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ORIGINAL PAPER

Stochastic Evaluation of Serial Multi-Purpose Solar System Using Gumbel–Hougaard Family Copula

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Abstract

Considering the high cost and safety risks linked with traditional energy sources such as fossil fuels, hydroelectricity, thermal electricity etc. this made it mandatory for the globe to investigate in renewable energy sector. In line with above mentioned issues, the present study concentrated on a multi-purpose solar system consisting of four subsystems: a solar panel, a controller, two USB charge points and four electric bulb points all merged together in a series–parallel conformation. Components/units failure is constant and in line with exponential function, copula and general repairs are available for the repair of partial and entire system failure. The transition diagram of the system is used to derived the partial differential equations of order one, and solved using supplementary variable and Laplace transformation procedures. Maple software package was used to generate expressions for availability, reliability, mean time to failure, sensitivity and cost. Results were justified using particular examples and presented in tables and figures.

Keywords Availability · Reliability · MTTF · Sensitivity · Cost · Solar

Introduction

It is vital to identify alternative energy sources to replace the current ones due to the issues of global warming that are facilitated by the emissions from the use of conventional energy sources including fossil fuels, hydroelectricity, thermal electricity, and others. However, extensive researches are conducted worldwide which eventually led to the existing of solar energy. Since then, simple and sophisticated solar-powered devices have been created in order to overcome the challenges of safety hazards and high cost of outdated energy sources.

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Solar energy as user-friendly, affordable, reliable, dependable, simple, low maintenance cost and efficiency made it popular renewable energy resource and recognizes by society over the world. These peculiarities made it possible for everyone to quickly switched to the use of solar energy for better performance of their gadgets and generate mega profit. Nevertheless, this made available provisions for individuals, groups, industrial and manufacturing companies, governments, non-governmental organizations (NGOs), business enterprises, etc. Even though, there are issues linked to the usage of solar energy, such as geographical location, timing (day and night), season, and so on. Yet, society accepted it. However, advanced researches are currently ongoing in an effort to overcome the aforementioned obstacles. The present advancements in science and technology have made it possible to eliminate the limitations placed on the usage of solar energy using backup systems, energy storage devices, and more complicated gadgets.

Considering the advantages of solar-power over the obsolete sources of energy like fossil fuels, hydroelectricity, thermal electricity, and others, these necessitated researchers from all over the world to double their effort in making simulations. Many systems were investigated using various procedures, different assumptions of parameters, different systems designed and configurations all in line to produce simple and strong solar-power systems that will satisfy the need of the society in general. The impact of discrete multi-arc rib roughness on the effective efficiency of a solar air heater was studied by Arwa et al. [1]. An analysis of the performance and economic feasibility of a hybrid solar cooling system that combines an ejector with vapor compression cycle powered by a photovoltaic thermal (PV/T) unit was examined by Ghassan and Mohammad [2] Molamohamadi and Talaei [3] focused on the analysis of a proper strategy for solar energy deployment in Iran using SWOT matrix. Mohamad et al. [4] carried out the performance analysis of solar absorption ice maker driven by parabolic trough collector. Assessment of the performance of solar water heater: an experimental and theoretical investigation was inquired by Naseer et al. [5]. Shateri et al. [6] investigated the optimization of heat transfer in an enclosure with a Trombe wall and solar chimney. Al-Widyana et al. [7] investigated the effect of alkaline nitrates and operating temperature on the performance of dye sensitized solar cells. An experimental and theoretical analysis of thermal losses in a flat plate solar heater with multi risers and headers was inquired by Al-Saiydee [8]. Harrabi et al. [9] examined the assessment of uncertainties in energetic and energetic performances of a flat plate solar water heater. Energy and economic analysis of curved and spiral flow flat-plate solar water collector was reported by Muthuraman et al. [10]. Shamshirgaran et al. [11] investigated the state of the art of techno-economics of nanofluid-laden flat-plate solar collectors for sustainable accomplishment. The energy and exergy evaluation of the evacuated tube solar collector using Cu2O/water nanofluid utilizing ANN methods was conducted by Sadeghi et al. [12].

However, advanced studies were carried out on sophisticated repairable systems where authors involved the use of copula procedures and assuming different failures and repairs to assessed the reliability elements. Reliability analysis of multi-workstation computer network configured as series–parallel system via Gumbel—Hougaard family copula was inquired by Isa et al. [13]. Ismail. [14] investigated the reliability and cost analysis of sachet water plant using Copula approach. Gupta et al. [15] focused on the behavioral analysis of cooling tower in steam turbine power plant using reliability, availability, maintainability and dependability investigation. Kumar et al. [16] conducted the analysis of a redundant system with priority and Weibull distribution for failure and repair. Singh et al. [17] focused on a performance analysis of a complex repairable system with two subsystems in series configuration with imperfect switch. Rawal et al. [18, 19] examined the reliability assessment of multi-computer system consisting n clients and k-out-of-n: g operational scheme with copula repair policy.

Mathematical modeling of sugar plant: a fuzzy approach was reported by Kumar and Saini [20]. Performance analysis of a computer system with imperfect fault detection of hardware was examined by Kumar et al. [21]. Yusuf et al. [22] investigated the use of copula approach to performance evaluation of manufacturing system. Saini et al. [23] carried out the availability optimization of biological and chemical processing unit using genetic algorithm and particle swarm optimization. Reliability and performance analysis of a series–parallel photovoltaic system with human operators using Gumbel-Hougaard family copula was examined by Maihulla et al. [24]. Yusuf et al. [22] reported on a Copula approach to performance evaluation of manufacturing system. Beside, many works similar to this study were previously presented by authors, nonetheless, no single author focused accurately on a multi-purpose solar system made up of four separate subsystems: a solar panel, a controller, two USB charge points, and four electric bulb points that were all merged together in a series–parallel conformation and studied using Gumbel–Hougaard family copula approach. Components/units failure is constant and in line with exponential function, copula and general repairs are available for the repair of partial and entire system failure. The transition outline of the system was used to derived the partial differential equations of order one, and solved using supplementary variable and Laplace transformation procedures. Maple software package was used to generate expressions for availability, reliability, MTTF, sensitivity and cost. Results were justified using particular examples as presented in Tables and Figures.

