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ORIGINAL PAPER

# On Solving a MFL Paradox in Linear Plus Linear Fractional Multi- Objective Transportation Problem Using Fuzzy Approach

Rachana Saini[1] · Vishwas Deep Joshi[1] · Jagdev Singh[1]

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## Abstract

In the present work, we have introduced the more-for-less paradox situation in the linear plus linear fractional multi-objective transportation problem (LPLF-MOTP). In this approach, more-for-less (MFL) situation solved using the fuzzy methodology. The paradoxical solution in multi-objective transportation problems gives us a less or equal compromise optimal solution when transfer more goods from source to destination. In this paper, we also discussed the comparison of MFL solutions between fuzzy programming and compromise solution using ranking procedure (Rizk-Allah). We observe that the result obtained using fuzzy approach shown the superiority over MFL approach. The presented approach has been illustrated with a numerical example.

**Keywords** Linear plus linear fractional · Multi-objective transportation problem · Fuzzy theory · More-for-less paradox

## Introduction

The real word problem with the transportation problem is a special case of linear programming. Joshi et al. [9] describe the paradoxical position of the multi-objective transportation problem with linear constraints and fractional constraints. Here paradox present in every objective is not necessary and by using a ranking procedure to compare the paradoxical with the compromise solution. In this article we discuss linear plus linear fractional multi-objective transportation problem with fuzzy programming problem uses MFL.

In the literature, the transportation problem has got better concentration. When transporting the same number of goods from each origin to each destination, the more-for-less (MFL)

✉ Vishwas Deep Joshi
vdjoshi.or@gmail.com

Rachana Saini
rachanasaini112@gmail.com

Jagdev Singh
jagdevsinghrathore@gmail.com

1 Department of Mathematics, Faculty of Science, JECRC University, Jaipur, Rajasthan, India

paradox allows more goods to be transported at a lower total cost. However, the transportation paradox is rarely mentioned in the many textbooks and materials dealing with transportation issues. Apparently, some researchers discovered the paradox independently of each other. However, most treatises on this subject refer to the treatises by Charnes and Klingman [4] and Szwarc [17] as their first treatises. In Charnes and Klingman [4], he named it more or less paradox, and they wrote: Some (Charnes and Cooper; [5]) are unknown to the majority of workers in the field of linear programming [6].

In many real-life situations, transportation problems using fractional objectives are widely used as a performance measure, such as analyzing the financial aspects of a transportation company and businesses, and managing transportation with individuals or groups facing difficulties. It has been maintain the proper ratio between some important parameters and crucial parameters related to the transportation of goods from a particular source to different destinations. Objective functions of the fractional functions include optimizations such as the ratio of total returns to total investment, the ratio of risk assets to capital, and total taxes on total public spending on goods [7].

Kumar et al. [11] presented equality type constraints and conflicting objectives of multi-objective transportation problems. Here objectives type is fuzzy of nature. Therefore they solved three methods using fuzzy programming. Joshi et al. [10] explained that the value of objective significance was below the optimal value and that transporting more quantities in a linear plus linear fractional transport problem could result in a decrease in lower values. They discussed the new heuristic conditions for the basic viable solution of open LPLFTP and the sufficient conditions to achieve this contradictory result. Li et al. [12] presented multi-objective (MO) transportation problems using a fuzzy compromise programming approach. The comprehensive review of various objective, have the marginal assessment of individual objectives, an overall assessment of all objectives. They use traditional optimization techniques to solve fuzzy compromise programming models and get uncontrollable compromise solutions. This solution maximizes the comprehensive membership of the global assessment for all purposes. Prochelvi et al. [15] developed an algorithm to find linear constraints of the paradoxical result of multi-objective transportation (MOT) problems. It obtains the best paradoxical pair and range of flow by using the sufficient condition of the existing paradox.

A new compromise algorithm for multi-objective transportation problems was discussed by Rizk-Allah et al. [16]. The characterized the NCPA by communicating three types of membership methods had objective namely, truth membership and indeterminacy membership respectively. The measuring validated the ranking degree used TOPSIS approach to the presentation of the NCPA.

Adlakha et al. [1] was analysis increasingly useless for administrator decisions (for example, efforts to increase factory capacity or increase advertising demand in a particular market). Sufficient conditions to determine the identity of the runner market and provide points. For large-scale transportation problems, almost the more-for-less paradoxical methods apply to this method and provide users with insight into the problem, making it an effective tool for administrators. The method used to solve a particular transport problem has developed an emotional alternative solution algorithm.

