-clay model was $2.1\times10^{-6}$, $1.44\times10^{-6}$, and $7.80\times10^{-7}$ $m^3/s$ for 3, 5, and 10 cm blankets, respectively. In general, using a blanket with small amount of nano-clay on the reservoir side of the earth dam could alleviate seepage problems.

**Keywords** Hydraulic conductivity · Montmorillonite nano-clay · Seepage · Sandy soil · Earth dam

## Introduction

For thousands of years, earth (embankment) dams have been used to control and store river water. They are typically constructed by placement and compaction of a complex semi-plastic mound of various compositions of soil, sand, clay, or rock. Stability and seepage in an earth dam are very important. Earth dams lose water through evaporation from the dam reservoir and seepage from the dam body. Little can be done for evaporation losses, but seepage losses can be reduced with good construction methods. Seepage happens due to the difference in upstream and downstream water height [1]. Seepage discharge is dependent on the soil medium, fluid type, and geometric conditions of the earth dam [2]. Water seepage in earth dams is one of the most important factors in structural designs [3]. Water seepage in homogeneous earth dams has caused many problems. Various studies have been carried out experimentally and numerically to calculate the seepage rate [4, 5] or its reduction [6]. Sivakumar Babu and Vasudevan investigated the effect of coir fiber on seepage velocity and piping resistance of soil [7]. The experiments were carried out for various hydraulic heads, fiber content, and fiber length. Results showed that seepage velocity was reduced and thus piping resistance of soil was enhanced. The influence of fiber-reinforced soil density and fiber parameters on the piping failure mode, hydraulic conductivity, and critical hydraulic gradient was investigated by Yang et al. [8]. Results showed that hydraulic conductivity was decreased and critical hydraulic gradient was increased as the fiber content increased. Haman et al. proposed methods such as blanket, bentonite, flexible membranes, and chemical materials for sealing of dams, and stated that the choice of proper sealing method depends

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on the coarse-grained or fine-grained nature of the soil [9]. Mousavi et al. studied the effect of seven different amounts of rice straw on some of the physical properties of paddy heavy soil and the formation of cracks [10]. Lentz examined the effect of polyacrylamide and biopolymer on flocculation, aggregate stability, and water seepage in soil [11].

In order to reduce soil hydraulic conductivity, nanomaterials are used in the new researches. Nano-clay is a form of plate-shaped smectite minerals. Due to their small dimensions, these nanoparticles have high specific surface area and very active interaction with other particles in the soil. Thus, the existence of even a very small amount of these materials has great influence on the soil engineering properties such as elasticity, permeability, modulus of elasticity, strength, and soil stability [12]. In various studies, nanomaterials have been used to improve geotechnical properties [13, 14] and hydraulic characteristics [15, 16] of soil. Kananizadeh et al. used these materials in a landfill, and concluded that adding 4% of nano-clay to clay soil reduced soil hydraulic conductivity from $3\times10^{-9}$ to $7.74\times10^{-11}$ cm/s under normal conditions [17]. Taha and Taha examined the effect of nanomaterials on inflation and contraction behavior of soil [18]. Neethu and Remya combined two different types of soils (lateritic soil and kaolinite clay) with different ratios of nano-clay (0, 0.25, 0.5, 1, 1.5, and 2%). The hydraulic conductivity coefficient decreased by 99% for lateritic soil and 96% for kaolinite clay [19]. Bahari and Shahnazari used nano-clay to increase soil physical properties. Results showed that addition of nano-clay increases soil strength and stability and decreases bed erosion and transportation of borrow materials [20].

In the present research, in order to reduce seepage from the body of a homogeneous earth dam, a mixed sandy soil blanket, similar in texture to the body of homogeneous dam, but containing different percentages of montmorillonite nano-clay, was used in upstream of the dam. The numerical model of this dam, built in SEEP/W software, was used to investigate the effect of blanket thickness on seepage rate.

