

# Multiple-Leader Vehicle-Following Behavior in Heterogeneous Weak Lane Discipline Traffic

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**Abstract** Traffic stream of developing countries have a characteristic heterogeneous traffic stream moving in a weak or no lane-discipline. Vehicle-following behavior is in a staggered fashion, with following gap changing as per amount of stagger (centerline separation in lateral direction), vehicle type of leading vehicle and following vehicles and also their speeds. In denser traffic streams with no lane-discipline, the presence of more than one leading vehicle (placed side by side) may influence the behavior and position of a following vehicle. The following vehicle may maintain its speed and position depending on the combined effect of individual leading vehicles. In this paper, behavior of a following vehicle (FV) with two leading vehicles (placed side by side) is studied and compared with behavior of a FV with only one leading vehicle. Change in the relationships of longitudinal gap with staggering and average speeds due to the introduction of a second leading vehicle is calculated for various vehicle pairs. Effect of variation of the lateral distance between two leading vehicles on average longitudinal gap is studied. An attempt is made to represent impact of two leading vehicles on FV in the form of a single effective vehicle. It is observed that longitudinal gap decreases with the introduction of a second leading vehicle. As lateral distance between two leading vehicles decreases, longitudinal gap between leading and following vehicles increases. Impact of simultaneous interaction with two leading vehicles is presented in this paper. The relationships developed are

useful for modeling car-following behavior in heterogeneous and dense traffic stream with no lane discipline.

**Keywords** Multiple vehicle-following · Weak lane discipline · Heterogeneous traffic

## Introduction

Traffic stream of developing countries have two characteristic features—(1) lack of lane discipline and (2) heterogeneous traffic. Since there is no concept of a lane, vehicles overtake whenever sufficient gaps are available to pass. Moreover, several vehicle types are generally observed on Indian roads which vary significantly in their sizes and operating characteristics. These different vehicles are generally classified in various classes like car, truck, light commercial vehicles (LCV), bus, auto-rickshaw (motorized three wheeler), motorized two wheelers and non-motorized vehicles, etc. In this traffic stream, position of a following vehicle (FV) is affected by several leading vehicles (LVs) in its front.

In order to analyze this traffic stream behavior, a study of interaction (longitudinal as well as lateral) between vehicles is necessary. In this scenario, vehicles do not follow each other in demarcated paths (or lanes). This is because the response of FV driver to any perturbation made by the LV is not just in the form of acceleration or braking, but also in the form of veering. Vehicle-following behavior is observed to proceed in a staggered fashion. Here, staggering is referred as centerline separation between two vehicles in lateral direction of the road. Therefore, it is important to study the relationship of staggering with vehicle-following parameters such as distance headway, speeds and also vehicle types of interacting pair of vehicles in this traffic stream. Some initial attempts were made by Gunay [8] and Budhkar and Maurya

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[4] in this regard. These attempts concluded that with the increase in staggering, following distance decreases.

However, in denser traffic streams with no lane-discipline, the presence of more than one LV (placed side by side) may influence the behavior and position of a FV. The driver of FV is subject to multiple sources of stimulus from each of LV's. The FV would maintain its speed and position depending on the combined effect of individual leading vehicles. The objective of this paper is to study the change in relationships of longitudinal gap with staggering and average speeds, due to the introduction of a second leading vehicle. For achieving this objective, traffic videos are captured at mid-block traffic streams, and trajectory points of interacting vehicles are extracted to obtain longitudinal gaps, amount of staggering and speeds of these vehicles. Obtained data are processed and modeled for various vehicle groups (one FV, and one or two LV's). The scope of this paper is limited to steady-state vehicle-following, that is when a vehicle is following another vehicle(s) for a longer duration and has stabilized behind them, and not on the verge of overtaking them.

The next section of this paper highlights the previous work related to study of vehicle-following behavior in weak lane discipline. The third section of the paper focusses on data collection and extraction methodology. The fourth section compares obtained results of single and multiple vehicle-following behavior. It also models the effect of amount of constraining (or restriction), effect of vehicle type and resultant representation of a vehicle-following behavior two vehicles. Last section concludes the research and opens future exploration in this regard.

## Previous Work

The previous research related to vehicle-following behavior in heterogeneous traffic with weak lane discipline can be grouped under studied related to car-following models for weak lane disciplined traffic stream, modeling the heterogeneous traffic, modelling the relationship between lateral and longitudinal distances, and multiple vehicle-following behavior. Since most studies were based in developed world where cars consist of majority of vehicle, the car-following term was popular. However, in this paper, the study is on heterogeneous traffic stream, thus the notation car-following is replaced with vehicle-following.

### Car-Following Models for Weak Lane Disciplined Traffic Stream

Car-following models describe the processes by which drivers follow each other in the traffic stream. They have been studied for more than half century (for e.g. Pipes' car

following model, [17]). A detailed description of all such car-following models can be obtained in Brackstone and McDonald [2] review paper.

However, unlike the basic car-following, the staggered vehicle-following widely exists in the developing countries due to lack of lane-discipline. It is a psychological tendency of drivers that they refrain from driving side by side with other vehicles for long periods of time [8]. Driver either passes the neighbouring vehicles or lag behind them to form a staggered queuing patterns. Hence, in modelling, consideration of an adjustment exercise for the calculation of the longitudinal positions of vehicles by taking the effect from the adjacent lane vehicles is important. Expanding modeling of the driver's attention to the two-dimensional arena is important. Some attempt in this regard is made by Tao et al. [19] where effect of neighboring vehicles and staggered headways are modeled by a modified GM model.

Gunay [9] defined the term lane-based driving discipline as the tendency to drive within a lane by keeping to the center as closely as possible. Jin et al. [12] developed a theory for staggered car-following based upon geometric analysis and proposed a general equation for time to collision based upon non-lane based traffic. This was obtained by using visual angle information that can be perceived by drivers directly. This approach is novel but data-collection from external source is difficult. Models are also proposed by using least distance between the vehicles (or pores) as a criterion. These include the continuum model by Nair et al. [16] and pore-space model by Ambarwati et al. [1]. However, as vehicle types increases, the models become more complicated. Moreover, the models assume uniformity in driver behavior of a particular vehicle type, which may not be true. Car-following may also be represented by fuzzy logic models as given by Kikuchi and Chakroborty [13].

### Modeling Vehicular Heterogeneity

For considering vehicular heterogeneity, there are important contributions such as Arasan and Koshy (2005) for transverse clearance between vehicles, and Ravishankar and Mathew [18] for evaluating parameters of Gipp's model [7] between various vehicle pairs in heterogeneous traffic. Grid-based [11] or strip based [15] approaches can also depict vehicle heterogeneity. Mallikarjuna and Rao [14] have modeled heterogeneous traffic which includes governing of sub-lanes based upon the dimensions of the smallest vehicle in traffic that is the two-wheeler. Gunay [10] attempted to explore the issue of two-dimensional headway analysis (lateral and longitudinal) in detail for better realism in traffic flow modelling.

