Transportation in Developing Economies (2021) 7:26
https://doi.org/10.1007/s40890-021-00135-2

ORIGINAL ARTICLE

A Model for Cordon Pricing Scheme Considering Spatial Equity

Shahriar Afandizadeh[1] · **Seyed Ebrahim Abdolmanafi**[2]

Received: 20 June 2020 / Accepted: 20 August 2021 / Published online: 29 September 2021

Abstract

This paper investigates the issue of spatial equity in the cordon pricing scheme. The spatial inequity is for drivers whose travel destinations are located within the cordon or should travel through it and finally they may face with higher travel time. Thus, to alleviate this inequity, a bi-level multi-objective optimization model is developed. Next, an algorithm is implemented according to the second version of the Strength Pareto Evolutionary Algorithm (SPEA2). Then, the developed model is applied to Sioux Falls network and the results is discussed. The results reveal that it seems reasonable to consider spatial equity as an objective function in cordon pricing. In addition, we can create a sustainable situation for the transportation system by improving spatial inequity with a relatively low reduction in social welfare. Moreover, there are spatial inequity impacts in real networks, which should be considered in the cordon pricing scheme. Furthermore, the developed model can increase the public acceptance of drivers and transportation authorities.

Keywords Cordon pricing · Spatial equity · Multi-modal network · Bi-level optimization model · SPEA2

Abbreviations

A	The set of links in the network
W	The set of OD pairs
x_a	The flow on link $a \in A$
t_a^C	Travel time of cars in link "a"
t_a^T	Travel time of taxis in link "a"
t_a^B	Travel time of buses in link "a"
t_a^0	Free flow travel time in link "a"
C_w^C	Minimum travel costs of cars between OD pair $w \in W$
C_w^T	Minimum travel costs of taxis between OD pair $w \in W$
C_w^B	Minimum travel costs of buses between OD pair $w \in W$
τ_a	Toll level in link "a"
$\tau_{\max}$	Maximum of toll level
$\theta_{\max}$	Maximum of price of P&R
d_w^m	Travel demand between OD pair $w \in W$ with mode "m"
D_w	Initial total travel demand between OD pair "w"
γ_w	Demand elasticity coefficient between OD pair "w"
μ_w	Minimum travel cost between OD pair "w"
C_{op}^C	Minimum travel costs by cars from origin "o" to P&R "p"
C_{pd}^C	Minimum travel costs by cars from P&R "p" to destination "d"
C_{pd}^T	Minimum travel costs by taxis from P&R "p" to destination "d"
C_{pd}^B	Minimum travel costs by buses from P&R "p" to destination "d"
θ_p	Price of P&R "p"
$(d_w^C)^{\text{old}}$	Initial travel demand by cars between OD pair "w"
$(d_w^C)^{\text{new}}$	Modified travel demand by cars between OD pair "w"
d_{op}^C	New travel demand by private cars from origin "o" to P&R "p"
d_{pd}^T	New travel demand by taxis from P&R "p" to destination "d"
d_{pd}^B	New travel demand by buses from P&R "p" to destination "d"
f_{rw}	The flow on route "r"
R_w	The set of all routes between OD pair $w \in W$
$D_w^{-1}(w)$	The inverse demand function
d_w	The demand between OD pair $w \in W$
d_w^C	Travel demand of cars between OD pair "w"

✉ Seyed Ebrahim Abdolmanafi
abdolmanafi@srbiau.ac.ir

[1] School of Civil Engineering, Iran University of Science and Technology (IUST), Tehran, Iran

[2] School of Civil Engineering, Islamic Azad University Science and Research Branch, Tehran, Iran

d_w^T	Travel demand of taxis between OD pair "w"
d_w^B	Travel demand of buses between OD pair "w"
x_a^C	Traffic flow for cars in link "a"
x_a^T	Traffic flow for taxis in link "a"
x_a^B	Traffic flow for buses in link "a"
$t_a(x_a)$	Travel cost on link "a", which is function of link flow x_a
C_a	The capacity of the link "a"
t_{pd}^C	Travel time by cars between P&R "p" and destination "d"
t_{pd}^T	Travel time by taxis between P&R "p" and destination "d"
t_{pd}^B	Travel time by buses between P&R "p" and destination "d"
u_w^C	Utility function of cars between OD pair $w \in W$
u_w^T	Utility function of taxis between OD pair $w \in W$
u_w^B	Utility function of buses between OD pair $w \in W$
$u_{\text{car only}}$	Utility function for no shifting from cars to other modes in P&Rs
$u_{\text{car-taxi}}$	Utility function for shifting from cars to taxis in P&Rs
$u_{\text{car-bus}}$	Utility function for shifting from cars to buses in P&Rs

Introduction

In developing countries in particular, cities have experienced a rapid growth in transport-related challenges, including pollution, congestion, accidents, environmental degradation, and energy depletion [1]. Traffic congestion as a worst side effect of transportation leads to enormous financial costs. Thus, to alleviate roadway congestion costs in some regions, cordon pricing has been introduced. Congestion pricing strategy has been first suggested by investigating a sample of a congested road and expressing some ideas about externalities and optimal congested charges by Pigou [2]. After Pigou's idea of using road pricing for adjusting road traffic congestion, intellectual and practical developments have occurred. Recently, the road pricing issue has widely attracted economists and transportation researchers [3–5].

