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ARTICLE

# Dynamical cross-correlation of multiple time series Ising model

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**Abstract** Using an Ising-based model extended to simulate multiple stock time series, we perform a large-scale simulation for a financial system with 100 stocks. We find that the financial system shows fat-tailed return distributions and the system volatility level measured as an average of absolute-returns changes over time. We investigate the dynamical properties of cross-correlation matrices among stocks and find that the eigenvalue distributions of the cross-correlation matrices deviate from those of the random matrix theory. It is found that the cumulative risk fraction (CRF) constructed from the largest eigenvalues changes at periods where the volatility level is high. The inverse participation ratio (IPR) and its higher-power version, IPR6, also exhibit the changes at the same high volatility periods. Therefore, the CRF, IPR, and IPR6 are expected to be useful measurements to identify abnormal states such as high-volatility periods.

**Keywords** Ising model · Cross-correlation · Random matrix theory · Cumulative risk fraction · Inverse participation ratio

**JEL Classification** C15 · G01 · G17

**Mathematics Subject Classification** 62P20 · 91B80

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## 1 Introduction

Asset price returns are known to exhibit some universal properties that are now classified as the "stylized facts," see, for example, Cont (2001). The notable properties included in the stylized facts are the fat-tailed distributions of returns and volatility clustering, which are not explained by the standard Gaussian process. A promising explanation on the asset price dynamics is the mixture of distribution hypothesis by Clark (1973), that is, the price return $R_t$ is described by a Gaussian process with time-varying volatility, $R_t = \sigma_t \epsilon_t$, where $\sigma_t^2$ and $\epsilon_t$ are the volatility and standard normal random variable at time $t$, respectively. Under the mixture of distribution hypothesis, the volatility changes according to the rate of information arrival. Since the rate of information arrival is latent in the real markets, Clark uses volume as a proxy of it. Empirically, the price return dynamics is examined by checking whether $R_t/\sigma_t$ recovers the standard normal random variable, and studies using the realized volatility claim that the price return dynamics is consistent with the Gaussian process with time-varying volatility (Andersen and Bollerslev 1998; Andersen et al. 2000, 2001a, b, 2007, 2010; Takaishi 2012; Takaishi et al. 2012).

To simulate a financial market, Bornholdt proposed an Ising-based model by including Ising spins that have either of the two states, namely "buy" state and "sell" state (Bornholdt 2001). It is shown that the model successfully captures major stylized facts such as fat-tailed distributions and volatility clustering (Bornholdt 2001; Yamano 2002; Kaizoji et al. 2002; Krause and Bornholdt 2013). The model was extended to a Potts-like model where three spin states are considered, and it is confirmed that the Potts-like model also exhibits the stylized facts (Takaishi 2005). Further, the return dynamics of the Ising-based models were checked by testing whether $R_t/\sigma_t$ recovers the standard normal random variable, and it is verified that the return dynamics is consistent with the Gaussian process with time-varying volatility same as the real financial markets (Takaishi 2013a, 2014).

The real financial market is a complex system that includes many stocks correlated with each other. Measuring correlations among stocks is of great importance to investigate the stability of financial markets, and a considerable number of studies are devoted to unveil properties of correlations among stocks (Plerou et al. 1999; Laloux et al. 1999; Plerou et al. 2000, 2002; Utsugi et al. 2004; Kim and Jeong 2005; Wang et al. 2011). While the original Bornholdt model simulates only one stock, the model was extended to simulate multiple stock time series in Takaishi (2015a, b), and simulations including up to three stocks were done. In this study, we make a large-scale simulation that includes 100 stocks and investigate the dynamical properties of cross-correlations among stocks. Further, we apply the principal component analysis and measure the cumulative risk fraction to monitor the states of the financial system. Finally, we calculate the inverse partition function (IPR) and its higher power version and claim that they are also useful to investigate states of the system.