## Relationship of Longitudinal Headway with Average Speed and Staggering

Relationship of longitudinal headway with average speeds and amount of staggering is studied by Budhkar and Maurya [4]. Data of interacting pairs of vehicles in a weak lane-disciplined traffic stream were extracted, and regression lines were modeled for relationship of longitudinal gap (LG) with average speed ( $\bar{v}$ ) and staggering or centerline separation (CS), which is the lateral distance between centerlines of FV and LV.

## Multiple Leading Vehicles Interaction with Following Vehicle

Multiple leading vehicle interaction involves the following of two or more leading vehicles placed side by side, simultaneously. In this case, either there is a priority LV which affects decision of the FV, or there is a combined effect of both the leaders on the following vehicle. In either case, it is not clear whether position of individual leader or all leaders affect the behavior of FV, because drivers' decision is latent, and can be perceived only in the form of their actions. Choudhury and Islam [5] have proposed a latent-leader approach, where leadership to a particular vehicle is assigned based upon utility parameters of average speeds, distances, vehicle types or relative speeds. This condition may be true for FV which are on the verge of overtaking and thus not a stable following case. However, in case of stable vehicle-following behavior, speeds do not differ by a large amount, and the position of FV has already been stabilized. Thus, driver behavior is reflected in gap-maintenance rather than acceleration, deceleration, or reaction based on significant relative speeds. Moreover, in a stable stream, there is an effect of all vehicles in the vicinity rather than individual vehicle's effect.

From the literature review it can be concluded that there has been limited study to model no lane-discipline traffic combined with vehicle heterogeneity. There has been little study which focus on FV following multiple LVs. This analysis is a crucial stage before going to model such a traffic stream of developing countries like India, Bangladesh, Vietnam or Nepal. The work in this paper is thus motivated to find out changes in driver behavior while following multiple vehicles. The scope of this paper is limited to mid-block sections with no external factors apart from vehicles affecting behavior of a particular vehicle's driver. Following are the external factors which may affect traffic stream in mid-block sections:

1. Horizontal and vertical curves within 1 km of section.
2. Gradients.
3. Merging or intersecting traffic.

4. Parked vehicles.
5. Significant pedestrian cross movements.
6. Effect of adverse weather conditions.
7. Pavement deteriorations to an extent that it affects speed of vehicles significantly.

To ensure that the above external factors do not have effect on traffic, precaution was taken during selection of sites and during actual data-collection. Sites were selected which do not have any significant impact of external effects from (1) to (4). Factors (5)–(7) were taken care during data collection and data extraction.

## Data Collection

For analyzing interactions of FV with LVs, initially, top-view video recording of traffic streams of mid-block sections is conducted by mounting the video camera at higher locations (such as high rise buildings). Lateral and longitudinal ( $x$  and  $y$ ) coordinates of vehicle (using roadway edge as reference point) are calculated by camera calibration techniques to obtain vehicle positions at given time.

## Data Collection Methodology and Extraction

Traffic videos for longitudinal gap analysis are recorded in five mid-block road sections located in different metro cities of India like Mumbai, Pune, Bangalore and Kolkata. Details of the traffic sections are mentioned in Table 1. These sites are so chosen based upon the prerequisites for camera calibration technique as described by Fung et al. [6]. Useful data are extracted from both carriageways from roads of each of the sections. Two kinds of data were extracted from the videos—(1) one vehicle following one leading vehicle, and (2) one vehicle following two leading vehicles. For extracting the first part, interacting leading-following vehicle pairs were identified and their trajectories are extracted. Two vehicles are identified as an interacting pair, if FV is unable to overtake LV (due to other vehicles travelling at significantly higher or lower speeds in the neighborhood), and follows it at steady state (speed difference less than 5 km/h). Data for several such pairs are obtained. In order to extract data from video for the second part, a group of three interacting vehicles (as shown in Fig. 1) are identified. Data for several such groups are extracted. Three vehicles in a stream are considered to be an interacting group, if presence of two LVs influence the behavior and position of the FV simultaneously. Due to the presence of two LVs, the FV is not able to overtake since lateral gap between the two LVs is insufficient to make an overtaking decision. Figure 1 shows a snapshot of such groups of interacting vehicles.

**Table 1** Details of location of data collection

S. no.	Road (city)	Location	Date	Time	Road width (m)	Trap length (m)
1	EM bypass (Kolkata)	Ruby hospital (Desun more)	18-Nov-2014	10:00–13:00	7.8	35
2	Bellary road (Bengaluru)	Mekhiri circle	30-Nov-2014	16:30–19:30	11.2	30
3	Sinhagad road (Pune)	Nilayam bridge	04-Dec-2014	11:00–13:00	10.4	35
4	Western Express Highway (Mumbai)	Ismail Yusuf College, Jogeshwari	07-Jul-2014	10:00–11:30	17.5	45
5	Link road (Mumbai)	Saidham complex (Malad West)	08-Jul-2014	18:30–20:00	14	40

**Fig. 1** Video snapshots showing two leading vehicles followed by one vehicle in different sections

### Data Extraction Technique

Image coordinates of vehicles are marked during vehicle interaction by means of a mouse click. These coordinates get recorded in a file. Camera calibration technique devised by Fung et al. [6] is used to convert image coordinates into actual field coordinates along and across length of the road. For a particular group of (two or three) interacting vehicles in the traffic stream, vehicle sizes and types are noted and marked by mouse click. Afterwards, 3–4 trajectory points for each of these vehicles are marked, by mouse clicking on one corner of each vehicle visible in the image at different time-stamps. Field coordinates corresponding to these image-coordinates can be calculated using camera

calibration techniques. The entire algorithm and procedure is explained in a research paper by Budhkar and Maurya [3].

The user finally obtains  $x$  and  $y$  coordinate of each of interacting vehicles in the field, their sizes and vehicle types at different time stamps. Speed of a vehicle is calculated based upon difference in timestamp and positions of two trajectory points of a vehicle. Centreline separation (CS) is the centre to centre distance measured between LV and FV at a given time. It provides the amount of staggering between two vehicles. Longitudinal gap (LG) is the distance between rear bumper of LV to front bumper of FV, calculated by subtracting vehicle length of LV from distance headway. Speeds of vehicles obtained by this method are used to calculate relative speeds. Data are considered only for those groups/

pairs which had a stable vehicle-following, or relative speed of FV and any of the LV were not more than 5 km/h. Thus, inspite of video recorded for a longer duration at each of section, useful data were limited.

Data for a total of 3761 LV-FV pairs, and 930 groups of three vehicles (LV-FV<sub>1</sub>-FV<sub>2</sub>, corresponding to 1860 LV-FV multiple vehicle-following pairs) were extracted and analysed. Vehicle types considered in this analysis include car, auto (motorized three wheeler), bike (motorized two wheelers), LCV (Light commercial vehicle), truck and bus (clubbed together as a single class of heavy vehicles). About 30% of these data are separated for validation purpose. Data from site number 2 and 3 in Table 1 are separated for validation purpose.

