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New Exact Solutions of the Davey–Stewartson Equation with Power-Law Nonlinearity

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Abstract This work obtains the soliton solutions of the generalized Davey–Stewartson equation with the complex coefficients. First, the extended Weierstrass transformation method is used to carry out the solutions of this equation, and some new solutions, known as Weierstrass elliptic function solutions, are obtained by this method. Then, the trial equation method is used to obtain the soliton solutions of this equation.

Keywords Weierstrass transformation method · Trial equation method · Soliton solutions

1 Introduction

The investigation of exact solutions of nonlinear evolution equations (NLEEs) plays an important role in the analysis of some physical phenomena. The types of solutions of NLEEs, that are integrated using various mathematical techniques, are very important and appear in various areas of physics, applied mathematics and engineering. In this paper, the Davey–Stewartson equation (DSE) that arises in the study of fluid dynamics

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will be studied. In fact, this equation particularly studies the long-wave–short-wave resonances and other patterns of propagating waves [1,2]. Also, this equation describes the evolution of a 3-dimensional wave packet on water of finite depth. Some solutions for this equation can be found in [3–6].

There are a lot of analytical methods of solving these NLEEs that have also been developed in the past few decades. Some of these methods are $\left(\frac{G'}{G}\right)$-expansion method [7–9], exp-function method [10,11], the tanh method [12], homogeneous balance method [13] and many more. In this paper the extended Weierstrass transformation method [14–17] will be applied to obtain Weierstrass function solutions to the DSE. Also, the trial equation method [17,18,18–26] will be applied to obtain the soliton solutions to this equation. Finally, we can say that the obtained solutions satisfy the equation.

2 Extended Weierstrass Transformation Method

In this section, the extended Weierstrass transformation method will be first described and then subsequently applied to solve the DSE.

2.1 Description of the Method

In this section, a brief description of the extended Weierstrass transformation method is represented. Consider the NLEEs, say in variables x_i , $i = 1, 2, 3$ and t as follows:

$$\phi(u, u_t, u_{x_i}, u_{x_i x_i}, u_{tt}, u_{x_i t}, ...) = 0, \tag{2.1}$$

where ϕ is, in general, a polynomial in $u(x_i, t)$ and its various partial derivatives. Seeking for travelling wave solution of (2.1), taking $u(x_i, t) = U(\xi)$ and $\xi = \sum_i k_i x_i + ct$ leads to an ordinary differential equation as

$$\phi(U, cU', k_i U', c^2 U'', ck_i U'', k_i^2 U'', ...) = 0. \tag{2.2}$$

In the next step, we suppose that the solution of (2.1) can be expressed in the general form

$$U(\xi) = \sum_{j=-N}^{N} a_j \, w^j(\xi), \tag{2.3}$$

where a_j are constants to be determined later, N is fixed by balancing the linear term of the highest order derivative with the highest nonlinear term in (2.2), while $w(\xi)$ satisfying the general elliptic equation [27]

$$w'(\xi) = \frac{d}{d\xi} w(\xi) = \sqrt{b_0 + b_1 w(\xi) + b_2 w^2(\xi) + b_3 w^3(\xi) + b_4 w^4(\xi)}. \tag{2.4}$$

Substituting (2.3) into (2.2) along with (2.4), equating the coefficients of all powers of $w^j(\xi)$ $(j = 0, 1, ...)$ to zero get a system of algebraic equations. Solving the system of nonlinear algebraic equations by Mathematica, we have explicit expressions for a_j, k_j and c. The success of algebraic methods depends on the solubility of the nonlinear algebraic system since trivial solutions only lead us to useless solutions. Further, the crucial step is to solve (2.1) in general, which is indeed, a difficult task. Whereby the solutions of (2.4) belong to solution classes of (2.1), some special cases of (2.1) depending on b_i-values are given in [14,16,17] and represented here as follows:

For $b_1 = b_3 = 0$:

The solutions of (2.4) in this case are

$$w_1(\xi) = \sqrt{\frac{3\psi(\xi; g_2, g_3) - b_2}{3b_4}}, \tag{2.5}$$

and

$$w_2(\xi) = \sqrt{\frac{3b_0}{\psi(\xi; g_2, g_3) - b_2}}, \tag{2.6}$$

where the invariants of the Weierstrass function $\psi(\xi; g_2, g_3)$ are expressed by

$$g_2 = \frac{4}{3}\left(b_2^2 - 3b_0b_4\right), \qquad g_3 = \frac{4b_2^2}{27}\left(9b_4 - 2b_2\right). \tag{2.7}$$