## Materials and Methods

### Experimental Set-up

In order to investigate the effect of adding nano-clay on seepage in a laboratory-made homogeneous earth dam, a suitable sandy soil according to FAO codal provision [21] was used. A thin soil blanket containing 0.25, 0.5, 0.75, and 1 wt% nano-clay was made in upstream of the dam. Then, the phreatic line was determined in the piezometers. Seepage rate was measured by the outlet water flow. Numerical model of the dam in SEEP/W software was verified to study the effect of soil blanket thickness on seepage rate.

### Physical Model

The experimental model consisted of a rectangular container and a sand model of the earth dam inside it (Fig. 1). The sand model was designed and built to investigate the variations of seepage discharge and position of phreatic line under the implementation of an upstream blanket containing different percentages of nano-clay. The container was 2.7 m long, 1.0 m wide, and 0.7 m high. The container was made of 2-mm-thick galvanized sheets in four sides and of 6-mm-thick plexiglass in one longitudinal side. Thin aluminum strips were used to strengthen the model and prevent galvanized-sheet distortion and plexiglass breakdown. The seams were sealed with a water-seal adhesive. The position of piezometers on plexiglass side was drilled by a laser cutter. The container was situated on a 0.5-m-high wooden frame and leveled in all directions. On the upstream side of this container, an elevated water tank was placed and connected to the container with a hose (Fig. 1). This tank provides water to make a reservoir on the left upper side of the earth dam. By drilling a hole in the upper and lower floor sections of the earth dam, and connecting these holes to the graduated water-barrel in the right-hand side of the model, variations of seepage rate could be monitored.

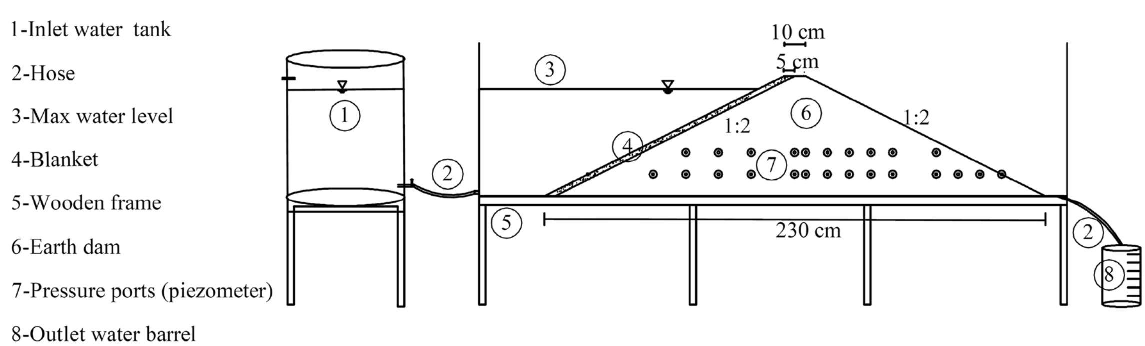


**Fig. 1** Layout of the experimental earth-dam model

The earth dam was constructed in the middle of the container (Fig. 1). The earth dam was 2.3 m long, 1.0 m wide, and 0.55 m high. Due to the soil characteristic which will be expressed, Slope of the upstream and downstream sides of the earth dam was 1:2 (vertical:horizontal). Dimensions of the earth dam are selected according to Fu and Jin [22].

In the earth dam, desired soil should be placed in layers not exceeding 200 mm (Manual of small earth dams). The earth dam was divided into six layers (five layers of 10 cm each and one layer of 5 cm). Given the maximum soil dry density, the specific gravity at 90% compaction was calculated, and according to the geometry of the earth dam, the required quantity of soil was calculated and used for each layer. For better compaction of the soil, a square ASTM standard hammer was made, and at each layer, the desired density was obtained according to the proper soil moisture content. Then, water was allowed to enter the container from the upstream water tank. Piezometers were read every 15 min to locate final position of the phreatic line. In real cases, the base of the earth dam should be stripped of all topsoil, silt, waste matter, vegetations, and then scratched. But, in our physical model, the galvanized plate acts as an impervious layer and all seepage happens from the main body of the dam. Therefore, seepage rate was calculated by measuring the outflow water collected each 15 min in the downstream graduated barrel. The readings were continued until the establishment of steady flow.

