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A Dunkl Analogue of Operators Including Two-Variable Hermite polynomials

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Abstract The aim of this paper is to introduce a Dunkl generalization of the operators including two-variable Hermite polynomials which are defined by Krech and to investigate approximating properties for these operators by means of the classical modulus of continuity, second modulus of continuity and Peetre's K-functional.

Keywords Dunkl analogue · Hermite polynomial · Modulus of continuity · Korovkin's type approximation theorem

Mathematics Subject Classification Primary 41A25 · 41A36 · Secondary 33C45

1 Introduction

Up to now, linear positive operators and their approximation properties have been studied by many research workers, see for example [3–6,8,9,14,15,22,25,27] and references therein. Also, linear positive operators defined via generating functions

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and their further extensions are intensively studied by a large number of authors. For various extensions and further properties, we refer for example Altin et al. [1], Dogru et al. [7], Olgun et al. [18], Sucu et al. [24], Tasdelen et al. [26], Varma et al. [28,29].

Recently, linear positive operators generated by a Dunkl generalization of the exponential function have been stated by many authors. In [23], Dunkl analogue of Szász operators by using Dunkl analogue of exponential function was given as follows

$$S_n^*(g;x)=\frac{1}{e_\nu(nx)}\sum_{k=0}^{\infty}\frac{(nx)^k}{\gamma_\nu(k)}g\left(\frac{k+2\nu\theta_k}{n}\right)\ ;\ n\in\mathbb{N},\ \nu,\ x\in[0,\infty), \tag{1.1}$$

for $g\in C[0,\infty)$, where Dunkl analogue of exponential function is defined by

$$e_\nu(x)=\sum_{k=0}^{\infty}\frac{x^k}{\gamma_\nu(k)} \tag{1.2}$$

for $k\in\mathbb{N}_0$ and $\nu>-\frac{1}{2}$ and the coefficients γ_ν are as follows

$$\gamma_\nu(2k)=\frac{2^{2k}k!\Gamma(k+\nu+1/2)}{\Gamma(\nu+1/2)}\text{ and }\gamma_\nu(2k+1)=\frac{2^{2k+1}k!\Gamma(k+\nu+3/2)}{\Gamma(\nu+1/2)} \tag{1.3}$$

in [20]. Also, the coefficients γ_ν verify the recursion relation

$$\frac{\gamma_\nu(k+1)}{\gamma_\nu(k)}=(2\nu\theta_{k+1}+k+1)\,,\ k\in\mathbb{N}_0, \tag{1.4}$$

where

$$\theta_k=\begin{cases}0, & if\ k=2p\\ 1, & if\ k=2p+1\end{cases} \tag{1.5}$$

for $p\in\mathbb{N}_0$. Similarly, Stancu-type generalization of Dunkl analogue of Szá sz-Kantorovich operators and Dunkl generalization of Szász operators via q-calculus have been defined in [10,11] and for other research see [16,17].

The two-variable Hermite Kampe de Feriet polynomials $H_n(\xi,\alpha)$ are defined by (see [2])

$$\sum_{n=0}^{\infty}\frac{H_n(\xi,\alpha)}{n!}t^n=e^{\xi t+\alpha t^2}$$

from which, it follows

$$H_n(\xi,\alpha)=n!\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{\alpha^k\xi^{n-2k}}{k!(n-2k)!}.$$

In a recent paper, Krech [13] has introduced the class of operators G_n^α given by

$$G_n^\alpha(f;x) = e^{-\left(nx+\alpha x^2\right)} \sum_{k=0}^{\infty} \frac{x^k}{k!} H_k(n,\alpha) f\left(\frac{k}{n}\right) \quad , \quad x \in [0,\infty) \,, \qquad f \in C[0,\infty),\ n \in \mathbb{N}\ , \alpha \geq 0 \tag{1.6}$$

in terms of two-variable Hermite polynomials and investigated approximation properties of G_n^α .

In the present paper, we first give the Dunkl generalization of two-variable Hermite polynomials and then we define a class of operators by using the Dunkl generalization of two-variable Hermite polynomials. We give the rates of convergence of the operators T_n to f by means of the classical modulus of continuity, second modulus of continuity and Peetre's K -functional and in terms of the elements of the Lipschitz class $Lip_M(\alpha)$.