Notations, Assumptions, System Description, State Description

Notations

N Represent time.

u_j Represnt failures of subsystems, and j = 1, 2, 3 and 4.

$g_1(h_1)/g_2(h_2)$ Represent repairs of subsystem 3 and 4.

$r_0\left(h_j\right)$ Represent repairs of whole system failure, and j = 1, 2, 3 and 4.

$Z_i(n)$ Represent the states of the system, and i = 0, 1, …, 8.

$\overline{Z}(s)$ Represent the Laplace conversion of $Z(n)$.

$Z_i\left(h_j, n\right)$ Represent states probability, repair variable and time for repair, where. i = 0, 1…, 8, and j = 1, 2, 3 and 4.

$E_g(n)$ Represent the expected gain within the range [0, n].

G_1, G_2 Cost essentials.

$S_g(h)$ Represent function such as $S_g(h)= g(h)e^{-\int_o^h g(h)dh}$.

$\overline{S}_h(s)$ Represent Laplace conversion of $S_g(h)$ as $\overline{S}_g(s) = \int_0^\infty e^{-sh} g(h)e^{-\int_0^h g(h)dh} dh$.

$$r_0(h) = \mathrm{C}_\theta\ (r_1(h)_, r_2(h))\ c_\theta(r_1(h), r_2(h)) = \exp\left(h^\theta + \left\{\log g(h)^\theta\right\}^{\frac{1}{\theta}}\right), 1 \le \theta \le \infty.$$

Represent copula repair facility, where $r_1 = g(h)$, and $r_2 = e^h$.

Assumptions

In the process of authenticating the model these were taken into consideration:

Normaly, everything in the system are satisfactory.

Table 1 States description

S/N	States	Descriptions
1	S_o	All subsystems/units are satisfactory
2	S_1	One unit of subsystem 4 fault, the backup unit turned to function for the replacement of the failed unit, therefore, the faulted unit undergone repair path, system operates
3	S_2	One unit of subsystem 3 fault, the backup unit turned to function for the replacement of the failed unit, therefore, the faulted unit undergone repair path, system operates
4	S_3	One unit of subsystem 3 fault, the backup unit turned to function for the replacement of the failed unit, therefore, the faulted unit undergone repair path, system operates
5	S_4	One unit of subsystem 4 fault, the backup unit turned to function for the replacement of the failed unit, therefore, the faulted unit undergone repair path, system operates
6	S_5	Second unit of subsystem 4 faulted, therefore, the entire system failed
7	S_6	Unit of subsystem 3 faulted for the second time, therefore, the entire system failed
8	S_7	System complete failure as a result of solar panel failure
9	S_8	System complete failure as a result of controller failure

For the system to fucntion the four subsystems are required.
There should be sunlight.
Failure of components is inevitable, though it can be addresses operating or otherwise.
Partial and whole system failure are tackled by mean of general and copula repairs.
System function satisfactory after been repaired.

System Description

Conventionally, the solar system consisting of four subsystems: a solar panel, a controller, two USB charge points and four electric bulb points all merged together in a series–parallel conformation. Initially, the system is performing satisfactory, instantly one unit of system 3 and 4 failed, the backup unit turned to function for the replacement of the failed unit, and the failed unit has undergone repair path, system operate, and the second unit of these subsystems failed, so the subsystems failed, and this led the entire system to stopped working. Similar to this, subsystem 1 and 2 failed, therefore, the whole system breaks down. Therefore, subsystem 1 and 2 are presumed to be delicate, and for that extra cares are required for system better performance. Eventually, partial failure is resolved using general repair, and system failure is fixed by mean of copula repair (Table 1; Figs. 1, 2).

States Description

Origination of Mathematical Model of the Solar System

The incoming set of partial differential equations are generated from the transition plan of the solar system.

$$\left(\frac{\partial}{\partial n} + 3u_1 + u_2 + u_3 + u_4\right)Z_0(n) = \int_0^\infty g_1(h_1)Z_1(h_1, n)dh_1 + \int_0^\infty g_2(h_2)Z_2(h_2, n)dh_2$$