Then, the soil was removed from the container and a new earth dam was built. In all the tests, the dam body was made of $S_0$ soil (no nano-clay). But, the 5-cm-thick upstream blanket was made of $S_{0.25}$, $S_{0.5}$, $S_{0.75}$, and $S_{1.0}$ soils (these codes refer to blankets with 0.25, 0.5, 0.75, and 1.0 wt% nano-clay). These models were coded as $D_{0.25}$, $D_{0.5}$, $D_{0.75}$, and $D_{1.0}$, respectively. Seepage discharge and phreatic-line position were also measured by piezometer for these four models. But, due to their low seepage rates, after the sixth hour, the readings were taken once each hour.

The top flow line of a saturated soil mass below which seepage takes place is called phreatic line. Hydrostatic pressure exists below the phreatic line, whereas atmospheric pressure exists above it. To measure the position of phreatic line in our earth dam, 25 piezometers of Casagrande type were installed and connected to the holes which were made in the plexiglass side.

## Sandy Soil of the Dam Body

The body of homogeneous earth dam filled with a sandy soil so that soil gradation is within the range set by FAO [21]. In Fig. 2, grading curve of the sandy soil and the standard range are shown. In this sand, 6.6% of the particles pass through the sieve with mesh No. 200; therefore, according to the gradation shown in Fig. 2, it is type SP-SC in the USCS classification and type A-3 in the AASHTO classification. Results of the AASHTO-T99 standard density test showed a maximum dry density of 1.95 g/cm$^3$ and also an optimal moisture content of 3.51%.

## Nano-clay

Nano-clays are minerals in the nanometer scale, which have an average specific surface area of about 250 m$^2$/g. This great specific surface causes a strong interaction between nano-clay and its surroundings. In the present study, nano-clay, which is a hydrophilic and powdery material, has been used as an additive. The nano-clay was modified montmorillonite clay. Physical and chemical properties of this nano-clay, as provided by the manufacturer (Sigma-Aldrich, Germany), are presented in Tables 1 and 2.

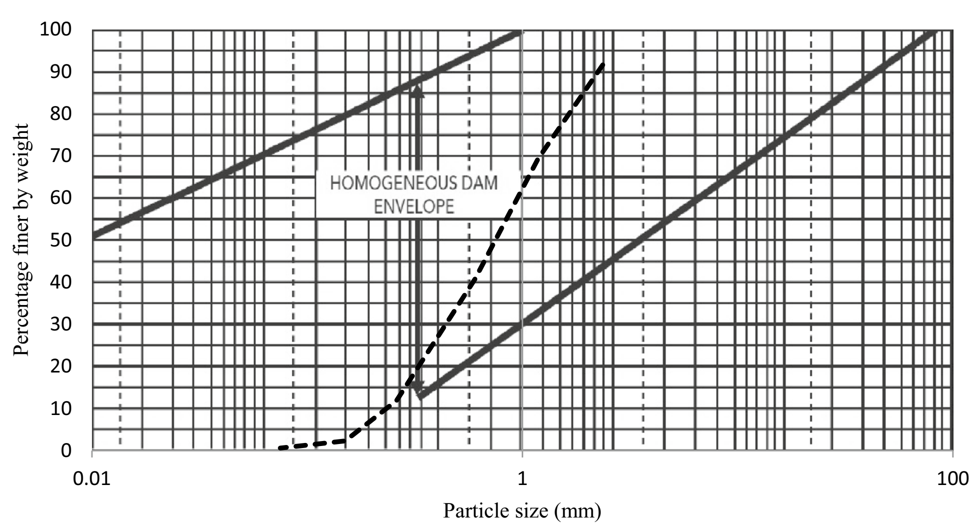


**Fig. 2** Grading curve of the sandy soil of this study

**Table 1** Physical properties of the nano-clay

| | |
|---|---|
| Density | 0.6 (g/cm$^3$) |
| Particle size | 1–2 nm |
| Specific area | 220–270 (m$^2$/g) |
| Empty gap between particles | 60 (Å) |
| Color | Light yellow |
| Moisture | 1–2% |