### Analysis and Discussions

This section deals with analysis of extracted data for individual pairwise (FV-LV) following, FV interacting with two LVs simultaneously and their comparison with each other. It also deals with effect of gap between both the LVs and their vehicle types on FV, and changes necessary if two LVs are represented as a single effective vehicle.

#### Analysis for Variation of Longitudinal Gap with Centerline Separation and Speed

From obtained dataset of interactions of pair of leading and following vehicles (i.e. FV following a single LV), Longitudinal gap (*LG*) is considered as dependent variable and centerline separation (*CS*) as well as average

speed  $\bar{v}$  is considered as independent variables. A trend of these variables was plotted for various vehicle types, and it is observed that *LG* increases with increase in  $\bar{v}$  and decrease in *CS*. It is observed that linear relationship gives a better fit and improving the degree of equation or any other relationship does not improve the  $R^2$  value substantially. Thus, general equation of this trend is mentioned in Eq. (1), which is first degree equation with *CS* and  $\bar{v}$ . In Eq. (1), *a* and *b* are coefficients for average speed of two vehicles and centerline separation respectively, and *c* is the intercept the regression-plane makes with *LG* axis. The residuals from this regression plane were calculated and found to be heteroskedastic. Appropriate transformations were made to make them homoscedastic. The distribution of residuals ( $\varphi$ ) in Eq. (1) about the regression plane is observed to be statistically similar to burr-distribution (as per Kolmogorov–Smirnov test), and its general equation is provided in Eq. (2). In Eq. (2),  $f(x)$  gives the distribution of residuals ( $x$ ) about the best fit regression line:

$$LG = a(\bar{v}) + b(CS) + c + \varphi, \tag{1}$$

$$f(x) = \frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{k+1}}. \tag{2}$$

In Eq. (2),  $\alpha$  and *k* are shape parameters,  $\beta$  is the continuous scale parameter and  $\gamma$  is the continuous location parameter. The results of the analysis for different LV-FV pairs are shown in Table 2. The work involved only pairwise extraction of data of interacting vehicles. Effect of multiple vehicle interaction was not evaluated.

**Table 2** Coefficients and residual parameters for plot of *LG* vs *CS* and speed

LV	FV	Sample size	Coefficients of Eq. (1) (modelled data)			Burr (4P) distribution parameters of modified residuals				
			<i>a</i>	<i>b</i>	<i>c</i>	<i>k</i>	$\alpha$	$\beta$	$\gamma$	<i>p</i> value
Car	Car	4356	0.11	-1.25	3.38	1.25	3.37	1.13	-1.15	0.091
Auto	Car	2187	0.03	-1.05	6.09	5.96	1.95	2.68	-1.02	0.145
Bike	Car	1885	0.04	-0.50	3.97	6.88	1.67	3.32	-1.00	0.409
LCV	Car	512	0.08	-1.42	4.26	3.44	1.95	1.91	-1.02	0.211
Heavy	Car	617	0.17	-0.80	1.78	1.55	3.19	1.14	-1.02	0.714
Car	Auto	345	0.08	-1.62	4.38	12.21	1.56	5.14	-0.96	0.048
Car	Bike	489	0.07	-0.30	2.50	15.47	1.63	5.87	-1.01	0.085
Car	LCV	220	0.11	-1.05	2.81	3.13	2.22	1.78	-1.07	0.331
Auto	Auto	608	0.01	-1.23	7.23	168.74	1.54	30.92	-1.01	0.896
Auto	Bike	291	0.09	-1.01	1.73	0.91	3.29	0.75	-0.94	0.423
Bike	Auto	117	0.04	-1.28	4.32	133.19	1.51	28.85	-1.02	0.035
Heavy	Heavy	108	0.26	-1.52	2.66	2.81	3.59	1.84	-1.34	0.064
Bike	Bike	89	0.03	-0.21	3.32	3.73	1.54	2.2	-0.99	0.101

### Determination of Position of FV in Vehicle-Following Scenario

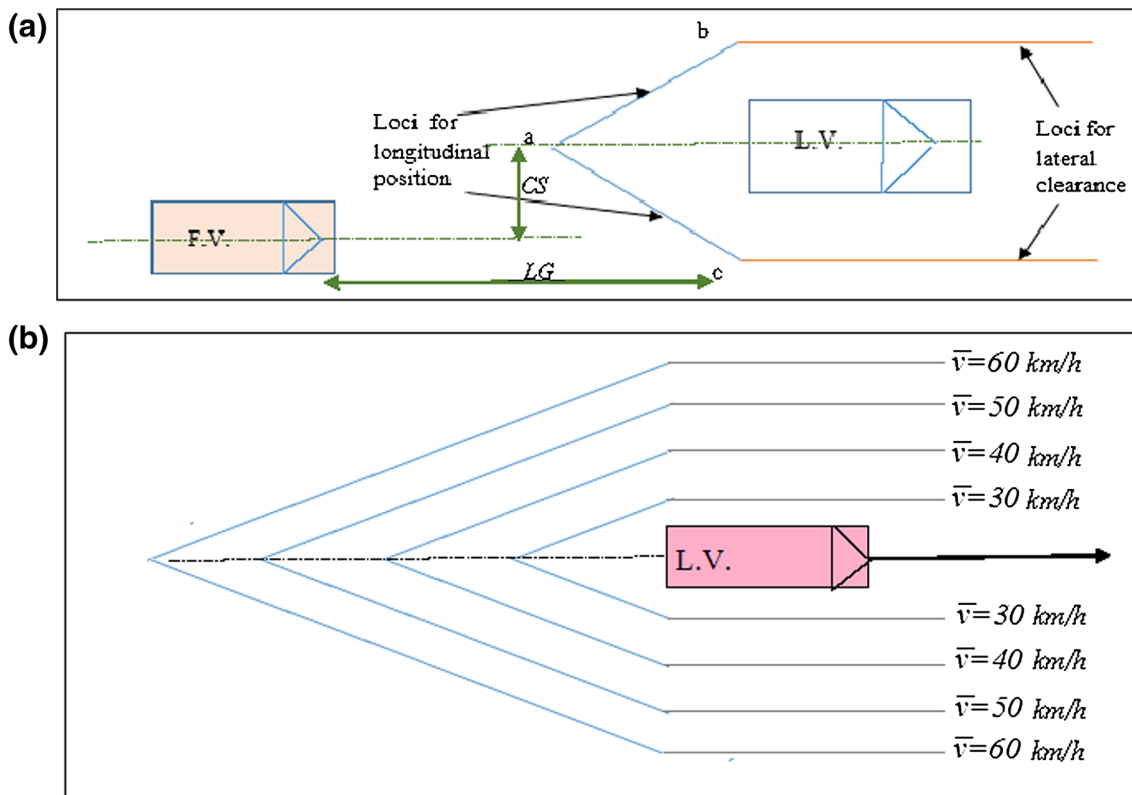
From discussions in previous sub-section, if position of LV, CS and average speed for an interacting vehicle pair is known, one can predict the position of its FV (i.e. LG can be estimated). In other words, using Eq. (1), midpoint position of FV's front bumper can be estimated, if FV and LV are in stable state of vehicle-following. If locus of all such possible points is found for different values of centerline separation, it will be a pair of straight lines with opposite slopes, intersecting at the centerline of LV (i.e. point A) as shown in Fig. 2a. This is because linearity is assumed between LG and CS. The intersection point of this pair of lines will represent zero CS and maximum LG. The domain of these straight lines will be restricted to the back edge of leading vehicle (i.e. points b and c which corresponds to zero LG and maximum CS). Beyond this position, lateral interactions will dominate and vehicle will maintain the required lateral clearances between itself and the interacting vehicle. Figure 2b describes the locus of different threshold lines. These thresholds are for a given average speed of interacting FV and LV pairs. On increasing the average speed, threshold loci move away from the LV as illustrated in Fig. 2b. This

