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ORIGINAL PAPER

Revisiting fixed capital models in the Sraffa framework

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Abstract
Von Neumann (Review of Economics Studies 13(1):1–9, 1945) and Sraffa (Production of commodities by means of commodities. Prelude to a critique of economic theory. Cambridge University Press, Cambridge, 1960) revived the classical ideas of treating fixed capital as a special case of joint production. Sraffa's model with a single machine that has constant efficiency has been widely generalised. Among such generalisations, Salvadori's contribution (Value, distribution and capital. Essays in honour of Pierangelo Garegnani, Routledge, London, pp 270–285, 1999) not only re-ignited research interest in the field, but also reshaped scholastic understanding of the importance of machines' efficiencies with regard to fixed capital models. Such novel insights make it necessary to revisit the development of fixed capital models in the Sraffa framework. In this paper these models are surveyed with a focus on the properties of the cost-minimising technique in each model.

Keywords Fixed capital · Sraffa · Cost-minimising technique · Transferable machines · Jointly utilised machines

JEL Classification B51 · C60 · D24

1 Introduction

It is well known that both von Neumann (1945)[1] and Sraffa (1960) revived the classical idea of treating fixed capital as a special case of joint production. Von Neumann expressed clearly that the "wear and tear of capital goods are to be described by introducing different stages of wear as different goods" (von Neumann 1945, p.

[1] The original paper by von Neumann was published in German in 1937 and translated into English in 1945. In this paper we refer to the English version.

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2), and although he did not mention any classical economist, scrutiny of the assumptions and methods which he used, such as the asymmetrical treatment of income distribution, the rules of free goods and the method of choice of technique, shows that his analysis more convincingly belongs to the classical tradition of economic thought rather than the neo-classical tradition (Kurz and Salvadori 1993). Sraffa also rediscovered the classical treatment of fixed capital as joint production independently,[2] and in his view, fixed capital is the "leading species" of joint production, an idea which dates back to the classical economists, Torrens, Ricardo, Malthus and Marx.[3]

The joint production treatment of fixed capital is not only different from the neo-classical treatment, but also more logically consistent. In neo-classical theory, fixed capital is usually treated as a stock, a "factor of production", which is either a given quantity of a single magnitude in the traditional version, or a given vector of capital goods in the modern or neo-Walrasian version. Either way, the neo-classical capital theory suffers from insurmountable difficulties that cannot be solved within its framework (Petri 2004). Even if we ignore the logical inconsistencies and the critiques raised by Sraffa's work and subsequent contribution in capital controversy to the neo-classical capital theory, it is still unsatisfactory to treat fixed capital as a stock. This is due to the fact that even though fixed capital usually lasts for several production periods, it is consumed (for instance, through physical depreciation) and reintegrated by new flows (for instance, by replacing parts) in almost every production period. Hence, fixed capital is "something of a hybrid" (Pasinetti 1980), which looks like a "stock" but not being entirely a stock. Such intricacies arising from the "stock" and "flow" can be easily avoided by the method of joint production, because the machines of the same type but different ages are treated as different commodities. As a consequence, all machines become flows, thereby the aforementioned intricacies disappear. Moreover, the classical treatment of fixed capital can analyse certain economic problems that the "fund-flow" approach, proposed by Georgescu-Roegen,

[2] For Sraffa's consecutive attempts in solving the problem of fixed capital in an objective way, see Kurz and Salvadori (2005).

[3] There exists a debate about whether the method of treating fixed capital as joint product was adopted by the classical economists and Marx. Moseley (2009) argues that Sraffa's attribution of the joint product method of treating fixed capital to Marx is misleading at best and totally wrong at worst, as is the same attribution to other classical economists like Torrens and Malthus. Gehrke (2011), in re-examining the "Reference to the literature" in Sraffa's book, argues that Moseley's view is difficult to sustain, and that it is absurd to think that the classical economists adopted the joint product method to treat fixed capital only if they used the method in exactly the same way as Sraffa, since the classical economists did not have the same analytical tools at their disposal. Moseley (2011) rebutted that his earlier conclusion still held, and further argued that Ricardo did not adopt the joint product method, and that Marx's "transformation of value" method was superior to the joint product method. Moseley's contention that Marx's theory was superior was based on his critique of the "flaw" (used machines are not sold on the market and as a consequence the rate of profit across machines of different ages in the same industry cannot be equal) and "unrealistic assumptions" of the latter. This argument is hard to sustain for the following reasons. First, even if there is no market for selling old machines, the prices of such machines determined in the Sraffa system are book values, which give the correct depreciation and annual charge of the machines. Second, those "unrealistic assumptions" (such as there is only one single fixed capital good in each industry) are not needed in some fixed capital models.

cannot deal with satisfactorily, irrespective of the fact that the latter may appear to have the same structure as the former (Kurz and Salvadori 2003). To provide a specific example, the "fund-flow" approach cannot satisfactorily provide answers to questions regarding why some machines become obsolete prematurely, or why some equipment is left idle rather than operated with the full degree of utilisation. Especially with regard to the problem of choice of technique, the "fund-flow" approach may yield misleading or incorrect answers when identifying the cost-minimising technique because this approach may unify several production processes within one entity and hide the fact that some of these processes are not cost-minimising (see the example provided by Kurz and Salvadori 2003). What is more important is that, unlike the "fund-flow" approach, the classical treatment of fixed capital does not make illegitimate prior assumptions about which of the underlying techniques will be used. In addition, the treatment of fixed capital as joint production is more general than other multi-sectoral fixed capital models (Lager 1997, 2006).

The seminal fixed capital model proposed by Sraffa is a simple one with a single machine that has constant efficiency. This model has been widely generalised, and such generalisations can be roughly categorised by two criteria according to the properties of old machines: joint utilisation and transferability. The former criterion concerns whether or not an old machine can be jointly utilised with other old machines in the same production process, and the latter criterion concerns whether or not an old machine is used in different sectors. Sraffa's model was first generalised into models with non-transferable and non-jointly utilised machines which have variable efficiencies (as in Baldone 1980; Schefold 1980; Varri 1980; Kurz and Salvadori 1994, 1995 chap. 7), and it seems that Sraffa's model was a special case within this category. Later contribution made by Salvadori (1988a, b) further generalised the above models into those with non-transferable and jointly utilised machines which have variable efficiencies, and the models within the former category become special cases of Salvadori's models. As a consequence, Sraffa's model seems to be an even more special case.