**Table 2** Chemical analysis of the nano-clay

| Parameter | (%) |
|---|---|
| $Na_2O$ | 0.98 |
| $K_2O$ | 0.86 |
| $TiO_2$ | 0.62 |
| $Al_2O_3$ | 19.6 |
| $SiO_2$ | 50.95 |
| LOI | 15.45 |
| MgO | 3.29 |
| $Fe_2O_3$ | 5.62 |
| CaO | 1.97 |
| Electrical conductivity | 25 MV |
| Ion exchange coefficient | 48 (meq/100 g) |

**Table 3** Hydraulic conductivity of different soil media

| Nano-clay (%) | Soil code | Hydraulic conductivity (cm/s) |
|---|---|---|
| 0 | $S_0$ | $1.1\times10^{-2}$ |
| 0.25 | $S_{0.25}$ | $3.67\times10^{-3}$ |
| 0.50 | $S_{0.50}$ | $7.3\times10^{-4}$ |
| 0.75 | $S_{0.75}$ | $9.12\times10^{-5}$ |
| 1.00 | $S_{1.0}$ | $1.24\times10^{-5}$ |

## Mixing of Sandy Soil with Nano-clay

The sandy soil was mixed with 0.25, 0.5, 0.75, and 1.0 wt% of nano-clay. Doubling method on dry soils was used for mixing the nano-clay with the sandy soil. The sample weight was calculated according to the specific gravity of the soil and considering cylinder volume of the hydraulic conductivity test. Soil was compacted up to 90% in all the tests. To determine hydraulic conductivity of the soil, constant-head permeability test was performed according to AASHTO-T215 standard. Results of these tests are presented in Table 3.

## Numerical Model

Flow of water in the saturated zone of positive pore pressure beneath the phreatic surface in the body of an earth dam follows Darcy Law. This law states that

$$Q = K \times i \times A, \tag{1}$$

where $Q$ is water discharge passing through the dam, $K$ is hydraulic conductivity (permeability coefficient) of the soil, $A$ is cross section of flow path, and furthermore $i$ is hydraulic gradient which is obtained from the following equation:

$$i = \frac{\partial h}{\partial s}, \tag{2}$$

where $s$ is length of the movement of water flow inside the porous medium and moreover $h$ is total head.

Seepage analysis depends on an equation that describes the phenomena of flow of water through a porous medium like soil. The three-dimensional Laplace equation forms the mathematical basis for most methods of seepage analysis:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0, \tag{3}$$

where $h$ is total hydraulic head at the desired point. Equation (3) is presented for heterogeneous soil conditions. In homogeneous porous media, we will have

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0. \tag{4}$$

The graphical solution of this equation in two-dimensions leads to drawing of two series of perpendicular lines, called flow and equipotential lines. These lines constitute the flow nets. Flow nets are perhaps the most widely known graphical technique for evaluating seepage rate.

Another solution of the Laplace equation is numerical solution. Numerical analysis, in the form of available softwares, is widely used to model seepage flow. The numerical methods are recommended because complex systems are modeled relatively easily, they use finite element or finite difference mesh, and sensitivity analysis is more convenient. Various softwares such as SEEP/W, FLAC, FRACMAN, FRACK, MODFLOW, and BIE are used in seepage issues [23].

One of the most widely used softwares, which is applied in the present study, is SEEP/W model [24]. This software is capable of modeling the saturated, unsaturated, steady-state, or transient flow problems in porous media by finite element method.