discussion is for a FV driver who is having behavior corresponding to the best fit regression line.

### Determination of FV Position Influenced by Multiple Leading Vehicles

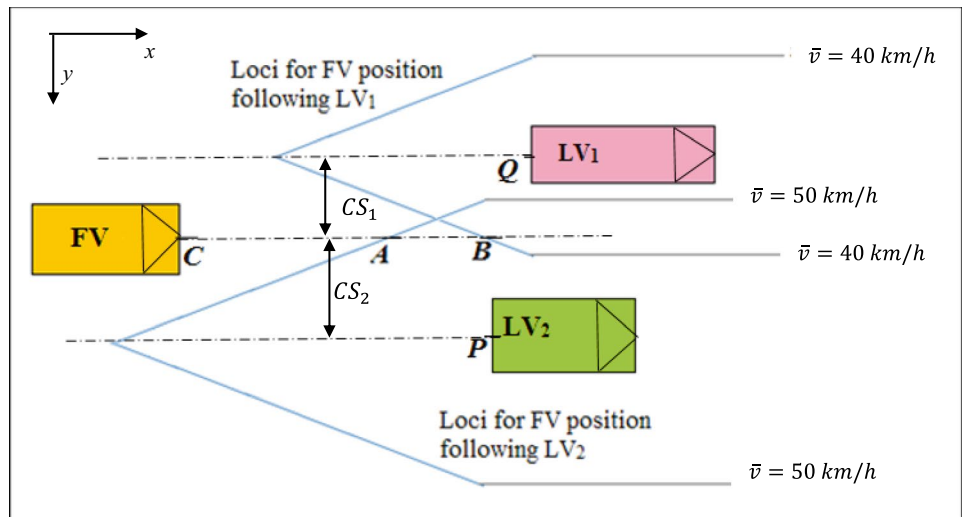
In previous section, interaction between a LV-FV pair under vehicle-following scenario and possible positions of FV under such scenario have been discussed. Identification of LV for a FV remains the most difficult part of the vehicle-following scenario. This difficulty level increases further for heterogeneous weak lane discipline traffic streams as a FV might be influenced by more than one LV simultaneously. However, most of the car-following theory developed till now considered a single LV affecting the behavior of FV.

In case of multiple vehicles interactions, following vehicle's position is affected by presence of all leading vehicles. In order to analyze the impact of multiple leading vehicles on following vehicle, two leading vehicles (referred as LV<sub>1</sub> and LV<sub>2</sub>) and one FV is considered which is represented in Fig. 3. Here, vehicle movement of FV is constrained due to presence of another LV (say, LV<sub>2</sub>). Longitudinal gaps (LG<sub>1</sub>, LG<sub>2</sub>), centerline separations (CS<sub>1</sub>, CS<sub>2</sub>), and average speeds



**Fig. 2** a Loci for longitudinal and lateral positions that can be maintained by front center of FV; and b loci of FV's front center, with varying average speeds

**Fig. 3** Loci of FV in presence of multiple LVs in vehicle-following case



$\bar{v}_1$  (between FV and  $LV_1$ ) and  $\bar{v}_2$  (between FV and  $LV_2$ ) are calculated.

During multiple vehicle interactions, when FV approaches  $LV_1$  and  $LV_2$ , it may have different  $CS_1$  and  $CS_2$  corresponding to vehicle  $LV_1$  and  $LV_2$  respectively. If only pairwise interactions are considered, then corresponding to individual vehicle pairs (FV- $LV_1$  pair, and FV- $LV_2$  pair) two LG's would be obtained, assuming that there is no second LV. It is assumed that vehicle will adjust to an absolute position which will enable itself to have minimum distance from both leading vehicles. This multiple vehicle-following behavior may be represented in the form of a locus diagram, shown as illustration example in Fig. 3. Loci of LG maintained with two vehicles (if there is individual following) are shown. Point A and point B correspond to locus from vehicle  $LV_2$  and  $LV_1$  respectively. It is speculated that vehicle will maintain its speed at point B instead of point A. This is because, if it maintains a position at point A, then it will not be under the impact of  $LV_1$  and will not be in a constrained position.

In this section, two values of LG's are compared:

- Corresponding to FV following only one LV ( $LV_1$  or  $LV_2$ ) assuming the other LV is absent. This is denoted by  $LG_{single}$ . The values of these LG's are obtained using the relationships are mentioned in Table 2. Since  $CS_1$ ,  $CS_2$ ,  $\bar{v}_1$ ,  $\bar{v}_2$  obtained from field data,  $LG_1^{single}$  and  $LG_2^{single}$  can be calculated using Eq. (2) and coefficients from Table 2. They are represented by  $(x_Q - x_B)$  and  $(x_P - x_A)$  respectively, in Fig. 3. Since it is a pairwise representation assuming the third vehicle (second LV) is absent, the data of  $LG_1^{single}$  and  $LG_2^{single}$  are merged. This condition is known as unconstrained condition.
- The second group of longitudinal gaps are the actual LG maintained by FV in the field with each of  $LV_1$  and  $LV_2$ .

They are denoted by  $LG_1^{double}$  and  $LG_2^{double}$  respectively. It is represented by  $(x_Q - x_C)$  and  $(x_P - x_C)$  respectively in Fig. 3.  $LG_1^{double}$  and  $LG_2^{double}$  are measured during data extraction from field. The data of  $LG_1^{double}$  and  $LG_2^{double}$  are merged. They are segregated depending upon different combinations of type of LV and FV, having significant number of data. This condition will be referred as constrained condition.

### Development of Model for Pairwise Following at Constrained Conditions

A linear model, similar to that fitted for FV following single LV is fitted for FV following two LV's (Eq. 1). Data of relationship of  $LG_1^{double}$  vs  $CS_1$  and  $\bar{v}_1$  and that of  $LG_2^{double}$  vs  $CS_2$  and  $\bar{v}_2$  are combined to make a model to represent constrained condition of vehicle-following behavior. The model consists of a deterministic part in the form of a best-fit line and a stochastic part in the form of residual distribution. Single degree polynomial functions are found to be reasonably fit the scatter plot. Increasing degree of equation does not improve  $R^2$  value substantially. General equation of curves is given in Eq. (1), where  $a$  and  $b$  are coefficients of  $\bar{v}$  and  $CS$  respectively,  $c$  is the constant term, and  $\varphi$  is residual term about the best fit curve.