2 The Dunkl Generalization of Two-Variable Hermite Polynomials

The Dunkl generalization of two-variable Hermite polynomials is defined by

$$\sum_{n=0}^{\infty} \frac{H_n^\mu(\xi,\alpha)}{n!} t^n = e^{\alpha t^2} e_\mu(\xi t) \tag{2.1}$$

from which, we conclude

$$H_n^\mu(\xi,\alpha) = n! \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{\alpha^k \xi^{n-2k}}{k! \gamma_\mu(n-2k)},$$

which gives the two-variable Hermite polynomials as $\mu = 0$. For our purpose, we denote

$$h_n^\mu(\xi,\alpha) = \frac{\gamma_\mu(n) H_n^\mu(\xi,\alpha)}{n!}$$

and we can write that the polynomials $h_n^\mu(\xi,\alpha)$ are generated by

$$\sum_{n=0}^{\infty} \frac{h_n^\mu(\xi,\alpha)}{\gamma_\mu(n)} t^n = e^{\alpha t^2} e_\mu(\xi t), \tag{2.2}$$

where

$$h_n^\mu(\xi,\alpha) = \gamma_\mu(n) \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{\alpha^k \xi^{n-2k}}{k! \gamma_\mu(n-2k)}.$$

In order to obtain some properties of $h_n^{\mu}(\xi,\alpha)$, we remind the following definition and lemma given in [20].

Definition 1 [20] Let $\mu \in \mathbb{C}_0$ $(\mathbb{C}_0 := \mathbb{C}\backslash\left\{-\frac{1}{2},-\frac{3}{2},...\right\}$, $x \in \mathbb{C}$ and let φ be entire function. The linear operator $\mathbb{D}_{\mu}$ is defined on all entire functions φ on $\mathbb{C}$ by

$$\mathbb{D}_{\mu}(\varphi(x)) = \varphi^{'}(x) + \frac{\mu}{x}(\varphi(x)-\varphi(-x)),\ x \in \mathbb{C}. \tag{2.3}$$

We use the notation $\mathbb{D}_{\mu,x}$ since $\mathbb{D}_{\mu}$ is acting on functions of the variable x. Thus, $\mathbb{D}_{\mu,x}(\varphi(x)) = \left(\mathbb{D}_{\mu}\varphi\right)(x)$.

Lemma 1 [20] *Let φ, ψ be entire functions. For the linear operator $\mathbb{D}_{\mu}$, the following statements hold*

(i) $\mathbb{D}_{\mu}^{j} : x^n \to \frac{\gamma_{\mu}(n)}{\gamma_{\mu}(n-j)}x^{n-j}$, $j=0,1,2,...,n$ $(n \in \mathbb{N})$; $\mathbb{D}_{\mu}^{j} : 1 \to 0$,

(ii) $\mathbb{D}_{\mu}(\varphi\psi) = \mathbb{D}_{\mu}(\varphi)\psi + \varphi\mathbb{D}_{\mu}(\psi)$, *where φ is an even function,*

(iii) $\mathbb{D}_{\mu} : e_{\mu}(\lambda x) \to \lambda e_{\mu}(\lambda x)$.

By using these definition and lemma, we can state the next result.

Lemma 2 *For the Dunkl generalization of two-variable Hermite polynomials $h_n^{\mu}(\xi,\alpha)$, the following results hold true*

$$(i)\ \sum_{n=0}^{\infty}\frac{h_{n+1}^{\mu}(\xi,\alpha)}{\gamma_{\mu}(n)}t^n = (\xi+2\alpha t)e^{\alpha t^2}e_{\mu}(\xi t),$$
$$(ii)\ \sum_{n=0}^{\infty}\frac{h_{n+2}^{\mu}(\xi,\alpha)}{\gamma_{\mu}(n)}t^n = (\xi^2+4\xi\alpha t+4\alpha^2t^2+2\alpha)e^{\alpha t^2}e_{\mu}(\xi t)+4\alpha\mu e^{\alpha t^2}e_{\mu}(-\xi t).$$

Proof Applying the linear operator $\mathbb{D}_{\mu}$ in view of Lemma 1 , we have

$$\begin{aligned}\mathbb{D}_{\mu}(te_{\mu}(\xi t)) &= (t\xi+1)e_{\mu}(\xi t)+2\mu e_{\mu}(-\xi t),\\ \mathbb{D}_{\mu}(e^{\alpha t^2}) &= 2\alpha te^{\alpha t^2}.\end{aligned} \tag{2.4}$$

Also applying the linear operator $\mathbb{D}_{\mu}$ to both side of generating function (2.2), we have

$$\sum_{n=0}^{\infty}\frac{h_n^{\mu}(\xi,\alpha)}{\gamma_{\mu}(n)}\mathbb{D}_{\mu}(t^n) = \mathbb{D}_{\mu}(e^{\alpha t^2}e_{\mu}(\xi t)).$$

