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ORIGINAL PAPER

Application of Hermite Wavelet Method and Differential Transformation Method for Nonlinear Temperature Distribution in a Rectangular Moving Porous Fin

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Abstract

We developed a novel technique called the Hermite wavelet collocation method (HWM) in the current work. Here, the variation of nonlinear temperature in a permeable moving fin of the rectangular domain is studied by the Hermite wavelet method and the Differential transformation method (DTM). Darcy's model is used to formulate the foremost heat transfer highly nonlinear ordinary differential equation (ODE). Numerical outcomes of the proposed method are compared with the exact and DTM. Comparison between calculated solutions showed that the Hermite wavelet collocation method is more suitable and correct than the Differential transformation method. The desire for exceptional flow constraints on the temperature distribution feature is concluded truthfully through graphs and tables. Graphic summaries are offered for the temperature distribution of various physical parameters in the present problem. The works are in extraordinary evidence for highly nonlinear ordinary differential equations in engineering applications.

Keywords Hermite wavelet method · Differential transformation method · Nonlinear ODE problems

Introduction

The majority of the engineering problems are expressed in terms of highly nonlinear differential equations. Obtaining accurate results for these types of engineering problems is relatively unhurt. However, in current years, numerical techniques have significantly been developed for highly nonlinear ODEs. Different numerical and analytical methods are applied to solve some problems. Practically all arithmetical modeling involves linear or nonlinear DEs. To determine those equations impeccably, approximately, or mathematically a few specialists employ various excellent scientific, semi-analytic plans alongside lasting labels of convergence. Various arithmetical techniques to find the solution of such highly nonlinear

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differential equations; nevertheless, a few critical information, including circular happening purpose, vanishes throughout finding the solution. Hence, analytical methods include emerged to find the solution of highly nonlinear ODEs such as the homotopy perturbation method and Collocation method [1], spectral element method [2], spectral homotopy analysis method [3], optimal homotopy analysis method [4], variational iteration method [5], differential transform method and finite difference method [6].

A growing number of engineering applications are concerned with energy transport by requiring the rapid movement of heat. To increase the heat transfer rate on a surface, fin assembly is commonly used. The heat transfer mechanism of the fin is to conduct heat from the heat source to the fin surface by its thermal conduction and then dissipate heat to the air by the effect of thermal convection. These extended surfaces are extensively used in various industrial applications. Conventional packages of warmth control in porous media consist of sun collectors, reactor cooling, warmness exchangers [7]. The idea of warmth control through permeable fins was delivered via Kiwan and Al-Nimr [8]. The warmth control version system through porous media changed into stepped forward using Darcy's version [9, 10]. Subsequently the paintings of kiwan and Al-nimr, several research have been accomplished to recognize the idea of warmth switch via porous fins. The thermal evaluation of finite length permeable fins through the insulated tip and established the force of governing physical parameters on nonlinear temperature distribution is studied by Patel and Meher [11]. It was concluded that permeable fins are more competent in dissipating heat to the ambient fluid than solid fins [12, 13]. Some analytical methods are applied to optimize permeable fins with various forecast models available by Kundu and Bhanja [14]. Taklifi et al. [15] studied rectangular permeable fin with magnetohydrodynamic (MHD) effects, explained the MHD effects near the fin tip and also heat-transfer rate in the permeable fin decreases. This type of application has been solved by some scientists applying various analytical and numerical methods [1–6]. Recently, various types of MHD thermal boundary layer of a Casson fluid flow problems have been analyzed [16, 17]. Nisar et al. [18] analyzed the steady free convective incompressible electrically conducting Jeffry fluid flow over a stretching surface. Ramzan et al. [19] are examined the role of modified Fourier law in the flow of an MHD Micropolar nano liquid flow with dust particles over a stretched surface. The MHD laminar flow, containing hybrid nanoparticles, with heat transfer phenomenon over a stretching sheet immersed in a porous medium, is investigated by Ali et al. [20]. New soliton solutions for generalized nonlinear differential equations using an effective analytical method explained by Ghanbari et al. [21].

As per our narrative review, we have not seen any research article on this model through wavelets. This impels us to propose the HWM for considered fin flow problem, and proficiency of the current technique is revealed through tables and graph simulation. Wavelet is a function, more precisely, is an arithmetical function applied to split a given function signal into various scale components. In the 1980s, they applied the French word "ondelette," which means "small wave." Later, it was transferred to English by translating "onde" into "wave," charitable "wavelet." Wavelet theory is developing tremendously due to arithmetical analysis by Stromberg, Grossmann, Morlet, Meyer, and Daubechies. Due to its unique features like compactly supported, orthogonality, and multiresolution analysis. Many mathematicians are applying wavelets in the field of numerical analysis; as a result presently, we have seen many diverse techniques on differential equations such as An investigation with the Hermite wavelets for accurate solutions for heat transfer problems [22, 23], high mass transfer via wavelet frames [24], Laguerre wavelet method [25].

To the best of our understanding, no study was performed temperature distribution in a rectangular moving porous fin using DTM and HWM. Moreover, we generated the new

Hermite wavelet operational matrix method and DTM to re-examine the above model. The highly nonlinear ordinary differential equation was tackled using DTM and HWM. The results are presented through tables and graphical plots for different known physical parameters. Subsequently, the obtained solutions compare with DTM and HWM and the exact solution of the highly nonlinear ODE. Comparison between resulting solutions showed that HWM is more suitable and correct than DTM. Furthermore, the essential physical quantities of real advantage and the control of unique physical known parameters on the temperature distribution's allotment were also incorporated in the present study.

