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Looking into the relationship between implied and realized volatility: a study on S&P CNX Nifty index option

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Abstract This paper looks into the robustness of implied volatility against the backward looking volatility of S&P CNX Nifty index option in India by considering both 'in the sample' and 'out of the sample' framework. The backward looking volatility is measured by employing Moving Average (MA), Exponential Weighted Moving Average (EWMA), Generalized Auto Regressive Conditional Heteroskedisticity (GARCH) and Exponential Generalized Auto Regressive Conditional Heteroskedisticity (EGARCH) model with Generalized Error Distribution (GED). The study computed the realized volatility of S&P CNX Nifty by using the overlapping database to match with the expiration of the corresponding option contract. This leads to the problem of minimizing the standard error of the estimator as per the OLS method. This problem has been resolved by employing Generalized Methods of Moments (GMM). The study period of the analysis is spanning over the period from 4th June 2001 to 23rd June 2011. The determinations of the study suggest that Conditional Volatility gives a superior forecast of realized volatility than forward looking volatility and other backward looking volatility. At the same time the analysis shows that implied volatilities are biased and inefficient estimates over the remaining life of the option contract. In 'out of the sample analysis', the family of ARCH models outperformed all other forecasting models with respect to predicting 30 days ahead volatility.

Keywords Implied volatility · Call and put option · Forecasting · Conditional volatility · Generalized error distribution · Error in variables · EWMA

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1 Introduction

Forecasting volatility is fundamental to the risk management operation in order to price derivatives, devise hedging strategies and estimate the financial risk of firm's portfolio positions. There are several methods to predict the future volatility. Nevertheless, generally, we can split volatility models into two parts: historical volatility and implied volatility. Historical volatility models use time series of past market prices while implied volatility depends on a particular model of the relationship between trading option prices and volatility. A theoretical option pricing model, such as Black–Scholes, will pass the theoretical price for an option as a function of implicit parameters such as strike price, risk free rate of interest, spot price, volatility, and time to maturity. Amongst all except volatility, all other parameters are either given or realized in the market. The implied volatility of an option is the volatility in which the Black–Scholes theoretical price equals to the market price. Implied volatilities are used to device market's expectation about future volatility. Thus, we can state that historical volatility is backward looking while implied volatility is a forward looking measure of volatility. Precisely measures and proper assessments of volatility are important for understanding the operation of the financial market. However, estimated volatilities vary based on the methodology, time period chosen and time horizon considered etc.

In the light of the above discussion the objective of this paper is to look into the predictive power of implied volatility against the backward looking volatility of S&P CNX Nifty index option in India. The present study differs from previous study in four perspectives—firstly, it has considered volatility data, sampled over a longer period (sample size 2513) than in previous studies which increases statistical power and allows for evolution of efficiency of the market dealing with S&P CNX Nifty index option that was presented in 2001. Secondly, the study shows the predictability of both call and put implied volatility. Thirdly, the study deals with overlapping (telescoping) data. Fourthly, in order to assess the validity of all the applied volatility measurement models, we compare them in an out of sample framework.

The major findings of the study propose that the conditional volatility measures provide a superior forecast of realized volatility than forward looking as well as other backward looking measures of volatility. The implied volatilities are biased and inefficient estimates over the remaining life of the option contract. The ARCH school of models outperformed all other prediction models with respect to predicting 1 month ahead volatility in the 'out of the sample' framework. The rest of this paper is structured as follows. Section 2 deals with the review of literature. In Sect. 3, we have presented the data and sampling process. The methodology for analysis that describes how volatility series are constructed is explained in Sect. 4. The empirical results are confronted in the Sect. 5 followed by the concluding remarks of the paper in Sect. 6.

2 Literature review

Volatilities implied in option prices are believed to be "the market's forecast" of future volatility during the option's remaining life. Early studies of the information content of ISDs (Implied Standard Deviations) show that implied volatility contains substantial information for future volatility. A few studies found that options contain volatility forecasts that are more accurate than historical measures of volatility by regressing future volatility on the weighted implied volatility across a broad sample of Chicago Board Options Exchange (CBOE) stocks (Latane and Redeleman 1976; Chiras and Manaster 1978; Beckers 1981). However, these studies performed shortly after the 1973 beginning of the CBOE option market, could only use a relatively short time span and, therefore, focused on cross sections rather than time series predictions. These studies essentially supported that stocks with higher implied volatilities also have higher ex-post realized volatility. Hull and White (1987) show that when volatility is constant, the Black–Scholes implied volatility of an ATM (at-the-money) option approximately equals to the expected future realized volatility during the option life.

Scott and Tucker (1989) reported some predictive ability in ISDs (Implied Standard Deviations) measured from PHLX currency options, but their methodology did not allow formal tests of theory. They ran OLS (Ordinary Least Square) regression with 5 currencies, 3 maturities, and 13 different dates. However, due to high correlations among observations, the usual OLS standard errors were severely biased, thereby invalidating the hypothesis test. Day and Lewis (1992) analyzed options on the S&P 100 index from 1983 to 1989, and found that the ISD (Implied Standard Deviation) contains significant information content for weekly volatility, although not necessarily higher than that of time series models. This approach, however, shortsighted the term structure of volatility since the return horizon did not match with the life of the option. Lamoureux and Lastrapes (1993), studied options on ten stocks with expirations from 1982 to 1984, found that implied volatility is biased and inefficient. Both of these studies used overlapping sample and additionally, are characterized by a 'maturity mismatch' problem. However, Lamoureux and Lastrapes examine a one-day-ahead and Day and Lewis examined one-week-ahead predictive power of implied volatilities computed from options that have a much longer remaining life. Thus, the results are not easy to interpret. To address this problem, Canina and Figlewisk (1993) regressed the volatility over the remaining contract life against the implied volatility of S&P 100 index options over 1983 to 1986. They found that implied volatility of S&P 100 index option contains no information about future volatility.

Empirical research conducted on currency options, on the other hand, in general concludes that the implied volatility on short maturity contracts performs better in predicting future volatility and contains information that is not present in historical volatility. Jorion (1995) investigated the information content of implied volatility from currency options traded on the Chicago Mercantile Exchange and found that statistical time series models are outperformed by the volatility implied in short-term options, although implied volatility appears to be a biased forecast. While utilizing the data on options traded in the 'over-the-counter' market, Galati and Tsatasaronic (1996) evaluated the predictive power of volatility implied in currency options. They conclude that the implied volatility of short-maturity options performs well in forecasting future volatility although it is a biased estimator. For longer horizons they found that neither historical nor implied volatility provides a good forecast of future volatility.

Christensen and Prabhala (1998) tested the prognostic power of implied volatility on S&P 100 index options. In contrast to previous studies, they found that implied volatility outperforms historical volatility in forecasting future volatility. The survey concluded that the deviation in their results compared to those of Canina and Figlewisk was a consequence of using longer time series and non-overlapping data. Fleming (1998), investigated whether the biasness that arises in the option market is purely due to measurement error and model misspecification, or whether the bias was also apparent in option market prices. His analysis also revealed that the bias is too large to be explained by skewness preference, but that it might be the result of market imperfections (e.g., transaction costs) and/or a premium demanded for volatility risk. He too found that the bias apparent through the trading strategies emerged only after the 1987 stock market crash.

Giot (2002), assessed the efficiency, information, content and unbiasedness of volatility forecasts based on the VIX/VXN implied volatility indices by employing RiskMetrics methods and GARCH type models at the 5, 10, and 22-day time horizon. His empirical application focused on the S&P100 and NASDAQ100 indices. He also deals with the information content of the competing volatility forecasts in a market risk (VaR type) evaluation framework. The performance of the models was evaluated using LR, independence, conditional coverage and density forecast tests. His results showed that volatility forecasts based on the VIX/VXN indices had the highest information content, both in the volatility forecasting and market risk assessment frameworks. Holger Claessen and Mittnik (2002) examined alternative strategies for predicting stock market volatility. In their analysis the out-of-sample forecasting experiments and implied-volatility information are derived from contemporaneously observed option prices or history-based volatility predictors such as GARCH models to find out if they are more appropriate for predicting future return volatility. Employing German DAX-index return data, they establish that past returns do not contain useful information beyond the volatility expectations already reflected in option prices. This supports the efficient market hypothesis for the DAX-index options market.

Soczo (2003) stated that for measuring market risk, we need to estimate the future behavior of financial instruments. For this purpose standard deviation and correlations could be quite useful to characterize individual behavior of instruments and their kinship. Then, he found two major concepts to estimate future volatility; one way was the assessment of historical data, while the other concept utilized option pricing theory to bring expected future volatility from option prices. His analysis revealed that the implied volatility derived from option prices is less biased. According to him, for the risk analysis of portfolios, correlation should also be calculated from past data alone. In case of financial markets having different holiday schedules, correlation calculations could be quite problematic with traditional methods unless the incomplete cases are ruled out. This solution would provide quite inefficient estimates; however, filling in missing data causes additional bias. Therefore, more improvement is required to implement data filling methods for more realistic financial processes. Taylor et al. (2010) assess the volatility information content of S&P index and 149 US firm stock options. They use ARCH and regression models to compare volatility forecasts defined by historical stock returns, at-the-money implied volatilities and model-free volatility expectations for 149 firms. According to them, for 1-day-ahead estimation, a historical ARCH model outperforms both of the volatility estimates extracted from option prices for 36 % of the firms, but the option forecasts are nearly always more informative for those firms that have the most actively traded options. When the prediction horizon extends until the expiry date of the options, the option forecasts are more informative than the historical volatility of 85 % of the firms. However, at-the-money implied volatilities generally outperform the model-free volatility expectations.