Road pricing theory is based on the fundamental economic principle of marginal cost pricing. It indicates that users who use congested roads have to pay a toll, which is equal to the difference between marginal social costs and marginal private costs in a way that social surplus increases [6]. The first-best pricing design or marginal cost pricing, unlike its full theoretical basis, is of little practical interest due to its operating costs and public acceptance. Therefore, the second-best pricing method, which is considered best from a practical perspective, has attracted many researchers recently [7]. In the second-best pricing method, only a toll is charged over a subset of links in the network. There are four types of toll charging schemes in a road network that seem to be popular [8]. They are travel-distance based charging, travel time based charging, link-based charging and cordon-based charging. Recently in some countries, a cordon pricing scheme has been used to reduce traffic demand in central congested urban areas [9–12]. In cordon pricing, simultaneous determination of toll locations and toll levels in a network is practically important [6, 13–15].

Despite the positive effects of congestion pricing, its implementation has faced with problems. Impacts of pricing policies have been a relevant issue of transportation research for many years. Most literature focuses on several impacts, such as effects on traffic congestion and mobility [8, 47, 48], on vehicle emissions [30, 48, 49], and equity effects [6, 22, 50–52]. Political and public disinterests are the reason why not enough attention has been paid to its equity impact [6]. The equity issue of the cordon pricing has diverse dimensions. The first equity issue in congestion pricing is social inequity that depends on the unequivocally distributional effects of pricing between rich and poor drivers who pay the same toll charge [16–22]. On the treatment of this equity issue in pricing, one important direction is to incorporate users' heterogeneity in designing congestion charging. In this body of literature, focus is given particularly with respect to value-of-time (VOT), where it was mentioned in early research that ignoring heterogeneity in VOT may bias the calculated welfare effects of pricing [53]. Many theoretical researches since have been carried out to explore the impact of pricing on users of different VOTs [54, 55]. The second equity issue in congestion pricing is spatial inequity in the sense that the changes of the generalized travel costs of drivers travelling between different origin–destination O-D pairs may be significantly different when tolls are charged at some selected links [23, 24]. Therefore, few studies have examined the spatial equity effect especially in designing of the cordon pricing. Despite the vast literature in social equity, research gaps can still be identified in the spatial equity.

We believe that the implementation of the cordon pricing may cause more inequity problems among drivers. It may change unequally the travel time of different OD pairs depending on the location of cordon. In other words, drivers whose destinations are within the cordon or pass through it may face with inequity problem. This paper considers this above-mentioned inequity problem among drivers in the network that they face. Thus, we believe that due to nature of pricing problem, we can find a new solution that ensure a more equitable distribution of travel time among drivers.

This paper is organized as follows. In "Description of Spatial Inequity Problem", spatial inequity problem by implementation of the cordon charging is described in an artificial network. Next, a bi-level multi-objective optimization model is developed in "Model Formulation". In "Solution Algorithm", a solution algorithm is presented based on SPEA2 method. Then, in 'Numeral Example and Discussion", the developed model is applied to Sioux Falls network as a numerical example and the results are illustrated and discussed. Finally, summary and concluding remarks are presented in "Conclusions".

Description of Spatial Inequity Problem

We believe that by the implementation of the cordon pricing scheme, the travel time of some drivers will increase and they will face with spatial inequity. To illustrate this spatial inequity problem among drivers, we utilize an artificial network with 4 nodes and 4 links, as shown in Fig. 1.

In addition, we consider 2 different OD pairs, from node 1 to node 4 and node 2 to node 4 with travel demand of 400 and 300 vehicles per hour, respectively. Moreover, the cost functions are assumed to be Eq. (1) to Eq. (4):

$$t_1(x_1) = 2.5 + \frac{x_1}{400}, \tag{1}$$

$$t_2(x_2) = 1 + \frac{x_2}{200}, \tag{2}$$

$$t_3(x_3) = 1 + \frac{x_3}{400}, \tag{3}$$

$$t_4(x_4) = 0.5 + \frac{x_4}{400}, \tag{4}$$

where $t_i(x_i)$ and x_i are travel time and flow in link "i", respectively.

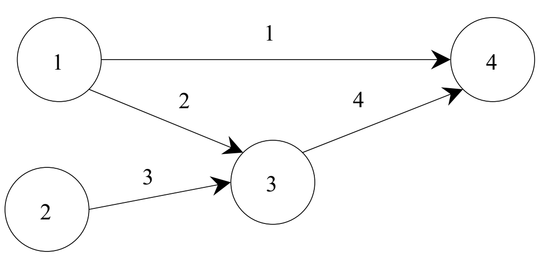

Fig. 1 An artificial network to show spatial inequity problem

To illustrate the spatial inequity problem, we assume two situations including a network without or with toll. Thus, in network with toll, we suppose charging a toll equal to 0.125 (min) in link 2 in Fig. 1. Then, the equilibrium travel cost for each OD pair is obtained by deterministic user equilibrium (DUE) in two situations. Next, we can calculate the travel cost in two situations. Table 1 present the travel cost and its changes in two situations.