Another type of solutions admits

$$w_3(\xi) = \sqrt{\frac{2b_0[6\psi(\xi; g_2, g_3) + 2b_2 + D_\pm]}{12\psi(\xi; g_2, g_3) + D_\pm}}, \tag{2.8}$$

where the quantity D is given by

$$D\pm = \frac{-5b_2 \pm \sqrt{9b_2^2 - 36b_0b_4}}{2}, \tag{2.9}$$

and the Weierstrass function invariants are

$$g_2 = -\frac{b_2}{12}(5D_\pm + 4b_2 + 33b_0b_4), \tag{2.10}$$

and

$$g_3 = \frac{1}{216}[b_2^2(21D_\pm + 20b_2) - 3b_0b_4(21D_\pm + 9b_2)]. \tag{2.11}$$

Also, there are two different solutions, which are found to be

$$w_4(\xi) = \frac{\sqrt{b_0}}{3}\frac{6\psi(\xi; g_2, g_3) + b_2}{\psi'(\xi; g_2, g_3)} \tag{2.12}$$

and

$$w_5(\xi) = \frac{3}{\sqrt{b_4}}\frac{\psi'(\xi; g_2, g_3)}{6\psi(\xi; g_2, g_3) + b_2}, \tag{2.13}$$

where $\psi'(\xi; g_2, g_3) = \frac{d\psi(\xi; g_2, g_3)}{d\xi}$ and the invariants of the Weierstrass function are

$$g_2 = \frac{b_2^2 - b_0 b_4}{12}, \qquad g_3 = \frac{36 b_0 b_4 - b_2^2}{216}. \tag{2.14}$$

2.2 Application to the DSE in (1 + 2) Dimensions

The dimensionless form of the DSE in (1+2) dimensions, with power-law nonlinearity, that is going to be studied in this paper is given by [1,28]

$$iq_t + a(q_{xx} + q_{yy}) + b|q|^{2n}q = \alpha q r, \tag{2.15}$$

$$r_{xx} + r_{yy} + \beta(|q|^{2n})_{xx} = 0. \tag{2.16}$$

Here, in (2.15) and (2.16), q and r are the dependent variables, while x, y and t are the independent variables. The first two of the independent variables are the spatial variables, while t represents time. The exponent n is the power-law parameter. It is necessary to have $n > 0$. In (2.15) and (2.16), q is a complex-valued function, while r is a real-valued function. Also, a, b, α and β are all constant coefficients. For solving the Eqs. (2.15) and (2.16) with the Weierstrass transformation method, using the wave variables

$$q = e^{i\theta}u(\xi), \quad r = v(\xi), \tag{2.17}$$

$$\theta = \theta_1 x + \theta_2 y + \theta_3 t, \quad \xi = \xi_1 x + \xi_2 y + \xi_3 t, \tag{2.18}$$

where $\theta_1, \theta_2, \theta_3, \xi_1, \xi_2$ and ξ_3 are real constants, converts (2.15) and (2.16) to the system of ODEs

$$(\xi_3 + 2a\theta_1\xi_1 + 2a\theta_2\xi_2)u(\xi) = 0, \tag{2.19}$$

$$-(\theta_3 + a{\theta_1}^2 + a{\theta_2}^2)u(\xi) + a({\xi_1}^2 + {\xi_2}^2)u''(\xi) + bu^{2n+1}(\xi) - \alpha u(\xi)v(\xi) = 0, \tag{2.20}$$

$$({\xi_1}^2 + {\xi_2}^2)v''(\xi) + \beta{\xi_1}^2(u^{2n})''(\xi) = 0 \tag{2.21}$$

where primes denote the derivatives with respect to ξ . Eq. (2.21) is then integrated term by term two times where integration constants are considered zero. This converts it into

$$v(\xi) = \frac{-\beta{\xi_1}^2}{{\xi_1}^2+{\xi_2}^2}u^{2n}(\xi). \tag{2.22}$$

Substituting (2.22) into (2.20) gives

$$\begin{aligned}&-(\theta_3+a{\theta_1}^2+a{\theta_2}^2)u(\xi)+a({\xi_1}^2+{\xi_2}^2)u''(\xi)\\&+\left(b+\alpha\beta\frac{{\xi_1}^2}{{\xi_1}^2+{\xi_2}^2}\right)u^{2n+1}(\xi)=0.\end{aligned} \tag{2.23}$$