Lonesco (2011) compared the predictive power of implied volatility, historical volatility, and exponential historical volatility, using monthly observations of S&P 500, FTSE 100 and DAX for the period of 2004 to 2010. The result shows that implied volatility is not only efficient estimator of future volatility, but also that its information content is at least good, if not much better than historical volatility. Yang (2012) explores the predictive ability of the volatility index (VIX) in emerging markets from December 2006 to March 2010. The study shows that the models including both the volatility indicator and the option market information have a stronger predictive power. The prognostic power of the models is improved by 88 % in explaining the future volatility of stock markets, much more serious than that of other models merely considering the volatility index. With respect to the trading information about different types of investors in option markets, the trading information from the foreign institutional investors in option markets demonstrates a significant positive relationship with the stock market volatility. In addition, the results also reveal that the volatility index (TVIX) of Taiwan stock index options is a strong indicator of future stock market volatility. The TVIX outperforms the historical volatility and the GARCH volatility forecast in assessing the activities of Taiwan's stock market. Misra et al. (2006), show that deeply in the money and out of the money options on CNX Nifty are having higher volatility than at the money options; the implied volatility of out of the money call options is more than in the money; implied volatility is higher for far the month contracts than for near the month contracts; deeply in the money and out of the money options with shorter maturity have higher volatility than those of with longer maturity; put options have higher volatility than call options; and implied volatility of more liquid options is more than that of less liquid options. Maheswaran and Ranjan (2005), study the ability of implied volatility to predict the volatility realized over the life of the option in Asian equity indices. They found that in Hong Kong and Taiwan, the implied volatility is an unbiased predictor of future realized volatility, whereas in South Korea and India, the implied volatility is a poor prognosticator. Panda et al. (2008) examined separately information content of call and put options on the S&P CNX Nifty index by using 1 month at-the-money options from June 4, 2001 to October 28, 2004. The result shows that call implied volatility is a better forecast than put implied volatility. According to them there is an error in variable problems in implied volatility. To correct it, they have used instrumental variable method. From both OLS and instrumental variables methods, the result shows that historical volatility does not add any information beyond what is already contained in the implied volatility. The results also confirm the theory that the implied volatility dominates historical volatility in forecasting realized volatility, i.e., all the information contained in historical volatility is being contemplated by the implied volatility, and the historical volatility has no incremental forecast ability. Therefore, they concluded that volatility implied by 1 month (near month) at-the-money call option price is efficient and slightly biased estimator of realized return volatility. This bias may be due to the fact that the implied volatility does not contain all the market information. Kumar (2008) investigates the information content of the implied volatility estimators and the historical volatility in forecasting future realized volatility by using non-overlapping near month option contracts of the S&P CNX Nifty Index with a time to maturity of 30 calendar days during the period from January 2002 to December 2006. To compare implied predictability power with historical volatility he uses regression methods. The results show that implied volatility estimators have information about the future volatility and implied volatility estimators dominate the historical volatility. The study also found that the implied volatility extracted from call options is far better than that computed from put options. Finally, he found that implied volatility estimators are unbiased and efficient estimators of the future realized volatility.

Chen et al. (2013) explored the relationship between VHSI and the future realized volatility of HSI, and predicts the future realized volatility of HSI with Kalman filter. The empirical findings of their study suggest that VHSI is an unbiased and efficient estimate of the future realized volatility and includes data about the future realized volatility when employing monthly data. They too reasoned that the predication performance of the Kalman filter is more serious than the linear regression model. Chen et al. (2014) undertook to analyze whether the implied volatility index can be predicted and the same can be used for option trading performances by checking the Hang Seng Index Volatility. They found that Hang Seng Index Volatility can be predicted more accurately when considering day-of-week effect and spillover effect. Kim et al. (2015) studies the hedging performance with mini gold futures traded on the Korean Exchange (KRX). They have considered the daily prices of gold and mini gold futures from September 13, 2010 to May 31, 2013. By employing the OLS model, VECM as well as the bivariate GJR-GARCH (1, 1) model, they conclude that the time varying GJR-GARCH (1, 1) model yields better hedging performance than time-invariant OLS or VECM models in the both in-sample and out-of sample periods.

In summary, most of the literatures offers clear evidence that option prices contain information about future asset return volatility that cannot be pulled up from past returns i.e., ‘Backward Looking’ volatilities. In this paper, we examine whether this conclusion also applies to call and put options implied volatilities on the S&P CNX Nifty index in the context of India, both in a ‘in-the-sample’ and ‘out-of-the sample’ framework or our analysis will show implied volatility is a biased and inefficient forecaster.

3 Data and sampling procedure

The present study considers S&P CNX Nifty index options which set about trading from June 4, 2001 under the National Stock Exchange, India. The index consists of 50 highly traded scripts drawn from diverse industries and marketplaces. The index options contracts have a uttermost of 3 months trading cycle (1st month-Near month, 2nd month-middle month and, 3rd month- Far month). New contracts are introduced on the trading day following the expiry of the near month contract. The exchange provides a minimum of seven strike prices for every option type (call and put) during the trading month. Our empirical analysis focuses on S&P CNX Nifty index options. The study is based on the daily closing prices of a net dividend of S&P CNX Nifty index and associated options on S&P CNX Nifty, traded on the National Stock Exchange (NSE) from May 31st, 2001 to Jun 30th, 2011 for the spot market. Nevertheless, the study period for the option market is spanning over the period from June 4th, 2001 to Jun 23rd, 2011. The daily spot returns are computed by differencing the natural logarithm of the prices on successive trading days. The option prices used are the nearest expiry call and put options in the near month with more than 6 days to expiry in order to ward off the shock of the rollovers of the contracts. All implied volatilities are calculated for the nearest-to-the money options. The one-month MIBOR (Mumbai Interbank Offer Rate) was taken as the risk free interest rate.

4 Methodology

4.1 Backward looking measure of volatility

4.1.1 Moving average method

One of the simplest approaches to calculate volatility involves estimating a historical moving average. To get a moving average estimate of volatility, the average is taken over a rolling window of historical volatility data. The order 'm' of a moving average process is characterized by the duration of the window that is chosen; hence processes are denoted by MA (m). A longer rolling window implies a moving average process that essentially retains a longer memory of past information. Here in this study of moving average process, we consider a 20-day rolling window. The daily calculation of volatility would be the variance of daily returns over the most recent 20 days. Assuming a zero mean daily return, the moving average volatility over a window of the last 20 days is calculated as follows:

$$\sigma^2_{t,MA(20)} = \frac{1}{20}\sum_{i=1}^{20} R^2_{t-i+1} \times 252 \tag{1}$$

Where $\sigma_{t, MA(20)}$ is the daily estimate of forecasted annualized volatility, expressed as a standard deviation in period t. Because moving-average volatility is calculated using equal weights for all observations in the historical time series, the computations are very bare. The result, however, is a smoothing effect that causes sharp changes in volatility to appear as plateaus over longer periods of time, failing to capture dramatic changes in volatility. This smoothing effect becomes more serious as the rolling window gets longer. A more advanced way of computing a moving average approximation is the Exponentially Weighted Moving average (EWMA) approach. This method has been invented by the Risk Metrics in calculating Value at Risk measurements.

4.1.2 Exponentially weighted moving average method (EWMA)

EWMA is essentially a simple extension of historical average volatility measure, which allows more recent observation to have a stronger impact on the forecast of volatility than older data points. Risk Metrics use EWMA to forecast variances and covariances (volatilities and correlations) of the multivariate normal distribution.

An attractive feature of the exponentially weighted estimator is that it can be written in recursive form which, in turn, will be utilized as a base for making volatility forecasts. In order to derive the recursive form, it is assumed that infinite amounts of information are usable. For instance, presuming again that the sample mean is zero, we can deduce the period t + 1 variance forecast, given data available at time t (single day earlier) as

$$\sigma^2_{t,EWMA} = \lambda\, {}_{\sigma t-1}^{\;2} + (1-\lambda)R_t^2 \times 252 \tag{2}$$

where $\sigma^2_{\mathrm{t,EWMA}}$ is the estimate of the variance for period t, which also becomes the forecast of future volatility for all period t, and λ (0 < λ < 1) is the 'decay factor', which determine how much weight is given to recent versus older observation. This parameter, λ determines the relative weights that are applied to the observations R_t (returns) and the effective amount of data used in estimating volatility. The decay factor could be estimated, but in many studies is set at 0.94 as recommended by RiskMetrics, producers of popular risk measurement software. It also assumed in risk metrics and many academic papers that mean of the return series is zero. For data that is of daily frequency or higher, this is not an unreasonable assumption, and is likely to lead to negligible loss of accuracy since it will typically be very small.

