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BRIEF TECHNICAL NOTE

A Conical Striker Bar to Obtain Constant True Strain Rate for Kolsky Bar Experiments

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Abstract

Many split Hopkinson pressure bar (SHPB) or Kolsky bar techniques use pulse shaping methods to obtain constant engineering strain rate for the specimen response. However, constitutive models for numerical simulations use the axial rate of deformation which is the true axial strain rate. In this study, we present an equation for the incident bar strain produced by a truncated conical striker bar. These incident bar strains are shaped so that we can obtain constant true strain rate for the specimen.

Keywords Conical striker bar · True strain rate · Kolsky bar · Pulse shaping

Introduction

Constitutive models obtained from large strain compression data are typically fit to curves at nearly constant engineering strain rates [1]. For Kolsky bar experiments, constant engineering strain rates can be obtained with pulse shaping techniques that produce the desired wave shapes in the incident bar. The most common method for pulse shaping is to place copper [2] or copper-steel [3] discs on the impact surface of the incident bar. After impact by the striker bar, the pulse shaper plastically deforms and spreads the pulse in the incident bar so that the specimen reaches dynamic force equilibrium and constant engineering strain rate.

However, as discussed in [4], constitutive models used for numerical simulations use the axial rate of deformation which is the true axial strain rate. Casem [5] presents a variable impedance, inverse approach [6] for the striker bar design. This method is used to design the striker bar cross-section to shape the incident strain pulse and obtain constant true strain rate for the specimen response. This wave shaping technique uses a thin copper disc or a small amount of grease on the incident bar and striker bar with a variable cross section. The striker bar consists of a series of cylindrical sections that decrease in diameter away from the impact end. Wave motion in the segmented striker bar is calculated with a detailed numerical analysis. Casem [5] demonstrates his procedure with experiments conducted on nylon specimens for a true strain rate of about 2500 1/s and axial strains that reach 0.60.

We observe that the striker bar with a series of cylindrical sections used in [5] could be closely approximated with a smooth, conical striker bar that simplifies the machining. More important, the conical striker bar wave motion could be analyzed with classical mathematical methods to obtain closed-form equations. For this study, we derive closed-form equations with the Laplace transform method for the stress and strain in the incident bar impacted by a conical striker bar. We compare our incident strain–time predictions with the predictions given by Casem [5] and show good agreement.

Conical Striker Model

A truncated conical striker bar with length L impacts a cylindrical incident bar with velocity V. As shown in Fig. 1, the larger striker bar end and the incident bar have diameter $2a$. The smaller striker bar end diameter is $2b$. Both bars have the same material properties given by density ρ and Young's modulus E. The wave velocity is given by $c = \sqrt{E/\rho}$. We solve for the incident stress pulse with the Laplace transform method.

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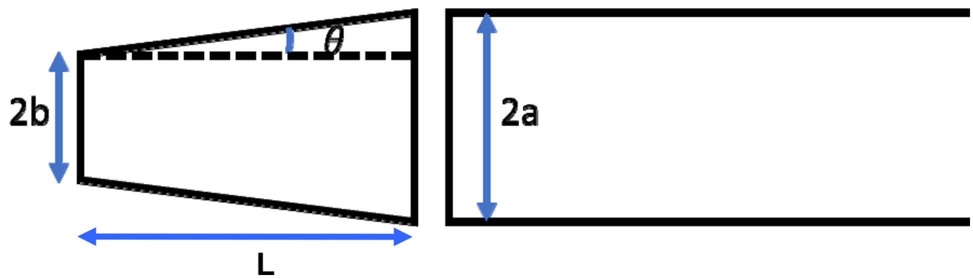

Fig. 1 Conical striker and incident bar before impact

Figure 2 shows the conical striker bar coordinates. The wave equation in terms of particle displacement u [7, 8] is

$$\frac{\partial^2 u}{\partial x^2} + \frac{2}{x}\frac{\partial u}{\partial x} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{1a}$$

where u is positive in the x-direction, and t is time. Axial stress and strain are related to particle displacement with

$$\sigma = E\varepsilon = E\frac{\partial u}{\partial x} \tag{2}$$

where σ is taken positive in tension.

We use the Laplace transform method [9] and transform Eq. (1) to

$$\frac{d^2\overline{u}}{dx^2} + \frac{2}{x}\frac{d\overline{u}}{dx} = \left(\frac{s}{c}\right)^2\overline{u} - \left(\frac{s}{c^2}\right)u(x,0) - \frac{1}{c^2}\frac{\partial u(x,0)}{\partial t}.$$

where $\overline{u}(x,s)$ is the Laplace transform of $u(x,t)$. Before impact

$$u(x,0) = 0 \quad \frac{\partial u(x,0)}{\partial t} = V,$$

and the transformed equation becomes

$$\frac{d^2\overline{u}}{dx^2} + \frac{2}{x}\frac{d\overline{u}}{dx} - \left(\frac{s}{c}\right)^2\overline{u} = -\frac{V}{c^2} \tag{3}$$

