ORIGINAL PAPER

A Dynamic Framework for Multiobjective Mixed-Integer Optimal Power Flow Analyses

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Abstract

This paper proposes a reliable and computationally efficient framework for solving multiobjective mixed-integer Optimal Power Flow problems. The main idea is to apply the interior point theory and the goal-attainment method to recast a generic OPF problem with both real and integer decision variables by an equivalent scalar optimization problem with equality constraints. Then, thanks to the adoption of the Lyapunov theory, an asymptotically stable dynamic system is designed such that its equilibrium points coincide with the stationary points of the Lagrangian function of the equivalent problem. Thanks to this approach, the OPF solutions can be promptly and reliably obtained by solving a set of ordinary differential equations, rather than using an iterative Newton-based scheme, which can fail to converge due to several numerical issues. Detailed numerical results are presented and discussed in order to prove the effectiveness of the proposed framework in solving real world problems.

Keywords Smart grid optimization · Power system computing · Power system analysis

Introduction

Optimal Power Flow (OPF) has been a widely used analysis tool by power system engineers, since its appearance in 1962 [1]. It aims at identifying a set of decision variables, which minimize one or multiple objective functions, such as the production costs, the greenhouse gas emissions, the power losses and the voltage deviations, satisfying both equality and inequality constraints. The equality constraints include the active and reactive power balances at each network bus, which are described by the power flow equations, while the inequality constraints describe both physical control limits, which cannot be violated (e.g. upper and lower bounds of real and reactive power generation), and operating limits, which are imposed to enhance security and do not represent physical bounds, but they can be relaxed temporarily, if necessary, to obtain feasible solutions [2].

OPF analyses can be formalised by a mixed integer non linear programming (MINLP) problem, which is a scalar, non-linear, non convex constrained optimization problem with both real and integer decision variables [3]. The scalar formulation relies on a single objective function, which is particularly useful in solving many planning and operation problems, as far as active power dispatch, load shedding, and market settlement are concerned.

As discussed in the literature [4], the application of conventional Newton-based iterative algorithms in solving scalar OPF problems could be hindered by several limitation factors, such as the limited capability in solving large-scale problems, the ineffectiveness in identifying global optimum solutions, and the difficulties in dealing with ill-conditioned problems.

To reliably solve these problems, several solution schemes could be employed. In particular, some of the authors of this paper proposed a challenging idea, which is based on the formulation of a scalar OPF problem as a set of ordinary differential equations, whose equilibrium points coincide with the problem solution. Starting from the Lyapunov Theory, in the authors' previous work, it was shown that this artificial dynamic model is asymptotically stable and it is quite insensitive to many factors which can cause numerical instability in conventional algorithms [5]. This framework has been widely adopted in the task of solving several

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OPF problems [6], and has been generalized in the task of solving scalar constrained optimization problems problems in different application domains [7].

A comparison between different power flow solution methods has been conducted in [8] and a multistage arrangement is introduced as an improvement to the continuous Newton's method. The authors of [9] proposed various robust solvers in current injection formulation. In particular, since this form is more prone to be ill-conditioned than the power mismatch form, they propose a technique to solve ill-conditioned power flow equations in current injection form. Also, they propose a robust and efficient Power Flow solution technique inspired by the Romberg's Integration Scheme in [10].

All these algorithms allow reliably solving OPF analyses characterized by scalar objective functions, improving the convergence to feasible solutions and avoiding the numerical instabilities of conventional iterative-based solution schemes.

On the other hand, evaluating power system performances through a single dominant index could limit the effectiveness of the OPF solution, especially in solving complex decision making problems with multiple and conflicting objectives. In these contexts, more advanced multi-objective formulations of the OPF problem (MOOPF), which are based on vector-valued objective functions, can be deployed in order to compute the non-inferior optimal solutions, which constitute the Pareto front of the multi-objective optimization problem [11].

In the scientific literature, the solution of MOOPF is mostly found through the weighting method, which consists in the scalarization of the vector of objective functions, through their weighted combination [12]. However, such method requires a precise knowledge about the necessary trade-offs between the objective functions, which is not always available. Furthermore, it suffers the convexity problem, which may not allow the analyst to explore the whole solution space [13].

Other solution methods try to compute non-inferior optimal solutions by employing iterative solution schemes, which may fail to converge in the presence of singularities in the Jacobian matrices, and may become unstable when the initial guess solution is far from the region of attraction of the optimal solution.

To solve these problems, many papers propose the deployment of metaheuristic algorithms, such as Genetic Algorithms [14], Evolutionary Programming, Particle Swarm Optimization [15]. A review of the wide literature on the role of these methods for solving multi-objective optimization is presented in [16]. As outlined in these papers, metaheuristic algorithms allow the analyst to effectively handle non-linear constraints, improving the solution space exploration, and reducing the probability of converging in local minima. Despite these benefits, metaheuristic techniques lack mathematical rigor and they might be equally challenging when considering computational burden.

Hence, the research for alternative solution schemes for the prompt and reliable solution of MOOPF problems represents a relevant issue to address.

To face this issue, this paper intends to enhance the theoretical framework conceptualized in [6] by proposing an improved dynamic-based formulation aimed at solving constrained multi-objective optimization problems with mixed real and integer decision variables. In particular, to deal with the multi-objective nature of the problem, a mathematical formulation based on the goal-attainment method is proposed. This formulation is based on the solution of multiple optimization problems, each one characterized by its own scalar objective function and constraints. The obtained solutions of these single-objective problems, called the *utopia points*, are employed to define proper inequality constraints describing the competitiveness of the objective functions [17].