The result of comparison shows that the travel cost will increase for drivers who travel from origin 1 to destination 4 and also will decrease for drivers who travel from origin 2 to destination 4. As a results, the drivers who travel from origin 1 to destination 4 will face with travel time increase. This inequity can be called as a spatial inequity.

Model Formulation

The issue of cordon pricing scheme is a transportation network optimization problem with user equilibrium constraints [25–29]. Therefore, cordon pricing scheme is a bi-level optimization problem. In other words, determining the values of the objective function of optimal design of the cordon scheme at upper level requires the travel behavior of users (flow and travel time), which is itself an optimization problem at lower level.

Lower Level Structure of Developed Model and its Solution Algorithm

As mentioned, the lower level problem in cordon pricing is user equilibrium. To solve the user equilibrium problem, we consider the following assumptions:

(1) Travel demand is elastic.
(2) There are three transportation modes consisting of private cars, taxis, and buses.
(3) There are Park-and-Rides (P&Rs) at the cordon boundary.

Moreover, to treat the elastic demand, we utilize an iterative diagonalization process and change it into fixed demand and then solve it; after each iteration, the convergence of the demand is examined. Steps of solution algorithm for lower-level problems are as follows [6, 30]:

Step 0 Assuming initial travel time using Eq. (5):

Table 1 The travel cost and its changes in two situations

OD	Situation 1 (without toll)	Situation 2 (with toll)	Change (min)	Change (%)
1–4	3.1875 (min)	3.21875 (min)	+0.03125	+
2–4	3.3125 (min)	3.28125 (min)	−0.03125	−

$$t_a^C = t_a^0, t_a^T = t_a^0, t_a^B = \alpha t_a^C = \alpha t_a^0, \tag{5}$$

where α is a constant coefficient that is assumed to be 1.2 [31].

It should be noted that all variables are defined at the end of the paper in the notational glossary.

Step 1 Calculating the minimum travel costs of each mode between OD pair "w" with respect to the path "k" assuming that taxis and buses do not pay tolls using Eqs. (6)–(8):

$$C_w^C = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^C + \delta_a\tau_a)\right], \tag{6}$$

$$C_w^T = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^T)\right], \tag{7}$$

$$C_w^B = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^B)\right], \tag{8}$$

where, if link "a" is tolled, δ_a is one; otherwise, it equals zero; if link "a" belongs to path "k" between origin "o" and destination "d", $\delta_{a,k}$ is one; otherwise, it is zero.

Step 2 Calculating the travel demand of modes (cars, taxis, and buses) between OD "w" assuming the independence of irrelevant alternatives using a Multinomial Logit Model Eq. (9):

$$d_w^m = D_w \exp(-\gamma_w \mu_w) \times \frac{\exp(a_m C_w^m + b_m)}{\sum_{m=C,T,B}\exp(a_m C_w^m + b_m)}, \tag{9}$$

where a_m and b_m are constant coefficients that are calibrated by network data; γ_w is demand elasticity coefficient between OD pair "w" that is related to the network condition; and μ_w is the minimum travel cost between OD pair "w" that is obtained by Eq. (10) [6, 30]:

$$\mu_w = \ln\left(\sum_{m=C,T,B}\exp(c_m C_w^m + d_m)\right), \tag{10}$$

where c_m and d_m are constant coefficients.

Step 3 Modifying car travel demand due to the existence of P&Rs at the cordon boundary; some drivers may shift from private cars to taxis and buses. The modification procedure includes:

(a) Identifying car travel demand whose destination is within the cordon.

(b) Determining the closest P&R to the origin "o" and destination "d" as a mid-location "p" ("p" is an index of P&R location).

(c) Calculating the minimum travel costs of modes with respect to the path "k". based upon the mid-location (P&R "p"), the minimum travel costs of modes are calculated under three conditions (car without mode change, car–taxi, and car–bus) using Eqs. (11)–(14):

$$C_{op}^C = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^C)\right], \tag{11}$$

$$C_{pd}^C = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^C + \tau_a)\right], \tag{12}$$

$$C_{pd}^T = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^T) + \theta_p\right], \tag{13}$$

$$C_{pd}^B = \text{Min}\left[\sum_{a\in A}\delta_{a,k}(t_a^B) + \theta_p\right], \tag{14}$$

where if link "a" belongs to path "k" between origin "o" and destination "d",$\delta_{a,k}$ is one; otherwise, it is zero.