4.1.3 Generalized ARCH (GARCH) models

Observing the natural prolongation of the ARMA process as parsimonious representations of a higher order AR process, Bollerslev (1986) continued the work of Engle to the Generalized ARCH or GARCH process. The GARCH (p, q) process defined as

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \cdots + \beta_p \sigma_{t-p}^2 \tag{3}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{4}$$

where, $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0.$

The conditional variance is a linear function of q lags of the squares of the error terms u_t^2 or the ARCH terms (also referred to as the news from the past) and p lags of the past values of the conditional variance (σ_t^2) or the GARCH terms, and constant ω. The inequality restrictions are imposed to assure a positive conditional variance, most certainly. The most elementary form of GARCH (1, 1) is as follows:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{5}$$

4.1.4 Exponential GARCH (EGARCH) model

The GARCH process fails to explain the then called "leverage effects" often mentioned in financial time series. The concept of leverage effects, first noted by Black (1976), refers to the tendency for changes in the livestock prices to be negatively correlated with changes in the stock volatility. In other words, the effect of a shock on the volatility is asymmetric, or to put it differently, the impact of a "good news" (positively lagged residuals) is different from the impact of the "bad news" (negative lagged residuals). A model that accounts for an asymmetric reaction to a shock was credited to Nelson (1991) and is called an exponential GARCH or EGARCH model.

$$\ln \sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \ln \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i |z_{t-i}| + \sum_{i=1}^{q} \theta_i z_{t-i}. \tag{6}$$

Where, $z_t = \frac{u_t}{\sigma_t}$. Note that the left-hand side of the Eq. (6) is the natural logarithm (Ln) of the conditional variance. The Ln form of the EGARCH (p, q) model ensures the non-negative of the conditional variance without the need to constrain the coefficients of the model. The asymmetric effect of positive and negative shocks is represented by the inclusion of the term z_{t-i}. If $\theta_i > 0$, volatility tends to rise (fall) when the lagged standardized shock, $z_{t-i} = \frac{u_{t-i}}{\sigma_{t-i}}$, is positive (negative). The persistence of shocks to the conditional variance is given by $\sum_{j=1}^{p} \beta_j$. Since negative coefficients are not ruled out, the EGARCH models allow for the possibility of cyclical behavior in volatility. The most elementary form of EGARCH (1, 1) is as follows:

$$\ln \sigma_t^2 = \omega + \beta_1 \ln \sigma_{t-1}^2 + \alpha_1 |z_{t-1}| + \theta_1 z_{t-1} \tag{7}$$

4.2 Forward looking measure of volatility

The implied volatility of an option is defined as the expected future volatility of the underlying asset over the remaining lifetime of the option that compares the average value of the option implied by a particular model to the option's actual market cost. The former studies in this line concluded that measures of option implied volatility are, indeed, the best predictor of future volatility.

Unlike time series measures of volatility that are completely backward-looking, option implied volatility is "backed-out" of actual option prices—which, in turn, are based on actual transactions and expectations of market participants—and, therefore, is inherently forward-looking. This measurement incorporates the most current market information and, therefore, it should reflect market expectations better than the historical measure.[1] Properly interpreted, implied volatility of an option, provides information about what market participants expect to happen with future asset returns.

Black and Scholes (1973) derived the value of a European call and put option on a non-dividend-paying-stock as a function of current stock price, the time to

[1] See Fama (1971, 1990).

expiration, the exercise price, the risk free rate of interest, and the volatility of the underlying stock.

For a European option, the current price of the call and put on a common stock or index is just

$$C = S_0 N(d_1) - Ke^{-rt} N(d_2) \tag{8}$$

$$P = Ke^{-rt} N(-d_2) - S_0 N(-d_1) \tag{9}$$

where C denotes the price of the call price, P denotes the put price N (d) denotes the cumulative normal distribution evaluated at d, S_0 is the Current asset price, T is the time to maturity of the option, K is the strike price, and r is the riskless interest rate, for this we have used the one month MIBOR (Mumbai Interbank Offer Rate).

$$d_1 = \frac{\ln(S_0/k) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/k) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where, σ is the volatility of the underlying asset.

Options on assets, paying a constant dividend yield can be priced using a modified form of Black–Scholes model.

In the Black–Scholes model, the one unobserved parameter is the volatility of the underlying stock. Theoretically, the proper volatility input in the Black–Scholes model is the instantaneous variance of asset returns, which means the variance of underlying asset's return over an infinitesimal time increment. Robert Merton (1973) concluded that under this interpretation, the volatility implied by the Black–Scholes model can be interpreted as the expected future instantaneous variance of the underlying asset's return over the remaining life of the option.

To value an option using the Black–Scholes model, we just substitute values of observed parameters and volatility into the Black–Scholes equation, which provides an exact link between the inputs to the model and the theoretical option price. To find out the implied volatility of European call and put option, with a given market price of the option, current stock price, strike price, time to maturity and interest rate, the value of volatility is derived by substituting those observed values into the Black–Scholes model. The resulting value of 'σ' is called the Black–Scoles implied volatility,[2] because that number represents the volatility of the underlying asset that is implied by quoted option price and the Black–Scholes model.

4.3 Realized volatility

In the last section we have explained the way to calculate both backward and forward looking volatilities. But now the question arises how to find out which one among them is the best, for that firstly we have to calculate the actual future volatility or realized volatility. Then we use this actual realized volatility as a

[2] Implied volatility for both call and put options.

dependent variable in the predictability regression. Since it is assumed that underlying prices are log normally distributed, the returns are calculated according to their log differences in prices and are hence continuously compounded. Therefore, a one-day behind return is calculated as

$$R_{t-1,t} = \ln(S_t) - \ln(S_{t-1}) \tag{10}$$

where, S_t denotes the closing price at time t. Similarly, the one-day ahead return is calculated as $R_{t,t+1}$. Both of these daily return series, then squared to serve as a basis for either input in a measurement technique or as a measure of volatility for the analysis. These series are also multiplied into square root of 252 assuming 252 trading days in a year, to make it annualized like the other measures of forecasting volatilities that are used in the analysis of our study. The resulting volatilities are expressed as standard deviations. Furthermore, it is always assumed that the mean of the return series is equal to zero, so that the daily variance is explained simply by the squared returns. The one-day behind return series was used to calculate all of the historical or backward looking measures of volatility such as MA (20), EWMA, GARCH and EGARCH. In the GARCH application, as demonstrated in the previous section, both the previous day's return and variance are regressors in determining the conditional volatility whereas the moving average processes are simply a function of the one-day behind annualized squared returns. All of the historical estimates were arrived at in the manner described in the previous section. It is important to note, however, that the available data in each final historical volatility series were truncated by the order of the moving average processes and the GARCH specifications.

The return series Eq. (10) is used to calculate the realized volatility and used as the dependent variable in the predictability regressions. The procedure for calculating this process was more complicated in that it was necessary to keep track of the lifetime of the option.

$$\sigma^2_{t,T} = \frac{1}{(T-t)} \sum_{i=1}^{T-t} R^2_{t+i} \times 252 \tag{11}$$

Where, $\sigma_{t,T}$ is the realized volatility, and $(T - t)$ are days to expiry the option contract. So, for example, if there are 22 days remaining in the lifetime of an option, the volatility of the futures contract is determined by averaging the squared returns over those 22 days. The square root of this average multiplied by the square root of 252 yields the actual realized volatility on an annualized basis. On the next day, the previous day's return would be dropped from the measurement since only 21 days remain to expiration and the procedure for calculating the volatility was repeated. This is the method is adopted from Jorion (1995).