As far as the discrete variables are concerned, they are modelled by introducing a proper set of equality constraints, which force them to assume a finite set of allowable values.

Detailed numerical results, obtained by applying the proposed paradigm on different power system benchmarks will be presented and discussed in order to prove its effectiveness in managing complex and real world problems.

The paper is organized as follows: Section "Elements of OPF Analysis" introduces the theoretical background, Section "Proposed OPF Framework" describes the proposed methodology to solve MOOPF problems through artificial dynamic systems. Section "Case Studies" presents the analyzed case studies and, finally, Section "Conclusions" draws the main conclusions.

Elements of OPF Analysis

OPF formulation can be classified according to different features: the design variables, the objective functions, the constraints and the mathematical formulation.

The most simplified OPF is formulated as a linear programming (LP) problem. It involves the simplification of the generic OPF problem by linearizing the model through specific assumptions about the system.

DC-OPF is an instance of such linear formulation, which assumes negligible values of power line's resistance over reactance. It can be employed in place of classical optimal power flow equations, when we are interested only in the active power. In this case, the accuracy of the model is reduced, however the results can be acceptable for applications requiring low accuracy but fast computation [18].

Quadratic programming, which is based on a quadratic cost function and linear constraints can improve the accuracy while retaining the low computational burden, however it is still based on the linearization of power flow constraints. Linear formulation can include discrete variables, for example to control transformer tap ratio or capacitors' banks. However, the most complex and representative way to model a power system is by a MINLP problem. Some examples of MINLP are described in [19] and security constrained unit commitment is one of the most important applications of such formulation. As [3] writes, there are few papers which give clear and complete MINLP formulations for the OPF, especially on how they are solved. This paper aims at filling this gap, introducing a method based on artificial dynamical systems to solve such problems.

OPF problems can also be classified according to their objective functions, variables and constraints. The most common objective function is the total cost, however OPF can be used also to minimize transmission losses, load shedding, environmental impact etc... In this paper multiple objectives are considered simultaneously, as it is done for MOOPF.

The main design variables are voltage magnitude and phase angles, active and reactive power generation, however further variables can be added depending on the application.

Mathematical Formalization

The classical OPF method can be formulated as follows [20]:

$$
\begin{aligned}
\min_{P,Q,V,\delta} \quad & f(P,Q,V,\delta) \\
\text{s.t.} \quad & PF(P,Q,V,\delta) = 0 \\
& P_{j,min} \le P_j \le P_{j,max} \quad j = 1,2,...,n \\
& Q_{j,min} \le Q_j \le Q_{j,max} \quad j = 1,2,...,n \\
& V_{j,min} \le V_j \le V_{j,max} \quad j = 1,2,...,n \\
& \delta_{j,min} \le \delta_j \le \delta_{j,max} \quad j = 1,2,...,n
\end{aligned} \quad (1)
$$

where $f(P,Q,V,\delta)$ is the objective function, which is a vector in case of multiobjective optimization. The variables P,Q,V,δ are the active power, the reactive power, the voltage magnitude and angle for each bus of the network. The equality constraints $PF(P,Q,V,\delta) = 0$ are the power flow equations, which in the polar form can be written as:

$$
P_j = \sum_{k=1}^{N} |V_j||V_k||Y_{jk}|cos(\delta_j - \delta_k - \theta_{jk}) \quad \forall j = 1,...,n \quad (2)
$$

$$
Q_j = \sum_{k=1}^{N} |V_j||V_k||Y_{jk}|sin(\delta_j - \delta_k - \theta_{jk}) \quad \forall j = 1,...,n \quad (3)
$$

The inequality constraints include minimum and maximum limits on design variables or transient stability constraints.

Emerging Needs in OPF Analyses

OPF analyses can play a fundamental role in decarbonized power systems, where the increasing network complexity, and the massive pervasion of distributed and dispersed generators drastically increase the number of decision variables, leading the components to operate close to their limits, and making the power system more vulnerable to external disturbances. All these issues hinder the solution of complex OPF analyses, especially in the presence of multiple-contingencies, which could affect the convergence and the stability of conventional schemes [3]. To this aim, it is incumbent upon the power system community to develop novel solution methods aimed at addressing the following critical issues:

- **Computational speed:** practical OPF algorithms require high speed for real time applications. With the increase of renewable power generation, the scheduling time is being reduced from hours to minutes and the response time must be lower;
- **Reliability:** a feasible solution has to be achieved even for ill-conditioned problems, contingency studies and other applications;
- **Robustness:** OPF solutions must have low sensitivity to initial points and it must be stable to changes in operating constraints;
- **Flexibility:** there must be the possibility to easily incorporate the solution algorithms in existing software tools, such as energy management systems;
- **Discrete modeling:** discrete variables may be needed in addition to continuous ones;
- **Incorporation of multiple objectives:** the method should allow both single and multi objective optimizations;
- **Introduction of inter-temporal constraints:** OPF may include time dependent constraints, a typical case is the presence of energy storage. OPF should be able to handle multiperiod optimization;
- **Probabilistic modelling:** with the increasing uncertainty, OPF methods should allow a probabilistic or uncertain counterpart.

Most of these issues can be effectively addressed by deploying the solution method proposed in this paper.