In Eq. (1),  $LG^{double}$  is the dependent variable and  $CS$  and  $\bar{v}$  as independent variables. The model is calculated for different LV-FV pairs of significant data (>50 data points). Residuals are calculated and they are found to be heteroscedastic. As LG increases, spread of residuals also increase, since longitudinal safety margin for following vehicles to follow increases. Necessary adjustment for heteroscedasticity was made. Residuals were divided by obtained value of LG i.e. divided by  $a(\bar{v}) + b(CS) + c$ . Modified residuals are

found to be Burr-distributed. Equation of Burr distribution is mentioned in Eq. (2). The model was developed for several vehicle pairs, namely each vehicle type followed by car, car followed by auto, bike and LCV, and autos and bikes following each other. The obtained set of parameters of model for each pair is mentioned in Table 3. Parameters of obtained model for FV following LV in constrained case are also validated. 30% field data are segregated for validation (not used in model development). Vehicle pairs (LV-FV) having significant dataset (>50 data-points) for validation include car-car, car-auto, auto-car, car-bike and auto-bike. Model is developed for the validation data and mentioned in Table 3.

From Table 3, following conclusions can be derived:

- Higher intercept values when autos, LCVs or cars are being followed.
- Lower intercept values when bikes follow other vehicles.
- Less sensitivity with CS when bikes follow or are being followed as compared to other vehicles, since bikes require lesser space to make a lateral movement to overtake.
- Lower intercept values as compared with the unconstrained behavior observed in Table 2.

The validation exercise is mentioned in Table 4. Model dataset is generated from developed model using coefficients of Eqs. (1) and (2) from Table 3, and this is compared with raw dataset of validation. Data of both these datasets are discretized into groups of different CS and  $\bar{v}$  levels. *t* test is

applied between the two datasets within these groups, for the null hypothesis that means of two samples are equal. Only the highlighted cells in Table 4 indicate the rejection of null hypothesis at 5% significance, and they are lesser in number than rest of the cells. A conclusion can be derived that, the datasets used for validation and modeling are significantly similar for majority of vehicle pairs at different CS and  $\bar{v}$  levels. Coefficients *a* and *b* of Eq. (1) for validation and modeling are also similar, (as observed from Table 3). There is difference in coefficient *c*, due to which the null hypothesis is rejected at lower CS and lower speeds (Table 4).

### Comparison of Relationships of Constrained and Unconstrained Vehicle-Following

Parameters obtained in Table 3 are compared with *LG* for individual vehicle pairs (assuming no effect of  $LV_2$ ) calculated in Table 2. For comparing whether the two datasets are statistically different, firstly the raw data of both the cases are discretized into groups of different CS and  $\bar{v}$  levels. *t* test is applied between datasets of constrained and unconstrained following, within each group. This exercise is conducted for different vehicle pairs. Table 5 mentions parameters of *t* test obtained. The null hypothesis is that means are equal.

From Table 5, it can be concluded that, for most vehicle pairs, the null hypothesis is rejected, thus there is observable difference at 5% significance levels. The difference is not significantly observed when bikes are followed, maybe because vehicles actually do not feel the constraining due

**Table 3** Coefficients of model equation for vehicle-following behavior in constrained case

LV	FV	Sample size	Coefficients of Eq. (1) (modelled data)			Burr (4P) distribution parameters of modified residuals			
			<i>a</i>	<i>b</i>	<i>c</i>	<i>k</i>	$\alpha$	$\beta$	$\gamma$
Car	Car	500	0.12	-1.13	1.18	3.83	4.31	21.23	-31.23
Auto	Car	134	0.09	-1.88	3.21	156.23	4.42	27.02	-7.85
Bike	Car	93	0.11	-1.24	1.13	63.84	3.50	22.91	-6.30
L.C.V	Car	88	0.06	-1.78	3.69	147.21	5.02	32.92	-11.21
Heavy	Car	55	0.13	-0.95	0.74	0.52	7.41	45.31	-74.54
Car	Auto	137	0.09	-1.64	2.24	21.53	5.65	37.82	-61.29
Car	Bike	138	0.04	-0.07	-0.56	4.16	55.43	97.84	-94.61
Car	L.C.V	55	0.07	-1.65	4.08	4.16	12.87	24.99	-21.71
Auto	Auto	102	0.06	-1.37	2.40	26.43	6.76	19.46	-11.23
Auto	Bike	124	0.10	-1.52	0.02	4.20	5.47	10.46	-7.63
Bike	Auto	54	0.20	-0.49	-3.32	35.11	26.47	55.71	-47.73
Bike	Bike	120	0.00	-0.96	1.46	35.95	9.92	21.31	-15.61
<i>Validation data</i>									
Car	Car	200	0.098	-1.75	3.64	3.25	11.94	25.37	-39.91
Car	Auto	55	0.137	-1.86	3.04	0.48	8.86	61.83	-14.41
Auto	Bike	50	0.029	-1.13	1.92	0.69	20.09	15.07	-8.19
Auto	Car	54	0.182	-1.45	1.22	41.71	8.009	20.52	-20.30
Car	Bike	55	0.053	-1.94	3.12	189.2	4.625	27.22	-7.99



**Table 4** p values of t test between model and validation dataset for different CS and  $\bar{v}$  levels for various vehicle pairs

LV-FV pair	(a) Car-car					(b) Car-auto			
	0-1	1-2	2-3	3-4	4-5	0-1	1-2	2-3	3-4
Speed, km/h									
0-12	0.00	0.86	0.89	N.D	N.D	N.D	0.07	0.35	N.D
12-24	0.14	0.34	0.04	0.81	N.D	N.D	0.00	0.34	0.29
24-36	0.00	0.13	0.37	0.59	N.D	0.39	0.56	0.01	0.09
36-48	0.60	0.79	0.00	0.68	N.D	N.D	0.80	N.D	0.26
48-60	0.40	0.00	0.09	0.25	0.07	N.D	N.D	N.D	N.D
LV-FV pair	(c) Car-bike			(d) Auto-bike			(e) Auto-car		
CS, m	0-1	1-2	2-3	0-1	1-2	2-3	0-1	1-2	2-3
Speed, km/h									
0-10	0.18	N.D	N.D	N.D	N.D	N.D	0.10	N.D	0.41
10-20	N.D	0.19	0.15	0.96	0.12	0.86	0.83	0.01	N.D
20-30	0.00	0.55	0.01	0.39	0.57	0.92	0.95	0.18	0.76
30-40	N.D	0.53	0.47	0.53	0.08	0.87	N.D	0.89	0.39
40-50	N.D	N.D	0.82	0.79	0.80	N.D	N.D	0.08	0.60