In the present research, dimensions of the experimental model were introduced to SEEP/W software. The mesh dimensions were selected by using sensitivity analysis as 5 cm (Fig. 3). Upstream boundary conditions were introduced to the model with respect to the water level at different

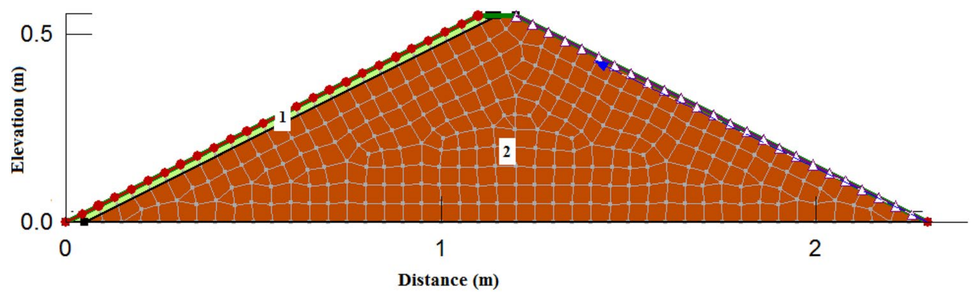


**Fig. 3** Meshed model of the earth dam (1—blanket, 2—earth-dam body filled with $S_0$ soil)

times. The boundary condition of downstream head was considered to be zero.

Hydraulic conductivity coefficients were introduced into the model and consequently after analyzing different methods, the van Genuchten method [25] was used to estimate hydraulic conductivity at various degrees of saturation. Time iteration was 15 min in transient analyses. Max of iteration and pressure head difference for convergence of transient model were 25 and 0.005, respectively. Homogeneous and isotropic earth dam materials were introduced. Validation of the numerical model carried out by comparing the results of the model and the experimental results, therefore calibration of the numerical model, was performed.

## Results and Discussion

### Experimental Results

The experimental model of the earth dam with no nano-clay blanket was a homogeneous porous medium and was treated as control. After 1.5 h, the first sign of seepage was observed at downstream of the dam, and after 1 h and 45 min, the first seepage discharge was generated. In this model, the upstream water level rose gently from zero to 49 cm after 4 h and reached a steady state after 4.5 h.

Results of the position of phreatic line in the transient conditions for 1.0-h intervals are shown in Fig. 4. Since the piezometers were installed at least at a height of 10 cm above the model floor, lower water heads were not measurable by the piezometers.

Results of the measured seepage rate showed that for control treatment ($S_0$ soil and no nano-clay), the steady-state seepage rate was 2.68 $m^3$/day. As was stated earlier, in order to reduce the seepage rate, a soil blanket, 5 cm thick, consisting of a mixture of sandy soil with different percentages of nano-clay was used in upstream of the earth dam. Results of these measures under steady-state conditions are shown in Fig. 5. As is shown in this figure, loss of hydraulic conductivity, by adding a small amount of nano-clay, makes significant changes in the water seepage line. The time required for these models ($D_{0.25}$ to $D_{1.0}$) to reach steady state was measured as 6, 8, 18, and 24 h, respectively. This time was obtained when the seepage rate in the experimental model became constant. The measured seepage rate in $D_{0.25}$ to $D_{1.0}$ models was 2.17, 0.88, 0.29, and 0.07 $m^3$/day, respectively, which showed a reduction of 19, 67, 89, and 97%, respectively, compared to the control model.

In Fig. 6, dimensionless diagram of seepage discharge ($Q/Q_{S0}$) is shown in terms of the amount of nano-clay in the blanket. As can be seen, increasing the nano-clay content of the blanket, sharply decreased this ratio.

Results indicated a delay of about 20 h for the steady flow in the 1% nano-clay model, as compared to the control model. For example, in piezometer at a distance of 1.5 m from the dam heel, the first reading of water appearance in the control model was taken after 45 min; while, in the 1% nano-clay model, it appeared after 8 h. Also, the phreatic