Proposed OPF Framework

Multiobjective OPF involves the simultaneous optimization of a vector of scalar functions $f_i(x)$, where x is the vector

of state variables and $\boldsymbol{u}$ the vector of control variables. The general multiobjective programming problem, requiring the optimization of n objectives may be formulated as follows:

$$
\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{u}} \quad & f_i(\boldsymbol{x},\boldsymbol{u}) \quad \forall\, i = 1, 2, ..., n \\
\text{s.t.} \quad & g_j(\boldsymbol{x},\boldsymbol{u}) = 0 \quad j = 1, 2, ..., m \\
& h_k(\boldsymbol{x},\boldsymbol{u}) \le 0 \quad k = 1, 2, ..., p \\
& \boldsymbol{x}_{min} \le \boldsymbol{x} \le \boldsymbol{x}_{max} \\
& \boldsymbol{u}_{min} \le \boldsymbol{u} \le \boldsymbol{u}_{max}
\end{aligned}
\tag{4}
$$

where g_j represents an equality constraint, h_k an inequality constraint and $x_{min}, u_{min}, x_{max}, u_{max}$ the minimum and maximum values of the state and control variables. In multiobjective optimization, system performance is described through a vector of objective functions. The optimal solution, in this case, is not the global optimum for each of the objective functions, rather a *Pareto optimal* point: i.e. no improvement in one of the objective functions can be achieved without the degradation of at least one of the other objective functions [21]. This sub-optimal point is called non-inferior point and it represents a trade-off between the objective functions.

The simplest algorithm to approximate a Pareto front is called weighting strategy or scalarization method [22]. It consists in merging a vector of objective functions into a scalar one, by positively weighting each function and summing them. Other methods proposed in the literature are the *ε constrained method* and the multilevel programming. [23]. Anyway, each of these methods requires a priori knowledge of the kind of problem being analyzed and they can be sensitive to the shape of Pareto front [24].

To overcome this limitation, this paper employs the goal attainment method, which allows finding the Pareto front by solving the following parametrized problem:

$$
\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{u},z} \quad & z \\
\text{s.t.} \quad & g(\boldsymbol{x},\boldsymbol{u}) = 0 \\
& h(\boldsymbol{x},\boldsymbol{u}) \le 0 \\
& f(\boldsymbol{x},\boldsymbol{u}) - wz \le \gamma
\end{aligned}
\tag{5}
$$

where z is an unrestricted scalar variable, $\boldsymbol{w}$ a vector of known weights, $\boldsymbol{\gamma}$ a vector of the desired levels to be achieved for each of the objective functions.

The quantity $w_i z$ is related to the degree of over or underattainment of the goal f_i. The weight w_i depends on the preferences of the analyst over the performance indices.

In particular, suppose that $\boldsymbol{\gamma}$ is an unattainable vector of objective functions. By choosing a small w_i, the level of underattainment of the function f_i, which is $f_i - w_i z \le \gamma_i$, will be low, and the function f_i will be closer to the desired value, opposed to other functions characterized by higher weights.

Some optimization problems might involve integer variables, for example in the case of mixed integer programming problems (MIP). In order to constraint a variable to be integer, a non linear equality constraint, which is equivalent, can be introduced.

For example, if a design variable x is defined in a set of integers $\mathbf{Z} = \{1, 2, 5\}$, the corresponding equality constraint can be written as:

$$
(x - 1)(x - 2)(x - 5) = 0 \tag{6}
$$

Solution Methodology

The multi-objective mathematical programming problem formulated in Eq. 5 can be recast as a system of non linear equations. The equation E_1 is obtained by observing that the minimization of the objective function in Eq. 5, is equivalent to finding the vectors of design variables which nullify the following error function:

$$
E_1 = z - q \tag{7}
$$

where q is an additional unknown variable representing the minimum of z.

As far as the inequality constraints are concerned, they can be converted into equality constraints by the introduction of non-negative slack variables, $\boldsymbol{s}$ and $\boldsymbol{t}$, according to the Interior Point theory [25, 26].

Therefore satisfying the inequalities and equalities means finding the zeroes of the following functions:

$$
\boldsymbol{E}_2 = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) - \boldsymbol{s} \tag{8}
$$

$$
\boldsymbol{E}_3 = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \tag{9}
$$

$$
\boldsymbol{E}_4 = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) - \boldsymbol{w}z - \boldsymbol{\gamma} - \boldsymbol{t} \tag{10}
$$

Similar equations can be introduced to take into account the minimum and maximum constraints on the variables.

By defining $\boldsymbol{\xi}$ as the vector of design and slack variables and $\boldsymbol{E}(\boldsymbol{\xi})$ as the set of equations previously introduced, the model can be written in a compact form:

$$
\boldsymbol{E}(\boldsymbol{\xi}) = 0 \tag{11}
$$

In order to compute the values of $\boldsymbol{\xi}$ which simultaneously nullify each component of the vector $\boldsymbol{E}(\boldsymbol{\xi})$, it is possible to minimize the sum of the squared residuals.

$$
W(\boldsymbol{\xi}) = \frac{1}{2}\boldsymbol{E}^T(\boldsymbol{\xi})\boldsymbol{E}(\boldsymbol{\xi}) \tag{12}
$$

The minimum of this function is reached when:

$$
\frac{dW(\boldsymbol{\xi})}{d\boldsymbol{\xi}} = \frac{d\boldsymbol{E}(\boldsymbol{\xi})}{d\boldsymbol{\xi}}\boldsymbol{E}(\boldsymbol{\xi}) = 0 \tag{13}
$$

An alternative approach to solve the multiobjective mathematical programming problem is based on the Lagrangian theory, which allows recasting the problem Eq. 5 to a scalar

unconstrained optimization problem, whose objective function $\mathcal{L}$ (namely the Lagrangian function) can be expressed as:

$$\mathcal{L} = W(\boldsymbol{\xi}) + \boldsymbol{\lambda}^T \boldsymbol{g}(\boldsymbol{\xi}) \tag{14}$$

The set of variables $\boldsymbol{\lambda}$ contains the lagrangian multipliers introduced to take into account the constraints in the optimization function.