(d) Modifying car travel demand based on the minimum travel costs by combining three conditions (car–car, car–taxi, and car–bus), travel demand by cars and other modes assuming the independence of irrelevant alternatives is modified using Eqs. (15)–(18):

$$(d_w^C)^{\text{new}} = (d_w^C)^{\text{old}} \times \frac{\exp(a_C(C_{op}^C + C_{pd}^C) + b_C)}{\exp(a_C(C_{op}^C + C_{pd}^C) + b_C) + \sum_{m=T,B}\exp(a_m(C_{op}^C + C_{pd}^m) + b_m)}, \tag{15}$$

$$d_{op}^C = (d_w^C)^{\text{old}} - (d_w^C)^{\text{new}}, \tag{16}$$

$$d_{pd}^T = d_{op}^C \times \frac{\exp(a'_T C_{\text{pd}}^T + b'_T)}{\exp(a'_T C_{\text{pd}}^T + b'_T) + \exp(a'_B C_{\text{pd}}^B + b'_B)}, \tag{17}$$

$$d_{pd}^B = d_{op}^C \times \frac{\exp(a'_B C_{\text{pd}}^B + b'_B)}{\exp(a'_B C_{\text{pd}}^B + b'_B) + \exp(a'_T C_{\text{pd}}^T + b'_T)}, \tag{18}$$

where a_i, b_i, a'_i, and b'_i are constant coefficients calibrated by network data.

Step 4 Solving the auto-assignment problem with fixed demand; if the demand between each OD is d_w, then the equilibrium model with fixed demand is formulated as follows [32]:

$$\text{Min} \sum_{a \in A} \int_0^{x_a} t_a(x_a)dx. \tag{19}$$

Subject to:

$$\sum_{r \in R_w} f_{rw} = d_w, \quad w \in W, \tag{20}$$

$$x_a = \sum_{w \in W} \sum_{r \in R_w} f_{rw} \delta_{ar}^w, \qquad a \in A, \tag{21}$$

$$f_{rw} \geq 0, \quad r \in R_w, \quad w \in W, \tag{22}$$

where δ_{ar}^w is one if route "r" between OD pair $w \in W$ uses link $a \in A$, and zero otherwise.

Step 5 Updating the travel time for private cars using the BPR equation.

Step 6 Assigning taxi demand based on the updated time of private cars using auto-assignment; next, taxi volume is determined in the network.

Step 7 Adding the equivalent taxis volume to the volume of private cars and estimating new travel time based on the BPR equation.

Step 8 Performing transit assignment using the Optimal Strategies method [31].

Step 9 Estimating the bus volume in the links; bus demand (person) in the links is converted into bus volume (vehicle) using a passenger coefficient.

Step 10 Adding the equivalent bus volume in the links; bus volume in the links is converted into bus equivalent volume and is then added to the previous equivalent volume.

Step 11 Updating the travel time for private cars; link travel time of private cars is updated based on new equivalent passenger car using the BPR equation.

Step 12 Verifying the convergence criterion for multi-modal assignment; if Eq. (23) is satisfied, proceed to step 13; otherwise, proceed to step 4:

$$\sum_a \left| \frac{X_a^{n+1} - \mathrm{X}_\mathrm{a}^\mathrm{n}}{\mathrm{X}_\mathrm{a}^\mathrm{n}} \right| \leq \varepsilon, \tag{23}$$

where X_a^n and X_a^{n+1} are the equivalent traffic flow in the link "a" in two successive iterations.

Step 13 Verifying the convergence criterion for demand; if Eq. (24) is satisfied, proceed to step 14; otherwise, proceed to step 1:

$$\sum_{m=C,T,B} \left| \frac{(d_w^m)^{n+1} - (d_w^m)^n}{(d_w^m)^n} \right| \leq \varepsilon, \tag{24}$$

where $(d_w^m)^n$ and $(d_w^m)^{n+1}$ are demand of mode "m" between OD pair "w" in the two successive iterations.

Step 14 Termination of multi-modal traffic assignment; the outputs of this step are traffic volumes of private cars, taxis, and buses in the links of the network.

Upper Level Structure of Developed Model

Upper Level Structure of Basic Cordon Pricing Model

The upper level of cordon pricing is to maximize the social welfare function by estimating the optimal values of a set of decision variables (with respect to the toll level and cordon location) as follows [33]:

$$F_1 = \text{Max} \left(\begin{array}{l} \sum_{w \in W} \int_0^{d_w} D_w^{-1}(w)dw - \sum_{a \in A} t_a^C x_a^C \\ - \sum_{a \in A} t_a^T x_a^T - \sum_{a \in A} t_a^B x_a^B \end{array} \right). \tag{25}$$

Subject to:

$$0 \leq \tau \leq \tau_{\max}, \tag{26}$$

$$0 \leq \theta \leq \theta_{\max}, \tag{27}$$

where constraint Eqs. (26) and (27) refer to the maximum and minimum values of the toll level and price of P&R, respectively. It should be noted that the values of the travel time and the flow for each mode are obtained from a lower level optimization problem.