Compared with the fruitiness of the models with non-transferable machines, models with transferable machines are rarely studied because most scholars believe that the introduction of transferable machines will cause some complexities of pure joint production. The lacuna in the research of transferable machines was filled by Salvadori (1999), who built a model with transferable and non-jointly utilised machines which have variable efficiencies but the efficiencies of machines are independent of the sectors in which machines are used. In Salvadori's model, it was shown that all the good properties of the models with non-transferable, non-jointly utilised machines still hold.

Salvadori's contribution not only re-ignited research interest in the field of fixed capital models, especially models with transferable machines (for example, see Huang 2015; Bidard 2016), but also reshaped the accademic understanding of the importance of machines' efficiencies with regard to fixed capital models. It turns out that if machines always work with constant efficiencies as assumed in Sraffa's model, then there is no need to assume non-transferability nor non-joint utilisation in order to get some good properties. Hence, Sraffa's model is not as special as it appears.

Given the novel insight gained from the studies on transferable machines, it is time to revisit this literature and to re-organise the fixed capital models in the Sraffa framework based on a new criterion: the efficiency of machine. Hence, fixed capital models can be divided into two groups: the models with constant efficiency machines and the models with variable efficiency machines. For the latter category, it is further possible to classify fixed capital models based on the above two mentioned criteria: joint utilisation and transferability. The purpose of this paper is to provide a new classification and to survey the properties of these fixed capital models.

With regard to fixed capital, there always exists a problem of choice of technique, which concerns the patterns of utilisation of machines and the choice of the economic lifetime of machines (Kurz and Salvadori 1995). In this paper only the latter problem is addressed,[4] and the attention is focused on the properties of the cost-minimising technique, as defined below.

This paper is organised as follows. Section 2 provides some definitions and a summary of some common assumptions used in fixed capital models. In Sect. 3 the model with machines of constant efficiencies is discussed. Thereafter models with machines of variable efficiencies are considered based on aforementioned classification criteria. Section 4 discusses the non-transferable, non-jointly utilised machines model, followed by Sect. 5, which discusses the non-transferable, jointly utilised machines model. In Sect. 6 we consider a model built by Salvadori (1999), a model with transferable, non-jointly utilised machines that have uniform efficiency paths (explained in further detail below). The subsequent section discusses a model with transferable and jointly utilised machines under the condition of a variation in the uniform efficiency paths. The paper's final section concludes.

2 Common assumptions and definitions

In order to distinguish models of fixed capital from those of pure joint production, some assumptions are needed. These common assumptions usually include:[5] first, all commodities are divided into two groups, namely finished goods and old machines. The former can be used as both consumption goods as well as means of production, while the latter are never used as consumption goods, but may be used as means of production. It should be noted that new machines belong to the category of finished goods. Second, each production process produces one and only one finished good, and may produce a quantity of old machines. In other words, there is no "pure" joint production: finished goods are not assumed to be jointly produced, and the only possible joint products are old machines. Therefore, it is possible to define a sector as

[4] For a systematic discussion on the patterns of utilisation of durable capital goods in the Sraffa framework, see Kurz (1986, 1990), Kurz and Salvadori (1995, Chap.7, Sect. 7).

[5] For a more formal representation of these assumptions, readers can refer to Kurz and Salvadori (1995, Chap. 7 and Chap. 9).

the set of processes that produce the same finished good. Third, old machines are assumed to be disposed of freely at any time with zero scrap value.[6] In addition, although a machine can be possibly used for a long time due to proper maintenance, it is assumed that perennial machines are excluded for the sake of convenience, which means that for each machine there exists a specific value for maximum physical life.

Next some basic definitions concerning production technology and the cost-minimising technique are provided. The problem of choice of technique can be dealt with by two equivalent approaches, namely the indirect approach and the direct approach (Kurz and Salvadori 1995). For the indirect approach, a single technique is first defined, and then a cost-minimising technique is chosen from the set of all techniques based on the cost-minimising criterion. By contrast, for the direct approach, a cost-minimising technique is chosen from the whole technology which comprises all production processes, and in this approach a cost-minimising technique can be determined without investigating non-cost-minimising techniques. In this paper the direct approach is used since compared with the indirect approach, the problem of joint production can be handled more easily with the former.

Assume that there are n commodities which can be produced by m constant-returns-to-scale processes, and each is represented by a triplet (a_i, l_i, b_i), $i = 1, 2, \dots, m$, where a_i is a semi-positive commodity input vector,[7] l_i is a non-negative labour input scalar, and b_i is a semi-positive commodity output vector. All commodities are reordered such that the first s commodities are finished goods, and the others are old machines. The whole technology is represented by the following matrices:

$$A = [a_1, a_2, \dots, a_m]^T$$
$$l = [l_1, l_2, \dots, l_m]^T$$
$$B = [b_1, b_2, \dots, b_m]^T$$

With regard to production technology, the following assumptions hold:

Assumption 1 It is impossible to produce commodities without any material inputs, or:

$$e_i^T A \geq 0, \quad i = 1, 2, \dots, m$$

Assumption 2 All commodities are producible, or:

$$Be_j \geq 0, \quad j = 1, 2, \dots, n$$

Assumption 3 Labour enters directly or indirectly into the production of all commodities, or:

$$\forall \epsilon > 0, (x \geq 0, x^T(B - \epsilon A) \geqq 0^T) \Rightarrow x^T l > 0$$

[6] Although this is a common assumption used in most fixed capital models, the problem of scrapped machines can also be dealt with, see Salvadori (1988a) or Sect. 5.

[7] All vectors are column vectors, and the transpose of a vector or a matrix is denoted by a superscript T.

Let p be the price vector, and w be the wage rate. The rate of profit r is assumed to be exogenous. Regarding quantity system, let x be the intensity vector, and d be the requirements for use. Specifically, d is assumed to have the following form:

$$d^T = gx^T A + c^T \tag{1}$$

in other words, the economy is assumed to undergo steady growth at rate g, which satisfies $g \leq r$, and c represents the consumption vector. Since old machines are not assumed to be consumed, some of the first s elements of c are positive and the others are zeros, or $c^T = (c_s^T, 0^T)$, where c_s is an s-dimensional semi-positive vector. The numéraire is defined by a vector f, which is any given semi-positive vector whose positive elements refer to commodities that are certainly produced. In the condition of free competition, the following system holds:

$$[B - (1 + r)A]p \leqq wl \tag{2a}$$

$$x^T[B - (1 + r)A]p = wx^T l \tag{2b}$$

$$x^T[B - (1 + g)A] \geqq c^T \tag{2c}$$

$$x^T[B - (1 + g)A]p = c^T p \tag{2d}$$

$$f^T p = 1 \tag{2e}$$

$$p \geqq 0, x \geqq 0, w \geq 0 \tag{2f}$$

Inequality (2a) means that no production process can obtain extra profits, and equation (2b) means that if there is one process which incurs extra costs, then the corresponding intensity of this process is zero. Inequality (2c) means that the amounts of commodities produced cannot be less than the amounts of commodities required by the production plus requirements for use, and equation (2d) means that if there exists one commodity that is overproduced, then its price is zero. Equation (2e) is the numéraire equation, and the meaning of inequalities (2f) is self-explanatory.