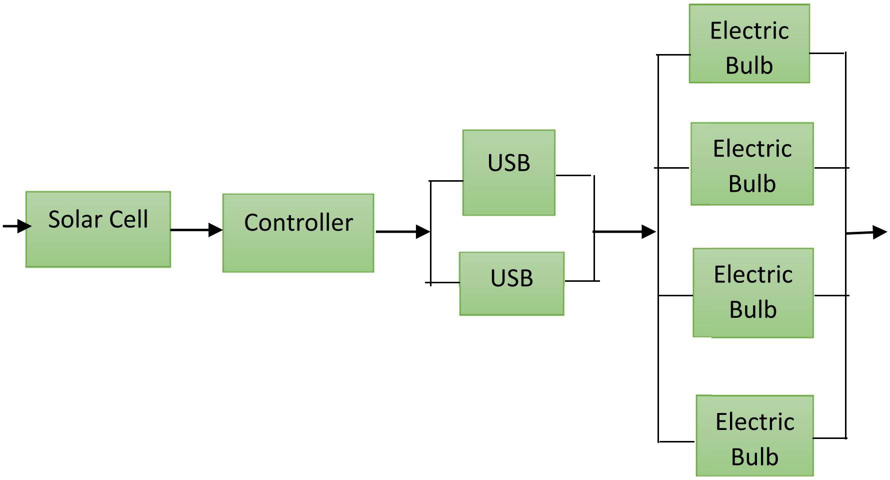

Fig. 1 Solar system block diagram

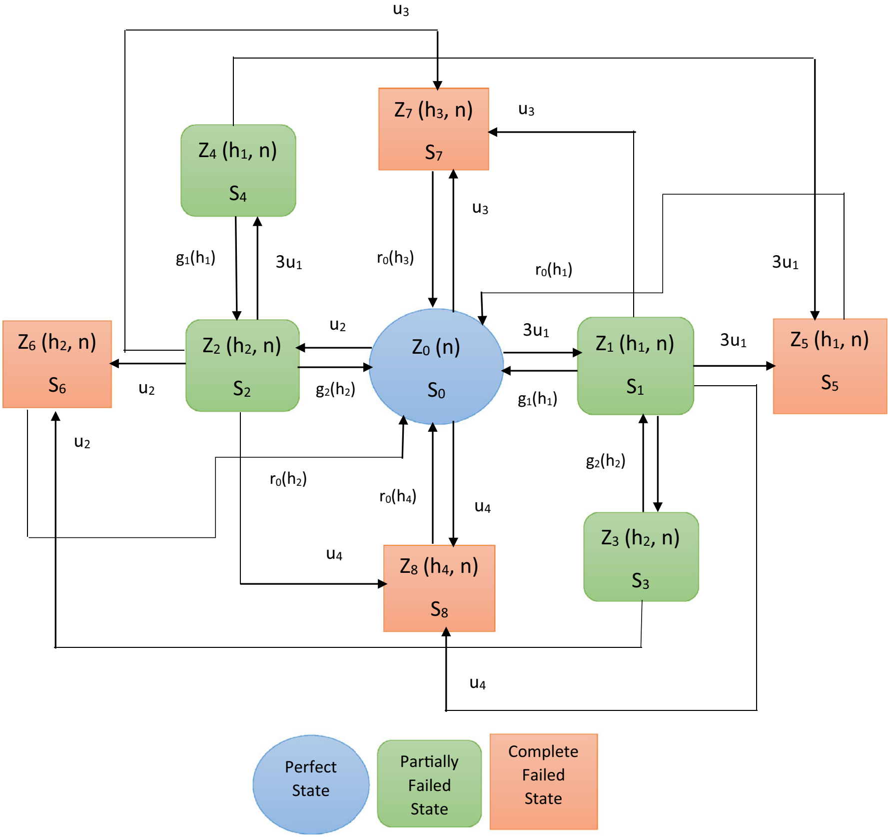

Fig. 2 Solar system transition plan

$$+\int_0^\infty r_0(h_1)Z_5(h_1,n)dh_1+\int_0^\infty r_0(h_2)Z_6(h_2,n)dh_2 + \int_0^\infty r_0(h_3)Z_7(h_3,n)dh_3+\int_0^\infty r_0(h_4)Z_8(h_4,n)dh_4 \tag{1}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_1}+3u_1+u_2+u_3+u_4+g_1(h_1)\right)Z_1(h_1,n)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_2}+3u_1+u_2+u_3+u_4+g_2(h_2)\right)Z_2(h_2,n)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_2}+u_2+g_2(h_2)\right)Z_3(h_2,n)=0 \tag{4}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_1}+3u_1+g_1(h_1)\right)Z_4(h_1,n)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_1}+r_0(h_1)\right)Z_5(h_1,n)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_2}+r_0(h_2)\right)Z_6(h_2,n)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_3}+r_0(h_3)\right)Z_7(h_3,n)=0 \tag{8}$$

$$\left(\frac{\partial}{\partial n}+\frac{\partial}{\partial h_4}+r_0(h_4)\right)Z_8(h_4,n)=0 \tag{9}$$

Boundary Conditions

$$Z_1(0,n)=3u_1Z_0(n) \tag{10}$$

$$Z_2(0,n)=u_2Z_0(n) \tag{11}$$

$$Z_3(0,n)=u_2Z_1(0,n) \tag{12}$$

$$Z_4(0,n)=3u_1Z_2(0,n) \tag{13}$$

$$Z_5(0,n)=3u_1(Z_1(0,n)+Z_4(0,n)) \tag{14}$$

$$Z_6(0,n)=u_2(Z_2(0,n)+Z_3(0,n)) \tag{15}$$

$$Z_7(0,n)=u_3(Z_0(n)+Z_1(0,n)+Z_2(0,n)) \tag{16}$$

$$Z_8(0,n)=u_4(Z_0(n)+Z_1(0,n)+Z_2(0,n)) \tag{17}$$

Initial Conditions

$$Z_0(0)=1,\ \text{the remaining state probabilities at n} = 0,\ \text{are zero,} \tag{18}$$

Solution of Mathematical Model of Solar System

The incoming equations are generated by mean of Laplace conversion of Eqs. (1) to (17) with assist of initial conditions.

$$\begin{aligned}(s + 3u_1 + u_2 + u_3 + u_4)\overline{Z}_0(s) &= 1 + \int_0^\infty g_1(h_1)\overline{Z}_1(h_1, s)dh_1 + \int_0^\infty g_2(h_2)\overline{Z}_2(h_2, s)dh_2 + \int_0^\infty r_0(h_1)\overline{Z}_5(h_1, s)dh_1 + \int_0^\infty r_0(h_2)\overline{Z}_6(h_2, s)dh_2 \\ &+ \int_0^\infty r_0(h_3)\overline{Z}_7(h_3, s)dh_3 + \int_0^\infty r_0(h_4)\overline{Z}_8(h_4, s)dh_4\end{aligned} \tag{19}$$

$$\left(s + \frac{\partial}{\partial h_1} + 3u_1 + u_2 + u_3 + u_4 + g_1(h_1)\right)\overline{Z}_1(h_1, s) = 0 \tag{20}$$

$$\left(s + \frac{\partial}{\partial h_2} + 3u_1 + u_2 + u_3 + u_4 + g_2(h_2)\right)\overline{Z}_2(h_2, s) = 0 \tag{21}$$

$$\left(s + \frac{\partial}{\partial h_2} + u_2 + g_2(h_2)\right)\overline{Z}_3(h_2, s) = 0 \tag{22}$$

$$\left(s + \frac{\partial}{\partial h_1} + 3u_1 + g_1(h_1)\right)\overline{Z}_4(h_1, s) = 0 \tag{23}$$

$$\left(s + \frac{\partial}{\partial h_1} + r_0(h_1)\right)\overline{Z}_5(h_1, s) = 0 \tag{24}$$

$$\left(s + \frac{\partial}{\partial h_2} + r_0(h_2)\right)\overline{Z}_6(h_2, s) = 0 \tag{25}$$

$$\left(s + \frac{\partial}{\partial h_3} + r_0(h_3)\right)\overline{Z}_7(h_3, s) = 0 \tag{26}$$

$$\left(s + \frac{\partial}{\partial h_4} + r_0(h_4)\right)\overline{Z}_8(h_4, s) = 0 \tag{27}$$