Formulation

The highly nonlinear ODE, which described heat-transfer in a rectangular moving porous fin, can be summarized as introduced by Joneidi et al. [26] and Ndlovu and Moitsheki [5]:

$$\frac{d}{dx}\left\{[1+\beta(y-y_a)]\frac{dy}{dx}\right\}-Nc(y-y_a)^{m+1}-Np(y-y_a)^2-Nr\left(y^4-y_a^4\right)-Pe\frac{dy}{dx}=0 \tag{2.1}$$

$$\frac{dy(0)}{dx}=0 \text{ and } y(1)=1 \tag{2.2}$$

where $0 \le x \le 1$. The important dimensionless parameters are introduced in Eq. (2.1).

Parameter	Notation
Temperature	y
Ambient temperature	y_a
Thermal conductivity gradient	β
Peclet number	Pe
convection-conduction parameter	Nc
radiation conduction parameter	Nr
porosity parameter	Np

Basic Idea of Hermite Wavelet Method and Differential Transformation Method

In this paper, we apply the Hermite wavelet method and Differential transformation method to the discussed problem.

Hermite Wavelet Method

The Hermite wavelet and function approximation are studied in detail [27–32]. To extract the functional matrix, here we use the following procedure. Consider a few Hermite wavelet basis at $k = 1$ as follows:

Table 1 The results of the numerical solution, HWM, and DTM of $y(x)$

x	$\beta = 0,\ Nc = 0.25, Np = Nr = Pe = y_a = m = 0$				
	Exact [26]	HWM	DTM	AE by HWM	AE by DTM
0.00	0.886818883970074	0.886818883970073	0.8868188841	$7.77e^{-16}$	$1.2992e^{-10}$
0.05	0.887096029305517	0.887096029305516	0.8870960292	$7.77e^{-16}$	$1.0551e^{-10}$
0.10	0.887927638536700	0.887927638536700	0.8879276383	$7.77e^{-16}$	$2.3670e^{-10}$
0.15	0.889314231446466	0.889314231446466	0.8893142310	$6.66e^{-16}$	$4.4646e^{-10}$
0.20	0.891256674700520	0.891256674700520	0.8912566748	$5.55e^{-16}$	$9.9480e^{-11}$
0.25	0.893756182389128	0.893756182389128	0.8937561827	$5.55e^{-16}$	$3.1087e^{-10}$
0.30	0.896814316785961	0.896814316785961	0.8968143173	$5.55e^{-16}$	$5.1403e^{-10}$
0.35	0.900432989324568	0.900432989324568	0.9004329897	$4.44e^{-16}$	$3.7543e^{-10}$
0.40	0.904614461793083	0.904614461793083	0.9046144623	$5.55e^{-16}$	$5.0691e^{-10}$
0.45	0.909361347747918	0.909361347747917	0.9093613475	$4.44e^{-16}$	$2.4791e^{-10}$
0.50	0.914676614147318	0.914676614147317	0.9146766135	$4.44e^{-16}$	$6.4731e^{-10}$
0.55	0.920563583205809	0.920563583205809	0.9205635842	$2.22e^{-16}$	$9.9419e^{-10}$
0.60	0.927025934470691	0.927025934470691	0.9270259345	$3.33e^{-16}$	$2.9309e^{-11}$
0.65	0.934067707121872	0.934067707121872	0.9340677074	$3.33e^{-16}$	$2.7812e^{-10}$
0.70	0.941693302496487	0.941693302496487	0.9416933025	$1.11e^{-16}$	$3.5130e^{-12}$
0.75	0.949907486839880	0.949907486839880	0.9499074872	0	$3.6012e^{-10}$
0.80	0.958715394284659	0.958715394284659	0.9587153946	$3.33e^{-16}$	$3.1534e^{-10}$
0.85	0.968122530059700	0.968122530059700	0.9681225298	$1.11e^{-16}$	$2.5970e^{-10}$
0.90	0.978134773931091	0.978134773931091	0.9781347745	$1.11e^{-16}$	$5.6890e^{-10}$
0.95	0.988758383877176	0.988758383877176	0.9887583840	0	$1.2282e^{-10}$
1	1.000000000000000	1.000000000000000	1.0000000000	0	0

$$
\begin{aligned}
\phi_{1,0}(x) &= \frac{2}{\sqrt{\pi}} \\
\phi_{1,1}(x) &= \frac{1}{\sqrt{\pi}}(8x - 4) \\
\phi_{1,2}(x) &= \frac{1}{\sqrt{\pi}}(32x^2 - 32x + 4) \\
\phi_{1,3}(x) &= \frac{1}{\sqrt{\pi}}(128x^3 - 192x^2 + 48x + 8) \\
\phi_{1,4}(x) &= \frac{1}{\sqrt{\pi}}(512x^4 - 1024x^3 + 384x^2 + 128x - 40) \\
\phi_{1,5}(x) &= \frac{1}{\sqrt{\pi}}(2048x^5 - 5120x^4 + 2560x^3 + 1280x^2 - 800x + 16) \\
\phi_{1,6}(x) &= \frac{1}{\sqrt{\pi}}(8192x^6 - 24576x^5 + 15360x^4 + 10240x^3 - 9600x^2 + 384x + 368)
\end{aligned}
$$