Afwat et al. [2] proposed a new way of product approach to solving the multi-objective transportation problem. Use fuzzy programming in different units to convert targets to membership and accumulate per product. Finding a solution that is close to the best solution is an easy and quick way. Bit et al. [3] proposed that all constraints are congruent equations and the goals are essentially inconsistent. This is a special case of vector minimums for linear multi-objective transportation problems. All existing methods create to build a compromised solution or a set of non-dominant solutions. Linear multi- objective transportation problems use fuzzy linear programming to provide effective solutions and optimal compromises. This is a comprehensive version of the Simplex algorithm. Nomani et al. [14] proposed a weighting method for goal planning to solve the multi-objective transportation problem. Depending on the expectations of decision-makers using this model, higher priority goals can be more satisfying. Taking into account the multi-objective transportation problem, the problem of this method is solved.

We have described a new way to resolve the MFL paradox in LPLF-MOTP using the fuzzy method when there is no common paradox in all targets. This paper is divided into the following sections: In the sect. "Mathematical Model", mathematical formulations are given; In the definition section, all the necessary definitions are discussed; Sect. "Step by Step MFL Algorithm for LPLF-MOTP" describes the MFL paradox and ambiguity handling process; and Sect. "Numerical Example" describes an example that supports the theory of the problem described in Sect. "Mathematical Model". The conclusions were explained in Sect. "Conclusion".

## Mathematical Model

Transportation problems involve distributing products from many supply points to many demand points with minimal total transportation costs. Consider m sources $S_1, S_2, \ldots, S_m$, n destinations points $D_1, D_2, \ldots, D_n$ and $K$ objectives $Z_1, Z_2, \ldots, Z_K$. We assume that minimized to all $K$ objectives. Suppose that $a_i (i = 1, 2, \ldots, m)$ supply points are $S_i$ sources available and $b_j (j = 1, 2, \ldots, n)$ demand points are $D_j$ destinations required level. Let a component of the goods from the source $S_i$ to the destination $D_j$ is a penalty $r_{ij}^k$ associated to transporting for each objective $Z_k$. Let the unknown quality of goods to be transported from source $S_i$ to destinations $D_j (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ represented by the variable $x_{ij}$.

### Linear plus linear fractional (LPLF) multi-objective transportation problem

Suppose LPLF-MOTP is following as:

$$[\mathbf{P_1}]\text{Min } Z_k = \textstyle\sum_{i=1}^m \sum_{j=1}^n r_{ij}^k x_{ij} + \sum_{i=1}^m \sum_{j=1}^n \frac{s_{ij}^k x_{ij}}{t_{ij}^k x_{ij}}, k = 1, 2, \ldots, K,$$

$$\text{Subject to} \begin{cases} \sum_{j=1}^{n} x_{ij} = a_i & \forall i = 1, 2, \ldots, m, \\ \sum_{i=1}^{m} x_{ij} = b_j & \forall j = 1, 2, \ldots, n, \\ x_{ij} \geq 0 \quad \forall i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \end{cases}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

This is a necessary and sufficient condition for the existence of a feasible solution called an equilibrium condition.where

$\sum_{i=1}^m \sum_{j=1}^n r_{ij}^k x_{ij} \geq 0; \sum_{i=1}^m \sum_{j=1}^n s_{ij}^k x_{ij} \geq 0; \sum_{i=1}^m \sum_{j=1}^n t_{ij}^k x_{ij} > 0; r_{ij}^k \geq 0; s_{ij}^k \geq 0$ and $t_{ij}^k \geq 0$.

$r_{ij}^k$= From supply point $i^{th}$ to destination $j^{th}$ capital in transporting quantities per unit,

$s_{ij}^{k}$= From supply point $i^{th}$ to destination $j^{th}$ depreciation in transporting quantities per unit,

$t_{ij}^{k}$= From supply point $i^{th}$ to destination $j^{th}$ profit earned in transporting quantities per unit.