## 2 Multiple time series Ising model

The multiple time series Ising model has been introduced in Takaishi (2015a, b) to simulate a financial system that includes many correlated stocks. Let us consider, a financial market, where $N$ stocks are traded. We assume that agents locate on sites of an $L \times L$ square lattice and the number of agents is $P = L \times L$. Each agent has a spin $s_i$ that takes two states, either $+1$ or $-1$, where $i$ stands for the $i$-th agent. We assign "$+1$" to the "buy" state and "$-1$" to the "sell" state. The agents flip their spin states probabilistically according to a local field. The local field of the $i$-th spin $h_i^{(k)}(t)$ at time $t$ for the $k$-th stock is defined by

$$h_i^{(k)}(t) = \sum_{\langle i,j \rangle} J s_j^{(k)}(t) - \alpha^{(k)} s_i^{(k)}(t) |M^{(k)}(t)| + \sum_{j=1}^{N} \gamma_{jk} M^{(j)}(t), \tag{1}$$

where $\langle i,j \rangle$ stands for a summation over the nearest neighbor pairs, $J$ is the nearest neighbor coupling, and in this study, we set $J = 1$. The first term on the right-hand side of Eq. (1) with $J > 0$ introduces the ferromagnetic effect that tends to align the nearest neighbor spins with the same sign. This effect corresponds to the herd behavior in financial markets, that is, the majority effect. $M^{(k)}(t)$ is the magnetization that measures an imbalance between "buy" and "sell" states, given by $M^{(k)}(t) = \frac{1}{P}\sum_{l=1}^{P} s_l^{(k)}(t)$ and the absolute value of $M^{(k)}(t)$, that is, $|M^{(k)}(t)|$ corresponds to the magnitude of the market bubble. The second term proportional to $|M^{(k)}(t)|$ introduces the effect that promotes a spin-flip, which corresponds to the minority effect. The third term describes the interaction with other stocks. More precisely, this interaction couples to the magnetization of other stocks and introduces an effect of imitating the states of other stocks. The magnitude of the interaction is given by the interaction parameters that form a matrix $\gamma_{lm}$ having zero diagonal elements, that is, $\gamma_{ll} = 0$. Although the parameter $\alpha^{(k)}$ can vary depending on the stock, in this study, we assume that all $\alpha^{(k)}$ have the same value, that is, $\alpha^{(k)} \equiv \alpha$. As in the Bornholdt model, the states of spins are updated according to the following probability $p$:

$$\begin{aligned} s_i^{(k)}(t+1) &= +1 \quad p = 1/(1 + \exp(-2\beta h_i^{(k)}(t))), \\ s_i^{(k)}(t+1) &= -1 \quad 1 - p. \end{aligned} \tag{2}$$

## 3 Simulation study

In previous studies (Takaishi 2015a, b), the model for a financial system with up to 3 stocks was simulated. In this study, we make a large-scale simulation for the model. Specifically, we consider a financial system trading 100 stocks. Each stock is traded by 14400 agents on a $120 \times 120$ lattice. Thus, in total, 1440000 agents are introduced in a simulation.

We perform simulations for two parameter sets. One is set to $(\beta, \alpha) = (2.0, 35)$ and all elements of the interaction matrix $\gamma$ are set to zero, that is, in this case, all 100 stocks are simulated independently and no correlation is introduced among stocks. This parameter set is denoted by "Set A." The other is set to $(\beta, \alpha) = (2.0, 55)$ and twenty percent of off-diagonal elements in $\gamma$ are set to non-zero values drawn from Gaussian random numbers with an average 0.05 and variance 0.1. The remaining elements are set to zero. Then, the other parameter set is denoted by "Set B."

Here, note that for the 2-dimensional Ising model, the critical temperature $T_c = 1/\beta$ is 2.269. Thus, the temperature $T = 1/\beta = 0.5$ for our simulations is below $T_c$ and our model with $\alpha = 0$ is in the ferromagnetic (ordered) phase. However, the $\alpha$-term introduces the effect that destroys the ferromagnetic phase and causes the paramagnetic (disordered) phase. Our model with $\alpha > 0$ results in a non-equilibrium where both the ferromagnetic and paramagnetic phases appear (Bornholdt 2001; Takaishi 2013a).