If there exists a solution (x^*, p^*, w^*) to system (2), then there is said to exist a cost-minimising technique. The following condition guarantees the existence of a solution (Kurz and Salvadori 1995, Chap. 8):

There exists a non-negative vector z such that the following inequality holds:

$$z^T[B - (1 + r)A] \geqq c^T \tag{3}$$

The price vector p^* (the wage rate w^* and intensity vector x^*) is called the long-period price vector (wage rate and intensity vector). A cost-minimising technique (A^*, l^*, B^*) is constituted by the processes which do not incur extra costs at the long-period prices p^* and wage rate w^* and which can produce the requirements for use with a positive intensity vector. More formally, we have the following system:

$$[B^* - (1 + r)A^*]p^* = w^* l^* \tag{4a}$$

$$\bar{x}^{*T}[B^* - (1 + g)A^*] = c^T \tag{4b}$$

where $\bar{x}^*$ is obtained from x^* by deleting any zero elements. In the following sections the properties of the cost-minimising technique of each model will be summarised.

Table 1 An example: a machine of constant effciency

	Inputs								Outputs					
	$S-1$	M_0	M_1	$\cdots$	M_{t-1}	$\cdots$	L		$S-1$	M_0	M_1	$\cdots$	M_{t-1}	$\cdots$
(1)	$a^T_{(s-1)}$	m_0	0	$\cdots$	0	$\cdots$	l_1	$\to$	$b^T_{(s-1)}$	0	m_1	$\cdots$	0	$\cdots$
(2)	$a^T_{(s-1)}$	0	m_1	$\cdots$	0	$\cdots$	l_1	$\to$	$b^T_{(s-1)}$	0	0	$\cdots$	0	$\cdots$
						$\vdots$	$\vdots$							
(3)	$a^T_{(s-1)}$	0	0	$\cdots$	m_{t-1}	$\cdots$	l_1	$\to$	$b^T_{(s-1)}$	0	0	$\cdots$	0	$\cdots$

Regarding the problem of choice of technique, it is well known that joint production causes many complexities: requirements for use, or "demand", may have an influence in determining the cost-minimising technique, the cost-minimising technique may not exist even if all techniques are feasible, the uniqueness of commodity prices may not hold if there is more than one cost-minimising technique, and so on (see for instance, Salvadori 1982, 1985; Salvadori and Steedman 1988; Schefold 1989; Kurz and Salvadori 1995). However, for fixed capital models, some such complexities do not exist.

3 Model with machines of constant efficiencies

The seminal model presented by Sraffa (1960) is a model with a single machine which has constant efficiency. By constant efficiency, Sraffa means that '[t]he quantities of means of production, of labour and of the main product [that is the finished good] are equal in the several processes in accordance with the assumption of constant efficiency during the life of the machine' (Sraffa 1960, p. 65).

Sraffa's model can be illustrated as follows. Assume that a machine M whose physical life is t years, is used in the production of finished good 1. Then there exist t processes producing finished good 1 using machine M of different ages as means of production. Let the sth commodity represent the new machine M_0, and let the $(s+1)$th commodity represent the 1-year-old machine M_1, and so on. Let the $(s+t-1)$th commodity be the $(t-1)$-year-old machine M_{t-1}. If machine M always works with constant efficiency, then for all processes producing finished good 1 using machine M, the first $s-1$ elements of commodity input and output vectors and the labour input scalar are the same. All these processes producing finished good 1 using machine M are listed in Table 1, where $a_{(s-1)}$ and $b_{(s-1)}$ denote the first $s-1$ elements of the commodity input vector and output vector, respectively.

If machine M always works with constant efficiency, then the problem of choice of technique only concerns whether or not it is profitable to use machine M in the production of finished good 1 at the given rate of profit. Assume that the processes listed in Table 1 are in the cost-minimising technique at the given rate of profit r. Then we have the following system producing finished good 1, a system similar to Sraffa's one (Sraffa 1960, p. 65):

$$(a^T_{(s-1)}p^*_{(s-1)} + m_0 p^*_{m_0})(1+r) + l_1 w^* = b^T_{(s-1)}p^*_{(s-1)} + m_1 p^*_{m_1} \tag{5a}$$

$$(a^T_{(s-1)}p^*_{(s-1)} + m_1 p^*_{m_1})(1+r) + l_1 w^* = b^T_{(s-1)}p^*_{(s-1)} + m_2 p^*_{m_2} \tag{5b}$$

$$\cdots \tag{5c}$$

$$(a^T_{(s-1)}p^*_{(s-1)} + m_{t-1} p^*_{m_{t-1}})(1+r) + l_1 w^* = b^T_{(s-1)}p^*_{(s-1)} \tag{5d}$$

where $p^*_{(s-1)}$ represents the first $s-1$ elements of p^*. Following Sraffa, if we multiply the above equations respectively by $(1+r)^{t-1}, (1+r)^{t-2}, \ldots, (1+r)$, 1, and add them, the old machines of different ages are cancelled out, then we get an integrated process producing finished good 1 without using old machines. Consequently, for the cost-minimising technique, some properties that are similar to those of single production can be obtained easily. For instance, determination of the cost-minimising technique is independent of the structure of requirements for use, and the prices in terms of the wage rate of finished goods are uniquely determined.

A less obvious fact is that neither non-joint utilisation nor non-transferability is crucial for these good properties if machines always work with constant efficiencies. If machines which always work with constant efficiencies are jointly utilised in the production of the same finished good, then for those processes using different types of machines and producing the same finished good, it is still possible to integrate these processes into one which does not use old machines, as is shown by Roncaglia (1978).