Balancing u'' with u^{2n+1} gives $N=\frac{1}{n}$. In order to obtain closed form solutions, we use the transformation

$$u(\xi) = V^{\frac{1}{n}}(\xi) \tag{2.24}$$

that will reduce (2.23) into

$$\begin{aligned}&-(\theta_3+a{\theta_1}^2+a{\theta_2}^2)n^2V^2+a({\xi_1}^2+{\xi_2}^2)(1-n)V'^2\\&+a({\xi_1}^2+{\xi_2}^2)nVV''+\left(b+\alpha\beta\frac{{\xi_1}^2}{{\xi_1}^2+{\xi_2}^2}\right)n^2V^4=0.\end{aligned} \tag{2.25}$$

Balancing VV'' with V^4 gives $N=1$. Therefore, we can write the solution of Eq. (2.25) in the form of

$$V(\xi) = a_{-1}w^{-1}(\xi)+a_0+a_1w(\xi) \tag{2.26}$$

where a_{-1}, a_0 and a_1 are constants to be determined later and $w(\xi)$ satisfies Eq. (2.4). Substituting Eq. (2.26) along with Eq. (2.4) into Eq. (2.25) and collecting all terms with the same order of $w^j(\xi)$ together, the left-hand side of Eq. (2.25) is converted into a polynomial in $w^j(\xi)$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for the constants $a_{-1}, a_0, a_1, b_0, b_1, b_2, b_3, b_4$ and θ_3 which can determine by using Mathematica. Thus, the solution functions $u_i(x,y,t)=V_i(\xi)$ where $(i=1,2,\ldots,5)$ can be given as follows. The nontrivial solutions of the algebraic system are obtained.

For $b_1=b_3=0$:

Case 1

$$\begin{aligned}&a_{-1}=a_0=b_0=0,\quad \theta_3=\frac{a\left(b_2(\xi_1^2+\xi_2^2)-n^2(\theta_1^2+\theta_2^2)\right)}{n^2},\\&a_1=\mp\frac{\xi_1^2+\xi_2^2}{n}\sqrt{\frac{-a(n+1)b_4}{(b+\alpha\beta)\xi_1^2+b\xi_2^2}},\end{aligned} \tag{2.27}$$

where $\xi_3=-2a(\theta_1\xi_1+\theta_2\xi_2)$ and b_2, b_4 are the free parameters. The Weierstrass function solution is given by (2.5)

$$w_{1,1}(\xi) = \sqrt{\frac{3\psi(\xi; g_2, g_3) - b_2}{3b_4}}. \tag{2.28}$$

where the invariants of the first Weierstrass function are given by $g_2 = \frac{4b_2^2}{3}$ and $g_3 = \frac{4b_2^2}{27}(9b_4 - 2b_2)$.

Also, the solution (2.13) can be written as

$$w_{1,5}(\xi) = \frac{3}{\sqrt{b_4}} \frac{\psi'(\xi; g_2, g_3)}{6\psi(\xi; g_2, g_3) + b_2}, \tag{2.29}$$

where the invariants of the fifth Weierstrass function are given by $g_2 = \frac{b_2^2}{12}$ and $g_3 = \frac{-b_2^2}{216}$.

Using these results and (2.24), (2.26) we can write the solutions of the DSE as

$$q_{1,i} = e^{i(\theta_1 x + \theta_2 y + \theta_3 t)} \left(\mp \frac{\xi_1^2 + \xi_2^2}{n} \sqrt{\frac{-a(n+1)b_4}{(b + \alpha\beta)\xi_1^2 + b\xi_2^2}} w_{1,i(\xi)} \right)^{\frac{1}{n}}, \quad i = 1, 5, \tag{2.30}$$

$$r_{1,i} = \frac{a\beta\xi_1^2(n+1)(\xi_1^2 + \xi_2^2)b_4}{n^2\left((b + \alpha\beta)\xi_1^2 + b\xi_2^2\right)} w_{1,i}^2(\xi), \quad i = 1, 5. \tag{2.31}$$