## MFL Paradox in Linear Plus Linear Fractional Multi-objective Transportation Problem

The linear plus linear fractional multi-objective transportation problem is MFL paradox transportation problem if the shipment volume from each supply point to all destinations is at least the same and more total goods can be shipped at a lower total cost, even if the shipping cost is not negative. The equality constraint for a given.

$$\textbf{[P}_\mathbf{2}\textbf{]}\ \text{Min}\ Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}^{k}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{s_{ij}^{k}x_{ij}}{t_{ij}^{k}x_{ij}},\ k = 1, 2, \ldots, K,$$

$$\text{Subject to}\begin{cases}\sum_{j=1}^{n} x_{ij} = a_i + l & \forall i = 1, 2, \ldots, m,\\ \sum_{i=1}^{m} x_{ij} = b_j + l & \forall j = 1, 2, \ldots, n,\\ x_{ij} \geq 0 & \forall i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.\end{cases}$$

$$\sum_{i=1}^{m} a_i + l = \sum_{j=1}^{n} b_j + l = F^0,$$

$$a_i + l > 0, b_j + l > 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.$$

where

$$\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}^{k}x_{ij} \geq 0; \sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij}^{k}x_{ij} \geq 0;$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^{k}x_{ij} > 0; r_{ij}^{k} \geq 0; s_{ij}^{k} \geq 0 \text{and} t_{ij}^{k} \geq 0$$

It is clear that an optimum feasible solution of [$P_1$] is a feasible solution of [$P_2$], cost-flow pair ($Z^0$, $F^0$) yield to the objective function [10].

## Linear Plus Linear Fractional Multi-objective Transportation Problem Using Fuzzy Linear Programming

In fuzzy set theory, fuzzy linear programming is suitable for linear multi-objective decision-making problems. In its theory, element $X$ is Membership in set A, indicated by Membership function $\varphi_k(X)$. [0, 1] is range of the membership function. In Multi-objective decision problems, the objective function defined via fuzzy set theory and the decision set is defined as intersection of all Fuzzy sets and constraints.

The Multi-objective transportation problem is considered the vector minimum problem. The first step is to assign two the values $\tilde{u}_k$ (Achievement of highest acceptable level) and $\tilde{l}_k$ (Expected achievement of the lower level) are used as upper and lower bounds Objective function $Z_k$. Let me $\tilde{d}_k = \tilde{u}_k$-$\tilde{l}_k$= Deterioration Marginal for objective k. All objectives are specified to the expected level and deterioration marginal, create a fuzzy model. The next step is to convert the fuzzy model to a "crisp" model, i.e. enter the traditional linear programming problem [3].

The starting fuzzy model is then given by the expected achievement of the lower level for each objective, follows as:

$$Z_k \leq \tilde{l}_k \qquad k = 1, 2, \ldots, K,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \forall i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad \forall j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0, \qquad \forall i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.$$

The membership function $\varphi_k(X)$ for the multi-objective transportation problem [3] is defined as:

$$\varphi_k(X) = \begin{cases} 1 & \text{if} \quad Z_k < \tilde{l}_k \\ 1 - \frac{Z_k - \tilde{l}_k}{\tilde{u}_k - \tilde{l}_k} & \text{if} \quad \tilde{l}_k \leq Z_k \leq \tilde{u}_k \\ 0 & \text{if} \quad Z_k > \tilde{u}_k \end{cases}$$

For the vector minimum problem the equivalent linear programming problem is:
Maximize $\alpha$
Subject to $\alpha \leq \frac{\tilde{u}_k - Z_k}{\tilde{u}_k - \tilde{l}_k}$, $k = 1, 2, \ldots, K$,

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \forall i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad \forall j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0, \qquad \forall i = 1, 2, \ldots, m; j = 1, 2, \ldots, n, \alpha \geq 0$$

This linear plus linear fractional programming problem can be further simplified as [3]:
**[P$_3$]** Maximize $\alpha$
Subject to

$$\left( \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij}^k x_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{s_{ij}^k x_{ij}}{t_{ij}^k x_{ij}} \right) + \alpha \left( \tilde{u}_k - \tilde{l}_k \right) \leq \tilde{u}_k, \qquad k = 1, 2, \ldots, K,$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \forall i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad \forall j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0, \qquad \forall i = 1, 2, \ldots, m; j = 1, 2, \ldots, n, \alpha \geq 0$$

## Definition

### Cost-Flow Pair

If the value of the objective function is $Z^0$ and the flow rate is $F^0$, then this corresponds to the feasible solution $X^0$ of the transportation problem and the pair corresponds to the feasible solution $X^0$[15].