The states of spin are updated according to Eq. (2) in random order and the periodic boundary condition is employed. After discarding the first $5 \times 10^3$ updates as thermalization, we collect data from $3 \times 10^4$ updates for analysis.

Following Kaizoji et al. (2002), the return of the $k$-th stock is defined by the difference of the magnetization, $R_k(t) = (M^{(k)}(t+1) - M^{(k)}(t))/2$, where $t$ is incremented in units of one update. Since the simulation generates 100 return time series, which are too many to show, we only show a representative return time series for Sets A and B in Fig. 1. It is found that the magnitude of returns for both Sets A and B changes over time, and we observe volatility clustering (Mandelbrot 1963; Fama 1970), which is one of the stylized facts often seen in the real financial markets (Mantegna and Stanley 1997; Gopikrishnana et al. 1999). To measure volatility persistency, we apply the exponential GARCH (EGARCH) model (Nelson 1991) for each representative return time series. The EGARCH model is described as follows:

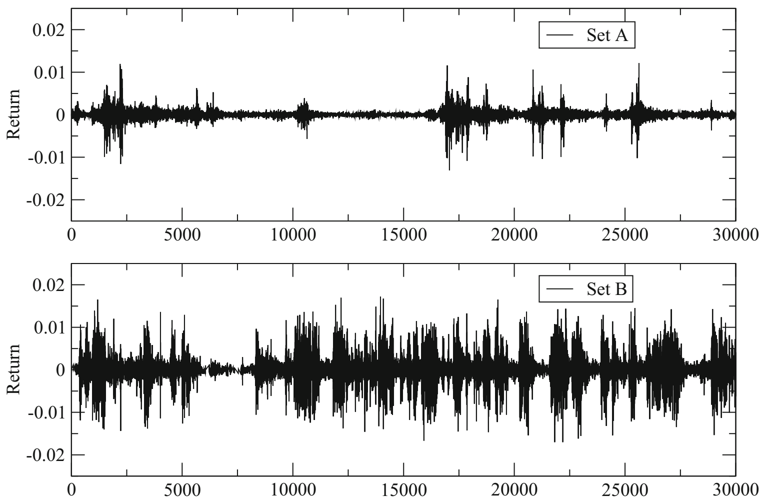


**Fig. 1** Representative return time series for Sets A and B

$$R(t) = \sigma_t \varepsilon_t, \tag{3}$$

$$\ln(\sigma_t^2) = \omega + \rho \ln(\sigma_{t-1}^2) + \theta z_{t-1} + \gamma(|z_{t-1}| - \sqrt{2/\pi}), \tag{4}$$

where $z_{t-1} = R_{t-1}/\sigma_{t-1}$, and $\omega, \rho, \theta$ and $\gamma$ are EGARCH parameters. We perform the parameter estimation of the EGARCH model by the Markov chain Monte Carlo method with the adaptive proposal density (Takaishi 2009a, b, c, 2010, 2013b; Takaishi and Chen 2012). Table 1 lists the results of the EGARCH parameters. Parameter $\rho$ measures the volatility persistency and we find that $\rho$ is very close to one for both Sets A and B, which means that volatility is very persistent, that is, the volatility is long-correlated.

Figure 2 shows the return distributions of Sets A and B. The return distributions are constructed from the normalized returns, that is, each return time series is normalized as $\bar{R}_k(t) = (R_k(t) - \mathrm{ave}_k)/\sigma_k$, where $\mathrm{ave}_k$ is the average value of $R_k(t)$ and $\sigma_k$ is the standard deviation of $R_k(t)$. We find that both the return distributions exhibit the fat-tailed property. The fat-tailed nature of return distributions in real financial markets has been investigated in literature, such as in Lux (1996) and Gopikrishnana et al. (1998, 1999), and the power law behavior for the return distributions is documented. Our results in Fig. 2 show an exponential behavior for the return distributions rather than the power law behavior. Appearance of the exponential behavior is not surprising, because the form of the return distributions may depend on the financial market we consider, and the exponential behavior for the return distributions has been found in the Indian stock market (Matia et al. 2004).