4.4 Predictability regressions

To compare the abilities of several methods of estimated measures of volatility in determining the future volatility over the remaining life of the option we used simple OLS regression. In order to do so, a time series of future volatility had to be created. This was done by calculating the average daily return over the remaining life of the option, as described in Sect. 4.3. This daily future volatility series is denoted by t, T where t represents the current date and T represents the future date of option expiration. A typical predictability regression is an ordinary least squares regression that may be expressed as:

$$\sigma_{t,T} = \alpha + \beta\hat{\sigma}_{t,i} + \varepsilon_{t,T} \tag{12}$$

where the $\sigma_{t.,T}$ is the realized volatility and estimated volatility forecast, $\hat{\sigma}_{t,i}$ may again include the implied volatility (i.e., $\sigma_{t,IV)}$[3] from option prices both for call and put, Moving average ($\sigma_{t,MA(20)}$), Exponentially weighted moving average method ($\sigma_{t,EWMA}$), Generalized autoregressive conditional heteroskedasticity ($\sigma_{t,GARCH}$) or the Exponential Generalized autoregressive conditional heteroskedasticity model ($\sigma_{t,EGARCH}$). If $\hat{\sigma}_t$ contains some information about future volatility then β should be nonzero. Second, if it is an unbiased forecast of realized volatility, then $\alpha = 0$ and $\beta = 1$. Finally, if implied volatility is efficient, the residuals ε_t should be white noise and uncorrelated with any variables in the market information set. Two types of predictability regression will run. In the first type we run all six individual forecasted volatility separately in a different regression with same dependent variable i.e., the realized volatility which we have already discussed. In the second step we will use different combinations of forecasted volatility as the independent variables. In this multiple regressions the implied volatility is the one of the independent variables as our objective is to know the predictability of implied volatility is more than the other backward looking volatility. The multiple regression to know the relative predictive power of alternative forecasts are as follows:

$$\sigma_{t,T} = \alpha + \beta_1\sigma_{t,CIV} + \beta_2\hat{\sigma}_{t,i} + \varepsilon_{t,T} \tag{13}$$

$$\sigma_{t,T} = \alpha + \beta_1\sigma_{t,PIV} + \beta_2\hat{\sigma}_{t,i} + \varepsilon_{t,T} \tag{14}$$

$$\sigma_{t,T} = \alpha + \beta_1\sigma_{t,CIV} + \beta_2\sigma_{t,PIV} + \varepsilon_{t,T} \tag{15}$$

where, $\sigma_{t,T}$ is the average realized volatility of the remaining life of the option and, $\sigma_{t,CIV}$ is the call implied volatility, $\sigma_{t,PIV}$ is the put implied volatility and $\hat{\sigma}_t$ are, respectively, the forecasted volatility from MA, EWMA, GARCH and EGARCH model at the period t. If implied volatility (both call and put) and other forecasted volatility contain independent information that is useful for predicting future volatility, the estimated β_1 and β_2 should both be nonzero. Alternatively, if the information in one forecast is a subset of the information contain in the other forecast, the estimated coefficient of the former forecast should be nonzero.

The above series does have a high serial correlation.[4] Serial correlation will not affect the unbiasedness or consistency of OLS estimators, but it affects its efficiency. It arrives by using overlapping observations rather than restricting the analysis to independent observations. Since the possibility of measurement errors in

[3] $\sigma_{t,CIV}$ for call implied volatility and $\sigma_{t,PIV}$ for Put implied volatility.

[4] Serial correlation occurs in time series studies when the error associated with a given time period carry over into future time periods.

independent variables could be seen in these types of series, Fleming (1998) used GMM estimation in order to deal with some error in variables. We have also performed GMM estimation, using lagged independent variables as instruments.

Residual autocorrelation could also be seen because of using overlapping observations rather than restricting the analysis to independent observations, we would not have this problem. This could result in inefficient slope estimates and spurious explanatory power. We correct this by employing GMM of Hansen (1982) along with Newey and West (1987) approach to estimate hetroskedasticity and autocorrelation consistent variance–covariance matrix.

4.5 Out of sample methodology

In the last section we have discussed the methodology deals with the in the sample performances of the forecasting models. For measuring the predictability of the estimators, it is better for the analyzer to analyze both 'in the sample' and 'out of the sample' performances of the estimators. The focus of this section is on the forecasting accuracy of 'h' day's ahead implied volatility compare to other backward looking volatility in an out of sample framework. Here 'h' period is approximately 30 days or in other words a month. For the analysis we have taken the data from 1st June 2001 to 30th June 2011. We divided the entire sample (1st June 2001 to 30th June 2011) into two parts. The first part covers the data set from 1st June 2001 to 31st May 2006 and the second part covers data set from 1st June 2006 to 30th June 2011. First part of the data set is used to estimate the model parameters of GARCH and EGARCH model, which are used to construct an out of sample 'h' day ahead volatility data set. By this procedure, we get 61 average forecasted values of GARCH and EGARCH. We have already discussed six alternative models of volatility forecasting. All competing models generate a set of 'h' step ahead volatility forecast. All that is needed now is a method to compare all with realized volatility. Four error statistics are applied to assess the predictive ability of six different models.

4.6 Error statistics

MSE (mean square error):

$$\frac{1}{N}\sum_{T=1}^{N}(\sigma_T^{RV} - \sigma_T^{f})^2 \tag{16}$$

MAE (mean absolute error):

$$\frac{1}{N}\sum_{T=1}^{N}\left|\sigma_T^{RV} - \sigma_T^{f}\right| \tag{17}$$

RMSE (root mean square error):

$$\sqrt{\frac{1}{N}\sum_{T=1}^{N}\left(\sigma_T^{RV}-\sigma_T^{f}\right)^2} \tag{18}$$

MAPE (mean absolute percentage error):

$$\frac{1}{N}\sum_{T=1}^{N}\left|\frac{\sigma_T^{RV}-\sigma_T^{f}}{\sigma_T^{RV}}\right| * 100 \tag{19}$$

In the above expression σ_T^{RV} is the realized volatility and σ_T^{f} forecasted models, it may include call implied volatility σ_T^{CIV}, put implied volatility σ_T^{PIV}, Moving average volatility σ_T^{MA}, GARCH(1, 1) volatility σ_T^{GARCH} or EGARCH (1, 1) volatility σ_T^{EGARCH}. N denotes the number of forecast model using each method. The MSE is taking the square of the mean error and averaging it. The MAE treats large and small deviations equally while RMSE criteria penalized large deviations more severely and MAPE taking MA into a percentage value of realized volatility. Note that an optimal forecast will not have MAE = RMSE = 0 because realized volatility is only a point estimate of the index price volatility which is unobservable.

5 Empirical analysis

For calculating MA(20), EWMA and ARCH family of models, the data are converted into a continuously compounded rate of return (R_t) by taking the first difference of the log prices, i.e. $R_t = 100 \cdot \mathrm{Ln}\ (S_t/S_{t-1})$. The estimation procedure of MA (20) and EWMA has already explained in Sects. 4.1.1 and 4.1.2. Now, the volatility models we are going to estimate in this section are intended to capture the conditional variance of the stochastic components of the returns. This paper uses two widely accepted models in this sphere, viz, GARCH and EGARCH models.

For calculating ARCH family of models we have computed the descriptive statistics of the spot market return. The summary statistics are presented in the Table 1.

It can be seen from the Table 1, that both the closing price and its return are skewed and non-normal. Specifically, closing price, annualized return is negatively skewed, whereas closing price is positively skewed. The J-B test has rejected the null hypothesis of the normal distribution for index return with high level of statistical significance. The histogram for the return also confirms this from the Fig. 3 in "Appendix".

Therefore, the actual distribution of daily annualized returns for the S&P CNX Nifty index has a fat tail (and a narrow waist) compared to a fitted normal distribution. The kurtosis value of daily return is 11.834 demonstrating the presence of fat tails.

When we calculate GARCH and EGARCH model, instead of the assumption of a normal distributed innovations, we use the Generalized Error Distribution GED (Taylor 1994) for maximizing the likelihood function, from which the normal distribution is a special case.

Table 1 Basic statistics of closing price and its return

Variable	Mean	Max	Min	SD	Skewness	Kurtosis	J-B
Closing price	3115.912	6312.45	854.2	1685.55	0.2134	1.6232	217.4557 (0.01)
Closing price return	0.000603	0.1633	−0.13053	0.0165	−0.284	11.834	8205.78[a] (0.01)

[a] Indicates the rejection of the null hypothesis of normal distribution in the J-B test

Table 2 Estimated test statistics for ARCH effect (Ljung-Box 'Q' Statistics)

Estimated variable	Q (5)	Q (10)	Q (20)	Q^2 (5)	Q^2 (10)	Q^2 (20)	LM (4)
u_t	8.505 (0.05)	22.421 (0.01)	46.796 (0.01)	354.0 (0.00)	524.1 (0.00)	696.38 (0.00)	211.854 (0.01)
Return	19.108 (0.00)	33.712 (0.00)	58.279 (0.00)				
$(\text{Return})^2$	331.76 (0.000)	499.13 (0.000)	659.07 (0.000)				

Q and Q^2 are the LB Q statistics for the residuals and square residuals respectively. Figures in the parentheses () are respective lag lengths

To start with, the study fits an Auto Regressive (AR) model of order one. This is carried out primarily to eliminate the first degree auto correlation among the returns, which makes the data amenable for further analysis. After fitting the AR (1) model, we have tested for the presence of autocorrelation among the residuals as well as squared residuals from the fitted model. The results from Ljung Box 'Q' statistics, which are used to test the null hypothesis of 'No Autocorrelation' against the alternative of the existence of autocorrelation, are reported in Table 2 (Mishra et al. 2007).

From the Table 2 results, it is inferred that the null hypothesis is strongly rejected in the case of residual and squared residuals. Prima facie, this creates the case to apply GARCH models (Mishra et al. 2007). In order to confirm the presence of ARCH effect in the data, we go for a LaGrange Multiplier (LM) Test, and, the result shows that the null hypothesis of 'No ARCH Effect' is strongly rejected in the annualized index return.