Case 2

$$a_0 = 0, \quad \xi_3 = -2a(\theta_1\xi_1 + \theta_2\xi_2), \quad b_0 = \frac{b_2^2}{4b_4},$$
$$\theta_3 = -a\left(\frac{n^2(\theta_1^2 + \theta_2^2) + 2b_2(\xi_1^2 + \xi_2^2)}{n^2}\right), \tag{2.32}$$

$$a_{-1} = \mp \frac{b_2(\xi_1^2 + \xi_2^2)}{2n} \sqrt{\frac{-a(n+1)}{\left((b + \alpha\beta)\xi_1^2 + b\xi_2^2\right) b_4}},$$
$$a_1 = \mp \frac{\xi_1^2 + \xi_2^2}{n} \sqrt{\frac{-a(n+1)b_4}{(b + \alpha\beta)\xi_1^2 + b\xi_2^2}}, \tag{2.33}$$

where b_2 and b_4 are the free parameters. The Weierstrass function solutions (2.5) and (2.6) can be reduced to the solutions

$$w_{2,1}(\xi) = \sqrt{\frac{3\psi(\xi; g_2, g_3) - b_2}{3b_4}} \quad \text{and} \quad w_{2,2}(\xi) = \frac{b_2}{2} \sqrt{\frac{3}{b_4(\psi(\xi; g_2, g_3) - b_2)}}, \tag{2.34}$$

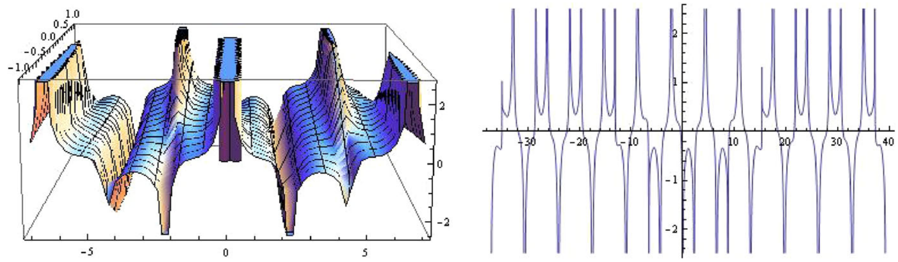

Fig. 1 The complex envelope $q(x,t)$ of the high-frequency wave [The imaginary part of equation (2.30)] where $\theta_1=\theta_2=\theta_3=\xi_1=\xi_2=b_4=a=b=\alpha=\beta=1, b_2=2$

where the invariants of the first and second Weierstrass functions are given by $g_2=\frac{b_2^2}{3}$ and $g_3=\frac{4b_2^2}{27}(9b_4-2b_2)$. Another type of solution admits

$$w_{2,3}(\xi)=b_2\sqrt{\frac{12\psi(\xi;g_2,g_3)-b_2}{2b_4(24\psi(\xi;g_2,g_3)-5b_2)}}, \tag{2.35}$$

where $D=-\frac{5b_2}{2}$ and the invariants of the third Weierstrass function are $g_2=\frac{-b_2^2}{48}(33b_2^2-34)$ and $g_3=\frac{b_2^3}{1728}$. The fourth and fifth Weierstrass function solutions are obtained as follows:

$$w_{2,4}(\xi)=\frac{b_2(6\psi(\xi;g_2,g_3)+b_2)}{6\sqrt{b_4}\psi'(\xi;g_2,g_3)},\quad w_{2,5}(\xi)=\frac{3}{\sqrt{b_4}}\frac{\psi'(\xi;g_2,g_3)}{6\psi(\xi;g_2,g_3)+b_2}, \tag{2.36}$$

where the invariants of these Weierstrass functions are $g_2=\frac{b_2^2}{16}$ and $g_3=\frac{b_2^2}{27}$. Finally, the Weierstrass function solutions to the DSE are given by

$$q_{2,i}=e^{i(\theta_1x+\theta_2y+\theta_3t)}\left[\mp\frac{\xi_1^2+\xi_2^2}{2n\sqrt{b_4}}\sqrt{\frac{-a(n+1)}{\left((b+\alpha\beta)\xi_1^2+b\xi_2^2\right)}}\left(b_2w_{2,i}^{-1}(\xi)+2b_4w_{2,i}(\xi)\right)\right]^{\frac{1}{n}},\quad i=1,...,5, \tag{2.37}$$

$$r_{2,i}=\frac{a\beta{\xi_1}^2(n+1)({\xi_1}^2+{\xi_2}^2)}{4n^2\left((b+\alpha\beta)\xi_1^2+b\xi_2^2\right)b_4}\left(b_2w_{2,i}^{-1}(\xi)+2b_4w_{2,i}(\xi)\right)^2,\quad i=1,...,5. \tag{2.38}$$

The solutions $q(x,t)$ (complex envelope of the high-frequency wave) (the real part and the imaginary part of the solution (2.30)) and $r(x,t)$ (the solution (2.31)) are displayed in Figs. 1, 2 and 3, respectively, with values of parameters listed in their captions.