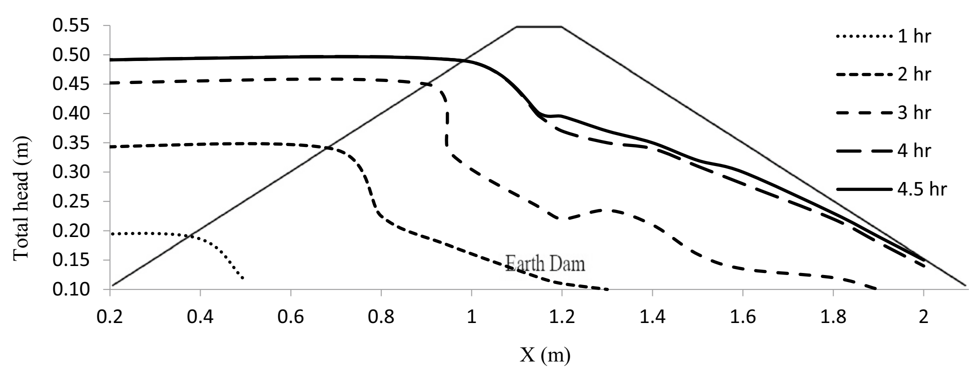


**Fig. 4** Transient phreatic-line position in the control model for different times

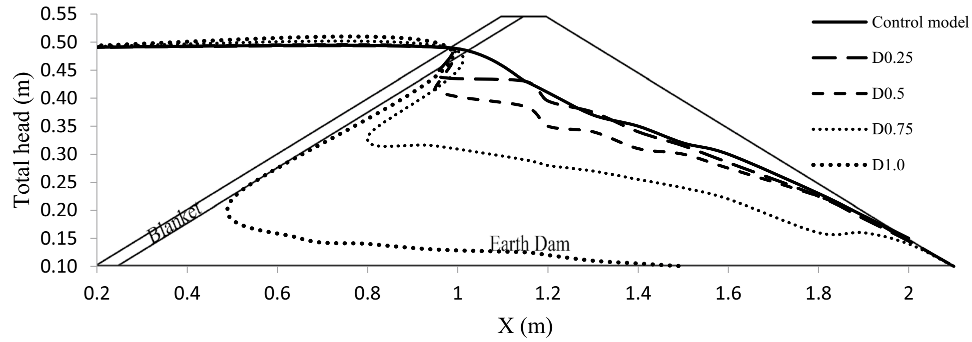


**Fig. 5** Steady phreatic line in the control and $D_{0.25}$ to $D_{1.0}$ models

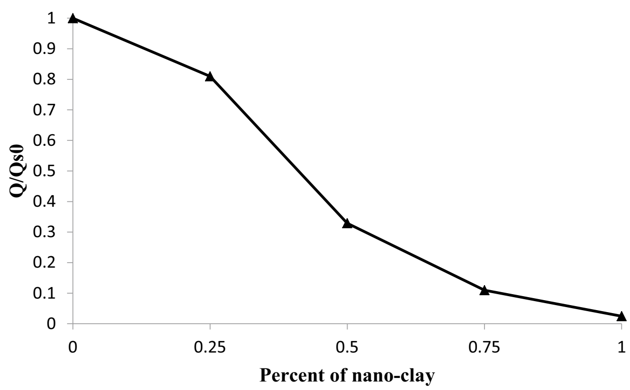


**Fig. 6** Dimensionless diagram of seepage rate versus the amount of nano-clay in the blanket

line dropped more by increasing the nano-clay content of the blanket. In addition to all these factors, a significant reduction in seepage discharge justifies the use of nano-clay materials as an impermeable blanket in the homogeneous earth dams.

## Numerical Results

Numerical model of this experimental earth dam was built in SEEP/W software. In Figs. 7, 8, and 9, phreatic water head in the piezometers located at 0.5, 0.95, 1.5, and 2 m from the water reservoir is compared for some of the real experiments and numerical models (control, $D_{0.5}$ and $D_{1.0}$ cases). Determination coefficients ($R^2$) for the experimental values versus numerical predictions for Figs. 7, 8, and 9 are 0.98, 0.92, and 0.91, respectively. Thus, the numerical software could model very well the water flow in the earth dam in transient state and unsaturated conditions.