To quantify the volatility level of the system, we use $|R_k(t)|$ as a proxy of the volatility and define a volatility index by an average of $|R_k(t)|$:

$$I(t) = \frac{1}{N} \sum_{k=1}^{N} |R_k(t)|. \tag{5}$$

Figure 3 shows the volatility index $I(t)$ for Sets A and B. While the volatility index of Set A is stable over time, that of Set B varies considerably through time. The average volatility is high in several periods. We denote such periods by E1–E4 as in Fig. 3.

## 4 Properties of cross-correlation

Let $R_k(t)$ be a return for stock $k$ $(k = 1, \ldots, N)$ at time $t$ $(t = 1, \ldots, T)$, where $N = 100$ and $T = 30000$. Further, we define the normalized return $m_k(t)$ as

**Table 1** Results of EGARCH parameters

| | $\omega$ | $\rho$ | $\gamma$ | $\theta$ |
|---|---|---|---|---|
| Set A | 0.147 (5) | 0.9969 (6) | −0.044 (9) | 0.011 (3) |
| Set B | 0.266 (6) | 0.9938 (7) | −0.072 (8) | 0.090 (38) |

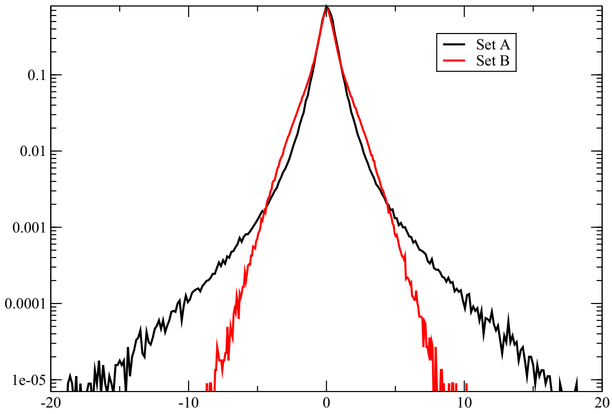


**Fig. 2** Return distributions of Sets A and B

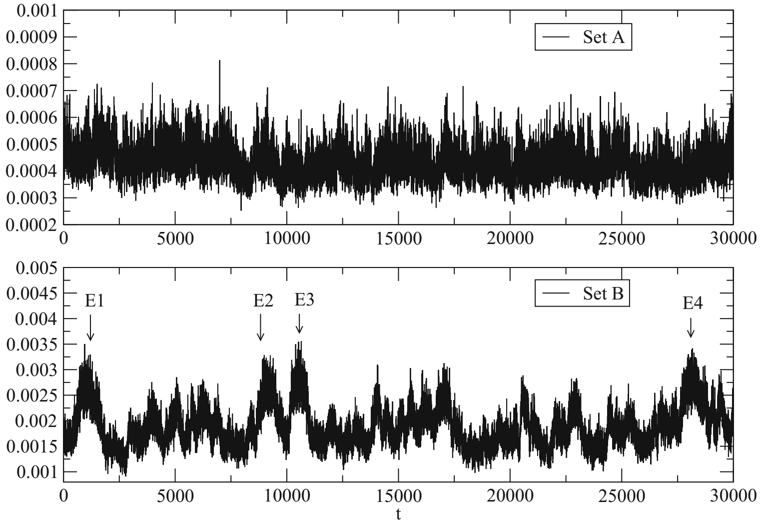


**Fig. 3** Volatility index for Sets A and B

$$m_k(t) = \frac{R_k(t) - \langle R_k \rangle}{\sigma_k}, \tag{6}$$

where $\langle \ldots \rangle$ indicates the time-series average and $\sigma_k$ is the standard deviation of $R_k(t)$. Using the normalized return $m_k(t)$, an equal-time cross-correlation matrix is calculated as $c_{kj} = \langle m_k m_j \rangle$. To study the dynamical properties of cross-correlation, the average $\langle \ldots \rangle$ is taken over a period of the rolling window and we consider a window size of 200. By definition, the elements of the cross-correlation matrix are restricted to $-1 \leq c_{kj} \leq 1$. Further, we do the same analysis for the absolute-return time series of $|R_k(t)|$.