For the purpose of illustration and without loss of generality, let us assume that two types of machine M and N, which always work with constant efficiencies and whose physical lives are 3 and 2, respectively, are jointly utilised in the processes producing finished good 1. Let the $(s-1)$th and sth commodities be new machines of type M and N, respectively. Further, assume that a process (a, l, b) which uses a 2-year-old machine of type M and a 1-year-old machine of type N as inputs belongs to the cost-minimising technique, or the following equation holds.

$$(a^T_{(s-2)}p^*_{(s-2)} + m_2 p^*_{m_2} + n_1 p^*_{n_1})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} \tag{6}$$

where $a_{(s-2)}$, $b_{(s-2)}$ and $p^*_{(s-2)}$ represent the first $s-2$ elements of a, b, p^*. Then it can be proved that the following system holds:

$$(a^T_{(s-2)}p^*_{(s-2)} + m_0 p^*_{m_0} + n_0 p^*_{n_0})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} + m_1 p^*_{m_1} + n_1 p^*_{n_1} \tag{7a}$$

$$(a^T_{(s-2)}p^*_{(s-2)} + m_0 p^*_{m_0} + n_1 p^*_{n_1})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} + m_1 p^*_{m_1} \tag{7b}$$

$$(a^T_{(s-2)}p^*_{(s-2)} + m_1 p^*_{m_1} + n_0 p^*_{n_0})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} + m_2 p^*_{m_2} + n_1 p^*_{n_1} \tag{7c}$$

$$(a^T_{(s-2)}p^*_{(s-2)} + m_1 p^*_{m_1} + n_1 p^*_{n_1})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} + m_2 p^*_{m_2} \tag{7d}$$

$$(a^T_{(s-2)}p^*_{(s-2)} + m_2 p^*_{m_2} + n_0 p^*_{n_0})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} + n_1 p^*_{n_1} \tag{7e}$$

$$(a^T_{(s-2)}p^*_{(s-2)} + m_2 p^*_{m_2} + n_1 p^*_{n_1})(1+r) + l_1 w^* = b^T_{(s-2)}p^*_{(s-2)} \tag{7f}$$

System (7) can be shown briefly as follows. Suppose that some of the equations in system (7) hold while others do not. For instance, suppose that only Eqs. (7a), (7c), (7d), and (7f) hold and others do not. If one of Eqs. (7b), (7e) pays extra profits, then a contradiction arises. If Eq. (7b) incurs extra costs, or:

$$(a^T_{(s-2)}p^*_{(s-2)} + m_0 p^*_{m_0} + n_1 p^*_{n_1})(1+r) + l_1 w^* > b^T_{(s-2)}p^*_{(s-2)} + m_1 p^*_{m_1} \tag{8}$$

From Eqs. (7a), (7d) and (8) we have:

$$(1+r)n_1 p^*_{n_1} > (1+r)n_0 p^*_{n_0} - n_1 p^*_{n_1} \tag{9a}$$

$$(a^T_{(s-2)}p^*_{(s-2)} + m_1 p^*_{m_1} + n_0 p^*_{n_0})(1+r) + l_1 w^* < b^T_{(s-2)}p^*_{(s-2)} + m_2 p^*_{m_2} + n_1 p^*_{n_1} \tag{9b}$$

Hence a contradiction arises again.[8] Therefore system (7) holds. Following Sraffa's approach, if we multiply the equations in system (7) respectively by $(1+r)^3$, $(1+r)^2$, $(1+r)^2$, $(1+r)$, $(1+r)$, 1, and add them, we then get an integrated process producing finished good 1 without using old machines. Therefore, joint utilisation causes no complexities if machines always work with constant efficiencies.

Regarding the case with transferable machines, generally transferability will cause some difficulties, as is suggested by Sraffa: if a machine is used in different industries (sectors), then this machine may have different working lives, and its efficiency may be different even if its working lives are the same.

However, as Sraffa wrote: " [i]f in all the industries the machine had *the same working life and constant efficiency*, the book-values for each age would be equal in all of them, since the annual charges would all be equal to the annuity described in § 75" (Sraffa 1960, section 78, p. 67, emphasis added.).

Hence, if transferable machines used in different sectors have the same working lives and constant efficiencies, then transferability will cause no problems. This can be illustrated using the example listed in Table 1 with some variations. Assume that the same type of old machine M which always work with constant efficiency can also be used in the production of finished good 2. The processes producing finished good 1 and 2 are distinguished by the superscripts (i), where $i = 1, 2$. Furthermore let us assume that in the cost-minimising technique, a 1-year old machine M_1 is produced by a process producing finished good 1, and the same old machine M_1 is used as an input in a process producing finished good 2, or the following equations hold:

$$(a^{(1)T}_{(s-1)}p^*_{(s-1)} + m_0 p^*_{m_0})(1+r) + l_1 w^* = b^{(1)T}_{(s-1)}p^*_{(s-1)} + m_1 p^*_{m_1} \tag{10a}$$

[8] Inequality (9a) is actually in contradiction with the constant efficiency of machine N, see Sect. 4.

$$(a^{(2)T}_{(s-1)}p^*_{(s-1)} + m_1 p^*_{m_1})(1+r) + l_2 w^* = b^{(2)T}_{(s-1)}p^*_{(s-1)} + m_2 p^*_{m_2} \tag{10b}$$

Then it can be proved that there exists a process which produces finished good 1 (2) using (producing) a 1-year-old machine as an input (output), or the following equations hold:

$$(a^{(1)T}_{(s-1)}p^*_{(s-1)} + m_1 p^*_{m_1})(1+r) + l_1 w^* = b^{(1)T}_{(s-1)}p^*_{(s-1)} + m_2 p^*_{m_2} \tag{11a}$$

$$(a^{(2)T}_{(s-1)}p^*_{(s-1)} + m_0 p^*_{m_0})(1+r) + l_2 w^* = b^{(2)T}_{(s-1)}p^*_{(s-1)} + m_1 p^*_{m_1} \tag{11b}$$

Based on system (10) and system (11), it becomes immediately clear that a similar integration to system (5) is also possible.

Given the above considerations, the model with machines of constant efficiencies is not as special as it appears. In what follows, we will discuss the models with machines of variable efficiencies based on the criteria mentioned above: transferability and joint utilisation.

4 Model without transferable or jointly utilised machines

Sraffa's model was first generalised into a model with non-transferable and non-jointly utilised machines that have variable efficiencies. This kind of fixed capital model, referred to as a single machine model in this survey, has been widely investigated (Baldone 1980; Schefold 1980; Varri 1980; Kurz and Salvadori 1994, 1995, chap. 7). A result of this generalisation is that the optimal life of a machine may not be the same as its physical life. If the efficiency of a machine is constant, there is no reason to stop using the machine until the end of its physical life. By contrast, if the efficiency of a machine is decreasing, then this machine may become economically obsolete before its physical life comes to an end. Actually, the optimal life for using the machine can be determined endogenously by the cost-minimising technique and it is not independent of income distribution.

Two specific assumptions are required to isolate this model from other fixed capital models. First, the transferability of old machines is ruled out. That is to say, for each old machine k which is produced by a process producing, for instance, finished good j, any process producing another finished good other than j does not use old machine k as an input, nor does it produce k as an output. Second, old machines are not jointly utilised. This assumption means that each process uses no more than one old machine, and each process produces no more than one old machine. Old machines produced by the processes that use finished goods alone are called 1-year-old machines, and those machines produced by the processes that use finished goods and 1-year-old machines are called 2-year-old machines, and so on.