The minimum of the Lagrangian function is reached when:

$$\begin{cases} \frac{d\mathcal{L}(\boldsymbol{\xi},\boldsymbol{\lambda})}{d\boldsymbol{\xi}} = 0 \\ \frac{d\mathcal{L}(\boldsymbol{\xi},\boldsymbol{\lambda})}{d\boldsymbol{\lambda}} = 0 \end{cases} \tag{15}$$

which is the system of non-linear equations describing the so called Karush-Kuhn-Tucker conditions for optimality.

Design of the Artificial Dynamic System

The following step is based on the design of a stable artificial dynamic system, whose equilibrium points represent the stationary points of $W(\boldsymbol{\xi})$.

The vector $\boldsymbol{\xi}$ can be assumed as a vector of state variables which evolve according to a parameter t, an artificial time. The solution corresponds to the final equilibrium point reached by this dynamical system.

Under this assumption, the scalar positive-semidefinite function $W(\boldsymbol{\xi})$ can be regarded as a Lyapunov function of the artificial dynamic system. Consequently, if the derivative of $W(\boldsymbol{\xi})$ is negative-definite or negative semidefinite along the trajectory of $\boldsymbol{\xi}(t)$, then the Lyapunov theorem would assure the existence of an asymptotically stable equilibrium point of the dynamical system which minimizes the error function.

In order to prove it, let us consider the time derivative of $W(\boldsymbol{\xi})$:

$$\frac{W(\boldsymbol{\xi}(t))}{dt} = \boldsymbol{E}^T(\boldsymbol{\xi}(t))\frac{\boldsymbol{E}(\boldsymbol{\xi}(t))}{dt} \tag{16}$$

Since:

$$\frac{d\boldsymbol{E}(\boldsymbol{\xi}(t))}{dt} = \frac{d\boldsymbol{E}(\boldsymbol{\xi}(t))}{d\boldsymbol{\xi}}\frac{d\boldsymbol{\xi}(t)}{dt} \tag{17}$$

we obtain:

$$\frac{d(W(\boldsymbol{\xi}(t)))}{dt} = \boldsymbol{E}^T(\boldsymbol{\xi}(t))\frac{d\boldsymbol{E}(\boldsymbol{\xi}(t))}{d\boldsymbol{\xi}}\frac{d\boldsymbol{\xi}(t)}{dt} \tag{18}$$

If $\boldsymbol{\xi}$ is changed according to the gradient of $W(\boldsymbol{\xi}(t))$:

$$d\boldsymbol{\xi}\frac{d\boldsymbol{\xi}(t)}{dt} = -k\frac{d(W(\boldsymbol{\xi}(t)))}{dt} \tag{19}$$

we obtain:

$$\frac{d(W(\boldsymbol{\xi}(t)))}{dt} = -k(\boldsymbol{E}^T(\xi(t))\frac{d\boldsymbol{E}(\boldsymbol{\xi}(t))}{d\boldsymbol{\xi}}\left(\frac{d\boldsymbol{E}(\boldsymbol{\xi}(t))}{d\boldsymbol{\xi}}\right)^T$$
$$(\boldsymbol{E}(\boldsymbol{\xi}(t))) \tag{20}$$

This is a quadratic form which is certainly negative-semidefinite.

The equilibrium point must satisfy the following condition:

$$\frac{dW(\boldsymbol{\xi})}{d\boldsymbol{\xi}} = \frac{d\boldsymbol{E}(\boldsymbol{\xi})}{d\boldsymbol{\xi}}\boldsymbol{E}(\boldsymbol{\xi}) = 0 \tag{21}$$

This condition is the same as Eq. 15, thus this is also a solution of the optimization problem.

The same computational scheme can be adopted for finding the stationary points of $\mathcal{L}$.

Case Studies

In this section, the proposed methodology has been applied in the task of solving the multi-objective voltage regulation problem for three test networks, which are based on the 14 bus, 30 bus, and 118 bus IEEE power systems, respectively. For all these case studies the minimisation of two conflicting objectives has been considered, namely the mean squared bus voltage magnitude deviations, which is computed as:

$$f_1 = \frac{1}{N}\sum_{k=1}^{N}(V_k - V_o)^2 \tag{22}$$

where V_k and V_o are the bus voltage magnitude at the kth bus and the nominal voltage magnitude, respectively, and the active power losses, which can be computed as:

$$f_2 = \sum_{k=1}^{N} P_k \tag{23}$$

where P_k is the active power injected at the kth bus. Moreover, in order to describe the discrete tap-changes at the generation buses $\mathcal{N}_g$, the corresponding voltage magnitudes are described by discrete variables ranging from 0.95 to 1.05 p.u., with a 0.01 step.