In Tables 3 and 4, we present the estimation results of AR (1)-GARCH (1, 1) and that AR (1)-EGARCH (1, 1) models. It may be pointed out that the study uses the GARCH and EGARCH models of order (1, 1) because this order has been found to provide the most parsimonious representation of ARCH class of models, and, at the same time empirically the acceptance of the order has been strongly proved. To test the degree of persistence in GARCH (1, 1) model we are using the Wald test in which the null hypothesis, $\alpha + \beta = 1$ (alternatively known as, 'the variance is integrated' or non-stationarity of the variance) is tested against the alternative $\alpha + \beta < 1$, using the estimated α, β coefficients of the variance equation. It can be seen from the Wald statistic value that the null hypothesis is rejected in favor of the alternative with a high degree of confidence interval. Thus, the test confirmed the stationary of the variance and since $\alpha + \beta = 0.90$ which is less than unity, the

Table 3 Parameter estimates of the GARCH (1, 1) model

Mean Equation	AR(1)		
	Coefficient	z-statistics	*P* value
α	0.0013	5.222	0.01
R_{t-1}	0.07357	3.4792	0.01
Variance equation	GARCH (1, 1)		
ω	0.000	4.839	0.01
u^2_{t-1}	0.1352	8.431	0.01
σ^2_{t-1}	0.8408	49.278	0.01
GED parameter	1.444	34.839	0.01
Log likelihood	7209.556		
AIC	−5.735		
SC	−5.721		
DW	1.99		
LM (4)		2.859	0.58

LM (4) represents LaGrange Multiplier statistics to test the presence of additional ARCH effect in the residuals from AR (1)-GARCH (1, 1)

Table 4 Parameter estimates of the EGARCH (1, 1) model

Mean equation	AR(1)		
	Coefficient	z-statistics	P-value
α	0.0009	3.9145	0.01
R_{t-1}	0.0894	4.1994	0.01
Variance equation	EGARCH (1, 1)		
ω	−0.594	−8.339	0.01
Ln σ^2_{t-1}	0.954	133.800	0.01
$\lvert z_{t-1}\rvert$	0.261	9.663	0.01
z_{t-1}	−0.118	−7.2428	0.01
GED parameter	1.464	35.776	0.01
Log likelihood	282.255		
AIC	−5.748		
SC	−5.732		
DW	2.03		
LM (4)		2.350	0.67

LM (4) represents LaGrange Multiplier statistics to test the presence of additional ARCH effect in the residuals from AR (1)—EGARCH (1, 1) models

volatility may decay rapidly and revert to its mean. The Tables 3 and 4 also includes the Lagrange multiplier (LM) test values for squared residuals. This shows that, the null hypothesis of ‘No ARCH Effect’ is strongly accepted both in GARCH (1, 1) and EGARCH (1, 1) model.

The descriptive statistics of all forecasting methods are present in Table 5. Before performing regression analysis, the study exercised a unit root test of all the series, because if volatility series possess a unit root, regressions specified as above are spurious. The Unit root test is carried out through Dickey-Fuller (DF), Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests. The results of unit root tests given in the Table 6 suggest that all the variables are stationary at level.

Table 5 Descriptive statistics

	$\sigma_{t,T}$	$\sigma_{t,CIV}$	$\sigma_{t,PIV}$	$\sigma_{t,GARCH}$	$\sigma_{t,EGARCH}$	$\sigma_{t,EWMA}$	$\sigma_{t,MA(20)}$
Mean	0.228	0.2442	0.295	0.238	0.235	0.237	0.231
Max	0.188	0.200	0.255	0.202	0.2071	0.199	0.192
Min	1.075	1.639	2.056	0.987	1.023	0.762	0.840
Std. Dev.	0.042	0.039	0.0536	0.125	0.1060	0.104	0.078
Skewness	0.134	0.155	0.150	0.111	0.1043	0.114	0.125
Kurtosis	2.266	3.286	2.758	2.478	2.1460	1.7599	2.049
Jarque–Bera	9.814	20.076	18.917	11.411	10.051	6.4226	8.0262
Probability	7011.33	35042.07	29706.03	9979.55	7132.112	2522.986	4403.445

$\sigma_{t,T}$ is realized volatility, $\sigma_{t,CIV}$ as call implied volatility, $\sigma_{t,PIV}$ as put implied volatility, $\sigma_{t,GARCH}$ as GARCH (1,1), $\sigma_{t,EGARCH}$ as EGARCH (1, 1) volatility, and $\sigma_{t,EWMA}$ as Exponential weighted Moving Average Volatility and $\sigma_{t,MA(20)}$ is the Moving average volatility

Table 6 Test of stationarity

Levels						
Variables	Without trend			With trend and intercept		
	DF	ADF	PP	DF	ADF	PP
$\sigma_{t,T}$	−5.774[a]	−6.324[a] (4)	−6.319[a] (10)	−5.841[a]	−6.407[a] (4)	−6.401[a] (8)
$\sigma_{t,CIV}$	−20.492[a]	−5.057[a] (13)	−34.052[a] (35)	−21.295[a]	−5.269[a] (13)	−35.476[a] (35)
$\sigma_{t,PIV}$	−20.121[a]	−4.563[a] (11)	−34.112[a] (35)	−20.586[a]	−4.643[a] (5)	−35.057[a] (35)
$\sigma_{t,GARCH}$	−6.577[a]	−7.040 (7)[a]	−6.739[a] (13)	−6.675[a]	−7.155 (7)[a]	−6.882[a] (12)
$\sigma_{t,EGARCH}$	−7.994[a]	−6.502[a] (13)	−7.848[a] (14)	−8.093[a]	−6.609[a] (13)	−7.969[a] (13)
$\sigma_{t,EWMA}$	−3.272[a]	−4.540[a] (10)	−4.169[a] (14)	−3.330[b]	−4.625[a] (10)	−4.255[a] (14)
$\sigma_{t,MA(20)}$	−3.663[a]	−3.969[a] (21)	−5.221[a] (25)	−3.668[b]	−4.015[a] (21)	−5.284[a] (25)

In PP test figures in the brackets are bandwidth selected using the Newey–West method

[a] Reject the null hypotheis of a unit root with 99 % confidence

[b] Reject the null hypotheis of a unit root with 95 % confidence Figures in the brackets aginst ADF statistics are the numbers of lags used to obtain white noise residuals, and these lags are selected using AIC

5.1 Simple predictability regressions

This section will analyze the results of simple predictability regression. For testing the null hypothesis of unbiasedness ($\alpha = 0$ and $\beta = 1$) in the regression, we have used a Wald test. Because of the overlapping in the dependent variable we have used GMM estimation in our analysis. Table 7 part-I, displays the results for six simple regressions involving each of the volatility forecast models. Starting with the regression involving the call implied volatility (CIV) as only independent variable, the coefficient is significant at 1 % level with a value of 0.54. This coefficient value is also higher than those which we arrived in the information content regression. Here the R^2 statistics is 0.19.

Table 7 Predictability regressions

	α	Estimates of slope coefficient β						R^2	Wald test
		$\sigma_{t,CIV}$	$\sigma_{t,PIV}$	$\sigma_{t,MA(20)}$	$\sigma_{t,EWMA}$	$\sigma_{t,GARCH}$	$\sigma_{t,EGARCH}$		
Part-I: Simple predictability regressions									
Coefficient	0.10[a]	0.54[a]						0.19	30.157[a]
Statistics	5.338	6.348							
Coefficient	0.071[a]		0.53[a]					0.17	109.342[a]
Statistics	3.569		6.796						
Coefficient	0.105[a]			0.53[a]				0.25	60.191[a]
Statistics	7.757			8.018					
Coefficient	0.080[a]				0.62[a]			0.29	35.246[a]
Statistics	5.831				9.484				
Coefficient	0.073[a]					0.65[a]		0.29	18.880[a]
Statistics	4.130					8.026			
Coefficient	0.063[a]						0.70[a]	0.29	13.799[a]
Statistics	3.518						8.621		
Part-II: Multiple predictability regressions									
Coefficient	0.084[b]	0.35[b]		0.25[b]				0.27	39.932[b]
Statistics	5.761	2.971		2.674					
Coefficient	0.074[b]	0.28[b]			0.36[b]			0.30	44.475[b]
Statistics	5.316	2.268			3.289				
Coefficient	0.066[b]	0.27[b]				0.40[b]		0.30	52.563[b]
Statistics	4.123	2.483				3.301			
Coefficient	0.057[b]	0.28[b]					0.43[b]	0.31	58.011[b]
Statistics	3.449	2.716					3.656		
Coefficient	0.068[b]		0.31[b]	0.29[b]				0.27	122.825[b]
Statistics	3.923		2.932	3.207					
Coefficient	0.063[b]		0.22[c]		0.42[b]			0.30	129.675[b]
Statistics	3.770		1.966		3.917				
Coefficient	0.053[b]		0.23[b]			0.44[b]		0.30	148.490[b]
Statistics	2.990		2.476			3.777			
Coefficient	0.043[c]		0.26[b]				0.46[b]	0.31	52.365[b]
Statistics	2.315		2.897				3.907		

Regression involving put implied volatility (PIV) as an independent variable has shown a significant coefficient 0.53 at the 1 % level of significance. The value of R^2 statistics is lower at 0.17 and it is also lower than the R^2 of predictability regression involving CIV. In both the cases the Wald test is rejecting the null hypothesis at a 1 % level of significance.