If we use t_1 to denote the number of old machines that are used in sector 1 (for instance, assume that two types of machines M and N, which last for $\tau_1 + 1$ and $\tau_2 + 1$

years, are used in the production of finished good 1, then $t_1 = \tau_1 + \tau_2$), and so on, and assume that there exist m_i processes that can produce commodity i, then after a proper reorder, the matrices A and B have the following forms:

$$A = \begin{array}{cc} & \begin{array}{ccccc} s & t_1 & t_2 & \cdots & t_u \end{array} \\ \begin{array}{c} m_1 \\ m_2 \\ \vdots \\ m_s \end{array} & \left(\begin{array}{c|c|c|c|c} A_{11} & A_{1t_1} & & & \\ \hline A_{21} & & A_{2t_2} & & \\ \hline \vdots & & & \ddots & \\ \hline A_{s1} & & & & A_{st_u} \end{array}\right) \end{array} \tag{12}$$

$$B = \begin{array}{cc} & \begin{array}{ccccc} s & t_1 & t_2 & \cdots & t_u \end{array} \\ \begin{array}{c} m_1 \\ m_2 \\ \vdots \\ m_s \end{array} & \left(\begin{array}{c|c|c|c|c} B_{11} & B_{1t_1} & & & \\ \hline B_{21} & & B_{2t_2} & & \\ \hline \vdots & & & \ddots & \\ \hline B_{s1} & & & & B_{st_u} \end{array}\right) \end{array} \tag{13}$$

where A_{i1} and B_{i1} represent the finished good inputs and outputs of sector i, respectively. In addition, only the ith column of B_{i1} is positive; other columns are nought. A_{it_i} (B_{it_i}) represents the old machines used in (produced by) sector i. If sector i does not use old machines, then the columns containing A_{it_i} (B_{it_i}) are void. Note that there may exist more than one type of old machine which can be used in the production of commodity i, yet not jointly utilised in a single process. Hence in each row of A_{it_i} (B_{it_i}), there exists at most one positive element, and the others are zeros.

Let (A^*, l^*, B^*) be a cost-minimising technique. For a single machine model, it can be proved that (A^*, l^*, B^*) has the following properties that are similar to those of single production (Baldone 1980; Schefold 1980; Varri 1980; Kurz and Salvadori 1994, 1995, chap. 7): first, determination of the cost-minimising technique is independent of the structure of requirements for use, provided that old machines do not enter into the consumption vector. Second, the prices in terms of the wage rate of actually produced finished goods are uniquely determined even if there exists more than one cost-minimising technique. Third, the prices in terms of the wage rate of finished goods are increasing functions of r.

These properties follow from the fact that there exists a vector $x_i(g)$ for each sector i using old machines such that $x_i^T(g)B_{i1}^* = e_i^T$ and $x_i^T(g)[B_{it_i}^* - (1+g)A_{it_i}^*] = 0$, where e_i is the ith unit vector. $(x_i^T(g)A_{i1}^*, x_i^T(g)l_{(i)}^*, e_i^T)^T$ is called a "core-process" (Kurz and Salvadori 1995) or an integrated process producing finished good i, where $l_{(i)}^*$ is a vector made up of the labour input scalars used in the processes producing finished good i and using old machines. Since the matrix formed by all these core processes has the same characteristics as the single production technique, the above properties can be obtained easily. After the prices of finished goods are obtained, the prices of old machines used in the cost-minimising technique can be determined sequentially.

Up to now we have not discussed the problems of depreciation and efficiency in this model. The depreciation of an old machine for one production period is nothing but the change in price of that machine over that period. Assume that one type of machine lasts for $(t+1)$ years in the cost-minimising technique, and let $p_0(r), p_1(r), \dots, p_t(r)$ be the prices of this machine type of ages $0, 1, \dots, t$, respectively. The depreciation of the i-year-old machine is (Kurz and Salvadori 1995, Chap. 7):

$$M_i(r) = p_i(r) - p_{i+1}(r) \quad (i = 0, 1, \dots, t-1) \tag{14a}$$

$$M_t(r) = p_t(r) \tag{14b}$$

The annual charge relative to the i-year-old machine is:

$$Y_i(r) = (1 + r)p_i(r) - p_{i+1}(r) \quad (i = 0, 1, \dots, t-1) \tag{15a}$$

$$Y_t(r) = (1 + r)p_t(r) \tag{15b}$$

which is the depreciation plus the profit earned by the capital in the form of the machine in the corresponding year. The efficiency of an i-year-old machine is defined as constant, increasing or decreasing if the annual charge of that machine is equal to, lower than, or higher than the annual charge of the $(i+1)$-year-old machine of the same type. Since the prices of machines are dependent on the condition of income distribution, depreciation, annual charge and efficiencies of machines are in general not independent of the condition of income distribution.

It should be noted that the above definition of constant efficiency is not in contradiction with Sraffa's definition. It can be checked that in system (5), the annual charges of machine M of different ages are always the same. Sraffa also wrote: " Supposing a machine 'm' to work with constant efficiency throughout its life, the annual charge to be paid for interest and depreciation in respect of it must be *constant*, if the price of all units of the product is to be uniform" (Sraffa 1960, section 75, p. 64, emphasis added.).

Indeed it is possible to prove (see Appendix) that in any (cost-minimising) technique if the annual charge relative to i-year-old machine is equal to the annual charge relative to the j-year-old machine, any $j \neq i$, at each rate of profit, then Sraffa's definition of constant efficiency holds.

In what follows we will adopt the definition of efficiency using the annual charge. As will be seen below, efficiency is very important when dealing with transferable machines.

5 Model with non-transferable and jointly utilised machines

Roncaglia (1978) presented a model with non-transferable, jointly utilised machines which have constant efficiencies.[9] It was Salvadori (1988a, b) who built a general model with non-transferable, jointly utilised machines.

Since the non-transferability assumption still holds, the technological matrices A and B still have the forms like Eqs. (12) and (13). However, the non-joint utilisation assumption is ruled out here. Therefore, each row of A and B can have more than one element that is positive.

We still focus on the properties of the cost-minimising technique. Salvadori (1988a; see also Kurz and Salvadori 1995, Chap. 9) proved that determination of the cost-minimising technique is independent of the structure of consumption, provided that old machines are not consumed, although the investment (rate of growth) may have an influence on the determination of the cost-minimising technique. The latter result follows from the fact that an old machine may be overproduced even if it is utilised in the production given that old machines can be jointly utilised, and as a consequence, the change of relative intensities of the actually operated processes producing the same finished goods matters in determining whether or not an old machine is overproduced. The rate of growth can affect the determination of the relative intensities of actually operated processes of finished goods, hence mattering in the determination of prices.