Remark 2.1 All the solutions obtained by using Weierstrass transformation method for Eqs. (2.15) and (2.16) have been checked by Mathematica. To our knowledge,

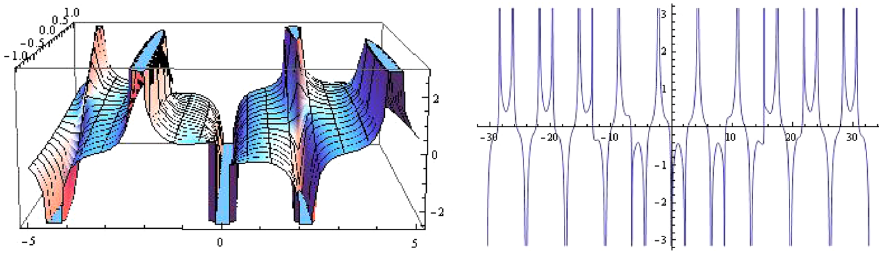

Fig. 2 The complex envelope $q(x, t)$ of the high-frequency wave [The real part of equation (2.30)] where $\theta_1 = \theta_2 = \theta_3 = \xi_1 = \xi_2 = b_4 = a = b = \alpha = \beta = 1, b_2 = 2$

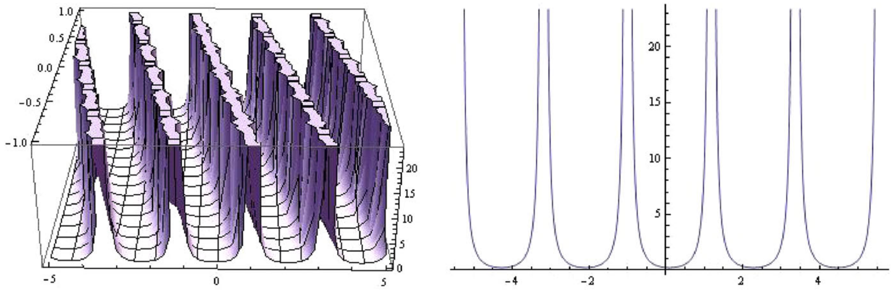

Fig. 3 The solution (2.31) is shown at $\theta_1 = \theta_2 = \theta_3 = \xi_1 = \xi_2 = b_4 = a = b = \alpha = \beta = 1, b_2 = 2$

the Weierstrass elliptic function solutions we found here to this nonlinear physical problem are not shown in the previous literature. These results are new exact solutions of Eqs. (2.15) and (2.16).

3 Trial Equation Method and its Applications

In this section, the trial equation method will be first described and then subsequently applied to solve the DSE equation. Take trial equation

$$(u')^2 = F(u) = \sum_{i=0}^{s} a_i u^i, \tag{3.1}$$

where s and a_i are constants to be determined. Substituting Eq. (3.1) and other derivative terms such as u'' or u''' and so on into Eq. (2.2) yields a polynomial $G(u)$ of u. According to the balance principle we can determine the value of s. Setting the coefficients of $G(u)$ to zeros, we get an ordinary differential equations system. Solving the nonlinear ordinary differential equation, we will determine c and values of $a_0, a_1, ...a_s$. Rewrite the Eq. (3.1) by integral form

$$\pm (\xi - \xi_0) = \int \frac{1}{\sqrt{F(u)}} du. \tag{3.2}$$

According to the complete discrimination system of polynomial, we classify the roots of $F(u)$, and solve the integral (3.2). Thus we obtain the exact solutions to Eq. (2.1). We refer the reader to [22] for details concerning the trial equation method.