Hence, the overall problem can be formalized as follows:

$$\min_{\boldsymbol{x},\boldsymbol{u},z} \quad z$$

$$\text{s.t.} \quad P_j - \sum_{k=1}^{N} V_j V_k Y_{jk}cos(\delta_j - \delta_k - \theta_{jk}) = 0 \quad \forall j \in \mathcal{N}$$

$$Q_j - \sum_{k=1}^{N} V_j V_k Y_{jk}|sin(\delta_j - \delta_k - \theta_{jk}) = 0 \quad \forall j \in \mathcal{N}$$

$$(V_i - 0.95)(V_i - 0.96)..(V_i - 1.05) = 0 \quad \forall i \in \mathcal{N}_g$$

$$Q_{i,min} \leq Q_i \leq Q_{i,max} \quad \forall i \in \mathcal{N}_g$$

$$V_{j,min} \leq V_j \leq V_{j,max} \quad \forall j \in \mathcal{N}$$

$$\delta_{j,min} \leq \delta_j \leq \delta_{j,max} \quad \forall j \in \mathcal{N}$$

$$f_1 - w_1 z \leq \gamma_1$$

$$f_2 - w_2 z \leq \gamma_2 \tag{24}$$

Where $\boldsymbol{x}$ is the vector of the state variables, namely the reactive power generated at each generator bus Q_i, and the voltage phasor $V_j \angle \delta_j$ of each bus; $\boldsymbol{u}$ is the vector of the decision variables, namely the voltage magnitude at the generation buses, $Y_{jk} \angle \theta_{jk}$ is the $(j,k)^{th}$ element of the admittance matrix.

The calculations are performed in Matlab 2020a, on an Intel Core i7 CPU.

IEEE 14 Bus Test Network

The utopia points computed for this network are $f_1^* = 0.1316$ and $f_2^* = 1.39 \cdot 10^{-4}$. Once the multi-objective formulation is completed, the optimization problem can be converted to an artificial dynamical system, as described by Eq. 21. Figures 1 and 2 show the equilibrium point reached by the state variables, voltage magnitude and angles respectively, which correspond to the optimal solution. The values of the objective functions are $f_{1,opt} = 0.5754$ and $f_{2,opt} = 0.0087$.

Figures 3 and 4 show the time-evolution of the objective functions, voltage regulation and losses, respectively. These values are portrayed together with the utopia points. As expected, multi-objective formulation leads to sub-optimal solutions. Both the objective functions are not corresponding to the utopia point. The aim of multi-objective optimization is, indeed, to find the optimal trade off between multiple objectives.

However, by managing the weights, different solutions can be attained, which may reduce the gap towards the utopia point for one function and increase it for the other function.
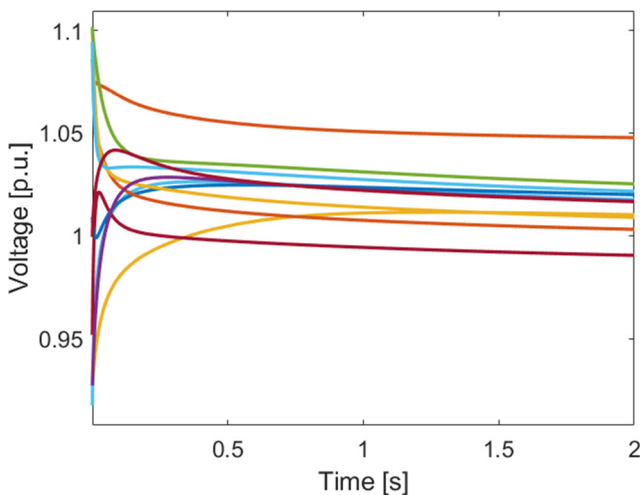
Fig. 1 Dynamic evolution of the state variables describing the voltage magnitude in p.u. . The curves represent the voltage magnitude time-evolution for each bus of the 14 bus test network
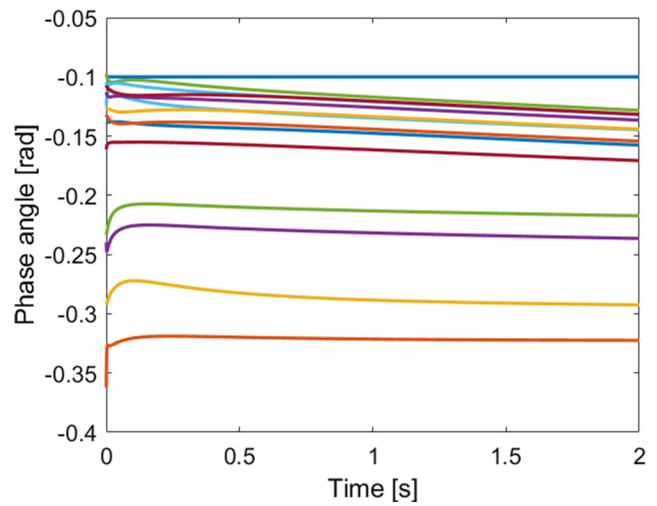

Fig. 2 Dynamic evolution of the state variables describing the voltage angles in radians. The curves represents the voltage angle time-evolution for each bus of the 14 bus test network

IEEE 30 Bus Test Network

The same procedure has been performed for the IEEE 30 bus test network.