### Paradoxical Solution

A solution $X^p$of [$P_2$] yielding the objective function–flow pair ($Z^0$, $F^0$) is called the 'Paradoxical Solution' if, for any other feasible solution of [$P_2$] yielding a flow pair ($Z$, $F$), we have

$$(Z, F) > \left(Z^p, F^p\right)$$

Or

$Z = Z^p$, but $F < F^p$

Or

$F = F^p$, but$Z > Z^p$

Let the optimum feasible solution of [P1] yield a value $Z^0 = r^0 + \frac{s^0}{t^0}$ of the objective function $r(x) + \frac{s(x)}{t(x)}$ [10].

## Step by Step MFL Algorithm for LPLF-MOTP

**Step 1:** Using [8, 14] solve the above problem and get the compromise optimal solution. Each objective using modified distribution method to get individual ideal and anti-ideal optimal solutions.
**Step 2:** Generate the combined shadow price matrix for the LPLF-MOTP.
**Step 3:** In the table recognize the position of negative shadow prices obtained by step 2. If no negative entries are established in the shadow price matrix go to step 6.
**Step 4:** In step 3 choose the most negative entry established for the MFL solution and fuzzy programming problem. Relax the demand and supply (max $(a_i, b_j)$) to getting the MFL solution for the LPLF-MOTP.
**Step 5:** Repeat the procedure for finding the other paradoxical solution.
**Step 6:** Solve the reduced problem as a regular unbalanced problem.

**Remark** In these MFL procedure and fuzzy condition, it is not necessary that both situations are present in every objective function.

## Numerical Example

In LPLF-MOTP, The linear function represents the transporting cost when goods are transported from different sources to their destinations, and the fractional part represents the ratio of sales tax to total public spending. Our objective is to determine a transporting schedule that minimizes the total sales tax paid by the sum of the ratio of total transporting costs to total public spending.

**Table 1** linear plus linear fractional objective matrix for multi-objective example

| | | | D1 | | D2 | | D3 | | D4 | | $a_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| O1 | $r_{ij}^k$ | $s_{ij}^k$ | 3, 5, 4 | 8, 3, 9 | 12, 3, 6 | 6, 7, 5 | 4, 8, 7 | 3, 8, 3 | 6, 8, 7 | 9, 8, 6 | 25 |
| | | $t_{ij}^k$ | | 13, 7, 9 | | 12, 8, 6 | | 8, 3, 6 | | 10, 8, 9 | |
| O2 | $r_{ij}^k$ | $s_{ij}^k$ | 7, 3, 8 | 2, 3, 8 | 2, 5, 5 | 2,3,5 | 9, 11, 2 | 11,12,2 | 5, 12, 6 | 5, 6, 9 | 40 |
| | | $t_{ij}^k$ | | 6, 7, 9 | | 7, 8 ,9 | | 13, 13, 5 | | 9, 8, 10 | |
| O3 | $r_{ij}^k$ | $s_{ij}^k$ | 6, 8, 9 | 7,8,9 | 4,10,8 | 3,2,7 | 3, 13, 4 | 5, 6, 8 | 1,5,14 | 7, 9, 4 | 35 |
| | | $t_{ij}^k$ | | 10,11,12 | | 6, 7, 8 | | 8, 9, 10 | | 8, 5, 8 | |
| $b_j$ | | | 35 | | 25 | | 15 | | 25 | | 100 |

**Table 2** Compromise optimal solution of shadow price matrix for MOLP-LFTP

| | | | D1 | | | D2 | | | D3 | | | D4 | | | $U_i^1$ | $U_i^2$ | $U_i^3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| O1 | | | 0 | 0 | 0 | − 1.42 | 7.37 | 1.67 | 3.3 | 21.57 | − 6.69 | 1.85 | 14.75 | 3.01 | − 3.71 | 2 | − 3.89 |
| O2 | | | 0 | 0 | 0 | 0 | 0 | 0 | 7.31 | 19.57 | − 2.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| O3 | | | 3.64 | − 2.47 | 17.35 | − 1.4 | − 0.53 | 13.16 | 0 | 0 | 0 | 0 | 0 | 0 | − 3.69 | − 5.9 | 7.6 |
| $V_j^1$ | $V_j^2$ | $V_j^3$ | 7.33 | 3.43 | 8.89 | 2.29 | 5.37 | 5.56 | 7.31 | 19.57 | − 2.8 | 5.56 | 12.75 | 6.9 | 100 | | |