Italicised cells indicate rejection of null hypothesis at 5% significance levels  
*N.D.* no data for comparison

**Table 5** p values of t test between constrained and unconstrained vehicle-following for different CS and  $\bar{v}$  levels for various vehicle pairs

LV-FV Pair		(a) Car-Car					(b) Auto-Car				(c) Bike-Car		(d) Car-Bike		
CS, m		0-1	1-2	2-3	3-4	4-5	0-1	1-2	2-3	3-4	0-1	1-2	0-1	1-2	2-3
Speed, km/h	0-12	0.01	0.00	0.00	0.00	N.D.	0.77	0.31	0.00	N.D.	0.34	0.63	0.07	0.13	N.D.
	12-24	0.00	0.03	0.05	0.00	N.D.	0.02	0.00	0.05	0.01	0.10	0.36	0.00	0.01	0.06
	24-36	0.00	0.26	0.01	0.02	0.00	0.39	0.00	0.03	0.01	0.26	0.81	0.00	0.00	0.00
	36-48	0.30	0.00	0.20	0.00	0.00	0.45	0.73	0.15	0.14	0.58	0.01	0.00	0.44	0.00
	48-60	0.01	0.00	0.00	0.03	0.00	N.D.	0.00	0.13	N.D.	0.02	0.13	N.D.	0.02	0.02
LV-FV Pair	(e) Car-Auto			(e) Auto-Auto			(f) Bike-Bike		(g) LCV-Car			(h) HV-Car			
CS, m		0-1	1-2	2-3	0-1	1-2	2-3	0-1	1-2	0-1	1-2	2-3	0-1	1-2	2-3
Speed, km/h	0-10	0.84	0.41	0.01	0.19	0.01	N.D.	N.D.	0.89	0.68	0.03	N.D.	0.84	0.44	N.D.
	10-20	0.00	0.00	0.05	0.03	0.00	0.01	0.00	0.02	0.46	0.28	0.62	0.27	0.45	N.D.
	20-30	0.29	0.78	0.00	0.03	0.01	0.05	0.00	0.00	0.11	0.03	0.01	0.56	0.06	0.27
	30-40	N.D.	0.60	0.37	0.46	0.01	0.50	0.00	0.00	0.45	0.79	0.06	N.D.	N.D.	N.D.
	40-50	0.75	0.52	N.D.	0.41	0.33	0.03	0.11	N.D.	0.67	0.99	0.56	0.30	0.69	0.65

Cells colored in shades of red or orange indicate the rejection of null hypothesis at 5% significance, whereas those in green and yellow indicate failing to reject the null hypothesis  
*N.D.* no data for comparison

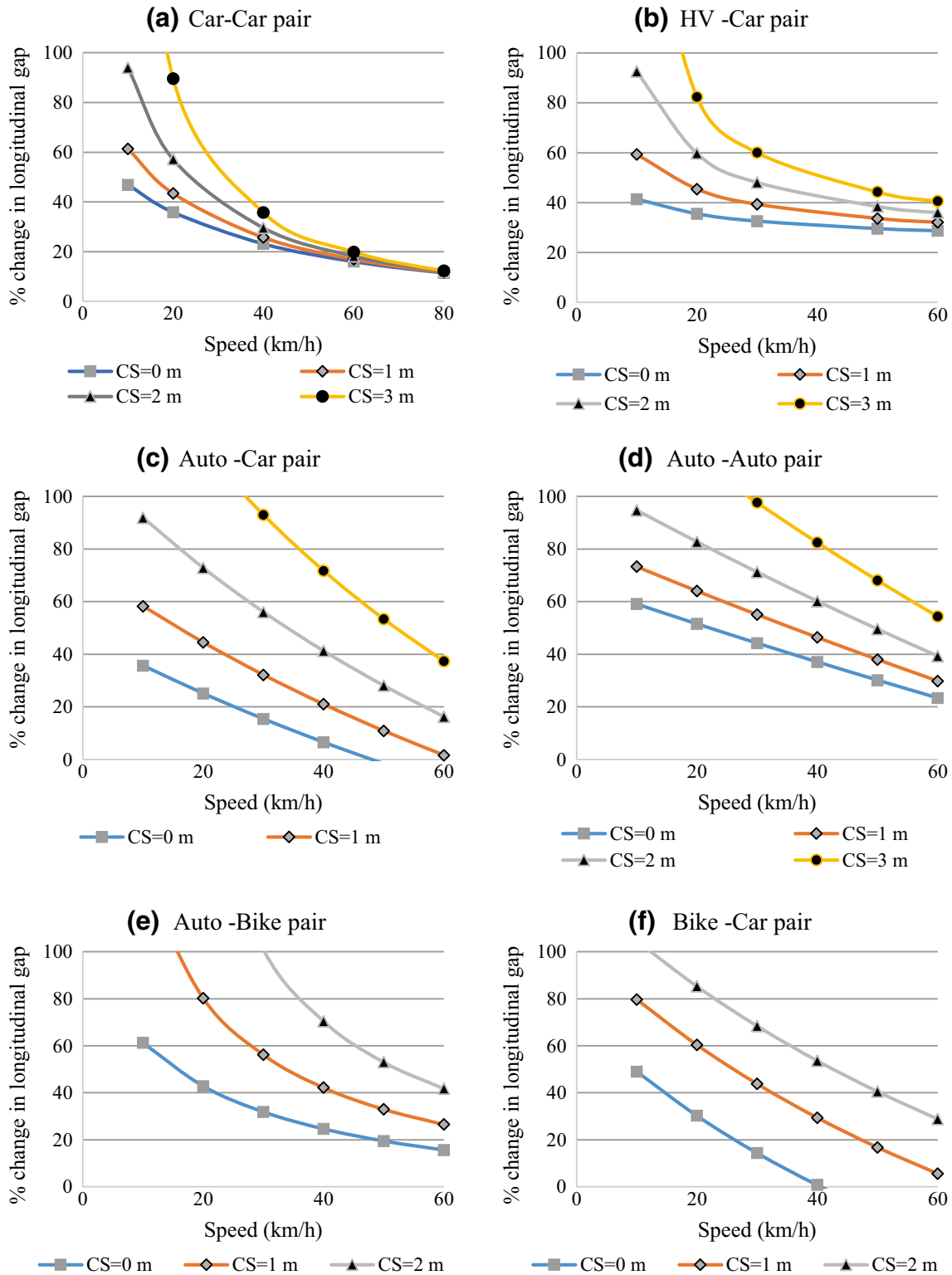
to bikes, since bikes are smaller in size. However, the difference is also not significantly observed in case of heavier size vehicles like truck, bus and LCV being followed. This maybe because of the large size, constraining remains consistent, influence to follower is dominated by heavy vehicle, thus drivers are more cautious when closing in

while following heavy vehicles and rather closely follow the other leading vehicle (if it is not a heavy vehicle).