Definition of Spatial Equity Function

As mentioned before, the spatial inequity problem is caused among drivers due to implementation of the cordon pricing scheme. In fact, it is almost impossible to ensure that the benefits and costs gained from cordon pricing scheme will be identical for all drivers. However, we can restrict the travel cost for each OD pair so that the spatial inequity problem cannot exceed a given level. Thus, we propose inequity (28) as a constraint and add it to the basic model:

$$\underset{w \in W}{\text{Max}} \left\{ \frac{\mu_w(\tau)}{\overline{\mu}_w} \right\} \leq \beta, \tag{28}$$

where $\mu_w(\tau)$ and $\overline{\mu}_w$ present the corresponding equilibrium travel cost for each car OD pair before and after cordon pricing scheme, respectively, which are obtained from a lower level optimization problem.

The parameter β is a given suitable positive constant that ensure spatial inequity problem does not exceed a given level. The smaller value of parameter β refers to a more equitable distribution of cost among all drivers. If $\beta \prec 1$, then all driver will enjoy a travel cost reduction at least by $100(1-\beta)\%$ derived from cordon pricing scheme. Moreover, if $\beta \succ 1$, it means that there may be some drivers who will suffer from a higher travel cost, but travel cost increase cannot be more than $100(\beta - 1)\%$.

It is to be noted that β should be selected to be $\beta \in \left[\text{Min}\left(\underset{w\in W}{\text{Max}}\left\{\frac{\mu_w(\tau)}{\bar{\mu}_w}\right\}\right), \text{Max}\left(\underset{w\in W}{\text{Max}}\left\{\frac{\mu_w(\tau)}{\bar{\mu}_w}\right\}\right)\right]$ or $\beta \in \left[\alpha_{\min}, \alpha_{\max}\right]$. In other words, $\alpha_{\min}$ and $\alpha_{\max}$ represent the limits of an increase in cost resulting from the implementation of a cordon pricing scenario (τ). When $\beta \prec \alpha_{\min}$, then the model cannot be solved. When $\beta \succ \alpha_{\max}$, the constraint (28) becomes an abundant constraint and the model is identical to the conventional bi-level optimization models. Thus, this limitation will have no effect. It is to be noted that the imposition of constraint (28) will lead to an increase in equilibrium OD travel cost between the private cars OD pairs which have the greatest decrease in equilibrium OD travel cost without an imposition of this constraint.

Upper Level Structure of Developed Model Considering Spatial Equity

Finally, the following optimization model with constraint considers the spatial equity impacts in the upper level of the model:

$$F_1 = \text{Max}\left(\begin{array}{l} \sum\limits_{w\in W} \int\limits_0^{d_w} D_w^{-1}(w)dw - \sum\limits_{a\in A} t_a^C x_a^C \\ - \sum\limits_{a\in A} t_a^T x_a^T - \sum\limits_{a\in A} t_a^B x_a^B \end{array}\right). \tag{29}$$

Subject to:

$$\underset{w\in W}{\text{Max}}\left\{\frac{\mu_w(\tau)}{\bar{\mu}_w}\right\} \leq \beta, \tag{30}$$

$$0 \leq \tau \leq \tau_{\max}, \tag{31}$$

$$0 \leq \theta \leq \theta_{\max}. \tag{32}$$

Moreover, we can change inequality Eq. (30) into Eq. (33) as a single objective function and add it to the main objective function in the model [34]:

$$F_2 = \text{Max}\left(\beta - \underset{w\in W}{\text{Max}}\left\{\frac{\mu_w(\tau)}{\bar{\mu}_w}\right\}\right). \tag{33}$$

Therefore, the optimization model with a constraint is converted into a conventional multi-objective optimization model as follows:

$$F_1 = \text{Max}\left(\begin{array}{l} \sum\limits_{w\in W} \int\limits_0^{d_w} D_w^{-1}(w)dw - \sum\limits_{a\in A} t_a^C x_a^C \\ - \sum\limits_{a\in A} t_a^T x_a^T - \sum\limits_{a\in A} t_a^B x_a^B \end{array}\right), \tag{34}$$

$$F_2 = \text{Max}\left(\beta - \underset{w\in W}{\text{Max}}\left\{\frac{\mu_w(\tau)}{\bar{\mu}_w}\right\}\right). \tag{35}$$

Subject to:

$$0 \leq \tau \leq \tau_{\max}, \tag{36}$$

$$0 \leq \theta \leq \theta_{\max}. \tag{37}$$

Thus, this model is used to derive the most favorable trade-off between social welfare and spatial equity by the implementation of the cordon pricing scheme.