For this network the utopia points $f_{1,opt} = 0.0094 \ p.u.$ and $f_{2,opt} = 0.4093 \ p.u.$, and, as in the previous case study, the solution computed by solving the multi-objective problem identifies a proper trade-off between these two extreme values. The state variables associated to this non-inferior solution are shown in Figs. 5, 6 which show the voltage magnitude and phase angle for each bus, respectively.
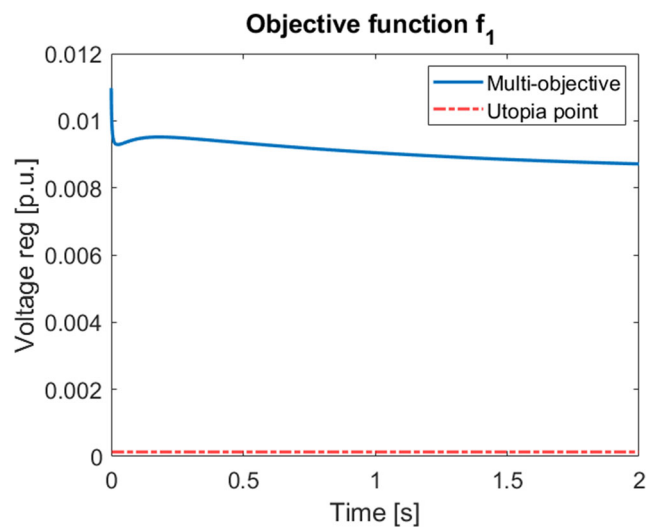

Fig. 3 Voltage regulation f_1, optimized through single objective optimization (red dashed line) and the proposed multi-objective optimization (blue continuous line) of 14 bus test network
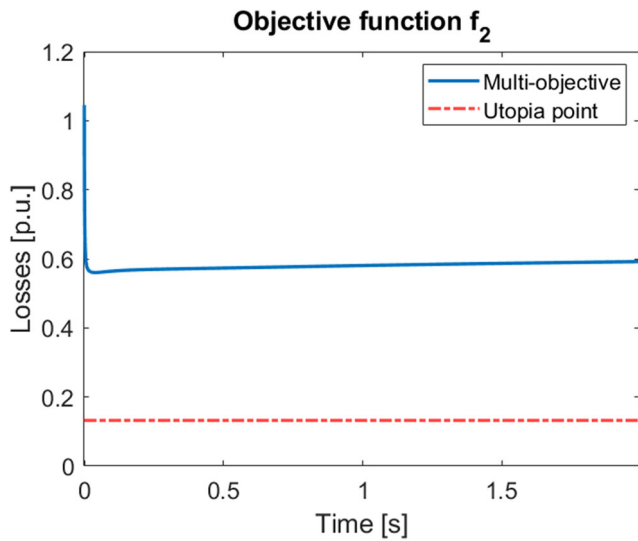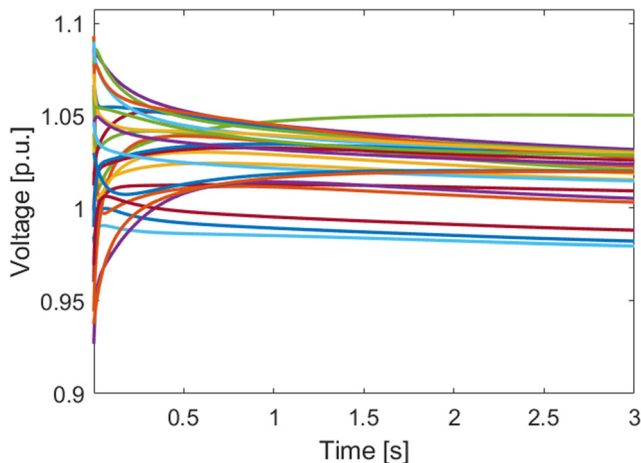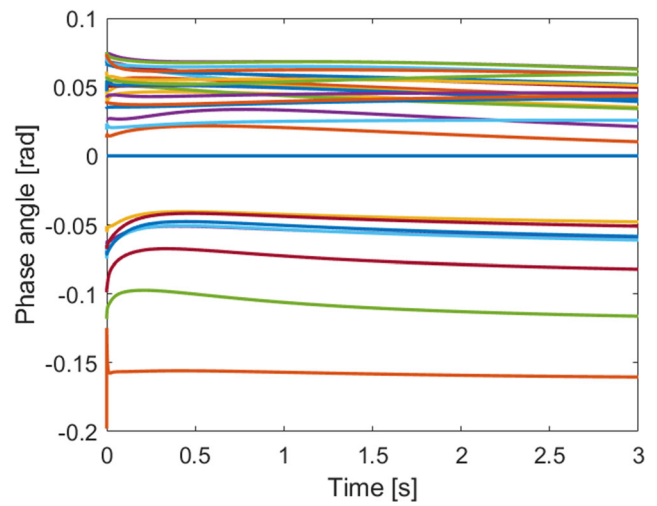
Fig. 4 Losses f_2, optimized through single objective optimization (red dashed line) and the proposed multi-objective optimization (blue continuous line) of 14 bus test network

Furthermore, Figs. 7 and 8 show the value of the objective functions for this system. As for the previous case, the objective functions are sub-optimal when compared to the single objective solution.