Further, percentage decrease in *LG* for constrained case when compared with unconstrained case, is calculated and presented for a few vehicle pairs (cars following autos, bikes, other cars, heavy vehicles, auto following auto, bike

following auto) for various values of  $CS$  and  $\bar{v}$ . This information is obtained from the coefficients of best fit regression lines for constrained and unconstrained cases, from Tables 2

and 3. The results are presented in Fig. 4. Similar results can be obtained for other vehicle type combinations too, mentioned in Table 3.



**Fig. 4** Decrease in longitudinal gap during constrained following, for various vehicle pairs, at different speed and CS levels

It is observed that, for all the vehicle pairs, there is a general decrease in intercept in constrained conditions, as compared with unconstrained conditions. There is a higher percentage decrease of  $LG$  with increase of  $CS$  and decrease of  $\bar{v}$ . Upon decrease in speed, there is higher percentage decrease of  $LG$  because at lower speeds, aggressiveness to achieve desired speed increases. Driver of FV tries to maximize speed at lower speed levels, so comes closer to LVs as compared with unconstrained situations. At higher  $CS$  levels, drivers feel more comfortable since they can perceive situations ahead of LVs, thus decreasing the  $LG$  levels. If decrease is more than 100%, it indicates that front bumper of FV has crossed rear bumper of one of LV's, but FV is still following other LV. Such cases are not shown in graphs of Fig. 4 since it will be ambiguous to say that it is a vehicle-following. Percentage decrease is more, when smaller size vehicles such as bikes and autos are followed. Moreover, it is observed that, in Fig. 3, vehicle moves closer to the vehicles than point B, which indicates that the constraining is present for both the vehicles (as against the speculation in "Determination of Position of FV in Vehicle-Following Scenario", that it will maintain its position at B, which indicates no constrain with one vehicle and constrain with another).

There is a decrease in  $LG$ , since a driver of FV following two LV's is able to clearly observe road space ahead of the LV's and anticipate the decisions of LV's with better precision or predictability. While moving in a traffic stream with no lane discipline, there is a general observance of a diamond-shape queuing pattern due to this phenomenon.

**Effect of Level of Constraining**

In "Development of Model for Pairwise Following at Constrained Conditions", no information (in terms of  $LG$  and vehicle type) about the third vehicle (i.e. second LV) was

considered. The level of constraining is measured here as the amount of lateral distance ( $CS_{LV}$ ) between centerlines of  $LV_1$  and  $LV_2$  which is equal to  $CS_1 + CS_2$ . Average longitudinal gap ( $LG^{double}$ ) of  $LG_1^{double}$  and  $LG_2^{double}$  is used as the dependent variable. Average speed is calculated as average of all three vehicles,  $\bar{v}$ . A linear relationship was found to give a better fit with  $\bar{v}$  and  $CS$  as the independent variables. The obtained regression equation is represented by Eq. (4), with Burr-distributed modified residuals,  $\varphi$ :

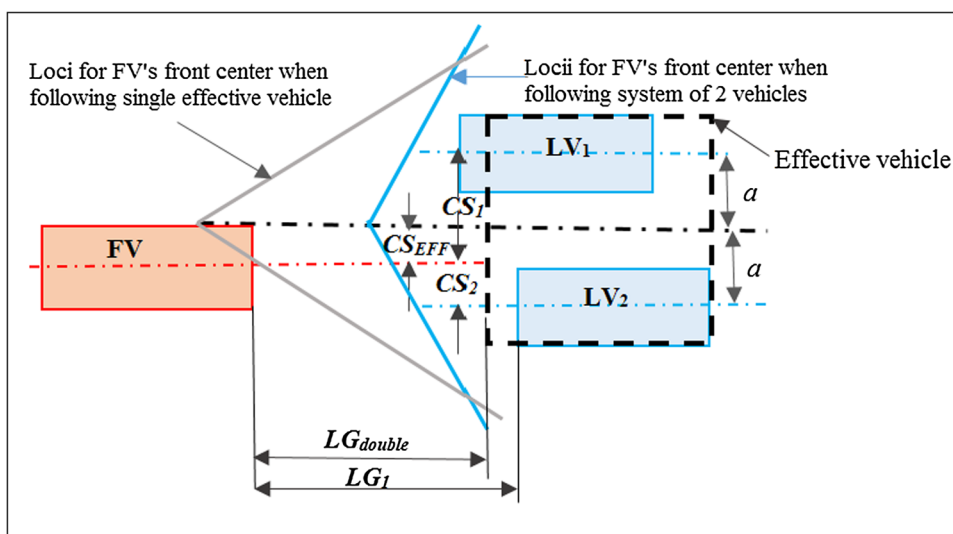
$$\overline{LG}^{double} = 0.061 \bar{v} - 0.577 CS_{LV} + 3.207 + \varphi. \tag{4}$$

A negative coefficient with  $CS$  implies a decrease of  $\overline{LG}^{double}$  with an increase of  $CS_{LV}$ . If lateral distance between two LV's increases, then driver of FV would try to squeeze in his/her vehicle in between the two LVs. Risk associated with sudden braking of LV's reduce if  $CS_{LV}$  increase. Further, due to higher  $CS_{LV}$ , FV can peep at the scenario ahead and anticipate the LV's action. If the gap becomes larger than the width of FV (plus a safety psychological margin), the FV would pass through the gap in the overtaking mode, if it decides to overtake.

**Representation of Two Leading Vehicles as a Single Vehicle**

An attempt is made to represent two LVs in the form of a single, combined vehicle system and it is checked whether its behavior matches with single vehicle-following behavior. In other words, it is verified whether the driver perceives a system of closely spaced LVs as a single vehicle or two different vehicles. Figure 5 shows this representation. For this purpose, effective  $CS$  is calculated as  $CS_{EFF} = 0.5 \times |CS_1 - CS_2|$ .  $CS_{EFF}$  gives the eccentricity between FV and combined vehicle system, or the lateral

**Fig. 5** Representation of system of two closely spaced leading vehicles as effective vehicle



**Table 6** Effect of vehicle type of constraining vehicle on  $LG$  maintenance

LV	FV	Constraining vehicle	Sample size	Coefficients of Eq. (1) (modelled data)		
				$a$	$b$	$c$
Car	Car	Car	228	0.0458	-1.616	4.279
Car	Car	Auto	72	0.2617	-1.827	-1.472

placement of centerline of FV with respect to centerline of system of LV's. Average longitudinal gap ( $LG^{double}$ ) of  $LG_1^{double}$  and  $LG_2^{double}$  is used as the dependent variable. Average speed is calculated as average of all three vehicles,  $\bar{v}$ . This exercise is conducted for all three vehicle types as cars. Data are considered upto a certain range of constraining, or  $(CS_1 + CS_2) < 3$  m, so that the two vehicles are considered as a group by the FV, and not individual vehicles. A linear relationship was found to give a better fit with  $\bar{v}$  and  $CS_{EFF}$  as the independent variables. The obtained regression equation is represented by Eq. (5), with Burr-distributed modified residuals,  $\varphi$ .