Reformulating Eq. (2.25), we obtain the following nonlinear ordinary differential equation

$$MVV'' + N(V')^2 - \left[\theta_3 + a{\theta_1}^2 + a{\theta_2}^2\right] PV^2 + RV^4 = 0, \tag{3.3}$$

where $M = an\left({\xi_1}^2 + {\xi_2}^2\right)^2$, $N = a(1-n)\left({\xi_1}^2 + {\xi_2}^2\right)^2, P = n^2\left({\xi_1}^2 + {\xi_2}^2\right)$ and $R = n^2\left[b({\xi_1}^2 + {\xi_2}^2) + \alpha\beta\xi_1^2\right]$.

Substituting trial equation (3.1) into Eq. (3.3) and using the balance principle we get $s = 4$. Using the solution procedure of trial equation method, we obtain the system of algebraic equations as follows:

$$\begin{gathered} 2a_0N = 0, \\ a_1M + 2a_1N = 0, \\ a_2M + a_2N - a{\theta_1}^2P - a{\theta_2}^2P - \theta_3P = 0, \\ 3a_3M + 2a_3N = 0, \\ 2a_4M + a_4N + R = 0. \end{gathered}$$

Solving the above system of algebraic equations, we obtain the following results:

$$\begin{aligned} &a_0 = 0, \quad a_1 = 0, \quad a_2 = a_2, \quad a_3 = 0, \quad a_4 = \frac{R}{2M+N}, \\ &\theta_3 = -a\left({\theta_1}^2 + {\theta_2}^2\right) + a_2(M+N) \end{aligned} \tag{3.4}$$

where a_2 and a_4 are free parameters. Substituting these results into Eq. (3.1) and (3.2), we have

$$\pm(\xi - \xi_0) = \int \frac{1}{\sqrt{a_2V^2 - \frac{R}{2M+N}V^4}} dV. \tag{3.5}$$

Integrating Eq. (3.5), we obtain the exact solutions of Eq. (3.3) as follows:

$$V(\xi) = 4\sqrt{a_2}\left[\exp\left(\sqrt{a_2}(\xi - \xi_0)\right) - 4a_4\exp\left(-\sqrt{a_2}(\xi - \xi_0)\right)\right]^{-1} \tag{3.6}$$

and

$$V(\xi) = 4\sqrt{a_2}\left[\exp\left(-\sqrt{a_2}(\xi + \xi_0)\right) - 4a_4\exp\left(\sqrt{a_2}(\xi + \xi_0)\right)\right]^{-1}. \tag{3.7}$$

Using the properties

$$\exp(\xi) - \exp(-\xi) = 2\sinh(\xi), \quad \exp(\xi) + \exp(-\xi) = 2\cosh(\xi), \tag{3.8}$$

when $a_4 = \pm\frac{1}{4}$ and $u = V^{\frac{1}{n}}$, it is easy to see that the solutions (3.6) and (3.7) can reduce to soliton solutions

$$u(\xi) = \frac{A}{\cosh^{\frac{1}{n}}\left[B(\xi_1 x + \xi_2 y + \xi_3 t \pm \xi_0)\right]}, \tag{3.9}$$

$$u(\xi) = \frac{A}{\left(\mp\sinh\left[B(\xi_1 x + \xi_2 y + \xi_3 t \pm \xi_0)\right]\right)^{\frac{1}{n}}}, \tag{3.10}$$

where $\xi_3 = -2a(\theta_1\xi_1 + \theta_2\xi_2)$, $A = (2\sqrt{a_2})^{\frac{1}{n}}$ and $B = \sqrt{a_2}$. Substituting (3.9) and (3.10) into (2.17) and (2.22), we have the travelling wave solution of the DSE, respectively,

$$q = e^{i(\theta_1 x + \theta_2 y + \theta_3 t)} \frac{A}{\cosh^{\frac{1}{n}}\left[B(\xi_1 x + \xi_2 y + \xi_3 t \pm \xi_0)\right]}, \tag{3.11}$$

$$r = \frac{C}{\cosh^2\left[B(\xi_1 x + \xi_2 y + \xi_3 t \pm \xi_0)\right]}, \tag{3.12}$$

and

$$q = e^{i(\theta_1 x + \theta_2 y + \theta_3 t)} \frac{A}{\left(\mp\sinh\left[B(\xi_1 x + \xi_2 y + \xi_3 t \pm \xi_0)\right]\right)^{\frac{1}{n}}}, \tag{3.13}$$