Figure 4 shows the dynamical evolution of the average off-diagonal matrix element $\langle c \rangle$ given by

$$\langle c \rangle = \frac{2}{N(N-1)} \sum_{k>j}^{N} c_{kj}, \tag{7}$$

for returns. It seems that the average off-diagonal matrix elements of Set A and those of B fluctuate around zero. The time-series average of off-diagonal matrix element for Set A is $-3.9\ (70) \times 10^{-5}$ and consistent with zero within the error range. On the other hand, that for Set B is found to be $7.4\ (11) \times 10^{-4}$ and, thus, non-zero.

Figure 5 shows the same figure as in Fig. 4 but for absolute returns. Although the average off-diagonal matrix elements fluctuate around zero for Set A, those of Set B deviate from zero and have an upward tendency.

## 5 Cumulative risk fraction

To further investigate the dynamical properties of cross-correlation matrices, the principal component analysis (PCA) is applied. Billio et al. (2012) suggested to use the PCA to quantify the systemic risk and introduced the cumulative risk fraction (CRF) as a risk measure. The CRF has also been studied in Kritzman et al. (2011), Zheng et al. (2012) and Ren and Zhou (2014). To calculate the CRF, we first compute the eigenvalues of the cross-correlation matrix, denoted as $\lambda_1, \lambda_2, \ldots, \lambda_N$, where all eigenvalues are sorted as $\lambda_1 > \lambda_2 > \cdots > \lambda_N$. Then, we calculate the CRF defined by Billio et al. (2012):