Table 2 The results of the exact solution, HWM, and DTM for $y(x)$

x	$\beta = 0,\ Nc = 1, Np = Nr = Pe = y_a = m = 0$				
	Exact [26]	HWM	DTM	AE by HWM	AE by DTM
0.00	0.648054273663885	0.648054273662149	0.6480542737	$1.7e^{-12}$	$3.6115e^{-11}$
0.05	0.648864510284163	0.648864510282524	0.6488645100	$1.63e^{-12}$	$2.8416e^{-10}$
0.10	0.651297246158581	0.651297246157072	0.6512972462	$1.50e^{-12}$	$4.1419e^{-11}$
0.15	0.655358564393981	0.655358564392577	0.6553585647	$1.40e^{-12}$	$3.0601e^{-10}$
0.20	0.661058620401396	0.661058620400088	0.6610586207	$1.30e^{-12}$	$2.9860e^{-10}$
0.25	0.668411667289873	0.668411667288660	0.6684116668	$1.21e^{-12}$	$4.8987e^{-10}$
0.30	0.677436091506664	0.677436091505546	0.6774360915	$1.11e^{-12}$	$6.6640e^{-12}$
0.35	0.688154458812923	0.688154458811896	0.6881544588	$1.02e^{-12}$	$1.2923e^{-11}$
0.40	0.700593570709864	0.700593570708924	0.7005935709	$9.39e^{-13}$	$1.9013e^{-10}$
0.45	0.714784531456473	0.714784531455619	0.7147845318	$8.54e^{-13}$	$3.4352e^{-10}$
0.50	0.730762825846359	0.730762825845589	0.7307628258	$7.70e^{-13}$	$4.6359e^{-11}$
0.55	0.748568407938217	0.748568407937529	0.7485684079	$6.88e^{-13}$	$3.8217e^{-11}$
0.60	0.768245800961792	0.768245800961184	0.7682458015	$6.08e^{-13}$	$5.3820e^{-10}$
0.65	0.789844208649138	0.789844208648607	0.7898442081	$5.31e^{-13}$	$5.4913e^{-10}$
0.70	0.813417638269582	0.813417638269129	0.8134176386	$4.53e^{-13}$	$3.3041\mathrm{e}^{-10}$
0.75	0.839025035676026	0.839025035675650	0.8390250359	$3.75\mathrm{e}^{-13}$	$2.2397\mathrm{e}^{-10}$
0.80	0.866730432700284	0.866730432699982	0.8667304332	$3.01\mathrm{e}^{-13}$	$4.9971\mathrm{e}^{-10}$
0.85	0.896603107266012	0.896603107265780	0.8966031067	$2.32\mathrm{e}^{-13}$	$5.6601\mathrm{e}^{-10}$
0.90	0.928717756619608	0.928717756619452	0.9287177570	$1.56\mathrm{e}^{-13}$	$3.8039\mathrm{e}^{-10}$
0.95	0.963154684112229	0.963154684112173	0.9631546839	$5.52\mathrm{e}^{-14}$	$2.1222\mathrm{e}^{-10}$
1	1.000000000000000	1.000000000000000	1.0000000001	0	$1\mathrm{e}^{-10}$

$$\phi_{1,7}(x) = \frac{1}{\sqrt{\pi}}(32768x^7 - 114688x^6 + 86016x^5 + 71680x^4 - 89600x^3 + 5376x^2 + 10304x - 928)$$

$$\phi_{1,8}(x) = \frac{1}{\sqrt{\pi}}(131072x^8 - 524288x^7 + 458752x^6 + 458752x^5 - 716800x^4 + 57344x^3 + 164864x^2 - 29696x - 3296)$$

$$\phi_{1,9}(x) = \frac{1}{\sqrt{\pi}}(524288x^9 - 2359296x^8 + 2359296x^7 + 2752512x^6 - 5160960x^5 + 516096x^4 + 1978368x^3 - 534528x^2 - 118656x + 21440)$$

$$\phi_{1,10}(x) = \frac{1}{\sqrt{\pi}}(2097152x^{10} - 10485760x^9 + 11796480x^8 + 15728640x^7 - 34406400x^6 + 4128768x^5 + 19783680x^4 - 7127040x^3 - 2373120x^2 + 857600x + 16448)$$