In the simple predictability regression MA (20) as an independent variable shows coefficient value of 0.53. It is significant at the 1 % level, but its R^2 statistic is high in comparison to CIV and PIV. Regression involving EWMA comes under the same

Table 7 continued

	α	Estimates of slope coefficient β						R^2	Wald test
		$\sigma_{t,CIV}$	$\sigma_{t,PIV}$	$\sigma_{t,MA(20)}$	$\sigma_{t,EWMA}$	$\sigma_{t,GARCH}$	$\sigma_{t,EGARCH}$		
Part-III									
Coefficient	0.077[b]	0.36[b]	0.21[c]					0.22	36.323[b]
Statistics	3.992	2.934	1.885						

[a] 1 % level of Significance and the GMM estimation of Hansen (1982) along with Newey–West (1987) variance and covariance estimation is used

[b] 1 % level of Significance

[c] 5 % level of Significance and the GMM estimation of Hansen (1982) along with Newey–West (1987) variance and covariance estimation is used

Predictability of regressions

$$\sigma_{t,T} = \alpha + \beta\hat{\sigma}_{t,i} + \varepsilon_{t,T} \quad (12)$$

$$\sigma_{t,T} = \alpha + \beta_1\sigma_{t,CIV} + \beta_2\hat{\sigma}_{t,i} + \varepsilon_{t,T} \quad (13)$$

$$\sigma_{t,T} = \alpha + \beta_1\sigma_{t,PIV} + \beta_2\hat{\sigma}_{t,i} + \varepsilon_{t,T} \quad (14)$$

$$\sigma_{t,T} = \alpha + \beta_1\sigma_{t,CIV} + \beta_2\sigma_{t,PIV} + \varepsilon_{t,T} \quad (15)$$

track as with MA (20) with a high R^2 statistic of 0.29. And its coefficient value is higher than the above three methods at 0.38. Both the slope and coefficient are also significant at the 1 % level.

Regression involving GARCH (1, 1) shows a higher coefficient value 0.65 than CIV, PIV MA (20) and EWMA. This shows that it predicts the realized volatility well than the implied volatilities and simple historical volatility forecasting models. The coefficient is also significant at the level of 1 %.

Regression involving EGARCH (1, 1) has shown a significant coefficient value 0.70 at the 1 % level. It also shows that it is outperformed all other forecasting volatility models. It has not only a high significant coefficient is higher than the coefficient value, but it has a high R^2 statistic of 0.29.

Now we can compare the coefficient of across all six of these regressions. First, it is observed that the EGARCH volatility has a more explanatory power than other volatility forecasting models. Second, among the ARCH family models EGARCH (1, 1) has more explanatory power than simple GARCH (1, 1) model. Third, Call implied volatility performed better than MA (20) and Put Implied volatility. Forth, the MA (20) volatility and put implied volatility performance are least with a very low coefficient value, 0.53.

As per the reported Wald test, all six regressions reject the null hypothesis of unbiased ness at 1 % level.

An obvious candidate explanation for the apparent bias and inefficiency of implied volatility is that implied volatility is measured with error. It is well known that error in the independent variable creates downward bias; the estimated coefficient is inconsistent, smaller in absolute value than the true coefficient. The biasedness in the implied volatility arises because of the errors-in-variables and because of that implied volatility's failure to subsume other forecasts. There are three types of measurement error: specification error from using the wrong options model, nonsynchronous trading, and a Jump in prices.[5] First the Black–Scholes formula in Eqs. (8) and (9) applies a European style call and put option on an asset that doesn't pay dividends and the transaction cost is zero prior to expiration. Whereas in the actual Indian financial market the stocks associate with S&P CNX Nifty pays dividends and there are a transaction costs. However, dividends do reduce call values, so implied volatility computed via Eqs. (8) and (9) minimizes true implied volatility. The difference should be roughly constant for all time periods since dividends are relatively uniform for the Nifty. Thus, in regressions that use implied volatility as an independent variable, estimates of the intercept term should be biased.

Secondly, infrequent trading of the option may cause measurement errors in the implied volatility series. For our analysis we use closing option prices and closing price of the S&P CNX Nifty index. Both these markets are close at the same hour and the methodology of calculation of closing prices is same, but the Nifty options are not fluctuating as frequently as the cash market in our sample period. The option closing price may stem from a trade taking place earlier during the day. The cash markets do not suffer from the same liquidity problem as the option. When the new information entering into the market, there is an interval of time effect. Suppose the news is good. Then the recoded spot price will be higher than the price the option price is based on, implying that the call and put implied volatility minimize (maximize) the true volatility. The opposite occurs when the market declines. Since good and bad news occurs randomly in the market, there is no reason to believe that the implied volatility will deviate consistently from the true volatility. The main reason for this problem is the multiple time series are not sampled simultaneously.

Finally, we note that the Black–Scholes formula in Eqs. (8) and (9) assumes that index levels follow a log-normal diffusion process with deterministic volatility. For example, if there are jumps in the price of the underlying asset, the model dose not price the options correctly. Consequently, the Black–Scholes implied volatilities can be misspecified. This introduces a systematic bias in the implied volatilities. Even if the Black–Scholes formula is correct, market microstructure effects may cause additional measurement errors (Harvey and Whaley 1991). As we can see there the measurement error of implied volatility has three parts: the true modeled implied volatility, a systematic bias and an idiosyncratic measurement error. Here in the empirical analysis, we are trying to control the idiosyncratic measurement error implied volatility.

5.2 Multiple predictability regressions

Table 7 part-II, displays the results for the predictability regressions involving multiple regressions.

In the regression involving CIV and MA (20) as independent variables shown that, both the coefficients are significant. CIV has a high coefficient value of 0.35 at

[5] Christensen and Prabhala (1998).

the 1 % level. The MA (20) failed beat CIV and displays a very low coefficient value 0.25. The R^2 statistics established a high value in the simple predictability regressions.

In the regression involving CIV and EWMA, the CIV is significant at the 1 % level with a coefficient value of 0.28, which, means a 10 % increase in CIV implies a 2.8 % increase in future volatility. The result also shows that the coefficient value of EWMA is high, positive and significant

Regression involving CIV and GARCH (1, 1) model shows that GARCH (1, 1) outperformed CIV model. The GARCH (1, 1) is significant at the 1 % level with a coefficient value of 0.40 with t statistics 3.301 whereas, CIV has a low coefficient value of 0.27. It has a positive value, but it is lower than the coefficient of GARCH (1, 1) and the corresponding t-statistics are also very low.

In the regression involving CIV and EGARCH (1, 1), the CIV replicates the same result, it has underperformed than that of EGARCH (1, 1). It shows a significant coefficient value of 0.28 at the 1 % level. Whereas, EGARCH (1, 1) shows a highly significant coefficient value 0.43. No doubt if we compare with MA (20), EWMA and GARCH (1, 1) in the same format, EGARCH (1, 1) is the best in high significant value of 0.43.

After comparing CIV, with different backward looking volatility forecasting models, we are now going to compare PIV to different backward looking volatility models.

While comparing PIV with MA (20) in a multiple regression, we found that PIV shows a significant coefficient value of 0.31 at the 1 % level, whereas MA (20) shows a low significant coefficient value of 0.29. Here the value of R^2 is at 0.27.

However, in comparison with the EWMA model, we have found the same short of result as in CIV. In this case also, the EWMA outperformed PIV. The coefficient value of EWMA is at 0.42 at the 1 % level of significance. The only difference is that here the PIV coefficient is significant at the 5 % level with a value of 0.22.

In comparison with GARCH (1, 1), the PIV performed inferior than GARCH (1, 1). It is significant at 1 % with a coefficient value 0.23. Whereas the coefficient of GARCH (1, 1) shows a better magnitude than MA (20) and EWMA at the 1 % level of significance, however, in comparison to information content regression, it has a low coefficient value.

As we have already discussed information content regression shows the EGARCH (1, 1) is the best model to explain 1 day ahead volatility and also the best method to predict the future volatility in a simple predictability regression. Now the question arises whether it is better in a multiple predictability regression. Comparing EGARCH with PIV we found that PIV has a significant coefficient value 0.26 at the 1 % level. Nevertheless the coefficient value of EGARCH (1, 1) is higher than PIV. It is significant at the 1 % level with a coefficient value of 0.46. It has a high R^2 statistic than the R^2 of the multiple predictability regression taking MA (20), EWMA and GARCH (1, 1) as one of the independent variable.

In the light of the above discussion, we found that time series volatility is the better forecasting models to predict the realized volatility in comparison to other forward looking volatility models. Now the question arises among the forward looking methods which one has more explanatory power.