The former result can be represented as follows. Assume that x^*, p^* and w^* are a solution to system (2) for a given consumption vector c_1. Then for another consumption vector $c_2 \neq c_1$, there exists another solution x^{**}, p^* and w^* to the same system. The proof of this result can be shown briefly as follows. Let x^* be partitioned as follows: $x^* = [x_1^{*T}, x_2^{*T}, \ldots, x_s^{*T}]^T$, where x_i^* is the intensities corresponding to sector i. Define a square matrix Q, whose ith row is $x_i^{*T} B_{1i}$, and a square matrix H, whose ith row is $x_i^{*T} A_{1i}$. Inequality (2c) can be represented as follows:

$$e^T[Q - (1+g)H] \geqq c_{1s} \tag{16a}$$

$$x_i^{*T}[B_{it_i} - (1+g)A_{it_i}] \geqq 0 \quad i = 1, \ldots, u \tag{16b}$$

where c_{1s} is the vector composed by the first s elements of c_1.

From inequality (16a) we know that matrix $[Q - (1+g)H]$ is invertible and the inverse is semi-positive (Kurz and Salvadori 1995, Mathematical Appendix, Theorem A. 3. 1). Hence there exists a vector v such that $v^T[Q - (1+g)H] = c_{2s}$, where c_{2s} is a vector composed by the first s elements of c_2. Further, a scalar multiplication of x_i will

[9] Roncaglia (1978) provided two generalisations to Sraffa's model: one is a model with non-transferable and non-jointly utilised machines with variable efficiency, and the other is a model with non-transferable, jointly utilised machines with constant efficiency. Although non-transferability is not explicitly assumed by Roncaglia (1978), he seems, like Sraffa, to suggest that if machines of the same type and the same age are used in the production of different finished goods, they should be treated as different machines which have different prices: "It is enough to think of the prices of machines of a given age and type -in general- as being different (because their circumstances of use have been different) in relation to the production process in which they have been used ⋯" (Roncaglia 1978, p. 46.). Hence, the non-transferability is implicitly assumed. However, if the efficiencies of machines are always constant, the assumption of non-transferability is unnecessary.

not change the inequalities (16b), i.e., $v_i x_i^*$ $(i = 1, \ldots, s)$ still satisfy (16b), where v_i is the ith element of v. The existence of vector v implies that there exist x^{**}, p^*, w^* as a solution to system (2) for a given consumption vector c_2, where $x^{**} = [v_1 x_1^{*T}, \ldots, v_s x_s^{*T}]^T$.

Salvadori (1988a) also proved that if the rate of growth equals the rate of profit ($g = r$), then the prices in terms of the wage rate of finished goods that are actually produced are uniquely determined even if there is more than one cost-minimising technique. This result may not hold if $g \neq r$. In addition, the uniqueness of prices in terms of the wage rate of old machines may not hold even if $g = r$.[10]

There is no need to introduce "ages" and "types" of machines in order to determine the cost-minimising technique. These two definitions are required when dealing with problems of depreciation and efficiency. Salvadori (1988a; see also Kurz and Salvadori 1995, Chap. 9) gave an assumption that defines the "ages" and "types" of machines. The analyses of depreciation and efficiency are the same as those in Sect. 4, and will not be restated here. There is only one point that needs emphasis: since old machines can be jointly utilised, the depreciation and efficiency of one particular machine is not only determined by the circulating capital goods, labour input utilised with it and the output produced with it, but is also influenced by the other machines jointly utilised with it. In other words, the efficiencies of jointly utilised machines are interdependent (Roncaglia 1978; Lager 1997) because the prices of old machines are interdependent.

Although the free disposal of old machines is commonly assumed, the problem of scrapped machines can be dealt with. Salvadori (1988a) also made a variant to the model mentioned above by replacing the assumption of free disposal with an assumption that the entire amount of scrapped machines is fully utilised, directly or indirectly, in the production of the finished goods with which they are produced, and he showed that the above results still hold.

6 Model with transferable and non-jointly utilised machines

In the models noted above, non-transferability is crucial for obtaining those good properties. In general, the existence of transferable machines will cause some complexities of pure joint production, because the system is "interlocked" (Schefold 1989).

[10] This can be briefly explained as follows: assume that there exist two processes involving the same commodities (including old machines) in two cost-minimising techniques (say technique 1 and 2) at the given rate of profit. Further assume that two types of old machines M and N are produced by these two processes. The quantities of M and N produced in technique i is m_i and n_i, respectively $(i = 1, 2)$. The annual charges of machines M and N are $C_M^{(i)}$ and $C_N^{(i)}$, where the superscript i represents technique i, $i = 1, 2$. The uniqueness of prices in terms of the wage rate of finished goods only implies that for these two processes, $(C_M^{(i)} m_i + C_N^{(i)} n_i)/w_i$ are equal in these two processes, where w_i is the wage rate of technique i. However, since old machines can be jointly utilised in the same process, the above equality does not guarantee that the prices of the same old machines are the same in these two processes.

Table 2 An example with one transferable machine

	Inputs						Outputs			
	Maize	Wheat	M_0	M_1	Labour		Maize	Wheat	M_0	M_1
(1)	1/15	1/5	1	0	1/2	→	1	0	0	1
(2)	3/20	1/10	0	1	1/2	→	1	0	0	0
(3)	4/15	2/5	1	0	1	→	0	1	0	1

As noted in Sect. 3, Sraffa was well aware of the difficulties caused by transferability, and he suggested that the difficulty could be overcome if the transferable machines used in different sectors have the same working lives and constant efficiencies (see Sraffa's quoted statement in Sect. 3). A problem is close at hand: what happens if the efficiencies of machines are not constant, but are independent of the sectors?

The suggestion made by Sraffa was developed by Salvadori (1999). Specifically Salvadori proposed a model with non-jointly utilised but transferable machines whose efficiencies are not constant but are still independent of the sectors in which they are used. Except for the assumptions mentioned in Sect. 2 and the non-joint utilisation assumption in Sect. 4, Salvadori dropped the assumption of non-transferability and made another assumption called the Uniform Efficiency Path Axiom which is fully quoted below.