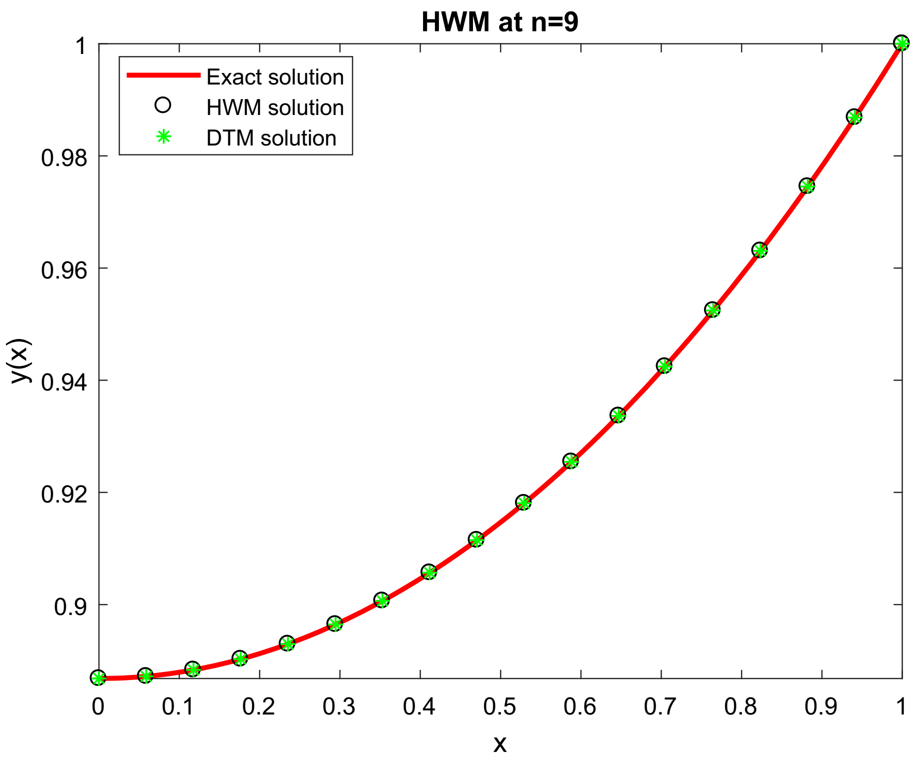

Fig. 1 Comparison of the solutions via exact solution, HWM, and DTM for $y(x)$ when $\beta = 0$, $Nc = 0.25$, $Np = Nr = Pe = y_a = m = 0$

$$\phi_{1,11}(x) = \frac{1}{\sqrt{\pi}}(8388608x^{11} - 46137344x^{10} + 57671680x^9 + 86507520x^8 - 216268800x^7 + 30277632x^6 + 174096384x^5 - 78397440x^4 - 34805760x^3 + 18867200x^2 + 723712x - 461696)$$

$$\phi_{1,12}(x) = \frac{1}{\sqrt{\pi}}(33554432x^{12} - 201326592x^{11} + 276824064x^{10} + 461373440x^9 - 1297612800x^8 + 207618048x^7 + 1392771072x^6 - 752615424x^5 - 417669120x^4 + 301875200x^3 + 17369088x^2 - 22161408x + 561536)$$

where, $\phi_9(x) = [\phi_{1,0}(x), \phi_{1,1}(x), \phi_{1,2}(x), \phi_{1,3}(x), \phi_{1,4}(x), \phi_{1,5}(x), \phi_{1,6}(x), \phi_{1,7}(x), \phi_{1,8}(x)]^T$.

At this time, integrate above first 9 basis regarding x limit from 0 to x, subsequently articulate as a linear-combination of Hermite-wavelet basis as,

$$\int_0^x \phi_{1,0}(x) = \left[\tfrac{1}{2}\ \tfrac{1}{4}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]\phi_9(x)$$

$$\int_0^x \phi_{1,1}(x) = \left[\tfrac{-1}{4}\ 0\ \tfrac{1}{8}\ 0\ 0\ 0\ 0\ 0\ 0\right]\phi_9(x)$$

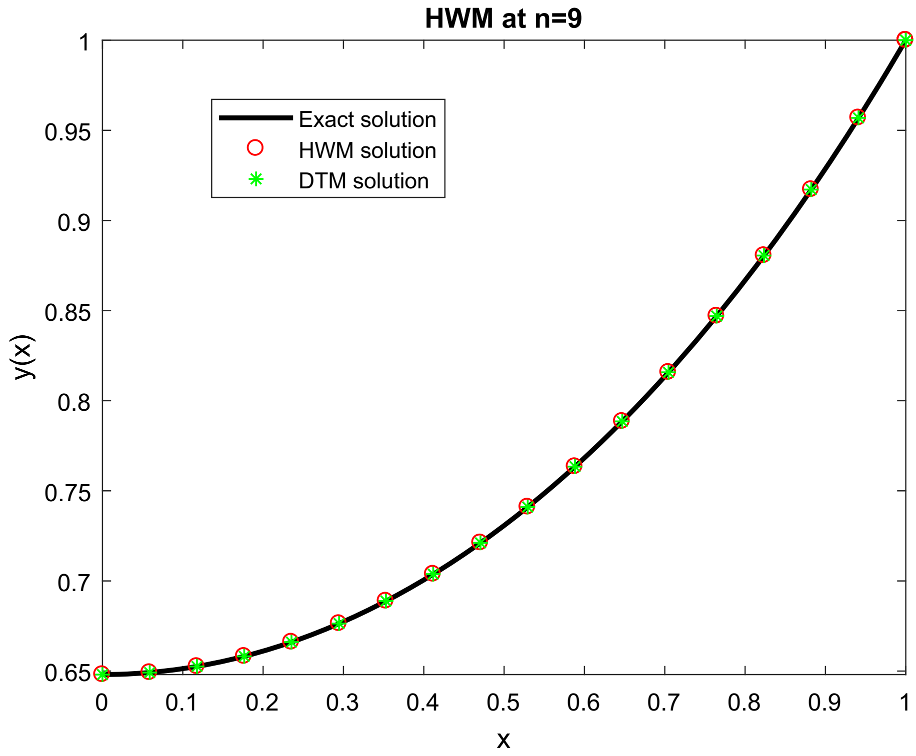

Fig. 2 Comparison of the results via exact solution, HWM, and DTM for $y(x)$ when $\beta = 0$, $Nc = 1$, $Np = Nr = Pe = y_a = m = 0$

$$\int_0^x \phi_{1,2}(x) = \left[\, \frac{-1}{3}\ 0\ 0\ \frac{1}{12}\ 0\ 0\ 0\ 0\ 0 \,\right] \phi_9(x)$$

$$\int_0^x \phi_{1,3}(x) = \left[\, \frac{5}{4}\ 0\ 0\ 0\ \frac{1}{16}\ 0\ 0\ 0\ 0 \,\right] \phi_9(x)$$

$$\int_0^x \phi_{1,4}(x) = \left[\, \frac{-2}{5}\ 0\ 0\ 0\ 0\ \frac{1}{20}\ 0\ 0\ 0 \,\right] \phi_9(x)$$

$$\int_0^x \phi_{1,5}(x) = \left[\, \frac{-23}{3}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{24}\ 0\ 0 \,\right] \phi_9(x)$$

$$\int_0^x \phi_{1,6}(x) = \left[\, \frac{116}{7}\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{28}\ 0 \,\right] \phi_9(x)$$

$$\int_0^x \phi_{1,7}(x) = \left[\, \frac{103}{2}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{32} \,\right] \phi_9(x)$$