**Table 3** Ranking Procedure [13] of comparison of solution using to proposed, MFL and fuzzy solution for flow 105

| 105 Flow | Solution of LPLFMOTP | $Z_1$ objective | $Z_2$ objective | $Z_3$ objective | $D_i^+$ | $D_i^-$ | R | Rank |
|---|---|---|---|---|---|---|---|---|
| Cell at (1, 2) | Ideal sol | 275.5759 | 540.8231 | 570.8528 | 0 | 580.8908 | 1 | 1 |
| | Anti-ideal sol | 640.5318 | 910.5394 | 830.7588 | 580.8908 | 0 | 0 | 5 |
| | MFL sol | 313.6035 | 617.7163 | 720.7168 | 172.6785 | 452.4784 | 0.723784 | 3 |
| | Com sol | 296.5755 | 742.6286 | 640.7642 | 214.6021 | 427.3148 | 0.665686 | 4 |
| | Fuzzy sol | 278.576 | 682.648 | 673.7484 | 175.245 | 455.6301 | 0.722219 | 2 |
| Cell at (1, 3) | Ideal sol | 300.5751 | 525.9098 | 550.849 | 0 | 596.6098 | 1 | 1 |
| | Anti-ideal sol | 590.5385 | 925.5569 | 885.5569 | 596.6098 | 0 | 0 | 5 |
| | MFL sol | 446.6251 | 619.7861 | 740.7254 | 257.2866 | 367.7415 | 0.58836 | 4 |
| | Com sol | 303.5753 | 757.6315 | 628.7782 | 244.4932 | 420.2049 | 0.632174 | 3 |
| | Fuzzy sol | 314.6153 | 682.709 | 677.7333 | 202.1948 | 422.3413 | 0.676248 | 2 |
| Cell at (3, 1) | Ideal sol | 300.5824 | 515.8243 | 600.8333 | 0 | 600.5434 | 1 | 1 |
| | Anti-ideal sol | 650.5581 | 915.578 | 880.7733 | 600.5434 | 0 | 0 | 5 |
| | MFL sol | 354.5823 | 619.706 | 788.7421 | 221.3981 | 428.4993 | 0.659334 | 3 |
| | Com sol | 333.5834 | 747.6512 | 678.7636 | 244.9135 | 412.4997 | 0.627459 | 4 |
| | Fuzzy sol | 318.583 | 687.6675 | 713.7523 | 206.4093 | 435.9434 | 0.678667 | 2 |
| Cell at (3, 2) | Ideal sol | 275.5792 | 525.8188 | 595.84 | 0 | 595.9258 | 1 | 1 |
| | Anti-ideal sol | 625.5549 | 925.5562 | 865.7558 | 595.9258 | 0 | 0 | 5 |
| | MFL sol | 343.5782 | 625.7066 | 778.7276 | 219.2015 | 420.7072 | 0.657449 | 3 |
| | Com sol | 305.5803 | 745.6503 | 670.7462 | 234.1728 | 415.6665 | 0.639645 | 4 |
| | Fuzzy sol | 290.5798 | 685.6667 | 705.7344 | 194.5589 | 441.9979 | 0.694357 | 2 |
| Cell at (2, 3) | Ideal sol | 320.5645 | 535.8768 | 560.8227 | 0 | 543.0378 | 1 | 1 |
| | Anti-ideal sol | 540.5305 | 915.5756 | 880.7193 | 543.0378 | 0 | 0 | 5 |

**Table 3** (continued)

| 105 Flow | Solution of LPLFMOTP | $Z_1$ objective | $Z_2$ objective | $Z_3$ objective | $D_i^+$ | $D_i^-$ | R | Rank |
|---|---|---|---|---|---|---|---|---|
| | MFL sol | 410.6198 | 625.7658 | 740.7143 | 220.3429 | 347.0849 | 0.611681 | 3 |
| | Com sol | 329.5651 | 771.6255 | 634.7774 | 247.2403 | 354.5637 | 0.589168 | 4 |
| | Fuzzy sol | 330.5947 | 690.6879 | 685.7405 | 199.1772 | 364.231 | 0.646478 | 2 |