Table 7, part-III, compare both implied volatilities as per the regression Eq. 15. The result shows that CIV has more explanatory power than PIV. It also shows that CIV has a significant coefficient value of 0.36 at the 1 % level where as PIV has a lower coefficient value 0.21 at the significance level of 5 %.

As per the reported Wald test, in the three panels of the table, all six regressions reject the null hypothesis of unbiased ness at 1 % level. The results show that the implied volatility are biased downside and also inefficient compare to other backward looking volatility methods.

5.3 Out of sampling forecasting model

We now consider the out of sample predictive power of the alternative forecasting models described in Sect. 4.4. Here, the specific interest is to examine whether implied volatility will show the same result as in the 'in the sample' framework. Below we evaluate the forecasting performances for 30 days horizons. For each horizon, we have constructed a series of non-overlapping forecasting window.

Average volatility of 30 days horizon is constructed from MA (20), EWMA, CIV and PIV on the day where there are approximately 30 days left to expiry of the option from 31st May 2006 to 30th June 2011. Using the in the sample data from 1st June 2001 to 31st May 2006, the GARCH (1, 1) GED and EGARCH (1, 1) GED models are estimated in order to compare out of sample forecasting performance with MA (20), EWMA, CIV and PIV.

Table 8 Comparison of out of sample forecasts with realized volatility

Error	σ_T^{EWMA}	σ_T^{MA}	σ_T^{CIV}	σ_T^{GARCH}	σ_T^{EGARCH}	σ_T^{PIV}
MSE	0.01548	0.01746	0.03175	0.01495	0.01608	0.02659
MAE	0.09077	0.09726	0.10658	0.09274	0.08916	0.12169
RMSE	0.12441	0.13213	0.17820	0.12228	0.12682	0.16307
MAPE	35.85669	37.35442	48.59276	39.15527	35.84027	56.90867

$\sigma_{t,T}$ is realized volatility, σ_T^{CIV} as call implied volatility, σ_T^{PIV} as put implied volatility, σ_T^{GARCH} as GARCH (1, 1), σ_T^{EGARCH} as EGARCH (1,1) volatility, and σ_T^{EWMA} as Exponential weighted Moving Average Volatility and σ_T^{MA} is the Moving average volatility in an out of sample framework. All the volatilities are 22 days ahead volatility

MSE mean square error, *MAE* mean absolute error, *RMSE* root mean square error, *MAPE* mean absolute percentage error

In out of sample, we have generated 61 one month forecasted value for each specified model. On each forecast point, the update GARCH (1, 1) and EGARCH (1, 1) forecast σ_T^{GARCH} and σ_T^{EGARCH} are constructed. Thus, by using the parameters estimated over the previous 1255 trading days by adding the approximately latest 22 observations and deleting the approximately first 22 observations in the previous sample each time. We are doing it for 61 times to get 61 forecasted values of GARCH and EGARCH. Figure 11 plots the six forecasted volatility models against realized volatility.

The result reported Table 8 presents the four error statistics for different models. The Table 8 summarized that out of six forecasted models, the ARCH models are clearly superior to other forecasting models in the case of all error statistics.

6 Conclusion

This paper investigates whether implied volatility predicts the future volatility beyond the backward looking volatilities. To examine this, we are using OLS with GMM techniques to overcome the overlapping data problem. For the analysis we use the daily return of S&P CNX Nifty from 1st June, 2001 to 24th Jun 2011 and its corresponding option markets spanning from the period 4th June, 2001 to 23rd June, 2011. There are several key conclusions that may draw from this analysis. First, in a simple predictability regression EGARCH (1, 1) model outperformed all other volatility forecasted models. Second, PIV is the worst performer in the predictability regression analysis. Third, the multiple predictability regression results show that both GARCH (1, 1) and EGARCH (1, 1) provide an informative forecast of future volatility that are superior to those of simple historical volatility and implied volatility. Fourth, among both implied volatilities CIV has more predictability power to explain the future volatility in comparison to PIV. Fifth, implied volatility are biased and inefficient estimator over the remaining life of the option. The biasedness can be explained by two possible interpretations, firstly, the error in variables problem, i.e., (1) specification error from using the options model with constraints assumption, (2) nonsynchronous trading in option market in India, and (3) Jump on index and option prices. Secondly the inefficiency in the Indian option market, the underlying asset is the spot market, but when investors are hedging their option positions then they use futures to hedge because there is a restriction on short selling in India and there is no traded option on futures contract. In out of sample analysis the family of ARCH models outperformed all other volatility forecast models in respect to predicting 30 days ahead volatility.

Appendix

See Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11.

Fig. 1 Trends in index closing price

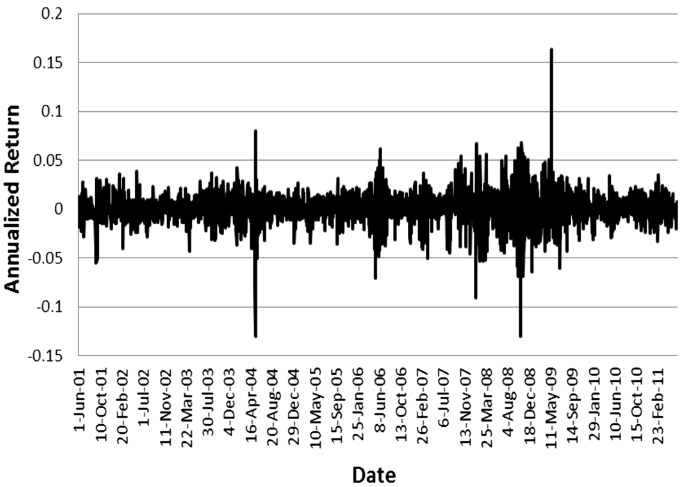

Fig. 2 Trends in return

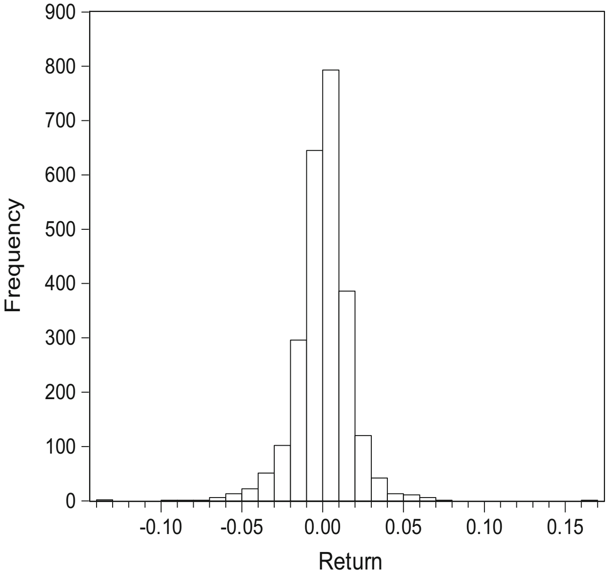

Fig. 3 Histogram showing the distribution of return series

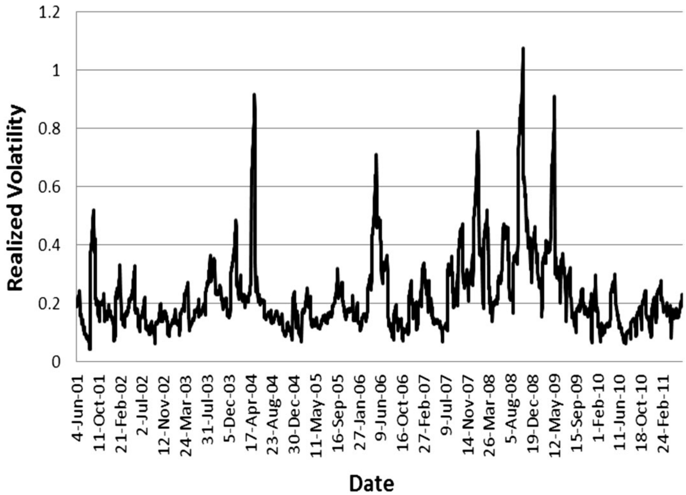

Fig. 4 Trends in realized volatility

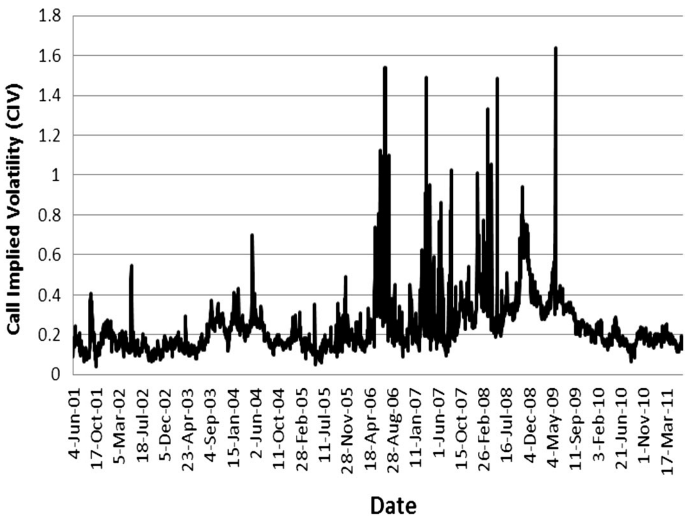

Fig. 5 Trends in call implied volatility (CIV)