Table 3 The results of the exact solution, HWM, and DTM for $y(x)$

x	$\beta = 0.4,\ Nc = 1, Np = Nr = Pe = y_a = m = 0$				
	Exact [26]	HWM	DTM	AE by HWM	AE by DTM
0.00	0.716046471814	0.716046471834	0.7160464622	2.00e-11	$9.6140e^{-9}$
0.05	0.716742314035	0.716742314054	0.7167422747	1.90e-11	$3.9335e^{-8}$
0.10	0.718830179624	0.718830179662	0.7188301611	3.80e-11	$1.8524e^{-8}$
0.15	0.722311474600	0.722311474632	0.7223114651	3.20e-11	$9.5000e^{-9}$
0.20	0.727188432514	0.727188432508	0.7271884199	6.00e-12	$1.2613e^{-8}$
0.25	0.733464148787	0.733464148754	0.7334641375	3.30e-11	$1.1287e^{-8}$
0.30	0.741142604274	0.741142604262	0.7411425952	1.20e-11	$9.0739e^{-9}$
0.35	0.750228627163	0.750228627154	0.7502286174	9.00e-12	$9.7629e^{-9}$
0.40	0.760727863879	0.760727863809	0.7607278540	7.00e-11	$9.8790e^{-9}$
0.45	0.772646763905	0.772646763914	0.7726467558	9.00e-12	$8.1050e^{-9}$
0.50	0.785992554268	0.785992554247	0.7859925453	2.10e-11	$8.9680e^{-9}$
0.55	0.800773195331	0.800773195343	0.8007731867	1.20e-11	$8.6310e^{-9}$
0.60	0.816997356422	0.816997356413	0.8169973500	9.00e-12	$6.4220e^{-9}$
0.65	0.834674381348	0.834674381309	0.8346743754	3.90e-11	$5.9479e^{-9}$
0.70	0.853814240004	0.853814240021	0.8538142337	1.70e-11	$6.3039e^{-9}$
0.75	0.874427488293	0.874427488245	0.8744274849	4.80e-11	$3.3929e^{-9}$
0.80	0.896525236609	0.896525236616	0.8965252359	7.00e-12	$7.0899e^{-10}$
0.85	0.920119098778	0.920119098734	0.9201190955	4.40e-11	$3.2779e^{-9}$
0.90	0.945221123165	0.945221123125	0.9452211304	4.00e-11	$7.2350e^{-9}$
0.95	0.971843786481	0.971843786431	0.9718438178	5.00e-11	$3.1318e^{-8}$
1	1.000000000000	1.000000000000	0.9999999996	0	$4.0000e^{-10}$

$$\int_0^x \phi_{1,8}(x) = \left[\, \tfrac{-2680}{9}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \,\right] \phi_9(x) + \frac{1}{36}\phi_{1,9}(x).$$

Hence,

$$\int_0^x \phi(x)\,dx = H_{9\times 9}\ \phi_9(x) + \ \overline{\phi}_9(x)$$

where,

Table 4 The results of the numerical solution, HWM, and DTM for $y(x)$

x	$\beta = 0.2,\ Nc = 0.25, Np = Nr = Pe = y_a = m = 0$				
	NS [26]	HWM	DTM	AE by HWM	AE by DTM
0.00	0.90344718163	0.90344718123	0.9034471796	4.00e-10	2.0300e^{-9}
0.05	0.90368630795	0.90368630746	0.9036863060	4.90e-10	1.9500e^{-9}
0.10	0.90440375553	0.90440375542	0.9044037536	1.10e-10	1.9299e^{-9}
0.15	0.90559972961	0.90559972995	0.9055997276	3.40e-10	2.0100e^{-9}
0.20	0.90727457224	0.90727457234	0.9072745703	1.00e-10	1.9399e^{-9}
0.25	0.90942876198	0.90942876112	0.9094287601	8.60e-10	1.8800e^{-9}
0.30	0.91206291354	0.91206291367	0.9120629117	1.30e-10	1.8400e^{-9}
0.35	0.91517777735	0.91517777721	0.9151777756	1.40e-10	1.7500e^{-9}
0.40	0.91877423912	0.91877423943	0.9187742374	3.10e-10	1.7200e^{-9}
0.45	0.92285331912	0.92285331954	0.9228533176	4.20e-10	1.5200e^{-9}
0.50	0.92741617157	0.92741617146	0.9274161700	1.10e-10	1.5700e^{-9}
0.55	0.93246408382	0.93246408373	0.9324640824	9.00e-11	1.4200e^{-9}
0.60	0.93799847545	0.93799847590	0.9379984741	4.50e-10	1.3500e^{-9}
0.65	0.94402089730	0.94402089721	0.9440208963	9.00e-11	1.0000e^{-9}
0.70	0.95053303043	0.95053303082	0.9505330293	3.90e-10	1.1299e^{-9}
0.75	0.95753668492	0.95753668431	0.9575366840	6.10e-10	9.1999e^{-10}
0.80	0.96503379868	0.96503379898	0.9650337981	3.00e-10	5.8000e^{-10}
0.85	0.97302643605	0.97302643663	0.9730264355	5.80e-10	5.5000e^{-10}
0.90	0.98151678645	0.98151678625	0.9815167860	2.00e-10	4.4999e^{-10}
0.95	0.99050716283	0.99050716241	0.9905071626	4.20e-10	2.2999e^{-10}
1	0.99999999999	1.000000000000	0.9999999996	1.00e-11	3.9000e^{-10}