IEEE 118 Bus Test Network

Finally, in order to prove the effectiveness of the proposed solution scheme in the task of solving the analyzed OPF problem for a larger power system, the IEEE 118 bus test network has been considered. The obtained results are summarised in Fig. 9, where the time-evolution of the state variables is depicted. By analyzing this figure it is worth noting that also for this case study the dynamic system

Fig. 6 Dynamic evolution of the state variables describing the voltage angles in radians. The curves represents the voltage angle time-evolution for each bus of the 30 bus test network

quickly converges to a stable equilibrium point, which represents the OPF solution. The effectiveness of this solution can be assessed by analysing Fig. 10, which reports the time-evolution of the error function, namely the gradient of the Lagrangian function of the problem formalised in Eq. 24, and in Fig. 11, where the time-evolution of the residual errors of the power flow equations is depicted. The corresponding time-evolution of the objective functions is depicted in Fig. 12, which confirms the effectiveness of the proposed method in the task of identifying a proper trade-off between the two problem objectives.

Furthermore, Fig. 12 shows the value of the objective functions for this system. As for the previous case, the
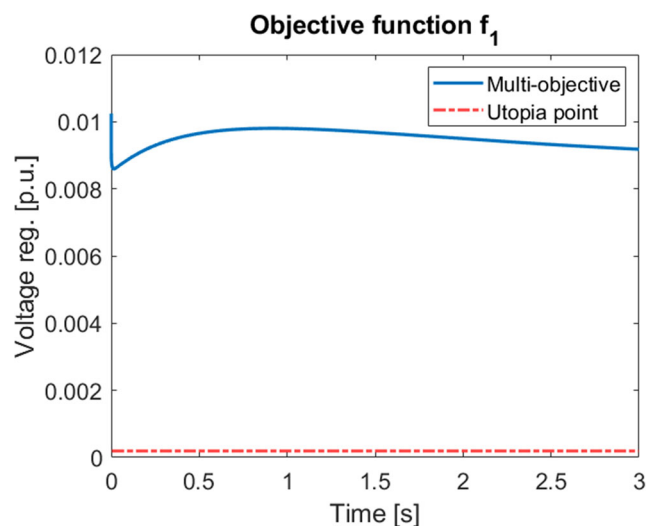

Fig. 5 Dynamic evolution of the state variables describing the voltage magnitudes in p.u.. The curves represents the voltage angle time-evolution for each bus of the 30 bus test network

Fig. 7 Voltage regulation f_1, optimized through single objective optimization (red dashed line) and the proposed multi-objective optimization (blue continuous line) of 30 bus test network
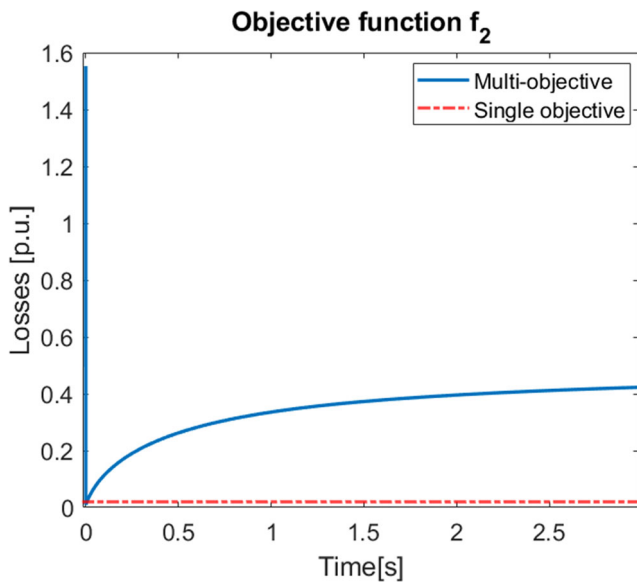
Fig. 8 Losses f_2, optimized through single objective optimization (red dashed line) and the proposed multi-objective optimization (blue continuous line) of 30 bus test network

objective functions are sub-optimal when compared to the single objective solution.

Finally, in order to compare the convergence performance characterising the proposed technique, the time-evolution of the error function is benchmarked with those obtained by solving the problem formalised in Eq. 24 by a conventional iterative scheme based on the interior-point method. The obtained results are reported in Fig. 13, which confirms the effectiveness of the proposed method in the task of promptly and reliably converge to a feasible OPF solution.
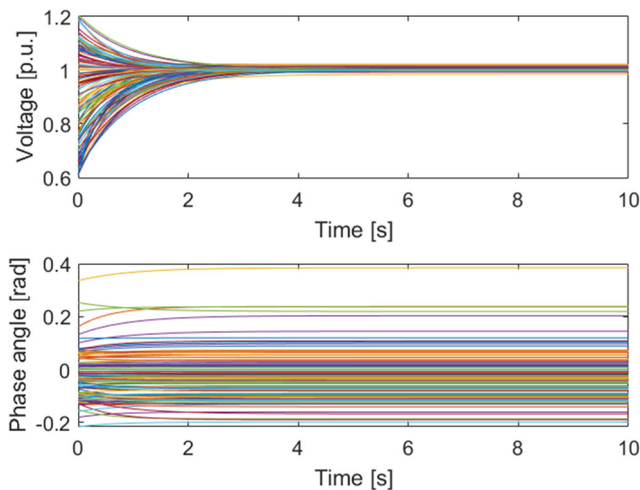