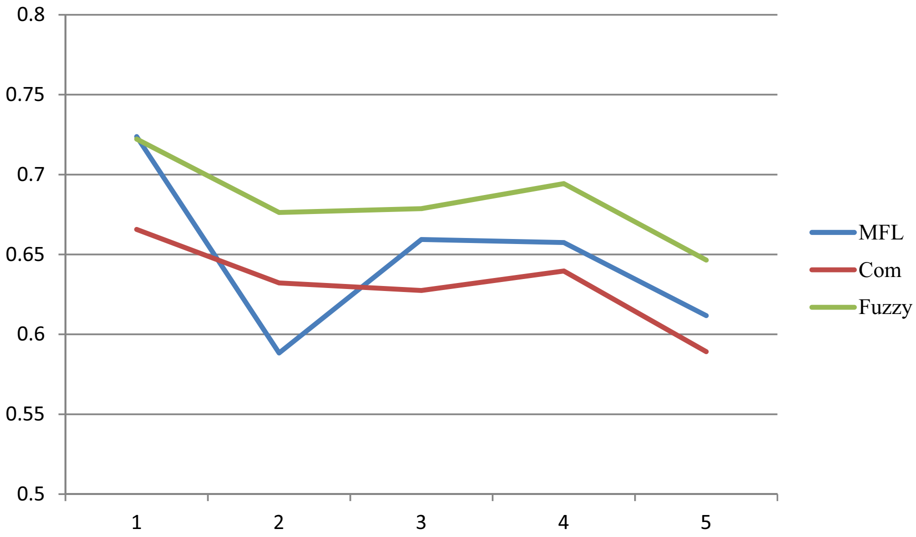


**Fig. 1** Comparison's ranking for LPLF-MOTP among MFL, compromise (com), fuzzy solution

Polyfilm companies process PET chips to produce polyfilm products such as transparent films, metalized films, and specialty PET films. The company has 3 branches and 4 depots in various locations in India. The company transports polyfilm from its branch offices by truck to the depot on the highway. Decision-makers want a ratio of shipping costs to sales tax to total public speed. The shipping cost is Rs per KM and the sales tax is Rs per KG. Decision-makers also want to find the amount of polyfilm that will be shipped from the $i^{th}$ branch to the $j^{th}$ depot to meet the requirements. Using MFL paradox [$P_2$] and MFL algorithm for LPLF-MOTP when transport more polyfilm from branches to destination then transportation cost and sales tax is minimized. Next we can use fuzzy linear programming [$P_3$] in LPLF-MOTP objective matrix in Table 1 represents the shipping cost and sales tax. Now we solve this example using Lingo 17.0 software.

We obtain the individual optimum solution for each objective for flow 100 as follows:

$X^1$=(25,0,0,0,10,25,0,5,0,0,15,20), $Z_1(X^1)$= 285.5763,

$X^2$=(0,20,5,0,35,5,0,0,0,0,10,25), $Z_2(X^2)$= 485.8667,

$X^3$=(25,0,0,0,0,15,0,25,10,10,15,0), $Z_3(X^3)$=555.8385.

Solving the LPLF-MOTP for flow 105 and weight (0.3, 0.3, 0.4), we get the compromise optimal solution as follows:

$X$ = (25,0,0,0,10,25,0,5,0,0,15,20),

$Z_1(X)$= 285.5763,

$Z_2(X)$= 635.6667,

$Z_3(X)$= 675.750.

Compromise optimal solution is represented in Table 2 (Shadow price matrix).

## At point (1,2) individual optimum solution for the flow 105

Where shadow prices are negative for each individual objective not to find any common cell. So we select the negative shadow price entry in three objectives. We got negative shadow price entries in cells (1,2), (1,3), (2,3), (3,1), (3,2). We increase the flow in demand and supply for the corresponding row and column by 5, the MFL, compromising, fuzzy solution are representing rank in Table 3 (Fig. 1).

## Conclusion

In this paper solve the MFL paradox and fuzzy method in LPLF-MOTP. Here approach allows easy identification of such MFL paradox cells in the objective matrix and calculation of the matrix and the calculation of the maximal allowable units and distribution of these excesses in a systematic approach. No common results in MFL, Fuzzy and compromise solutions in above table compare to same flow each-other give to ranking [13]. We found that our approach gives a result in ranking procedure to comparison with the compromise optimal solutions obtained by [8–10, 13]. The reader can see the graphic in the above figure. The contents of this article can open a new dimension to create a MFL paradox in linear plus linear fractional multi-level multi- objective transportation problem using fuzzy approach.

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**Conflict of interest** The authors declare that they have no conflict of interest.

**Ethical Approval** We have declared that the information about all the listed authors are correct. During the submission of the manuscript, we have followed all the rules of ethical standards.

**Informed Consent** The listed authors have confirmed their authorship.

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