$$
H_{9\times 9} = \begin{bmatrix}
\frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{4} & 0 & \frac{1}{8} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{3} & 0 & 0 & \frac{1}{12} & 0 & 0 & 0 & 0 & 0 \\
\frac{5}{4} & 0 & 0 & 0 & \frac{1}{16} & 0 & 0 & 0 & 0 \\
\frac{-2}{5} & 0 & 0 & 0 & 0 & \frac{1}{20} & 0 & 0 & 0 \\
\frac{-23}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{24} & 0 & 0 \\
\frac{116}{7} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{28} & 0 \\
\frac{103}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{32} \\
\frac{-2680}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad \overline{\phi_9}(x) = \begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{36}\phi_{1,9}(x)
\end{bmatrix}.
$$

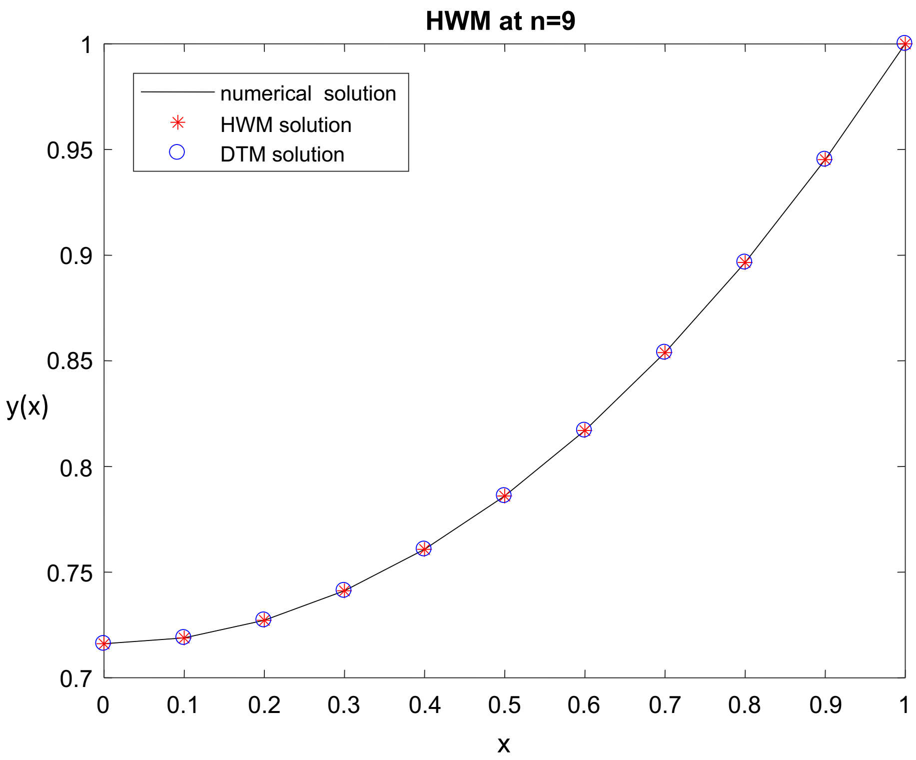

Fig. 3 Comparison of the results via Numerical solution, HWM, and DTM for $y(x)$ when $\beta = 0.4$, $Nc = 1$, $Np = Nr = Pe = y_a = m = 0$

Subsequently, two times integration of above 9 basis we get

$$\int_0^x \int_0^x \phi_{1,0}(x)\,dxdx = \left[\tfrac{3}{16}\ \tfrac{1}{8}\ \tfrac{1}{32}\ 0\ 0\ 0\ 0\ 0\ 0\right]\phi_9(x)$$

$$\int_0^x \int_0^x \phi_{1,1}(x)\,dxdx = \left[\tfrac{-1}{6}\ \tfrac{-1}{16}\ 0\ \tfrac{1}{96}\ 0\ 0\ 0\ 0\ 0\right]\phi_9(x)$$

$$\int_0^x \int_0^x \phi_{1,2}(x)\,dxdx = \left[\tfrac{-1}{16}\ \tfrac{-1}{12}\ 0\ 0\ \tfrac{1}{192}\ 0\ 0\ 0\ 0\right]\phi_9(x)$$

$$\int_0^x \int_0^x \phi_{1,3}(x)\,dxdx = \left[\tfrac{3}{5}\ \tfrac{5}{16}\ 0\ 0\ 0\ \tfrac{1}{320}\ 0\ 0\ 0\right]\phi_9(x)$$

$$\int_0^x \int_0^x \phi_{1,4}(x)\,dxdx = \left[\tfrac{-7}{12}\ \tfrac{-1}{10}\ 0\ 0\ 0\ 0\ \tfrac{1}{480}\ 0\ 0\right]\phi_9(x)$$

$$\int_0^x \int_0^x \phi_{1,5}(x)\,dxdx = \left[\tfrac{-22}{7}\ \tfrac{-23}{12}\ 0\ 0\ 0\ 0\ 0\ \tfrac{1}{672}\ 0\right]\phi_9(x)$$