Solution Algorithm

The pricing problem is a non-convex optimization problem and NP-Hard problem, for which it is difficult to find the optimum solution using standard optimization methods. Therefore, it is necessary to apply a global optimization method to solve the developed model. In addition, multi-objective optimization models are more complex than single-objective optimization models and different methods of solution should be applied [35]. Although there are various ways to approach a multi-objective optimization problem, most studies in the area of evolutionary multi-objective optimization have concentrated on the approximation of the Pareto set. The goal of approximating the Pareto set is itself multi-objective. Evolutionary algorithms (EAs) are one of the popular algorithms to solve multi-objective optimization. The first actual implementation of what is now called a multi-objective evolutionary algorithm (MOEA) is Schaffer's vector evaluation genetic algorithm (VEGA), which was introduced in the mid-1980s, mainly aimed to solve problems in machine learning [36–38]. Since then, a wide variety of algorithms have been proposed in the literature [39–41].

SPEA2 is a member of Pareto-based approach group. The SPEA algorithm was introduced by Zitzler and Thiele [42]. This approach is conceived as a way of integrating different MOEAs. SPEA uses an archive containing non-dominated solutions that are previously found (the so-called external non-dominated set). In each generation, non-dominated individuals are copied to the external non-dominated set. For each individual in this external set, a strength value is computed. This strength is similar to the ranking value of a multi-objective genetic algorithm (MOGA), since it is proportional to the number of solutions a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external non-dominated solutions that dominate it. In addition, a clustering technique called the “average linkage method” [43] is used to maintain diversity. However, the SPEA2 approach has three main differences with respect to its predecessor [44]: (1) It incorporates a fine-grained fitness assignment strategy, which takes into account the number of individuals that dominate it and the number of individuals by which it is dominated for each individual; (2) It uses the nearest neighbor density estimation technique, which guides the search more efficiently; and (3) It has an enhanced archive truncation method that guarantees the preservation of boundary solutions. In fact, SPEA2 is an improved version of SPEA.

The two distinct objectives in multi-objective optimization are (1) to find solutions as close to the Pareto-optimal solutions as possible and (2) to discover solutions as diverse as possible in the obtained non-dominated front. Comparison of some previous studies shows that the non-dominated solutions obtained by SPEA2 are better than other methods both in terms of convergence and diversity but at the expense of computational time (e.g. [56]). In the present study, computational time is not very important because the cordon pricing scheme is a design problem. Therefore, we found SPEA2 as the best method due to its capabilities for the present study. According to the developed model and using the SPEA2 method [44, 45], the Pseudo Code of the algorithm for the solution of the developed model are as follows:

```
1: Input.
        A: transportation network
        Ne: population size
        N: archive size
        T: maximum number of generations.
2:  Initialization. Generate an initial population P(0) and create an empty archive (external archive) A(0). Set t=0.
3:    For i=1 to T Do
4:      User Equilibrium. Solve the lower level problem.
5:      Fitness assignment. Calculate fitness values of individuals in P(t) and A(t).
6:      Environment selection. Copy all non-dominated individuals in P(t) and A(t) to A(t+1).
7:        If size of A(t+1) exceeds N then
8:        Reduce A(t+1) by means of the truncation operator; otherwise
9:        If size of A(t+1) is less than N then
10:       Fill A(t+1) with dominated individuals in P(t) and A(t) by means of the sorting operator.
11:        End If
12:     Termination.
13:        If t>T is satisfied, then
14:        Stop and output non-dominated set. Otherwise,
15:        Continue.
16:        End If
17:     Mating selection. Perform binary tournament selection with replacement on A(t+1) in order to fill mating pool. The size of
mating pool is Ne.
18:      Reproduction. Apply diversity operator consists of recombination and mutation operators to the mating pool P(t+1) to the
resulting population. Set t=t+1; go to line 4.
19:    End For
20:  End
21: Output. non-dominated set.
```

Numeral Example and Discussion

To apply the developed model and present the discussion, the Sioux Falls network is used as a numeral example as shown in Fig. 2.

The Sioux Falls network consists of 24 nodes and 76 links. Travel cost function is as follows:

$$\mathrm{t_a}(x_a) = t_a^0\left[1.0 + 0.15\left(\frac{x_a}{C_a}\right)^4\right]. \tag{38}$$

All data of network including free flow travel time and link capacity presented in study of Afandizadeh and Abdolmanafi [30]. In addition, four bus lines are considered. Features of the bus lines service are shown in Table 2 [30].