Boundary conditions

$$\overline{Z}_1(0, s) = 3u_1\overline{Z}_0(s) \tag{28}$$

$$\overline{Z}_2(0, s) = u_2\overline{Z}_0(s) \tag{29}$$

$$\overline{Z}_3(0, s) = u_2\overline{Z}_1(0, s) \tag{30}$$

$$\overline{Z}_4(0, s) = 3u_1\overline{Z}_2(0, s) \tag{31}$$

$$\overline{Z}_5(0, s) = 3u_1\left(\overline{Z}_1(0, s) + \overline{Z}_4(0, s)\right) \tag{32}$$

$$\overline{Z}_6(0, s) = u_2\left(\overline{Z}_2(0, s) + \overline{Z}_3(0, s)\right) \tag{33}$$

$$\overline{Z}_7(0, s) = u_3\left(\overline{Z}_0(s) + \overline{Z}_1(0, s) + \overline{Z}_2(0, s)\right) \tag{34}$$

$$\overline{Z}_8(0, s) = u_4\left(\overline{Z}_0(s) + \overline{Z}_1(0, s) + \overline{Z}_2(0, s)\right) \tag{35}$$

The incoming equations are generated with aid of boundary conditions

$$\overline{Z}_0(s) = \frac{1}{F(s)} \tag{36}$$

$$\overline{Z}_1(s) = \frac{3u_1}{F(s)}\left(\frac{1-\overline{s}_{g_1}(s+3u_1+u_2+u_3+u_4)}{s+3u_1+u_2+u_3+u_4}\right) \tag{37}$$

$$\overline{Z}_2(s) = \frac{u_2}{F(s)}\left(\frac{1-\overline{s}_{g_2}(s+3u_1+u_2+u_3+u_4)}{s+3u_1+u_2+u_3+u_4}\right) \tag{38}$$

$$\overline{Z}_3(s) = \frac{3u_1u_2}{F(s)}\left(\frac{1-\overline{s}_{g_2}(s+u_2)}{s+u_2}\right) \tag{39}$$

$$\overline{Z}_4(s) = \frac{3u_1u_2}{F(s)}\left(\frac{1-\overline{s}_{g_1}(s+3u_1)}{s+3u_1}\right) \tag{40}$$

$$\overline{Z}_5(s) = \frac{\left(9u_1^2+9u_1^2u_2\right)}{F(s)}\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right) \tag{41}$$

$$\overline{Z}_6(s) = \frac{\left(u_2^2+3u_2^2u_1\right)}{F(s)}\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right) \tag{42}$$

$$\overline{Z}_7(s) = \frac{(u_3+3u_1u_3+u_2u_3)}{F(s)}\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right) \tag{43}$$

$$\overline{Z}_8(s) = \frac{(u_4+3u_1u_4+u_2u_4)}{F(s)}\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right) \tag{44}$$

where $F(s)$ is generated as:

$$F(s) = \left(s+3u_1+u_2+u_3+u_4 - \left(\begin{array}{l} 3u_1\overline{S}_{g_1}(s+3u_1+u_2+u_3+u_4) \\ +u_2\overline{S}_{g_2}(s+3u_1+u_2+u_3+u_4) \\ +\left(\begin{array}{l} 9u_1^2+9u_1^2u_2+u_2^2+3u_1u_2^2+u_3+3u_1u_3 \\ +u_2u_3+u_4+3u_1u_4+u_2u_4 \end{array}\right)\overline{S}_{r_0}(s) \end{array}\right)\right) \tag{45}$$

The working time of the solar system is obtained as;

$$\overline{Z}_{up}(s) = \frac{1}{F(s)}\left(\begin{array}{l} 1+3u_1\left(\dfrac{1-\overline{s}_{g_1}(s+3u_1+u_2+u_3+u_4)}{s+3u_1+u_2+u_3+u_4}\right) \\ +u_2\left(\dfrac{1-\overline{s}_{g_2}(s+3u_1+u_2+u_3+u_4)}{s+3u_1+u_2+u_3+u_4}\right)+ \\ 3u_1u_2\left(\dfrac{1-\overline{s}_{g_2}(s+u_2)}{s+u_2}\right)+3u_1u_2\left(\dfrac{1-\overline{s}_{g_1}(s+3u_1)}{s+3u_1}\right) \end{array}\right) \tag{46}$$

Similar to this, the off time of the solar system was generated as:

$$\overline{Z}_{off}(s) = 1-\overline{Z}_{up}(s) \tag{47}$$

Table 2 The availability of the solar system with esteem to time

n	Copula repair facility's availability	General repair facility's availability
0	1.0000	1.0000
1	0.9985	0.9957
2	0.9981	0.9934
3	0.9972	0.9918
4	0.9961	0.9904
5	0.9950	0.9892
6	0.9938	0.9879
7	0.9926	0.9867
8	0.9913	0.9855
9	0.9901	0.9843
10	0.9889	0.9831

Assessment of the Mathematical Model of Solar System for Copious States

Availability Analysis of the Solar System via General and Copula Repair Facilities

Allowing the copula repair facility as: $c_\theta(r_1(h), r_2(h)) = \exp\left(h^\theta + \left\{\log g(h)^\theta\right\}^{\frac{1}{\theta}}\right)$, $\overline{s}_g(s) = \frac{g}{s+g}$, failure rates as: $u_1 = 0.0123$, $u_2 = 0.0213$, $u_3 = 0.0031$, $u_4 = 0.0045$, and all repairs are taken to one, however, these values were used in Eq. (46), the consequences were converted using the inverse Laplace transformation to obtained availability expressions for copula and general repair facilities as:

$$\overline{Z}_{up}(n) = \begin{pmatrix} -0.000344e^{-1.02130n} - 0.000261e^{-1.03690n} \\ +0.003743e^{-2.72854n} - 0.004273e^{-1.12013n} \\ +1.001136e^{-0.00122n} \end{pmatrix} \tag{48}$$

$$\overline{Z}_{up}(n) = \begin{pmatrix} -0.000180e^{-1.12517n} + 0.005333e^{-1.00521n} \\ +0.995211e^{-0.00121n} - 0.000470e^{-1.02130n} \\ -0.000289e^{-0.03690n} \end{pmatrix} \tag{49}$$

By means of Eqs. (48) and (49), the availability of the system was determined as presented in Table 2 (Fig. 3).