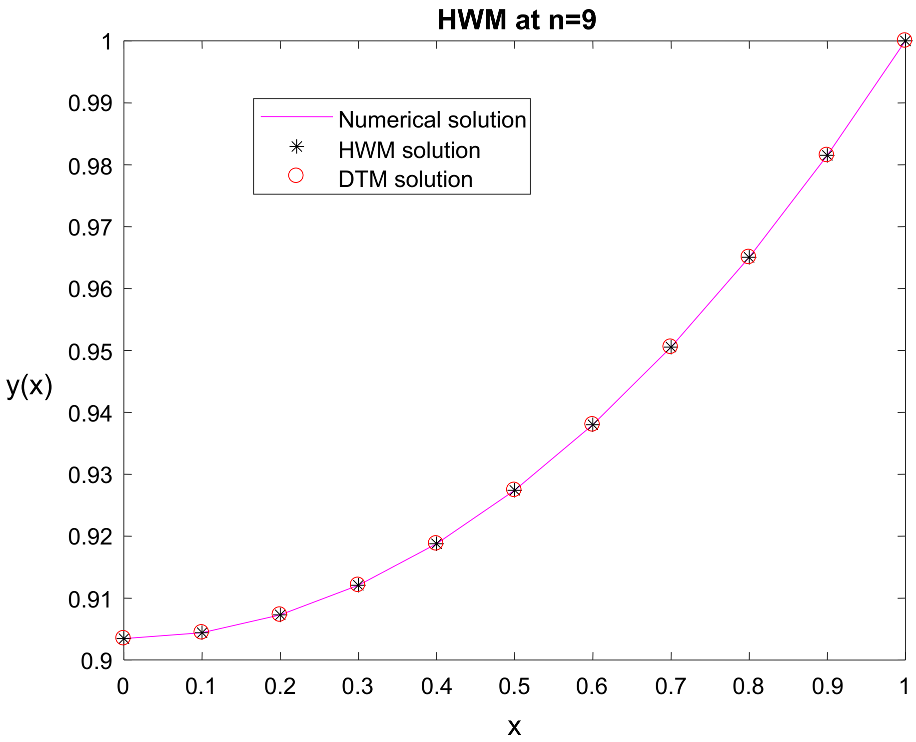

Fig. 4 Comparison of the results via Numerical solution, HWM, and DTM for $y(x)$ when $\beta = 0.2$, $Nc = 0.25$, $Np = Nr = Pe = y_a = m = 0$

$$\int_0^x \int_0^x \phi_{1,6}(x)\,dx dx = \left[\, \tfrac{81}{8} \;\; \tfrac{29}{7} \;\; 0\;0\;0\;0\;0\;0 \;\; \tfrac{1}{896} \,\right] \phi_9(x)$$

$$\int_0^x \int_0^x \phi_{1,7}(x)\,dx dx = \left[\, \tfrac{148}{9} \;\; \tfrac{103}{8} \;\; 0\;0\;0\;0\;0\;0\;0 \,\right] \phi_9(x) + \frac{1}{1152}\phi_{1,9}(x)$$

$$\int_0^x \int_0^x \phi_{1,8}(x)\,dx dx = \left[\, \tfrac{-773}{5} \;\; \tfrac{-670}{9} \;\; 0\;0\;0\;0\;0\;0\;0 \,\right] \phi_9(x) + \frac{1}{1440}\phi_{1,10}(x).$$

Hence,

$$\int_0^x \int_0^x \phi(x)\,dx dx = H'_{9\times 9}\;\phi_9(x) + \overline{\phi}'_9(x)$$