By using (2.4) and Lemma 1 (i), we get the first relation. Similarly, if we apply the linear operator $\mathbb{D}_{\mu}$ to the relation in (i), we get

$$\sum_{n=0}^{\infty}\frac{h_{n+1}^{\mu}(\xi,\alpha)}{\gamma_{\mu}(n)}\mathbb{D}_{\mu}(t^n) = \mathbb{D}_{\mu}\left[(\xi+2\alpha t)e^{\alpha t^2}e_{\mu}(\xi t)\right].$$

From (2.4) and Lemma 1, it follows

$$\sum_{n=0}^{\infty} \frac{h_{n+2}^{\mu}(\xi, \alpha)}{\gamma_{\mu}(n)} t^n = (\xi^2 + 4\xi\alpha t + 4\alpha^2 t^2 + 2\alpha) e^{\alpha t^2} e_{\mu}(\xi t) + 4\alpha\mu e^{\alpha t^2} e_{\mu}(-\xi t).$$

□

Definition 2 With the help of the Dunkl generalization of two-variable Hermite polynomials given in (2.2), we introduce the operators $T_n(f; x)$, $n \in \mathbb{N}$ given by

$$T_n(f; x) := \frac{1}{e^{\alpha x^2} e_{\mu}(nx)} \sum_{k=0}^{\infty} \frac{h_k^{\mu}(n, \alpha)}{\gamma_{\mu}(k)} x^k f\left(\frac{k + 2\mu\theta_k}{n}\right), \tag{2.5}$$

where $\alpha \geq 0, \mu \geq 0, \ f \in C[0, \infty)$ and $x \in [0, \infty)$. Operators (2.5) are linear and positive. In the case of $\mu = 0$, it gives G_n^{α} given by (1.6).

Lemma 3 *For the operators $T_n(f; x)$, we can obtain the following equations:*

(i) $T_n(1; x) = 1$,
(ii) $T_n(t; x) = x + \frac{2\alpha x^2}{n}$,
(iii) $T_n(t^2; x) = x^2 + \frac{4\alpha}{n^2} x^2 + \frac{4\alpha}{n} x^3 + \frac{4\alpha^2}{n^2} x^4 + \frac{x}{n} + \frac{2\mu x}{n} \frac{e_{\mu}(-nx)}{e_{\mu}(nx)}$.

Proof By using the generating function in (2.2), the relation (i) holds. For the proof of (ii), in view of the recursion relation in (1.4), we get

$$T_n(t; x) = \frac{1}{n e^{\alpha x^2} e_{\mu}(nx)} \sum_{k=1}^{\infty} \frac{h_k^{\mu}(n, \alpha)}{\gamma_{\mu}(k-1)} x^k.$$

When we replace k by $k + 1$, we obtain (ii) by use of Lemma 2 (i). For the proof of (iii), by using (1.4), we have

$$T_n(t^2; x) = \frac{x}{n^2 e^{\alpha x^2} e_{\mu}(nx)} \sum_{k=0}^{\infty} (k + 1 + 2\mu\theta_{k+1}) \frac{h_{k+1}^{\mu}(n, \alpha)}{\gamma_{\mu}(k)} x^k.$$

From the equation

$$\theta_{k+1} = \theta_k + (-1)^k, \tag{2.6}$$

it yields

$$T_n(t^2;x)=\frac{x}{n^2e^{\alpha x^2}e_\mu(nx)}\sum_{k=0}^{\infty}(k+2\mu\theta_k)\frac{h_{k+1}^{\mu}(n,\alpha)}{\gamma_\mu(k)}x^k$$
$$+\frac{x}{n^2e^{\alpha x^2}e_\mu(nx)}\sum_{k=0}^{\infty}(1+2\mu(-1)^k)\frac{h_{k+1}^{\mu}(n,\alpha)}{\gamma_\mu(k)}x^k.$$

Using the recursion relation in (1.4) in the first series, it follows

$$T_n(t^2;x)=\frac{x^2}{n^2e^{\alpha x^2}e_\mu(nx)}\sum_{k=0}^{\infty}\frac{h_{k+2}^{\mu}(n,\alpha)}{\gamma_\mu(k)}x^k+\frac{x}{n^2e^{\alpha x^2}e_\mu(nx)}\sum_{k=0}^{\infty}\frac{h_{k+1}^{\mu}(n,\alpha)}{\gamma_\mu(k)}x^k$$
$$+\frac{2\mu x}{n^2e^{\alpha x^2}e_\mu(nx)}\sum_{k=0}^{\infty}(-x)^k\frac{h_{k+1}^{\mu}(n,\alpha)}{\gamma_\mu(k)}.$$