Uniform Efficiency Path Axiom (Salvadori 1999): If a type of machine is used in the production of finished goods i and j $(i \neq j)$, then there is a vector $(a_{ij}^T, b_{ij}^T, l_{ij})$ such that for each process (a_s^T, b_s^T, l_s) producing finished good i using a machine of that type (old or new) there is a process (a_t^T, b_t^T, l_t) producing finished good j such that the vector (a_t^T, b_t^T, l_t) is a linear combination of vectors (a_s^T, b_s^T, l_s) and $(a_{ij}^T, b_{ij}^T, l_{ij})$.

A simple example is provided in Table 2. Assume that there exists only one transferable machine: a tractor (M), which lasts for 2 years, is used in the production of maize and wheat (two different finished goods). Furthermore, assume that the processes listed in Table 2 are in the cost-minimising technique. M_0 represents a new tractor, which is a finished good, and M_1 is a 1-year-old tractor (an old machine). Assume that land is not scarce and that the rate of profit is 1/4.

Since M_0 is a finished good, whose price is determined by other processes that are not listed here, it is assumed that its price is determined to be 1/3. Taking maize as the numéraire, the above processes are able to determine the prices of wheat (p_2), M_1 (p_{M_1}), and the wage rate (w). It can be checked that $p_2 = 2$, $p_{M_1} = 1/4$ and $w = 1/2$.

If the Uniform Efficiency Path Axiom holds, then there exists a vector $(a_{ij}^T, b_{ij}^T, l_{ij})$ and another process [specifically, process (4) which is represented by (a_4, b_4, l_4)] producing wheat such that the following equations hold:

$$(4/15, 2/5, 1, 0, 0, 1, 0, 1, 1) = \lambda_1(1/15, 1/5, 1, 0, 1, 0, 0, 1, 1/2) + \lambda_2(a_{ij}^T, b_{ij}^T, l_{ij}) \tag{17}$$

Table 3 An example with one transferable machine-continued

	Inputs						Outputs			
	Maize	Wheat	M_0	M_1	Labour		Maize	Wheat	M_0	M_1
(4)	7/20	3/10	0	1	1	→	0	1	0	0

$$(a_4^T, b_4^T, l_4) = \lambda_3(3/20, 1/10, 0, 1, 1, 0, 0, 0, 1/2) + \lambda_4(a_{ij}^T, b_{ij}^T, l_{ij}) \tag{18}$$

The vector $(a_{ij}^T, b_{ij}^T, l_{ij})$ is not normalised. For the sake of simplicity, let $\lambda_1 = 1$, and let process (4) be normalised such that it produces 1 unit of wheat. Then we have:

$$(a_{ij}^T, b_{ij}^T, l_{ij}) = (1/5, 1/5, 0, 0, -1, 1, 0, 0, 1/2)$$

Process (4) is listed in Table 3, and it can be shown that this process also belongs to the cost-minimising technique. We can explain the existences of the vector $(a_{ij}^T, b_{ij}^T, l_{ij})$ and process (4) as follows. For processes (1) and (3), when the tractor is transferred from the sector producing maize to the sector producing wheat, a change in output (b_{ij}^T) requires a corresponding change in inputs (a_{ij}^T, l_{ij}). If the efficiency of the machine M is independent of the sectors, then we change the same input to process (2), and we can also produce wheat (process (4)).

From the analysis of the non-transferable, non-jointly utilised machines model we know that there exists a vector $x(g)$ such that multiplication of $x^T(g)$ to processes (1) and (2) will yield a core process producing maize. Using the same $x^T(g)$ to multiply processes (3) and (4), we can also get a core process producing wheat. Hence the properties of the non-transferable, non-jointly utilised machines model follow.

The Uniform Efficiency Path Axiom guarantees that if we treat the transferable machine M as two different machines (for instance tractor 1 and tractor 2), then these two machines have the same prices, hence the same path of depreciation and efficiency. In other words, the efficiency of the transferable machine is independent of the sector. Transferability causes no trouble if the Uniform Efficiency Path Axiom holds.

In a recent contribution, Bidard (2016) discussed the models with transferable machines from a different perspective, pointing out that any model with transferable machines which has neat properties (the same as those of single production) can be associated to a model with non-transferable machines in which the prices of machines of the same ages but different types that are used in the production of different finished goods, are the same at each rate of profit. Bidard therefore argues that what matters is that the processes producing different finished goods but using the machines of the same type have equiprofitability. Hence he maintains that instead of the Uniform Efficiency Path it would be appropriate to say that there is equiprofitability. This, of course is not in contradiction with the argument developed by Salvadori (1999).

As stated earlier, Salvadori's argument generalises Sraffa's suggestion that transferable machines always have the same lives and constant efficiencies, into one such that the efficiencies of machines are not constant but are still independent of the sectors in which they are used. The assumption concerning efficiency independent of the sector is recognised as consisting in the fact that if a type of machine whose economic lifetime is n years and is used in the production of m finished goods, then the linearly independent processes must be $(m + n - 1)$, whereas if efficiency depends on the sectors, then the linearly independent processes may be $(m \times n)$. Hence equiprofitability is not a better explanation than, but rather a result of, the Uniform Efficiency Path Axiom.

The term equiprofitability can be applied to any kind of model (for instance, it may occur under certain circumstances in the case of single production or pure joint production), and it may yield a mistaken impression that in the model with transferable machines, the property that processes using transferable machines producing different finished goods are equally profitable only holds by chance and consequently that the model has no economic relevance. However, under the circumstance that transferable machines have uniform efficiency paths, a circumstance which has its rationale and which is suggested by Sraffa, such equiprofitability is a definite, rather than an accidental result. In fact, what is more important is not to assert that if there is equiprofitability then the model preserves good properties, but to identify the conditions such that equiprofitability and these good properties hold.

Yet Bidard goes further, suggesting that there is no need to develop models with transferable machines with such a restrict assumption (the Uniform Efficiency Path Axiom) and that attention should be focused on applying the non-transferable machine model with only the remark that the model with transferable machines can be associated with one with non-transferable machines. However, even if this remark may be useful to understand the model and its properties, it does not undermine Salvadori's contribution. On the contrary, this remark further strengthens the idea that if the Uniform Efficiency Path Axiom holds, then the non-transferability assumption is unnecessary to obtain the neat properties that are similar to those of single production.

7 Model with transferable and jointly utilised machines

Despite limiting his analysis to the case of transferable but non-jointly utilised machines, Salvadori conjectured that a similar formalism can be constructed for the case in which machines are used jointly, and this conjecture is correct. The Uniform Efficiency Path Axiom with some modifications can be applied to the case with transferable and jointly utilised machines, and it can be shown that under such an assumption the properties of the cost-minimising technique of the non-transferable jointly utilised machines model still hold (Huang 2015).