$$r = \frac{C}{\sinh^2\left[B(\xi_1 x + \xi_2 y + \xi_3 t \pm \xi_0)\right]}, \tag{3.14}$$

where $\xi_3 = -2a(\theta_1\xi_1 + \theta_2\xi_2)$, $\theta_3 = -a(a_2(\xi_1^2 + \xi_2^2)^2 - \theta_1^2 - \theta_2^2)$, $C = \frac{-4\beta{\xi_1}^2 a_2}{{\xi_1}^2 + {\xi_2}^2}$. From (2.18), ξ_1 and ξ_2 are the widths of the solitons in the $x-$ and $y-$ directions, respectively, while ξ_3 is the velocity of the soliton. From the phase component given by θ, θ_1 and θ_2 are the phase frequencies in the x- and y-directions, respectively, while θ_3 is the wave number of the soliton. Also, Eqs. (3.13) and (3.14) represent singular soliton solutions for Eqs. (2.15) and (2.16). In (3.11)–(3.14), A and C are the amplitudes of the solitons.

The solutions $q(x, t)$ (the imaginary part of the solutions (3.11) and (3.13)) and $r(x, t)$ (the bright 1-soliton solution (3.12) and the singular soliton solution (3.14)) are displayed in Figs. 4 and 5, respectively, with values of parameters listed in their captions.

Remark 3.1 If we let the corresponding values for some parameters, solutions (3.11) and (3.12) are, respectively, in full agreement with the solutions (2.21) and (2.24) and the solutions (3.11) and (3.12) mentioned in Refs. [3,4].

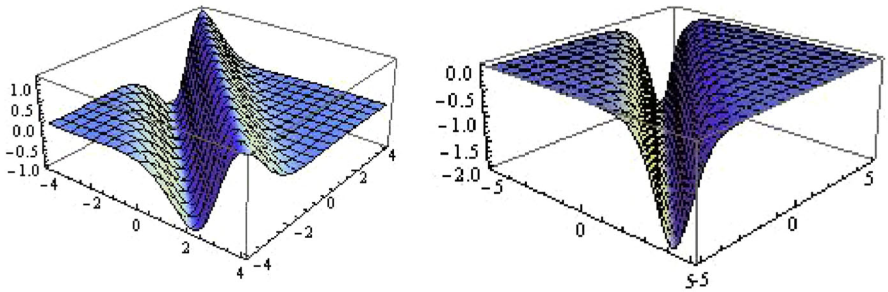

Fig. 4 The complex envelope $q(x, t)$ of the high-frequency wave (The imaginary part of equation (3.11)) and the bright 1-soliton solution (3.12) where $\theta_1 = \theta_2 = \theta_3 = \xi_1 = \xi_2 = \xi_3 = a_2 = a = \beta = n = 1$

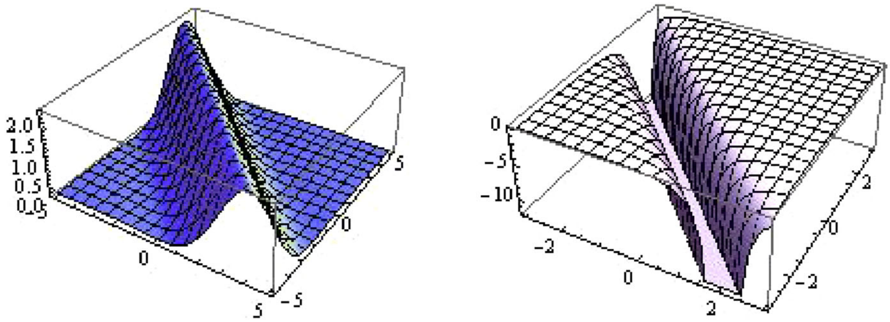

Fig. 5 The complex envelope $q(x, t)$ of the high-frequency wave (The imaginary part of equation (3.13)) and the singular soliton solution (3.14) where $\theta_1 = \theta_2 = \theta_3 = \xi_1 = \xi_2 = \xi_3 = a_2 = a = \beta = n = 1$,with imaginary

4 Conclusion

In this paper, the extended Weierstrass transformation method and trial equation method are used to carry out the integration of the DSE. Some new soliton solutions are obtained using these methods. The obtained solutions are very useful in the field of nonlinear science. These methods can be also applied to solve other types of the generalized NLEEs with complex coefficients.

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