$$\overline{LG}^{double} = 0.061 \bar{v} - 0.187 CS_{EFF} + 1.745 + \varphi \quad (5)$$

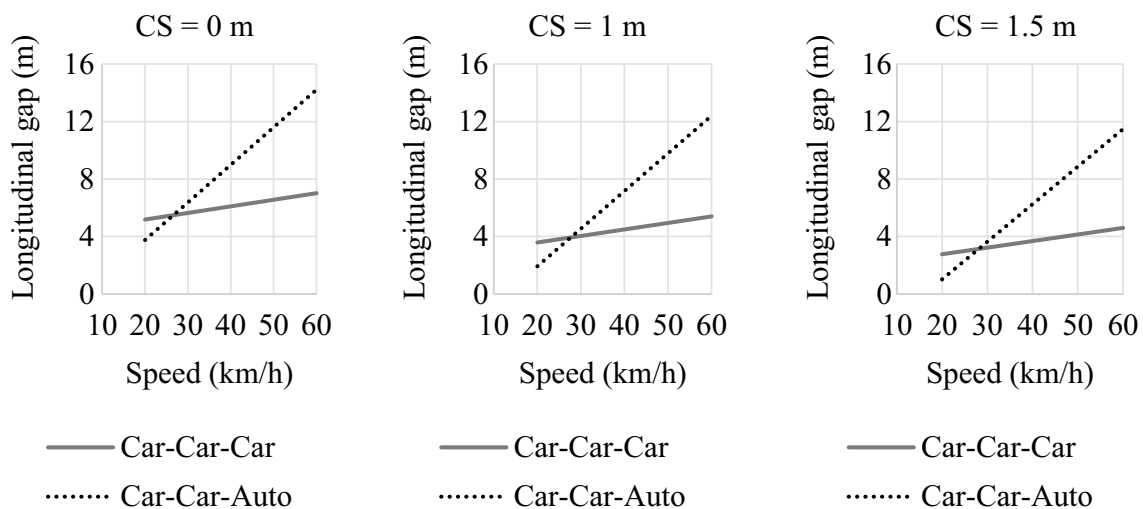
A negative coefficient with  $CS_{EFF}$  implies a decrease of  $\overline{LG}^{double}$  with an increase of effective  $CS$ . It is observed that  $\overline{LG}^{double}$  obtained for combined system at lower  $CS_{EFF}$  values ( $< 2$  m) is significantly less than  $LG_{single}$  (obtained from Table 2) for the same range of  $CS$  values. However at higher  $CS_{EFF}$  levels ( $> 3$  m), the result matches with that obtained from  $LG_{single}$  at similar levels of  $CS$ . There is a rise in slope coefficient of  $CS$ .

The result can be interpreted as the situation in Fig. 5. Consider two cases, case 1, in which a car is following a system of two closely-spaced cars, and second case in which it is following a single hypothetical vehicle equal to

width of two cars plus the gap maintained in between them (represented with dotted lines in Fig. 5), with other characteristics similar to a car. Since the driver of FV will try to overtake the system, he/she will feel safer at higher  $CS_{EFF}$  and thus close-in to the system at higher values of  $CS_{EFF}$ . Hence with the increase in  $CS$ , the  $LG$  will decrease. At higher  $CS_{EFF}$  levels ( $> 3$  m), driver of vehicle maintains similar  $LG$  as with case 2. However, at lower values of  $CS$ , in case 1, driver is able to peep in the gap and perceive situation of traffic with better precision, thus maintains lesser  $LG$  with the system than it will maintain with a single vehicle of equivalent width. Thus, a driver will perceive the system of closely spaced vehicles as a single vehicle when  $CS_{EFF} > 3$  m. In simulation models, if vehicle-following behavior with a group of similarly behaving vehicles is to be represented as a combined (aggregate) behavior with a large vehicle with average properties of all those vehicles, then necessary adjustments to  $LG$  need to be provided at lesser  $CS$  values.

**Effect of Vehicle Type of Constraining Vehicle**

In this subsection, using the data for one vehicle following two vehicles, vehicle type of  $LV_1$  and FV are kept the same (car and car) whereas vehicle type of  $LV_2$  is varied (between car and auto) to check the effect of type of constraining



**Fig. 6** Comparison of longitudinal gap maintained when constraining vehicle is a car or auto, at different  $CS$  levels

vehicle.  $LG_1^{double}$  is compared in both the cases by varying  $LV_2$ . The observed values are mentioned in Table 6. It indicates significant difference in behavior when constraining vehicle is an auto. When one of the cars is replaced with auto, driver of FV follows the other car more closely at speeds above 30 km/h. The illustration is shown in Fig. 6. One of the reasons maybe because he/she can predict behavior of his/her own vehicle type with more confidence. For robust conclusion, this exercise needs to be conducted on a number of such combinations (such as auto-auto-auto with auto-auto-car, or bike-bike-bike with bike-bike-auto, etc). Due to lack of sample size for other groups of vehicles, the authors leave this as an exercise for future research.

## Conclusion, Application and Future Scope

In this paper, driver behavior of a following vehicle (FV) following two leading vehicles (LV) simultaneously in a weak lane-disciplined traffic is studied as a constrained condition of vehicle-following behavior. It is compared with unconstrained vehicle-following behavior, where only one LV is present.

The following conclusions can be drawn from analysis presented in this paper:

- Relationship between longitudinal gap with centerline separation and speed can be represented by a straight regression line as deterministic part (positive slope with speed and negative slope with centerline separation) and Burr-distributed modified residuals about regression line as stochastic part.
- Longitudinal gap ( $LG$ ) decreases for constrained conditions for all vehicle pairs. This reduction in  $LG$  is due to better anticipation and confidence of LV's action by FV after peeping at the traffic scenario ahead. This decrease is significant for most vehicle pairs.
- This reduction of  $LG$  is higher when smaller size vehicles are followed by larger size vehicles.
- The reduction of  $LG$  is higher at lower speeds and higher centerline separations.
- If lateral separation between two LV's decreases, the  $LG$  maintained by FV increases due to reduction in opportunity to peep at the traffic scenario ahead by FV.
- Two leading vehicles can be represented as a combined, single vehicle with  $LG$  equaling almost half the  $LG$  maintained for individual single LV at zero centerline separation, and equaling the  $LG$  maintained at higher ( $>3$  m) centerline separations.
- There is a significant effect of type of constraining vehicle which can be studied in detail as a future scope.

Results of this study can help in better understanding of vehicle interactions under weak lane-discipline with heterogeneous traffic conditions. When a vehicle follows two vehicles simultaneously, necessary corrections need to be incorporated as against a vehicle following a single vehicle, as discussed in this paper. This further can help in better modeling of vehicle interactions in dense traffic streams with weak lane-discipline. If the effect of sidewise interactions with neighboring vehicles in dense traffic stream is combined with this model, it may lead to the development of new traffic simulation model, incorporating the effect of simultaneous interactions of neighboring vehicles.

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