Reliability Analysis of the Solar System

Supposing the failure rates as; $u_1 = 0.0123$, $u_2 = 0.0213$, $u_3 = 0.0031$, $u_4 = 0.0045$, repairs to zero, and these were used in Eq. (46), the consequences were converted using

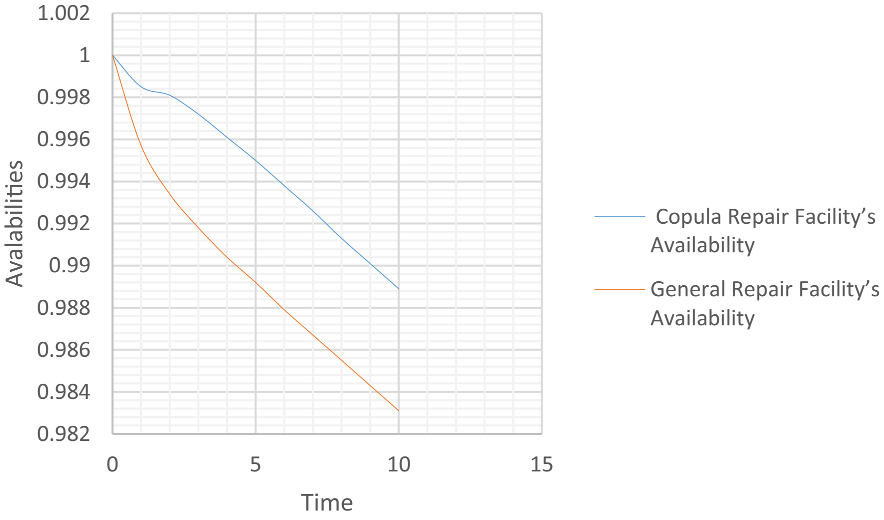

Fig. 3 shows availabilities of the solar system against time

inverse Laplace transformation to produce reliability expression as:

$$R(n) = \begin{pmatrix} 0.017662e^{-0.02130n} \\ +0.027196e^{-0.03690n} \\ +1.555149e^{-0.06580n} \end{pmatrix} \tag{50}$$

Via Eqs. (50), the reliability of the system was determined as shown in Table 3 (Fig. 4).

MTTF Analysis of the Solar System

Repairs were assumed to zero and s approaches 0, these values were used in Eq. (46), the consequence is MTTF expression as:

$$MTTF = \lim_{s \to 0} \overline{Z}_{up}(s) = \frac{1}{3u_1 + u_2 + u_3 + u_4} \begin{pmatrix} 1 + \dfrac{3u_1}{3u_1 + u_2 + u_3 + u_4} \\ + \dfrac{u_2}{3u_1 + u_2 + u_3 + u_4} \\ +3u_1 + u_2 \end{pmatrix} \tag{51}$$

Via Eq. (51), MTTF of each failure rate was determined and presented in Table 4 (Fig. 5).

Sensitivity Analysis of the Solar System

Sensitivity expression of the system was obtained as a result of differentiating the MTTF expression partially with esteem to failure rate. Therefore, the sensitivity of each failure rate was calculated and depicted in Table 5 (Fig. 6).

Table 3 The reliability of the solar system with esteem to time

n	0	1	2	3	4	5	6	7	8	9	10
Reliability	1.0000	0.9923	0.9815	0.9682	0.9527	0.9352	0.9162	0.8958	0.8744	0.8521	0.8291

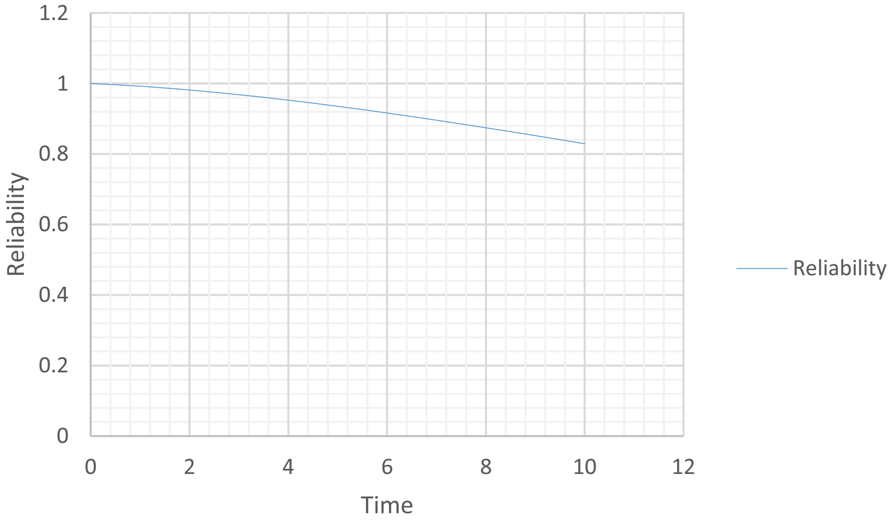

Fig. 4 depicted the reliability of the solar system against time

Table 4 The MTTF of the solar system with esteem to failure rate

Failure Rate	MTTF for u_1 $u_2 = 0.0213$ $u_3 = 0.0031$ $u_4 = 0.0045$	MTTF for u_2 $u_1 = 0.0123$ $u_3 = 0.0031$ $u_4 = 0.0045$	MTTF for u_3 $u_1 = 0.0123$ $u_2 = 0.0213$ $u_4 = 0.0045$	MTTF for u_4 $u_1 = 0.0123$ $u_2 = 0.0213$ $u_4 = 0.0045$
0.0001	60.3193	41.8519	31.6075	32.6723
0.0002	59.8058	41.7690	31.5338	32.5941
0.0003	59.3008	41.6865	31.4604	32.5163
0.0004	58.8042	41.6043	31.3874	32.4387
0.0005	58.3157	41.5224	31.3146	32.3616
0.0006	57.8352	41.4409	31.2422	32.2847
0.0007	57.3625	41.3596	31.1700	32.2082
0.0008	56.8973	41.2787	31.0981	32.1320
0.0009	56.4396	41.1982	31.0266	32.0561