After verification of the numerical model, effect of blanket thickness on seepage rate was investigated. Numerical models of $D_{0.5}$ and $D_{1.0}$ cases were analyzed in SEEP/W software for 3-, 5-, and 10-cm blankets. Seepage rate in $D_{0.5}$ was $9.46\times10^{-6}$, $8.93\times10^{-6}$, and $8.01\times10^{-6}$ m$^3$/s and in $D_{1.0}$ was $2.17\times10^{-6}$, $1.44\times10^{-6}$, and $7.80\times10^{-7}$ m$^3$/s for 3-, 5-, and 10-cm blankets, respectively. Therefore, comparison of the above seepage rates shows that increasing blanket thickness is more effective in blankets with higher nano-clay content.

Results of the flow lines in $D_{1.0}$ model for 3-cm and 10-cm blankets are shown in Fig. 10. Results showed that increasing the blanket thickness reduces seepage rate and lower position of the phreatic line.

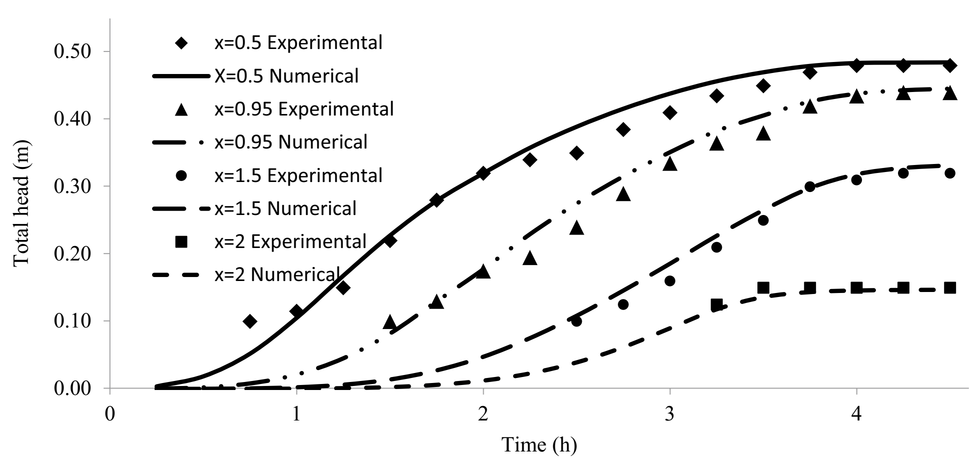


**Fig. 7** Phreatic water level in the piezometers in the experimental and numerical models (control case)

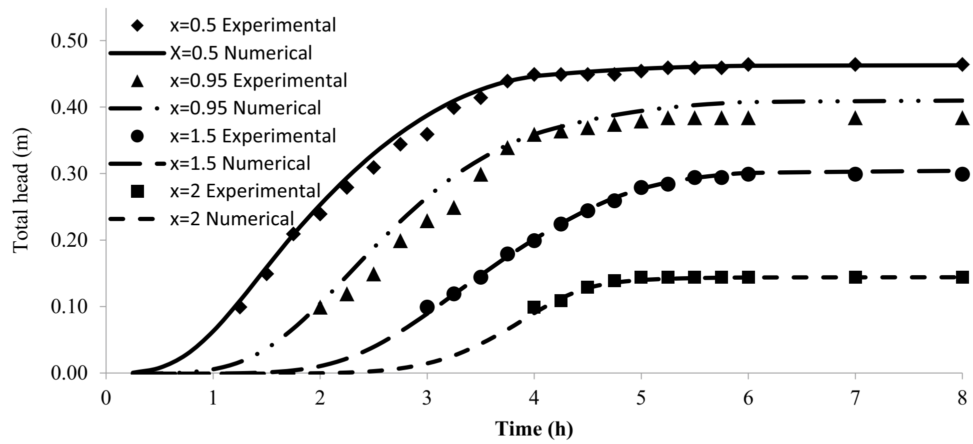


**Fig. 8** Phreatic water level in the piezometers in the experimental and numerical models ($D_{0.5}$ case)

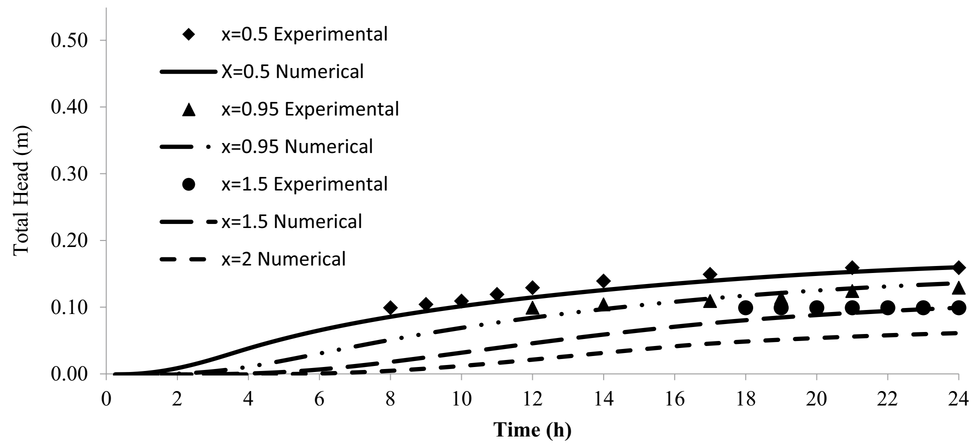


**Fig. 9** Phreatic water level in the piezometers in the experimental and numerical models ($D_{1.0}$ case)

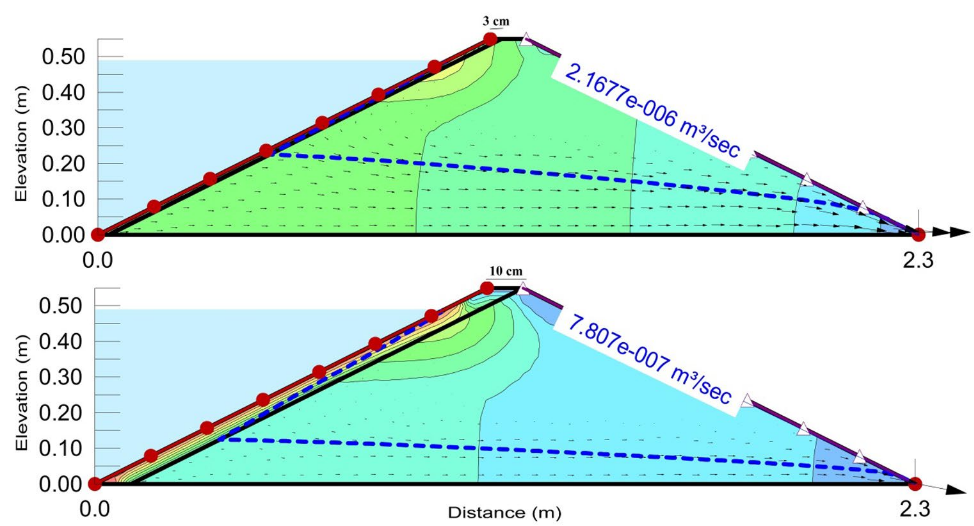


**Fig. 10** Results of flux and phreatic line in SEEP/W software in $D_{1.0}$ case for 3-cm and 10-cm blankets

## Conclusions

Nano-clay properties decrease the hydraulic conductivity of the sandy soil. The small amount of nano-clay could reduce the seepage rate in homogeneous earth dams significantly. Experimental and modeled phreatic lines are very close to each other. Increasing the blanket thickness has a profound effect on the seepage rate and position of the phreatic line. For verification of the results, SEEP/W software with van Genuchten method can simulate seepage in a dam with mixed nano-clay blanket. So, SEEP/W software can model seepage for nano-clay mixed material. Nano-clay properties do not change over time and nano-clay does not move through the dam, as expected for transient result.

In this paper, seepage through the dam was investigated and the mechanical properties of the dam like stability have not been studied. Hence, in future research, these properties can be verified and in case the nano-clay can lead to improve in dam strength, this material can be utilized for dam construction.

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