From Lemma 2 (i) and (ii), we complete the proof of (iii). □

Lemma 4 *As a consequence of Lemma 3, we can give the next results for T_n operators*

$$\Delta_1=T_n(t-x;x)=\frac{2\alpha x^2}{n},$$
$$\Delta_2=T_n((t-x)^2;x)=\frac{1}{n^2}x\left(4x^3\alpha^2+4\alpha x+n\right)+\frac{2\mu x}{n}\frac{e_\mu(-nx)}{e_\mu(nx)}. \quad (2.7)$$

Theorem 1 *For T_n operators and any uniformly continuous bounded function g on the interval $[0,\infty)$, we can give*

$$T_n(g;x)\overset{uniformly}{\rightrightarrows}g(x)$$

on each compact set $A\subset[0,\infty)$ when $n\to\infty$.

Proof From Korovkin Theorem in [12], when $n\to\infty$, we have $T_n(g;x)\overset{\text{uniformly}}{\rightrightarrows}g(x)$ on $A\subset[0,\infty)$ which is each compact set because $\lim_{n\to\infty}T_n(e_i;x)=x^i$, for $i=0,1,2$, which is uniformly on $A\subset[0,\infty)$ with the help of using Lemma 4. □

Theorem 2 *The operator T_n maps $C_B[0,\infty)$ into $C_B[0,\infty)$ and $\|T_n(f)\|\le\|f\|$ for each $f\in C_B[0,\infty)$ where C_B is the space of uniformly continuous and bounded functions on $[0,\infty)$.*

3 Convergence of Operators in (2.5)

In what follows, we give some rates of convergence of the operators T_n. Firstly, we recall some definitions as follows. Let $Lip_M(\alpha)$ Lipschitz class of order α. If $g\in Lip_M(\alpha)$, the inequality

$$|g(s)-g(t)| \leq M|s-t|^{\alpha}$$

holds where $s, t \in [0,\infty)$, $0<\alpha\leq 1$ and $M>0$. $\widetilde{C}[0,\infty)$ is the space of uniformly continuous on $[0,\infty)$. The modulus of continuity $g \in \widetilde{C}[0,\infty)$ is denoted by

$$\omega(g;\delta) := \sup_{\substack{s,t\in[0,\infty) \\ |s-t|\leq\delta}} |g(s)-g(t)|. \tag{3.1}$$

We first estimate the rates of convergence of the operators T_n by using modulus of continuity and in terms of the elements of the Lipschitz class $Lip_M(\alpha)$.

Theorem 3 *If $h \in Lip_M(\alpha)$, we have*

$$|T_n(h;x)-h(x)| \leq M(\Delta_2)^{\alpha/2},$$

where Δ_2 is given in Lemma 4.

Proof Since $h \in Lip_M(\alpha)$, it follows from linearity

$$|T_n(h;x)-h(x)| \leq T_n(|h(t)-h(x)|;x) \leq MT_n\left(|t-x|^{\alpha};x\right).$$

From Lemma 4 and Hölder's famous inequality, we can write

$$|T_n(h;x)-h(x)| \leq M[\Delta_2]^{\frac{\alpha}{2}}.$$

Thus, we find the required inequality. □

Theorem 4 *The operators in (2.5) verify the inequality*

$$|T_n(g;x)-g(x)| \leq \left(1+\sqrt{\frac{1}{n}x\left(4x^3\alpha^2+4x\alpha+n\right)+2\mu x\frac{e_\mu(-nx)}{e_\mu(nx)}}\right)\omega\left(g;\frac{1}{\sqrt{n}}\right),$$

where $g \in \widetilde{C}[0,\infty)$.

Proof The proof is clear from the result of Shisha and Mond in [21]. □

Let $C_B[0,\infty)$ denote the space of uniformly continuous and bounded functions on $[0,\infty)$. Also

$$C_B^2[0,\infty) = \{g \in C_B[0,\infty) : g', g'' \in C_B[0,\infty)\} \tag{3.2}$$

with the norm

$$\|g\|_{C_B^2[0,\infty)} = \|g\|_{C_B[0,\infty)} + \left\|g'\right\|_{C_B[0,\infty)} + \left\|g''\right\|_{C_B[0,\infty)}$$

for all g in $C_B^2[0,\infty)$.

Lemma 5 *For $h \in C_B^2[0,\infty)$, the following inequality holds true*

$$|T_n(h;x) - h(x)| \leq [\Delta_1 + \Delta_2] \|h\|_{C_B^2[0,\infty)}, \tag{3.3}$$

where Δ_1 and Δ_2 are given by in Lemma 4.