Cost Analysis of the Solar System

Cost Analysis of the Solar System Using Copula Repair

$$E_g(n) = G_1 \int_0^n Z_{up}(n)dn - G_2 n \tag{52}$$

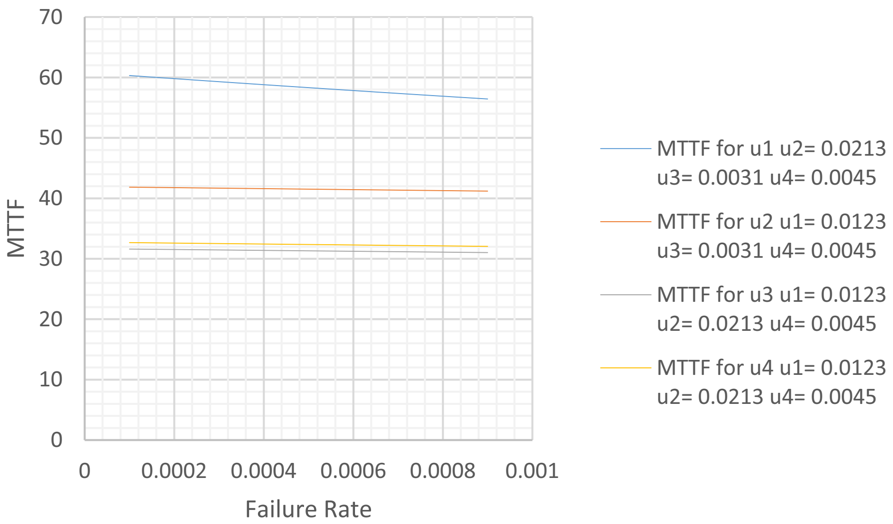

Fig. 5 displays MTTF of the solar system against failure rate

Table 5 The sensitivity of the solar system with esteem to failure rate

Failure rate	Ə(MTTF)/u_1 $u_2 = 0.0213$ $u_3 = 0.0031$ $u_4 = 0.0045$	Ə(MTTF)/u_2 $u_1 = 0.0123$ $u_3 = 0.0031$ $u_4 = 0.0045$	Ə(MTTF)/u_3 $u_1 = 0.0123$ $u_2 = 0.0213$ $u_4 = 0.0045$	Ə(MTTF)/u_4 $u_1 = 0.0123$ $u_2 = 0.0213$ $u_4 = 0.0045$
0.0001	− 5178.6875	− 830.2966	− 738.2914	− 783.5535
0.0002	− 5092.1428	− 826.9673	− 735.2009	− 780.1924
0.0003	− 5007.6558	− 823.6578	− 732.1287	− 776.8517
0.0004	− 4925.1662	− 820.3679	− 729.0746	− 773.5311
0.0005	− 4844.6156	− 817.0973	− 726.0385	− 770.2305
0.0006	− 4765.9476	− 813.8461	− 723.0202	− 766.9497
0.0007	− 4689.1074	− 810.6139	− 720.0196	− 763.6886
0.0008	− 4614.0421	− 807.4008	− 717.0366	− 760.4471
0.0009	− 4540.7006	− 804.2064	− 714.0710	− 757.2250

$$E_g(n) = G_1 \left\{ \begin{array}{l} 0.000337e^{-1.02130n} + 0.000252e^{-1.03690n} \\ -0.001372e^{-2.72854n} + 0.003815e^{-1.12013n} \\ -818.858878e^{-0.00122n} + 818.855852 \end{array} \right\} - G_2(n) \tag{53}$$

Using Eq. (53), revenue was fixed to a unit while service cost and time were varied, therefore, the expected gain was computed and presented in Table 6 (Fig. 7).

Similar to this, using Eq. (53) such that the service cost was fixed to a unit while revenue and time were varied, expected gain was calculated as shown in Table 7 (Fig. 8).

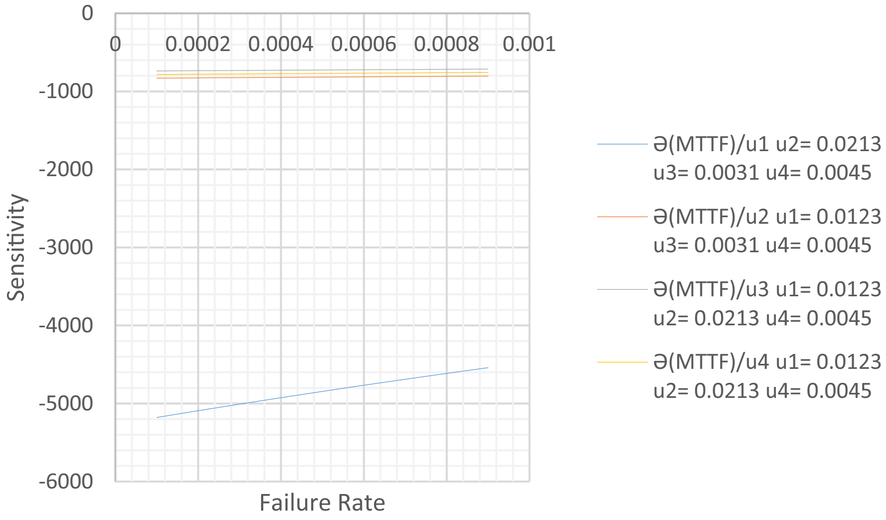

Fig. 6 shows sensitivity of the solar system against failure rate

Table 6 The expected gain of the solar system with esteem to time

n	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.1$	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.2$	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.3$	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.4$	$E_g(a)$ for $G_1 = 1$ $G_2 = 0.5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8988	0.7988	0.6988	0.5988	0.4988
2	1.7972	1.5972	1.3972	1.1972	0.9972
3	2.6950	2.3950	2.0950	1.7950	1.4950
4	3.5917	3.1917	2.7917	2.3917	1.9917
5	4.4873	3.9873	3.4873	2.9873	2.4873
6	5.3818	4.7818	4.1818	3.5818	2.9818
7	6.2750	5.5750	4.8750	4.1750	3.4750
8	7.1670	6.3670	5.5670	4.7670	3.9670
9	8.0578	7.1578	6.2578	5.3578	4.4578
10	8.9473	7.9473	6.9473	5.9473	4.9473