For a wave traveling in the negative x-direction, Eq. (3) has solution

$$\overline{u}(x,s) = A\frac{1}{x}\exp\left(\frac{xs}{c}\right) + \frac{V}{s^2} \tag{4}$$

where A is a constant. The transformed stress $\overline{\sigma}$ and particle velocity υ are

$$\overline{\sigma} = E\frac{\partial\overline{u}}{\partial x} = EA\exp\left(\frac{xs}{c}\right)\left[\frac{s}{xc} - \frac{1}{x^2}\right] \tag{5}$$

$$\upsilon = \frac{\partial u}{\partial t}, \overline{\upsilon} = s\overline{u} = A\frac{s}{x}\exp\left(\frac{xs}{c}\right) + \frac{V}{s} \tag{6}$$

The wave equation for the incident bar in terms of particle displacement u [7, 8] is

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{7}$$

where u is positive in the x-direction. Axial stress and strain are related with

$$\sigma = E\varepsilon = E\frac{\partial u}{\partial x} \tag{8}$$

where σ is positive in tension.

We transform Eq. (7) and obtain

$$\frac{d^2\overline{u}}{dx^2} - \left(\frac{s}{c}\right)^2\overline{u} = 0 \tag{9}$$

where the incident bar is initially at rest. For a wave traveling in the x-direction

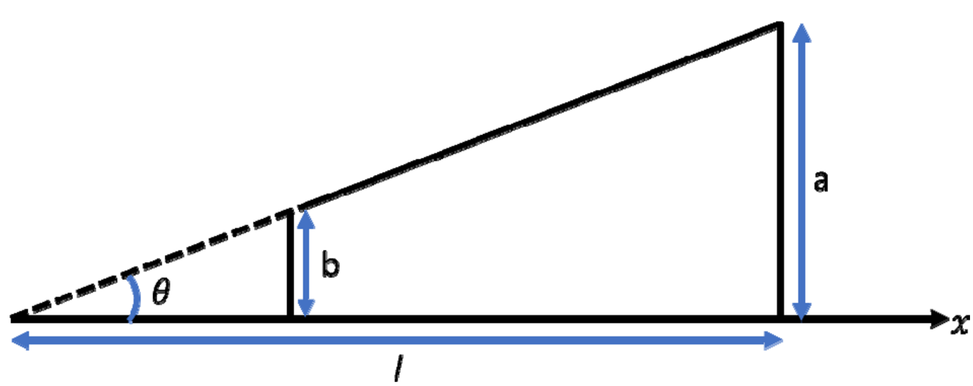

Fig. 2 Coordinate system for the conical striker bar

$$\overline{u} = D \exp\left(-\frac{xs}{c}\right) \tag{10}$$

where D is a constant. The transformed stress and particle velocity are

$$\overline{\sigma} = E\frac{d\overline{u}}{dx} = -\left(\frac{s}{c}\right) ED \exp\left(-\frac{xs}{c}\right) \tag{11}$$

$$v = \frac{\partial u}{\partial t}, \overline{v} = s\overline{u} = sD \exp\left(-\frac{xs}{c}\right) \tag{12}$$

Now, we solve for the constant D needed in Eq. (11). At $x=l$, the cross-sectional bar areas are equal, so for equal particle velocities and stresses at the interface

$$A\frac{s}{l} \exp\left(\frac{ls}{c}\right) + \frac{V}{s} = sD \exp\left(-\frac{ls}{c}\right) \tag{13a}$$

$$EA \exp\left(\frac{ls}{c}\right)\left[\frac{s}{lc} - \frac{1}{l^2}\right] = -\left(\frac{s}{c}\right) ED \exp\left(-\frac{ls}{c}\right) \tag{13b}$$

We obtain the constant D from Eq. (13a) and (13b). The transformed incident bar stress is

$$\overline{\sigma}(x, s) = -\frac{EV}{c}\left[\frac{l}{2ls - c} - \frac{c}{s(2ls - c)}\right] \exp\left(-\frac{(x-l)}{c}\right) \tag{14}$$

We take the inverse transform of Eq. (14) and obtain

$$\sigma(x, t) = -\frac{EV}{c}\left[1 - \frac{1}{2} \exp\left(\frac{ct}{2l} - \frac{x-l}{2l}\right)\right] H\left(t - \frac{(x-l)}{c}\right), \tag{15}$$

where H is the Heaviside unit function. At the striker bar-incident bar interface, $x=l$, and

$$\sigma(l, t) = -\frac{EV}{c}\left[1 - \frac{1}{2} \exp\left(\frac{ct}{2l}\right)\right] H(t) \tag{16}$$

At the wave front,$t = (l - x)/c$

$$\sigma\left(t = \frac{(l-x)}{c}\right) = -\frac{EV}{2c} \tag{17}$$

For applications, the length l in the incident bar equation must be related to the conical striker geometry. From Figs. 1 and 2

$$\tan\theta = \frac{a-b}{L} = \frac{a}{l}, \quad l = a\left(\frac{L}{a-b}\right) \tag{18}$$

The stress wave in the incident bar is non-dispersive. Thus, the stress-time response given by Eq. (16) moves in the x-direction at propagation velocity c. This is expressed mathematically by Eq. (15). The incident stress pulse duration is

$$t_d = \frac{2L}{c} \tag{19}$$

In addition, we point out that the stress equation for a cylindrical striker bar [10, 11] is the same as that given by Eq. (17) for the wave front stress. In the next section, we show that the stress-time response in the incident bar decreases behind the wave front for a conical striker bar.