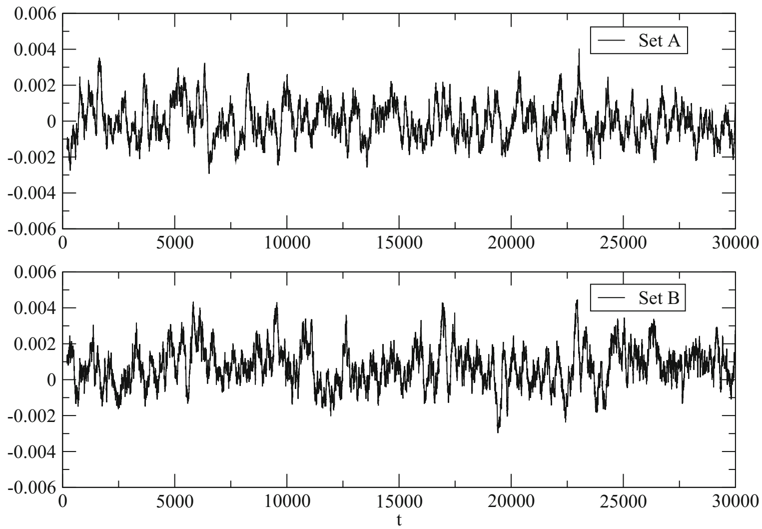


**Fig. 4** Average off-diagonal elements from returns

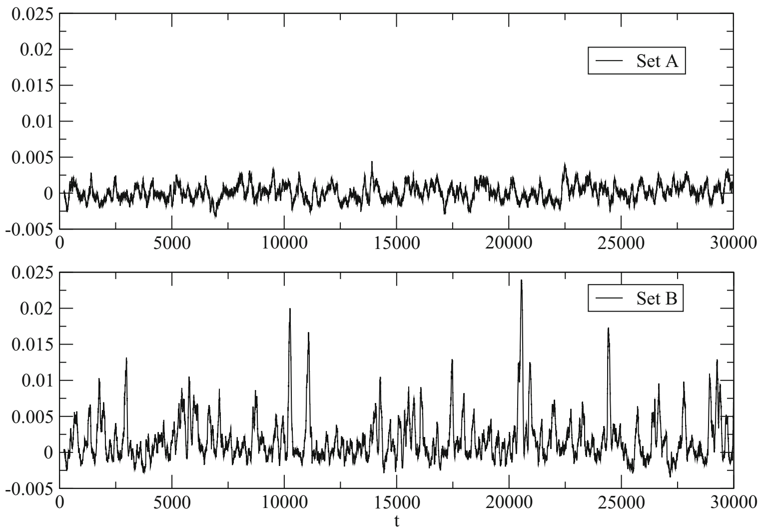


**Fig. 5** Average off-diagonal elements from absolute returns

$$\mathrm{CRF}_m = \frac{\omega_m}{\Omega}, \tag{8}$$

where $\Omega$ is the total variance of the system given by $\Omega = \sum_{j=1}^{N} \lambda_j$ and $\omega_m$ is the risk associated with the first $m$ principal components, given by $\omega_m = \sum_{j=1}^{m} \lambda_j$. The CRF quantifies the portion of the system variance explained by the first $m$ principal components (Kritzman et al. 2011). Usually, the first few principal components explain most of the system variance. In the periods of financial crisis, many stocks are highly interconnected and their prices easily move together. Therefore, the volatility of stocks also increases. In such financial crisis periods, the CRF is expected to increase considerably because the system variance also increases.

Figure 6a shows the CRF1 to CRF5 from the cross-correlation matrices of returns for Set B. We find that there are four bulges that correspond to E1–E4 as in Fig. 3. Figure 6b shows the CRF1 to CRF5 of absolute returns for Set B. We also find that considerable decreases exist in the CRF corresponding to E1–E4. These findings may suggest that for changes in the CRF, it is more important to locate the places where the potential risk is high and an empirical study on changes of the CRF has been already done in Zheng et al. (2012).

## 6 Comparison with random matrix theory

Let $y_k(t)$ be an independent, identically distributed random variable with $k = 1, \ldots, N$ at time $t = 1, \ldots, T$. The normalized variable is defined by

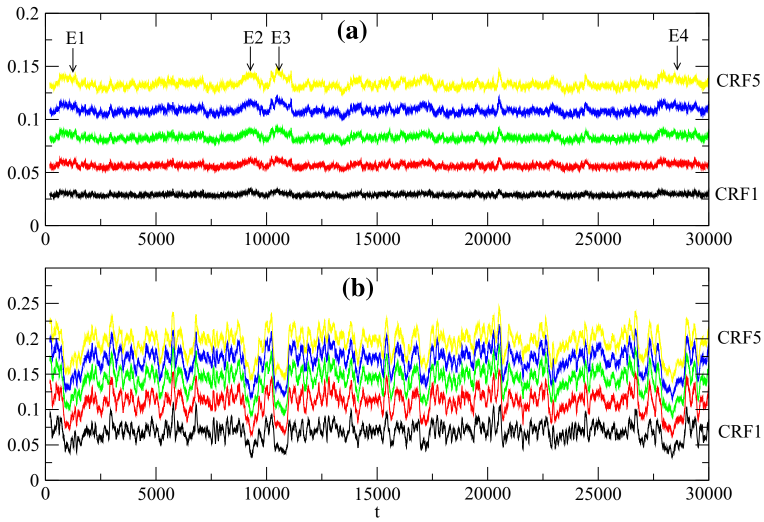


**Fig. 6** Dynamical evolution of cumulative risk fraction from returns (**a**) and absolute returns (**b**) for Set B

$$w_k(t) = \frac{y_k(t) - \langle y_k \rangle}{\sigma_{y_k}}, \tag{9}$$

where $\sigma_{y_k}$ is the standard deviation of $y_k$. The equal-time cross-correlation between variables $y_k(t)$ is given by $W_{kj} = \langle w_k w_j \rangle$. The matrix $W$ is called the Wishart matrix. For $N \to \infty$ and $T \to \infty$ with $Q = T/N > 1$, an eigenvalue distribution of the random matrix $W$ is theoretically given by Edelman (1988) and Sengupta and Mitra (1999):

$$\rho(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda}, \quad \lambda_\pm = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}. \tag{10}$$