Cost Analysis of the Solar System Using General Repair

$$E_g(n) = G_1 \int_0^n Z_{up}(n)dn - G_2 n \tag{54}$$

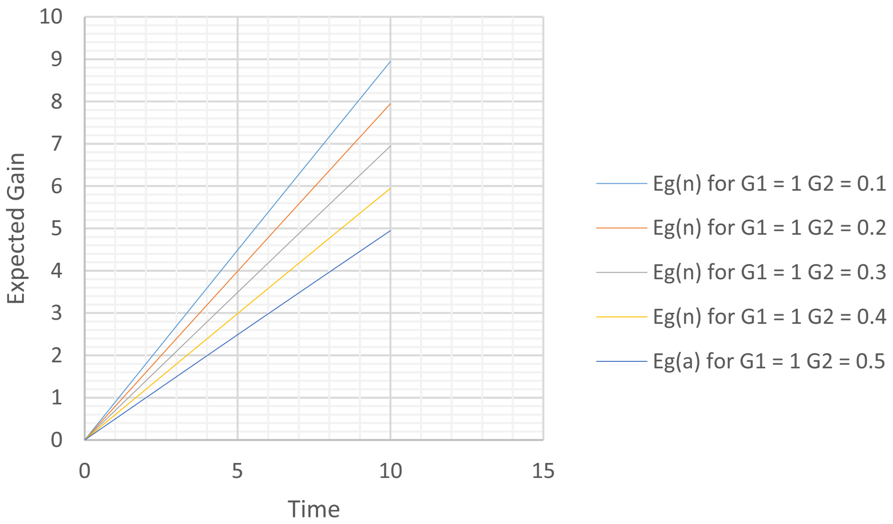

Fig. 7 depicted expected gain of the solar system against time

Table 7 The expected gain of the solar system with esteem to time

n	$E_g(n)$ for $G_1 = 2$ $G_2 = 1$	$E_g(n)$ for $G_1 = 3$ $G_2 = 1$	$E_g(n)$ for $G_1 = 4$ $G_2 = 1$	$E_g(n)$ for $G_1 = 5$ $G_2 = 1$	$E_g(n)$ for $G_1 = 6$ $G_2 = 1$
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9977	1.9965	2.9954	3.9942	4.9931
2	1.9945	3.9918	5.9890	7.9863	9.9836
3	2.9900	5.9851	8.9801	11.9751	14.9702
4	3.9835	7.9753	11.9871	15.9589	19.9507
5	4.9747	9.9621	14.9495	19.9369	24.9243
6	5.9636	11.9454	17.9272	23.9090	29.8908
7	6.9500	13.9250	20.9000	27.8751	34.8501
8	7.9340	15.9010	23.8680	31.8351	39.8021
9	8.9156	17.8734	26.8312	35.7890	44.7468
10	9.8947	19.8421	29.7895	39.7369	49.6842

$$E_g(n) = G_1 \left\{ \begin{array}{l} -0.000160e^{-1.12517n} - 0.005306e^{-1.00521n} \\ -818.851084e^{-0.00121n} + 0.000479e^{-1.02130n} \\ +0.000279e^{-1.03690n} + 818.8558443 \end{array} \right\} - G_2(n) \tag{55}$$

By the mean of Eq. (55) and fixing the revenue to one, however, service cost and time were varied, and the expected gain was generated as displayed in Table 8 (Fig. 9).

Conversely, using Eq. (55) and fixing the service cost to one, therefor, revenue and time were varied, the expected gain was obtained and presented in Table 9 (Fig. 10).

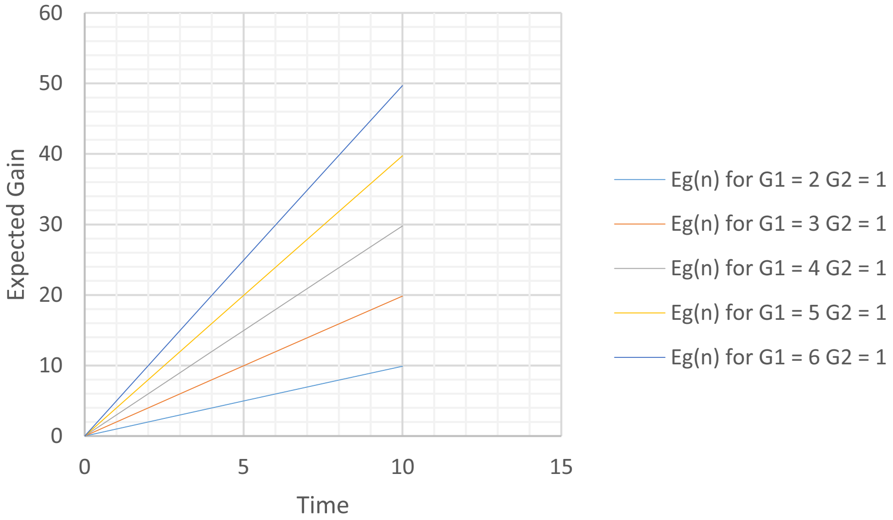

Fig. 8 shows expected gain of the solar system against time

Table 8 The expected gain of the solar system with esteem to time

n	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.1$	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.2$	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.3$	$E_g(n)$ for $G_1 = 1$ $G_2 = 0.4$	$E_g(a)$ for $G_1 = 1$ $G_2 = 0.5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8976	0.7976	0.6976	0.5976	0.4976
2	1.7921	1.5921	1.3921	1.1912	0.9921
3	2.6847	2.3847	2.0847	1.7847	1.4847
4	3.5758	3.1758	2.7758	2.3758	1.9758
5	4.4656	3.9656	3.4656	2.9656	2.4656
6	5.3543	4.7543	4.1543	3.5543	2.9543
7	6.2416	5.5416	4.8416	4.1416	3.4416
8	7.1278	6.3278	5.5278	4.7278	3.9278
9	8.0128	7.1128	6.2128	5.3128	4.4128
10	8.8966	7.8966	6.8966	5.8966	4.8966

Results Discussion

A thorough analysis of the solar system was conducted, and the following results were obtained. Firstly, the availability or readiness of the system was determined and presented in Tables and Figures, and the findings indicated that the availability is sufficient to performs the mission or purpose it was designed for. And from the side of managers, the availability of the solar system is encouraging as big returns are anticipated. Secondly, the reliability of the system, since the rate at which it varies with time is very slow, this shows that it is