Fig. 2 The Sioux Falls network

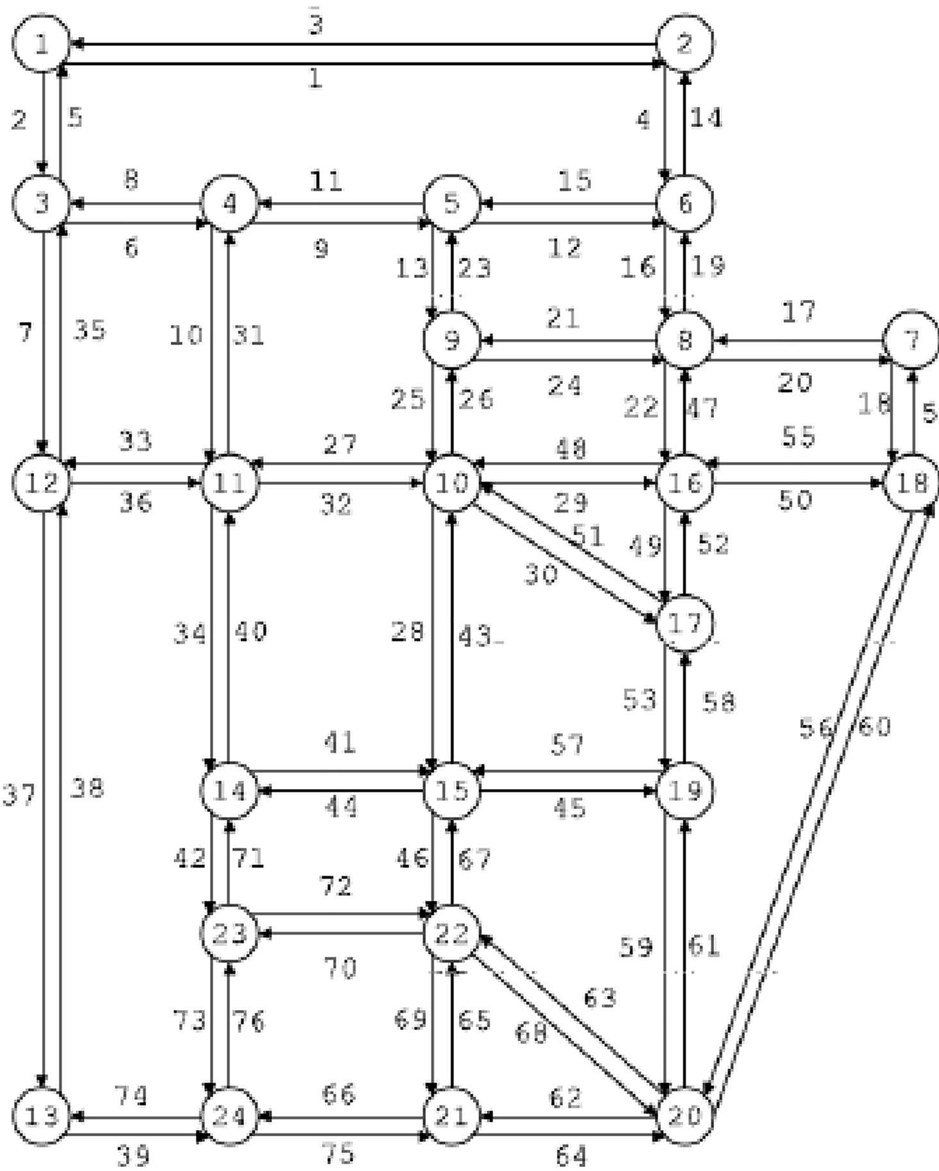

Table 2 Features of bus lines

Lines	Line 1	Line 2	Line 3	Line 4
Headway (min)	5	15	5	10
Speed (km/h)	20	15	25	15
Stations (nodes)	11, 10, 16, 17, 19	2, 6, 8, 16	8, 9, 10, 15, 22, 21	1, 3, 12, 11, 10

Travel demand and characteristics of network of the Sioux Falls presented in Wang et al.'s study is considered [46].

Moreover, Eqs. (39)–(41) are used as the utility functions of modes between OD pairs [6, 30]:

$$u_w^C = -0.0101\, t_w^C, \tag{39}$$

$$u_w^T = -0.2613 - 0.1096\, t_w^T, \tag{40}$$

$$u_w^B = -0.6936 - 0.1257\, t_w^B. \tag{41}$$

To consider drivers' behavior change due to the existence of P&R at the cordon boundary, Eqs. (42)–(44) are used as the utility function for shifting cars to taxis or buses (car only, car–taxi, and car–bus) [6, 30]:

$$u_{\text{car only}} = -0.0284\, t_{ps}^C, \tag{42}$$

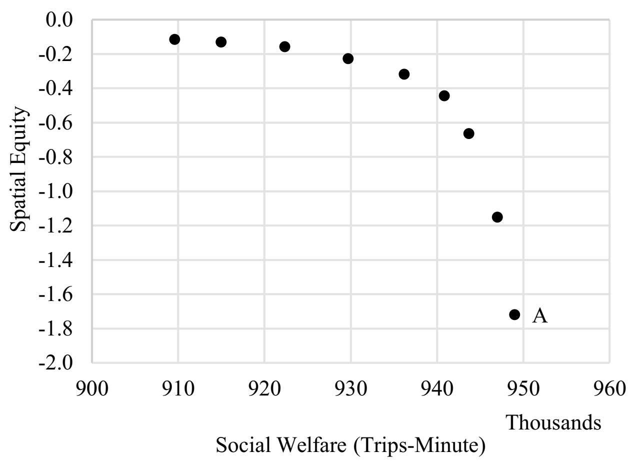

Fig. 3 Pareto front in the objective space (F_1 and F_2)