Figure 7 shows the eigenvalue distribution of the random matrix $W$ with $Q = 200/100 = 2$ and that of the cross-correlation matrix for Set B. The eigenvalue distributions for Set B deviate from the random matrix theory (RMT), especially at large eigenvalues. The largest eigenvalue calculated from return (absolute return) data is 3.46 (10.71), which is beyond the largest eigenvalue, 2.91 from the RMT. It is known that in the real financial markets, the largest eigenvalue is far beyond that from the RMT (Plerou et al. 1999; Laloux et al. 1999). Although we find a similar deviation for Set A, the deviation from the RMT for Set B is more sizeable than that for Set A.

Next, we dynamically calculate the IPR that characterizes the eigenvectors, defined by

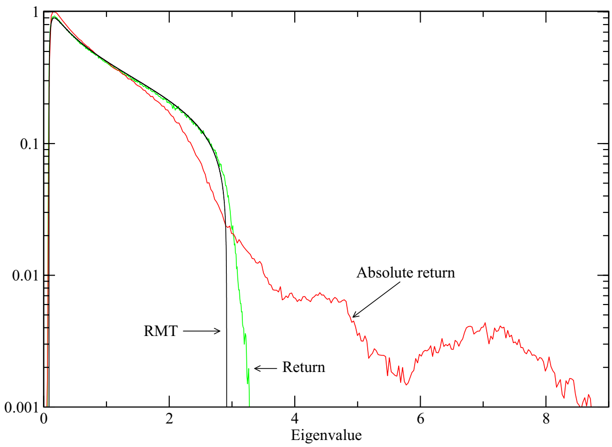


**Fig. 7** Eigenvalue distributions of cross-correlation matrix for Set B and the result from the RMT

$$\mathrm{IPR}(l) = \sum_{j=1,}^{N} (v_l^j)^4, \tag{11}$$

where $v_l^j$ is the $j$-th component of the eigenvector for the $l$-th eigenvalue. In the RMT, the eigenvector components are de-localized and distributed as a Gaussian distribution $\sim \sqrt{\frac{N}{2\pi}} \exp\left(-\frac{N}{2}(v_l^j)^2\right)$. In such a case, the expectation of the IPR is $3/N$. On the other hand, when the eigenvector components are localized, for example, only one component has a non-zero value, the expectation of the IPR would be 1. Further, one can also extend the IPR to the higher power version such as IPR6 defined by

$$\mathrm{IPR6}(l) = \sum_{j=1,}^{N} (v_l^j)^6. \tag{12}$$

The expectation of the IPR6 in the RMT is calculated to be $15/N^2$.

Figure 8 shows the IPR(1) and IPR6(1) from the return cross-correlation for Set B, that is, those for the largest eigenvalue. Although both the IPR(1) and IPR6(1) deviate from the RMT indicated by the lines of $3/N$ and $15/N^2$, no structural change corresponding to E1–E4 is seen. Thus, in this model, the IPR and IPR6 from the return cross-correlation are found to be insensitive to the volatility level.

Figure 9 shows the IPR(1) and IPR6(1) from the absolute-return cross-correlation for Set B. We find that the IPR(1) and IPR6(1) deviate largely from the RMT and that large increases exist at locations corresponding to E1–E4. Although we also calculated the IPR($l$) and IPR6($l$) for $l > 1$, that is, for lower eigenvalues, no structural change corresponding to E1–E4 was seen. Therefore, it seems that only the IPR and IPR6 for the largest eigenvalue are sensitive to the volatility (risk) level of the system.