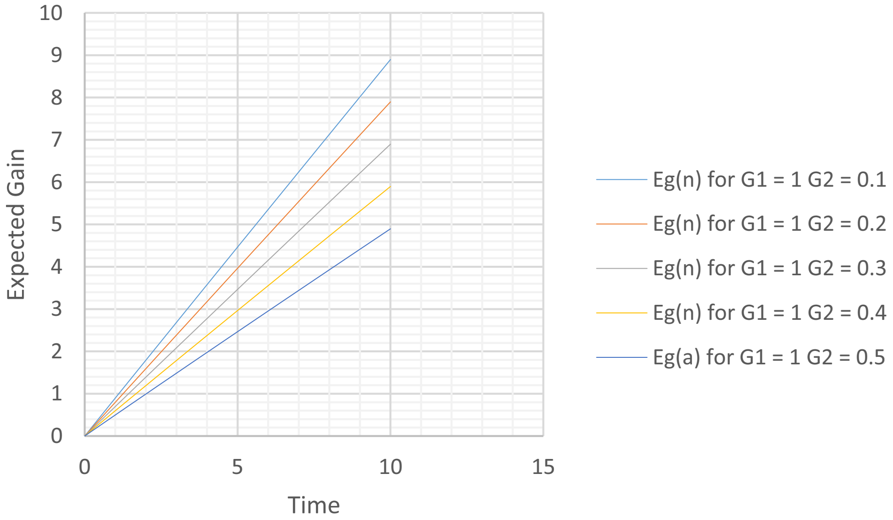

Fig. 9 depicted the expected gain of the solar system against time

Table 9 The expected gain of the solar system with esteem to time

n	$E_g(n)$ for $G_1 = 2$ $G_2 = 1$	$E_g(n)$ for $G_1 = 3$ $G_2 = 1$	$E_g(n)$ for $G_1 = 4$ $G_2 = 1$	$E_g(n)$ for $G_1 = 5$ $G_2 = 1$	$E_g(n)$ for $G_1 = 6$ $G_2 = 1$
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9952	1.9928	2.9905	3.9881	4.9857
2	1.9842	3.9763	5.9685	7.9606	9.9527
3	2.9694	5.9541	8.9389	11.9236	14.9083
4	3.9517	7.9275	11.9034	15.8793	19.8551
5	4.9313	9.8970	14.8627	19.8284	24.7941
6	5.9086	11.8629	17.8172	23.7715	29.7258
7	6.8833	13.8250	20.7667	27.7084	34.6501
8	7.8557	15.7836	23.7114	31.6393	39.5672
9	8.8257	17.7385	26.6514	35.5642	44.4771
10	9.7932	19.6899	29.5865	39.4832	49.3798

going to last for long period of time. However, managers expect mega incomes mobilization. Thirdly, MTTF of the system, as a maintenance metric which estimate the amount of time system works to failure was determined and from the findings it is very interesting to note that spontaneous change of failure rate reduces MTTF, and each failure rate coincided to its MTTF. Fourthly, the sensitivity of the system, it is another maintenance measures which describe the level of failure rate, such that if the failure is strong enough implies the strong of its sensitivity, and the system performance capacity is reduces to certain level, and the consequences lead to malfunction of the system or low incomes generation to the management. Fifthly, the cost

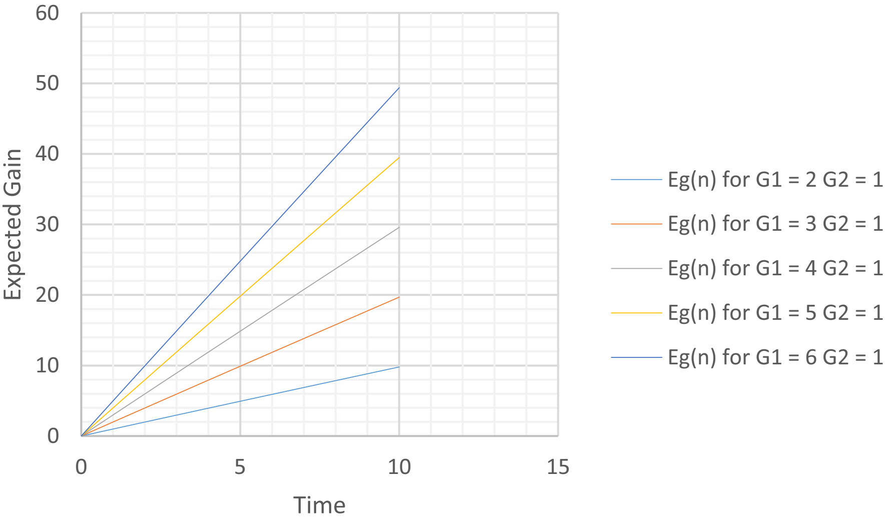

Fig. 10 shows the expected gain of the solar against time

analysis, it is studied and the results is presented in Tables and Figures, and from the findings it is carefully observed that the gain increases. In general overview, the results obtained using general repair is less than that of copula repair, therefore, copula repair is suggested for system optimal performance.

As observed from the study, subsystems such as the solar cell, the controller are seeming to be delicate, therefore, adequate care need to be provide in order to avert their failure, because failure of one may cause the failure of the entire system. Nevertheless, those subsystems could be improving by cooperating more backups units.

Conclusion

This study presents a multi-purpose solar system consisting of four subsystems: a solar panel, a controller, two USB charge points and four electric bulb points all merged together in a series–parallel conformation. The transition diagram of the system was used to derived the partial differential equations of order one, and solved using supplementary variable and Laplace transformation procedures. Maple software package was used to generate expressions for availability, reliability, MTTF, sensitivity and cost. The findings were justified using particular examples and presented in Tables and Figures.

The results of study are forecasted to be of beneficial to system managers, as it provides adequate maintenance strategies for improving reliability, profitability, and efficiency of their devices and generate mega incomes mobilization. The credit also goes to maintenance managers, designing engineers and so on.

This study future direction could be feature using other reliability optimization approaches, such as PSO, generic algorithms, genetic algorithms, grey wolf and so on.

Author contributions ALI-Developed the model edited the manuscript. AA-Solved the model of the manuscript. IY-Analyzed the model of the munuscript.

Data availability The authors declared that all data and material supporting the findings of the research are available in the article.

Declarations

Conflict of interests The authors declare no competing interests.

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