Incident Strain Pulses

Casem [5] presents a method to determine the shape of the incident pulse needed to obtain a nearly constant true strain rate of 2500 1/s for a nylon specimen. Then, he uses a detailed numerical method to obtain the shape of a segmented aluminum striker bar to produce this pulse in an aluminum incident bar. In this section, we compare results from our conical striker bar model with the desired incident pulse presented by Casem [5].

Experiments [5] were conducted with 7075-T651 aluminum bars with Young's modulus, density, and wave velocity given by $E=73.9$ GPa, $\rho=2810$ kg/m^3, and $c=5130$ m/s. The incident bar and segmented striker bar at the impact end had a diameter of 19.05 mm. The segmented striker bar had a series of 20 cylindrical sections with decreasing diameters away from the impact end. The segmented striker bar had a length of 680 mm and a diameter of 14 mm at the end of the striker bar.

For our model, the geometry for the conical striker bar is shown in Fig. 1. We take $2a=19.05$ mm and $L=680$ mm to have the same pulse duration as that given in [5]. From Eq. (19), the incident pulse duration is 0.265 ms. To design the conical striker bar, we have one adjustable parameter, namely $2b$ as shown in Fig. 1. We start with $2b=14$ mm as given in [5] and vary the parameter $2b$ in Eqs. (18) and (16) until we have a close fit to the desired incident pulse given in [5] which has a wave front strain of $V/2c = 1360\ \mu\varepsilon$.

Figure 3 shows the desired incident pulse [5] and the pulse predicted by the conical striker bar model given by Eqs. (18) and (16) with value of $2b=14$ mm. In Fig. 3, time is measured from the arrival of the wave front of the pulse traveling at $c=5130$ m/s. Clearly, predictions from the conical striker bar model are in good agreement with the desired pulse presented by Casem [5].

Summary and Discussion

We derived a closed form equation that predicts the incident stress pulse produced by a truncated conical striker bar. This equation can be used as a convenient alternative to the detailed numerical method presented by Casem [5] in his

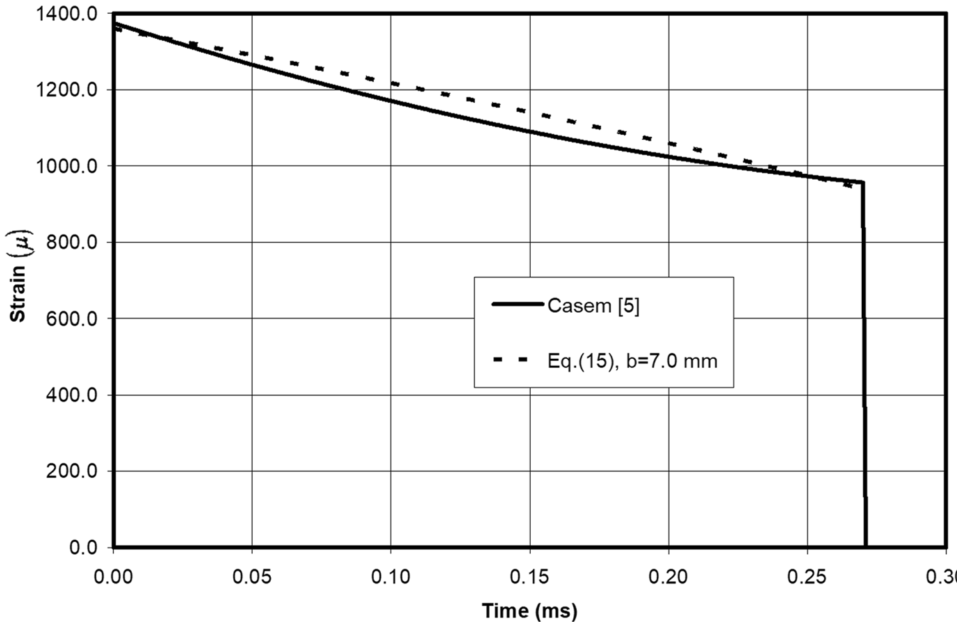

Fig. 3 Incident strains versus time

procedure to obtain a constant true strain rate for Kolsky bar experiments.

Reference [4] presents a procedure that uses data with constant engineering strain rate to obtain a constitutive equation in terms of true strain rate. By contrast, Casem [5] present an experimental method to directly obtain true strain rate, However, both studies showed only minor differences between true stress versus true strain predictions from experiments conducted with engineering and true strain rates. These comparisons are presented for 4340 Rc 45 steel for true strains to 0.15 [4] and nylon for true strains to 0.60 [5].

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work done in this paper.

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