To be more specific, in the model with transferable and jointly utilised machines, except for those assumptions listed in Sect. 2, both the assumptions of non-transferable and non-jointly utilised machines are ruled out and replaced by the Uniform

Efficiency Path Axiom with some variations. The reasons why the Uniform Efficiency Path Axiom cannot be directly applied to the case with jointly utilised machines are as follows: first, the vector $(a_{ij}^T, b_{ij}^T, l_{ij})$ mentioned in the axiom may not exist. This is due to the fact that when old machines are not jointly utilised, the difference between two processes producing different finished goods and using an old machine of the same type and the same age only involves finished goods, while when old machines are jointly utilised, one transferable machine may be jointly utilised with old machines of different types and ages in the processes producing different finished goods. Hence the difference between the processes producing different finished goods and using the same transferable machine may involve old machines of different types and ages. Second, as pointed out in Sect. 5, the efficiencies of machines are interdependent when they can be jointly utilised. Hence the efficiency path of one particular transferable machine may not be uniform due to the influences of other non-transferable machines jointly utilised with it in the same sectors.

In order to deal with jointly utilised machines, the Uniform Efficiency Path Axiom needs some modifications, and it is maintained in the following way: the efficiency path of a transferable machine is uniform if other machines jointly utilised with it are assumed to be non-existent (see Assumption 7 in Huang 2015). This modified Uniform Efficiency Path Axiom has both strengths and weaknesses: on the one hand, it generalises the Uniform Efficiency Path Axiom because it allows jointly utilised machines (it is exactly the same as the Uniform Efficiency Path Axiom if joint utilisation of old machines is excluded). On the other hand, explanation of this assumption with the fact that transferable machines have uniform efficiency paths needs caution due to the fact that the efficiencies of machines are interdependent when they are jointly utilised. Only in one of the following situations can this assumption be explained as that transferable machines have uniform efficiency paths: first, each transferable machine and other machines jointly utilised with it form a plant;[11] second, transferable machines of different ages are jointly utilised with other machines of the same ages and the same types; third, all other machines jointly utilised with transferable machines have constant efficiencies; fourth, the transferable machine itself has constant efficiency.

It can be shown that if the assumptions in Sect. 2 and the modified Uniform Efficiency Path Axiom hold, then determination of the cost-minimising technique is independent of the structure of consumption, provided that the old machines are not consumed, and that the prices in terms of the wage rate of finished goods are positive. In addition, if the rate of growth equals the rate of profit ($g = r$), then the prices in terms of the wage rate of finished goods are uniquely determined even if there exists more than one cost-minimising technique. Furthermore, the modified Uniform Efficiency Path Axiom is not only a sufficient condition, but also a necessary condition for the determination of the cost-minimising technique being independent of the structure of consumption.

[11] For the definition of a plant, see Kurz and Salvadori (1995, p. 207, p. 266). In this case, the efficiency path of the whole plant is uniform.

If the modified Uniform Efficiency Path Axiom holds, and if a type of machine whose economic life is n years and is jointly utilised with other k non-transferable machines (whose economic lives are t_k, respectively) in the production of m finished goods, then the linearly independent processes must be $(m + n + \sum_{j=1}^{k} t_j - k - 1)$, whereas if efficiency depends on the sectors, then the linearly independent processes may be $(m \times n + \sum_{j=1}^{k} t_j - k)$.

8 Concluding remarks

This paper briefly surveyed the development of fixed capital models in the Sraffa framework. The importance of efficiency of a machine is emphased based on the new insight gained from recent studies on transferable machines. It emerged that if machines always work with constant efficiencies, then neither non-joint utilisation nor non-transferability of old machines is crucial to obtain some neat properties. Hence a new classification regarding the fixed capital models should be made. This paper also summarises the properties of the cost-minimising technique in each model.

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Appendix

In this appendix, we will prove that for a cost-minimising technique, if a machine always works with constant efficiency irrespective of the rate of profit, then the definition of efficiency used in Sect. 4 is equivalent to Sraffa's one. Given that, if machines are jointly utilised, the efficiencies of machines of different types are interdependent, which further complicates this issue, we will prove the equivalence of these two definitions in a case with only non-jointly utilised machines.

Proposition 1 *Let (A^*, l^*, B^*) be a cost-minimising technique for any given r that belongs to $[\underline{r}, \bar{r}]$. If the annual charge relative to the i-year-old machine M, defined by system (15), is always the same as the annual charge of the $(i+1)$-year-old machine of the same type for any rate of profit r which belongs to $[\underline{r}, \bar{r}]$, $i = 0, 1, \ldots, t-1$, where t is the maximum age of machine M produced by the processes producing finished good j in the cost-minimising technique, then for all processes producing finished good j using machine M as an input, the quantities of finished good inputs except the new machine and of labour input in order to produce one unit of finished good j are the same.*

Proof Let (a_i, l_i, b_i) and $(a_{i+1}, l_{i+1}, b_{i+1})$ represent two processes producing finished good j using machine M of age i and $i+1$ $(i = 0, 1, \ldots, t-1)$, respectively, and let the new machine M be the sth finished good. These two processes are normalised in such way that they produce one unit finished good j. The annual charge relative to the i-year-old machine is:

$$\begin{aligned} Y_{M_i}(r) &= (1+r)p^*_{M_i}(r) - p^*_{M_{i+1}}(r) \\ &= \frac{1}{m}\left[b^T_{i(s-1)}p^*_{(s-1)} - (1+r)a^T_{i(s-1)}p^*_{(s-1)} - l_i w^*\right] \end{aligned} \tag{19}$$

where m is the quantity of machine M in order to produce one unit of finished good j, $b_{i(s-1)}$, $a_{i(s-1)}$ and $p^*_{(s-1)}$ represent the first $s-1$ elements of b_i, a_i and p^*, respectively.

Since $b_{i(s-1)} = b_{i+1,(s-1)}$, if $Y_{M_i}(r) = Y_{M_{i+1}}(r)$, we have the following equation:

$$(1+r)a^T_{i(s-1)}p^*_{(s-1)} + l_i w^* = (1+r)a^T_{i+1,(s-1)}p^*_{(s-1)} + l_{i+1}w^* \tag{20}$$

or:

$$(1+r)\left[a^T_{i(s-1)} - a^T_{i+1,(s-1)}\right]\frac{p^*_{(s-1)}}{w^*} = l_{i+1} - l_i \tag{21}$$

From Sect. 4 we know that the prices in terms of the wage rate are increasing functions of r, and an increase in r will increase the level of $(1+r)\frac{p^*_{(s-1)}}{w^*}$. Since the right hand of (21) is constant, we have $l_i = l_{i+1}$ and $a_{i(s-1)} = a_{i+1,(s-1)}$. □

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