Proof From the Taylor's series of the function h,

$$h(s) = h(x) + (s-x) h'(x) + \frac{(s-x)^2}{2!} h''(\varrho), \ \varrho \in (x,s).$$

Applying the operator T_n to both sides of this equality and then using the linearity of the operator, we have

$$T_n(h;x) - h(x) = h'(x)\Delta_1 + \frac{h''(\varrho)}{2}\Delta_2.$$

From Lemma 4, it yields

$$\begin{aligned} |T_n(h;x) - h(x)| &\leq \frac{2\alpha x^2}{n} \left\|h'\right\|_{C_B[0,\infty)} \\ &\quad + \left[\frac{1}{n^2} x\left(4x^3\alpha^2 + 4\alpha x + n\right) + \frac{2\mu x}{n}\frac{e_\mu(-nx)}{e_\mu(nx)}\right] \left\|h''\right\|_{C_B[0,\infty)} \\ &\leq [\Delta_1 + \Delta_2] \|h\|_{C_B^2[0,\infty)}, \end{aligned}$$

which finishes the proof. □

Now we recall that the second order of modulus continuity of f on $C_B[0,\infty)$ is given as

$$\omega_2(f;\delta) := \sup_{0<s\leq\delta} \|f(.+2s) - 2f(.+s) + f(.)\|_{C_B[0,\infty)}.$$

Peetre's K-functional of the function $f \in C_B[0,\infty)$ is as follows

$$K(f;\delta) := \inf_{g \in C_B^2[0,\infty)} \left\{\|f-g\|_{C_B} + \delta \|g\|_{C_B^2}\right\}. \tag{3.4}$$

The relation between K and ω_2 is as

$$K(f;\delta) \leq M\left\{w_2\left(f;\sqrt{\delta}\right) + \min(1,\delta)\|f\|_{C_B}\right\} \tag{3.5}$$

for all $\delta > 0$. Here M is a positive constant. Now, we can give the important theorem.

Theorem 5 *For the operators defined by (2.5), the following inequality holds*

$$|T_n(g;x) - g(x)| \leq 2M\left\{\min\left(1, \frac{\chi_n(x)}{2}\right)\|g\|_{C_B[0,\infty)} + \omega_2\left(g; \sqrt{\frac{\chi_n(x)}{2}}\right)\right\} \tag{3.6}$$

where for all g in $C_B[0,\infty)$, $x \in [0,\infty)$, M is a positive constant which is independent of n and $\chi_n(x) = \Delta_1 + \Delta_2$.

Proof For any $f \in C_B^2[0,\infty)$, from the triangle inequality, we can write

$$\Theta = |T_n(g;x) - g(x)| \leq |T_n(g-f;x)| + |T_n(f;x) - f(x)| + |g(x) - f(x)|$$

from Lemma 5, which follows

$$\begin{aligned}\Theta &\leq 2\|g-f\|_{C_B[0,\infty)} + \chi_n(x)\|f\|_{C_B^2[0,\infty)} \\ &= 2\left\{\|g-f\|_{C_B[0,\infty)} + \frac{\chi_n}{2}(x)\|f\|_{C_B^2[0,\infty)}\right\}.\end{aligned}$$

From (3.4), we have

$$\Theta \leq 2K\left(g; \frac{\chi_n(x)}{2}\right),$$

which holds

$$\Theta \leq 2M\left\{\min\left(1, \frac{\chi_n(x)}{2}\right)\|g\|_{C_B[0,\infty)} + \omega_2\left(g; \sqrt{\frac{\chi_n(x)}{2}}\right)\right\}$$

from (3.5). □

Similar to the proof of above theorem, simple computations give the next theorem.

Theorem 6 *If $g \in C_B[0,\infty)$ and $x \in [0,\infty)$, we get*

$$\begin{aligned}|T_n(g;x) - g(x)| \leq M\omega_2&\left(g; \frac{1}{2}\sqrt{\frac{1}{n^2}x\left(8x^3\alpha^2 + 4x\alpha + n\right) + \frac{2\mu x}{n}\frac{e_\mu(-nx)}{e_\mu(nx)}}\right) \\ &+\omega\left(g; \frac{2\alpha x^2}{n}\right)\end{aligned}$$

where M is a positive constant.

Remark 1 Similar results to Theorem 6 can be obtained using the theorem by Paltanea (see [19]).

Remark 2 The case of $\mu = 0$ in Theorem 6 gives the result given in [13].

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