Fig. 9 Dynamic evolution of the state variables describing the voltage magnitudes and angles. The curves represent the time-evolution of the voltage angle and magnitude for each bus of the 118 bus test network
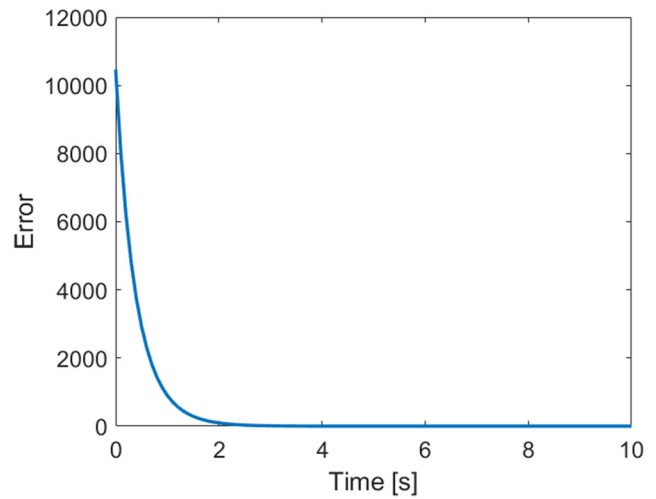

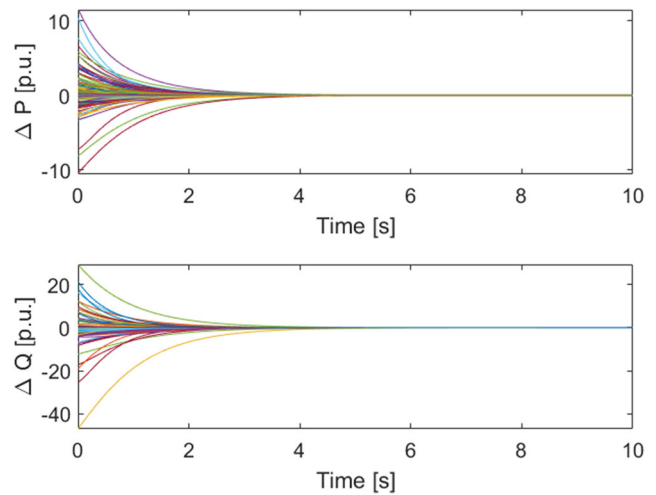**Fig. 10** Time-evolution of the Error function for the 118 bus test network

Fig. 11 ΔP and ΔQ for each bus of 118 bus test network
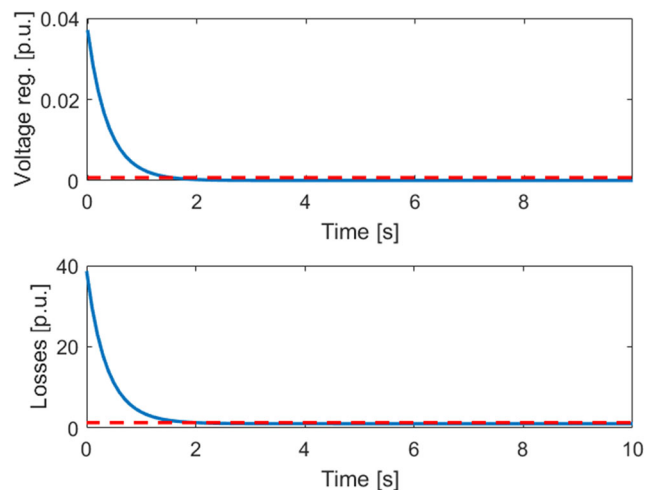

Fig. 12 Voltage regulation f_1 and losses f_2, optimized through single objective optimization (red dashed line) and the proposed multi-objective optimization (blue continuous line) of 118 bus test network
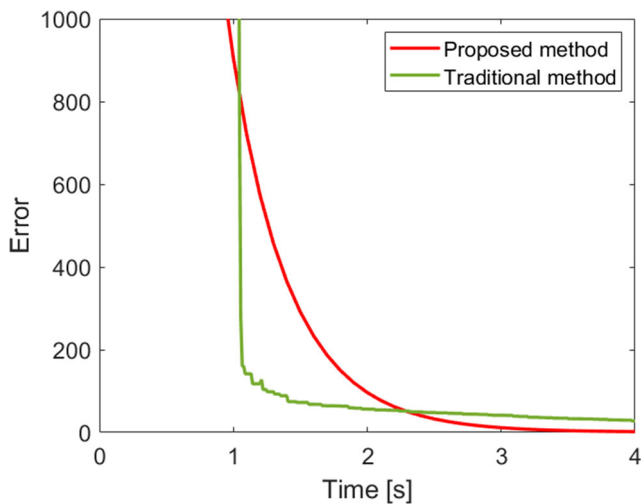
Fig. 13 Comparison between the proposed dynamic optimization method (red line) and a conventional optimization method (green line)

Conclusions

In this paper, a dynamic framework is proposed as an alternative method to solve multi-objective mixed integer non linear optimal power flow problems. Through Lyapunov theory, it has been shown that the artificial dynamic system converges to a local solution even in presence of large non-linearities, and it is capable of handling ill-conditioned cases, where other methods fail.

Furthermore, by casting integrity constraints as non-linear constraints, and combining the artificial dynamic system theory to Lagrangian multipliers and goal-attainment method, multi-objective formulation has been introduced.

The numerical simulations on several test networks showed the results obtained through this method, which can be extended to networks of any dimensions just by increasing the number of state variables of the dynamic system and introducing further differential equations related to the additional constraints.

Further research might involve the application of the proposed method to other power system's applications, such as the management of virtual power plants, the unit commitment, demand side management or energy hubs.

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