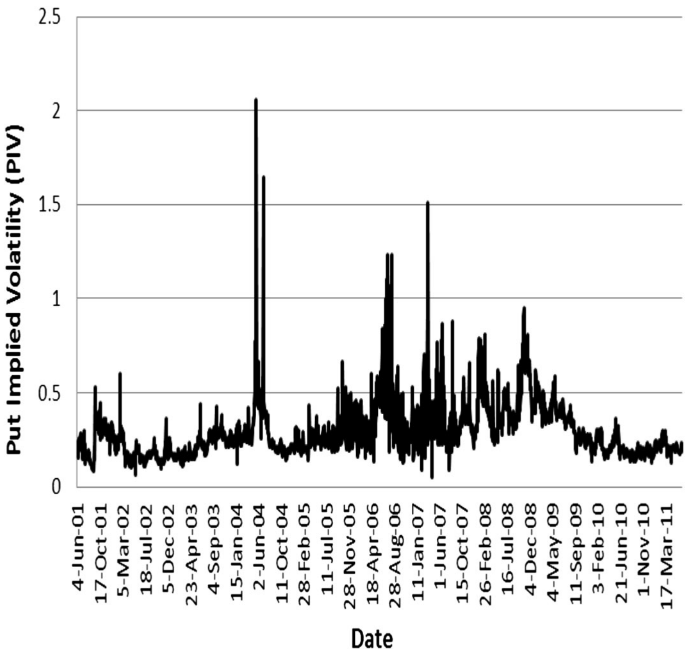

Fig. 6 Trends in put implied volatility (PIV)

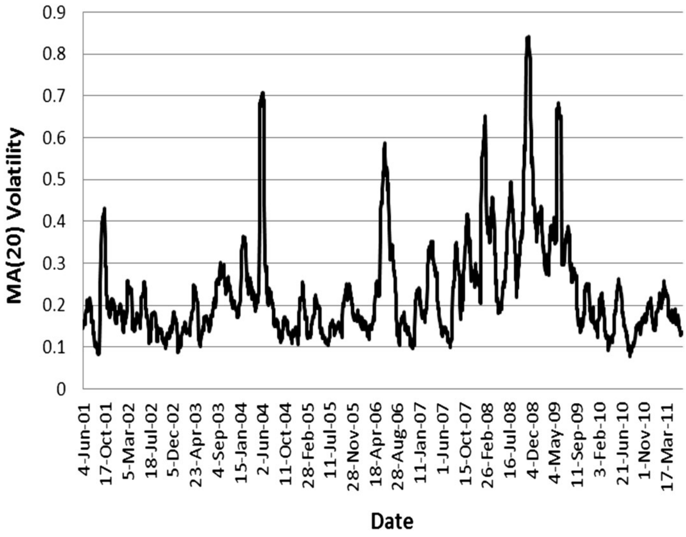

Fig. 7 Trends in MA(20) volatility

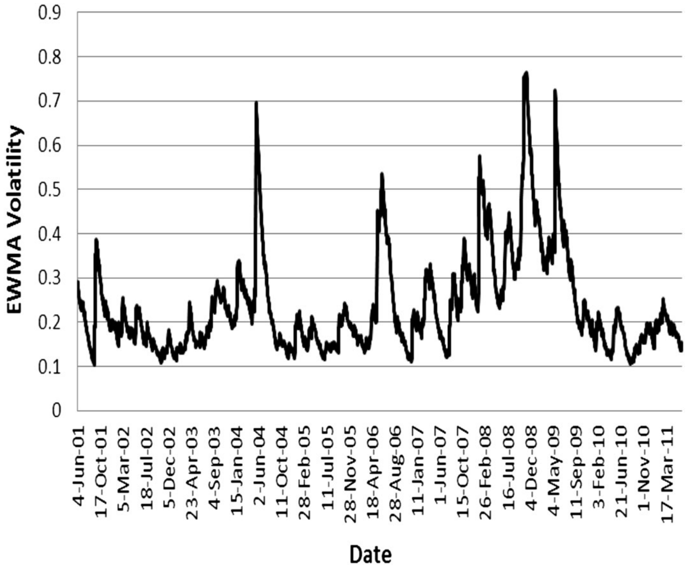

Fig. 8 Trends in EWMA volatility

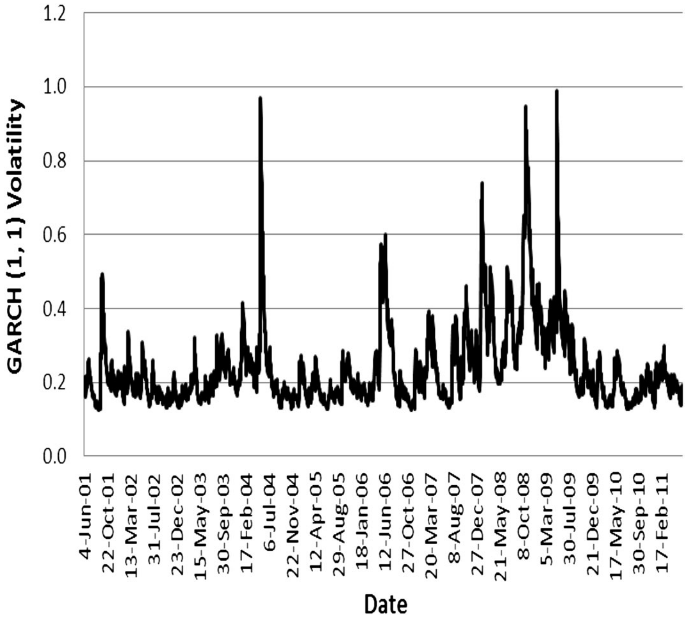

Fig. 9 Trends in GARCH (1, 1) volatility

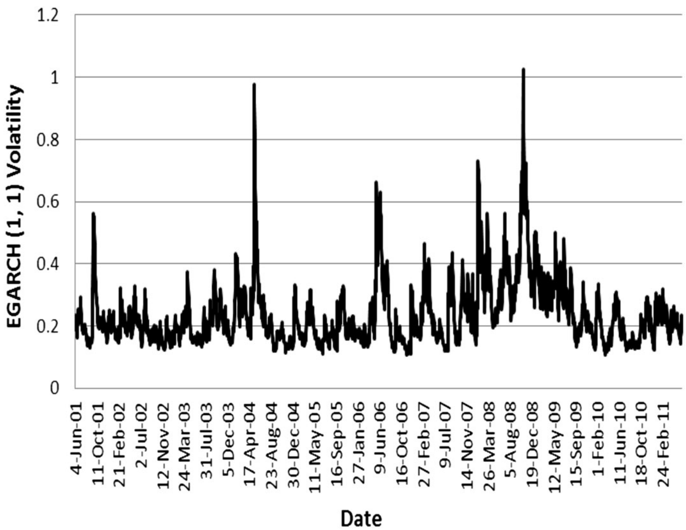

Fig. 10 Trends in EGARCH (1, 1) volatility

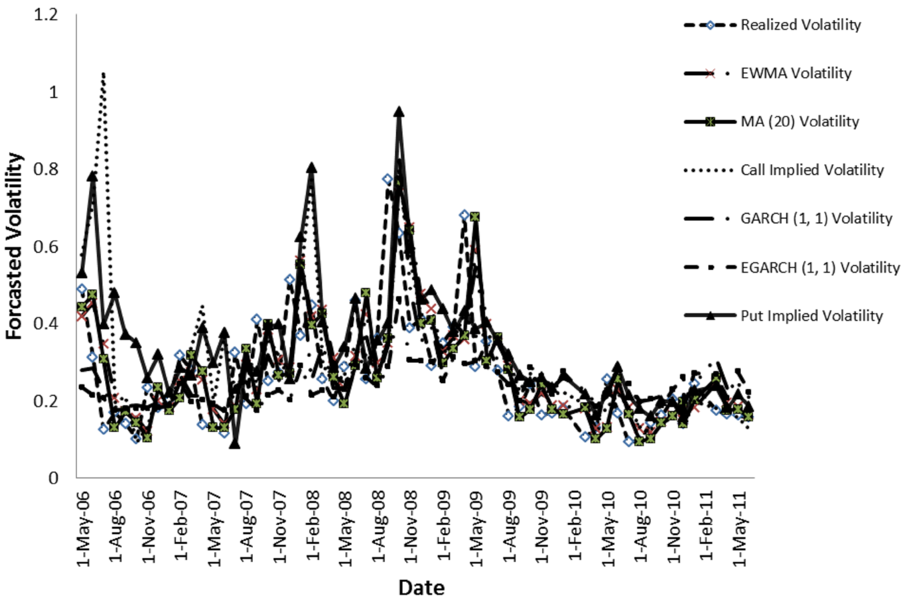

Fig. 11 Comparison of out-of-sample forecast with realized volatility: a 1 month forecast

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