$$u_{\text{car-taxi}} = 1.21 - 0.0451\, t_{ps}^{T}, \tag{43}$$

$$u_{\text{car-bus}} = 1.24 - 0.0432\, t_{ps}^{B}. \tag{44}$$

Moreover, for the spatial equity function,β is assumed to equal 1.05. Therefore, 5% increase in travel time (or spatial inequity) after implantation of the cordon pricing is permitted. Given the expressed assumptions, the algorithm of the developed model is implemented using Matlab software. Next, two objective functions (F_1 and F_2), which are the social welfare and the spatial equity, are considered simultaneously. Then, non-dominated results (a set of optimal results) are extracted based on SPEA2 method (with the maximum number of generation: 300). Finally, the position of non-dominated results in the objective space or Pareto front is depicted in Fig. 3.

The concave curve is formed by non-dominated results, which confirms the validity of the developed model. This curve reveals that the cordon pricing scheme is a multi-objective problem, indicating that spatial equity will not necessarily increase due to an increase in social welfare. Based on the results of the developed model with two objective functions (F_1 and F_2), we have:

(1) The social welfare objective function (F_1) changes in the range of 909,597 to 949,001 trips-minute. The best situation of social welfare objective function (F_1: 949,001 trips-minute) is equivalent to − 1.115 in the spatial equity objective function (F_2) (result "A" in Fig. 3).
(2) The spatial equity objective function (F_2) changes in the range of − 1.719 to − 0.115. The best situation of spatial equity objective function (F_2: − 0.115) is equivalent to 909,597 trips-minute in social welfare objective function (F_1) (result "B" in Fig. 3).

Therefore, the formation of the Pareto front in the objective function confirms that near the point of maximum social welfare there is a tradeoff between social welfare and spatial equity. Indeed, such a tradeoff is inevitable given two objective functions unless they achieve a maximum at the same point. Let X_I be the value of the decision variable (or variables) that maximizes objective function "I", and define X_J similarly for objective function "J". Starting at X_I, and moving toward X_J will surely reduce the value of objective function "I". In other words, it seems reasonable to consider cordon pricing as a multi-objective problem by considering spatial equity. If the social welfare objective function is more important than the spatial equity objective function for decision makers, he/she can choose result "A". If the spatial equity objective function is more important than the social welfare objective function, he/she can select result "B".

In addition, according to the non-dominated results of the developed model, we can conclude:

- By choosing another result (changing from result "A" to result "B") in the objective space, we can create the best situation for the spatial equity objective function (F_2), while the social welfare objective function (F_1) is only reduced to 4.15%.

Therefore, we can create a sustainable situation for the transportation system by improving the spatial inequity with a reduction in social welfare. It should be noted that the value of this reduction depends on the value of time (VOT) of users and how much they value spatial equity.

Moreover, results of "A" and "B" in the objective function correspond to the specific features of cordon location, toll level, and price of P&R, as presented in Table 3.

A comparison of the results (changing from result "A" to result "B") shows that:

- Nodes number in the cordon decreases by 28.57%;
- Toll level increases by 9.09%;
- Price of P&R increases by 35.71%.

Table 3 Features of solutions (results "A" and "B")

Result	Nodes in the cordon	Toll level (h)	Price of P&R (h)
A	4, 9, 10, 15, 17, 19, 22	1.10	0.14
B	4, 9, 10, 15, 19	1.20	0.19

Therefore, we can improve the spatial equity by selecting another result (result “B”). Hence, by the selection of this result, we should decrease the cordon area (nodes in the cordon) and increase toll level and price of P&R. Note that these changes are not fixed and depend on the network (supply) and demand.

Conclusions

This paper investigated the issue of spatial inequity among some drivers in the cordon pricing scheme. The spatial inequity resulting from the introduction of a cordon pricing scheme in the artificial network was illustrated. The spatial inequity is for drivers whose travel destinations are located within the cordon or should travel through it and finally they may face with higher travel time. We called this inequity as a spatial inequity. To reduce the spatial inequity, a multi-objective bi-level optimization model was developed. Then, an algorithm was presented according to the second version of Strength Pareto Evolutionary Algorithm (SPEA2) for solving the developed model. The developed model was applied to the Sioux Falls network as a numerical example and then the results of the model were analyzed.

The results showed that this model can be a useful tool for equitable designing of the cordon pricing scheme. Further, the formation of the Pareto front in the objective function confirms the point of maximum social welfare and there is a tradeoff between social welfare and spatial equity. Indeed, such a tradeoff is inevitable given two objective functions unless they achieve a maximum at the same point. In other words, it seems reasonable to consider cordon pricing as a multi-objective problem by considering spatial equity. In addition, the results revealed that by searching in the solution space, it can reduce spatial inequity with relatively low reduction in social welfare. Therefore, we can create a sustainable situation for the transportation system by improving spatial inequity with a relatively low reduction in social welfare. Moreover, there are spatial inequity impacts in real networks, which should be considered in the cordon pricing scheme.

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