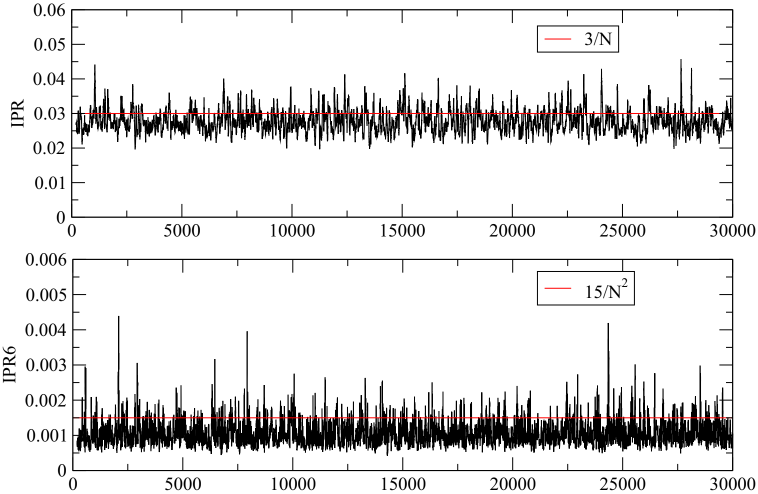


**Fig. 8** IPR and IPR6 from return eigenvalues for Set B

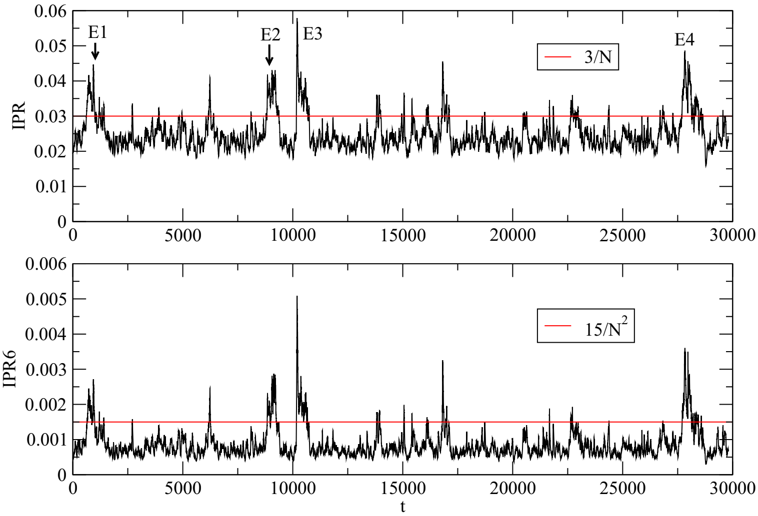


**Fig. 9** IPR and IPR6 from absolute-return eigenvalues for Set B

## 7 Conclusions

Using the multiple time series Ising model, we performed a large-scale simulation with 100 stocks on a 120 $\times$ 120 lattice and investigated the dynamical properties of cross-correlation between stock returns. The simulation with a non-zero interaction matrix shows that there exist several periods where the volatility is especially high.

Further, we investigated the cumulative risk fraction (CRF) that is expected to measure the system risk and found that the CRF changes considerably at the high-volatility phases. The inverse partition ratio (IPR) and its higher power version, IPR6, were also investigated, and we found that the IPR and IPR6 from absolute-returns change at the same periods where the volatility is high. Our findings suggest that the CRF, the IPR, and IPR6 are useful measurements to identify abnormal states such as high-volatility periods.

Our model successfully reproduced major stylized facts such as the volatility clustering and fat-tailed return distribution. However, there exist some differences between our model and the real financial markets. For instance, the largest eigenvalue for the return cross-correlation matrix is only slightly beyond that for the random matrix theory and not far beyond that in the real financial markets. Moreover, the IPR and IPR6 from returns seem to have no sensitivity on the market status, as shown in Fig. 8. Therefore, further studies are needed to clarify the difference between both and to obtain some insights to improve our model.

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