where,

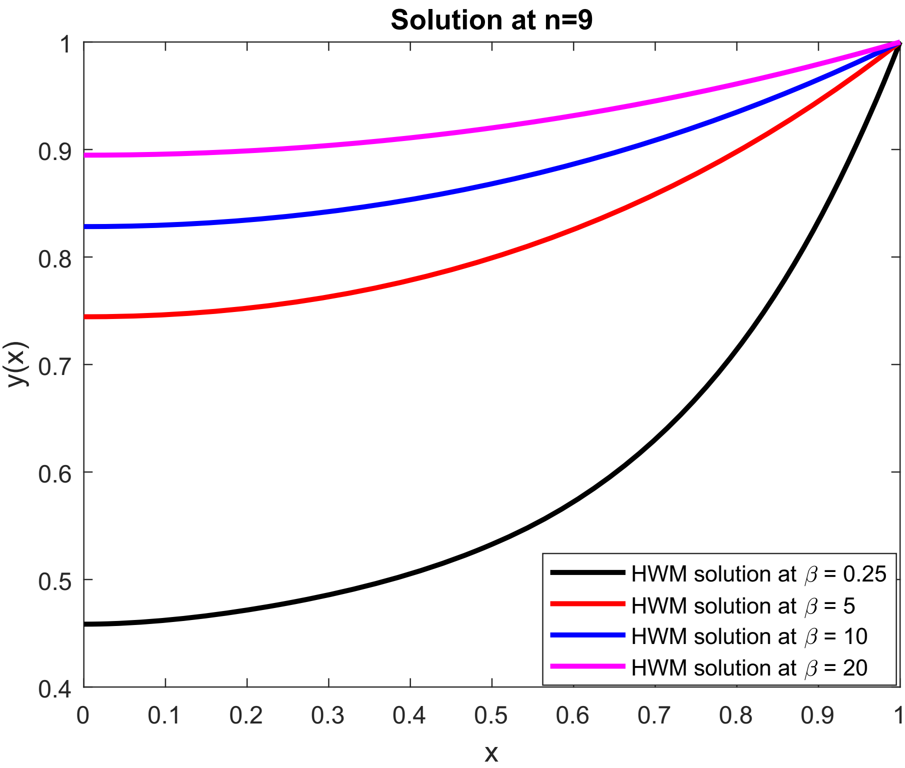

Fig. 5 The effect of β on $y(x)$

$$H^{'}_{9\times 9}=\begin{bmatrix} \frac{3}{16} & \frac{1}{8} & \frac{1}{32} & 0 & 0 & 0 & 0 & 0 & 0\\ \frac{-1}{6} & \frac{-1}{16} & 0 & \frac{1}{96} & 0 & 0 & 0 & 0 & 0\\ \frac{-1}{16} & \frac{-1}{12} & 0 & 0 & \frac{1}{192} & 0 & 0 & 0 & 0\\ \frac{3}{5} & \frac{5}{16} & 0 & 0 & 0 & \frac{1}{320} & 0 & 0 & 0\\ \frac{-7}{12} & \frac{-1}{10} & 0 & 0 & 0 & 0 & \frac{1}{480} & 0 & 0\\ \frac{-22}{7} & \frac{-23}{12} & 0 & 0 & 0 & 0 & 0 & \frac{1}{672} & 0\\ \frac{81}{8} & \frac{29}{7} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{896}\\ \frac{148}{9} & \frac{103}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \frac{-773}{5} & \frac{-670}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \overline{\phi^{'}_9}(x)=\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ \frac{1}{1152}\phi_{1,9}(x)\\ \frac{1}{1440}\phi_{1,10}(x) \end{bmatrix}.$$

For instance, the modeled equation is of second order. So, we created 9*9 matrices up to the second order-functional matrix of integration.

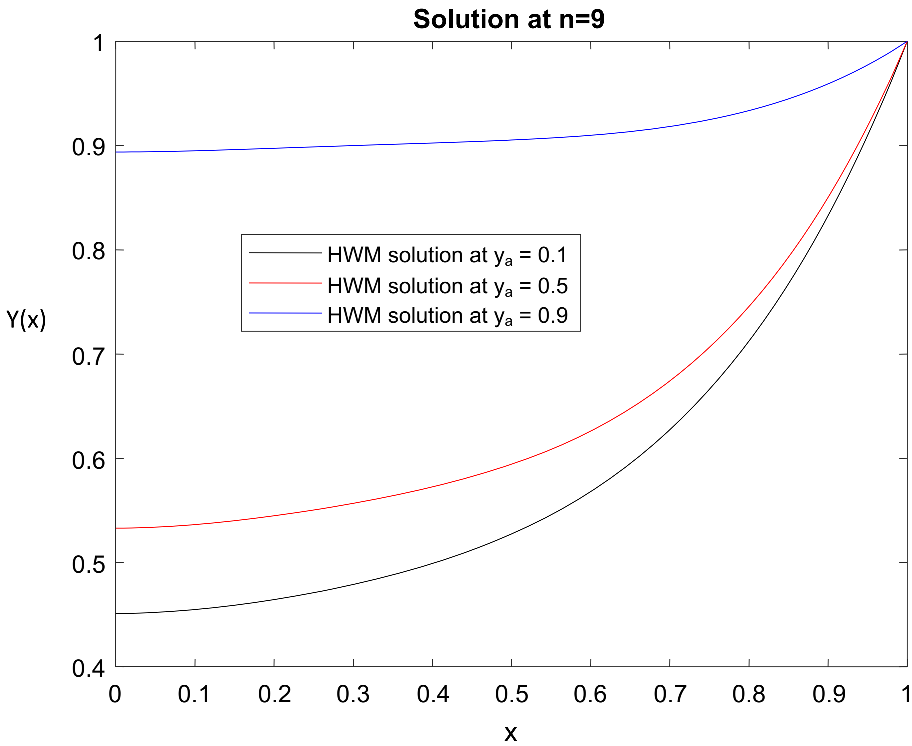

Fig. 6 The effect of y_a on $y(x)$

Basics of Differential Transformation Method

We assume $y(x)$ to be an analytic function in a domain Ω and $x = x_i$ signify some point in Ω. The solution $y(x)$ is subsequently assumed by one power-series whose center is positioned at $x = x_i$. The Taylor-series development solution of $y(x)$ is of the outline [26]

$$y(x) = \sum_{k=0}^{\infty} \frac{(x - x_i)^4}{k!} \left[\frac{d^k y(x)}{dx^k} \right]_{x=x_i} \forall x \in \Omega$$

In particular, $x_i = 0$ then above equation becomes

$$y(x) = \sum_{k=0}^{\infty} \frac{x^4}{k!} \left[\frac{d^k y(x)}{dx^k} \right]_{x=0} \forall x \in \Omega$$

As given in [26], the differential transformation of the solution $y(x)$ is written as:

$$Y(k) = \sum_{k=0}^{\infty} \frac{\mathrm{M}^4}{k!} \left[\frac{d^k y(x)}{dx^k} \right]_{x=0}$$

where novel solution $y(x)$ and transformed solution $Y(k)$. The differential spectrum of $Y(k)$ is restricted inside the distance $x \in [0, \mathrm{M}]$ and as well as M is some constant. The differential

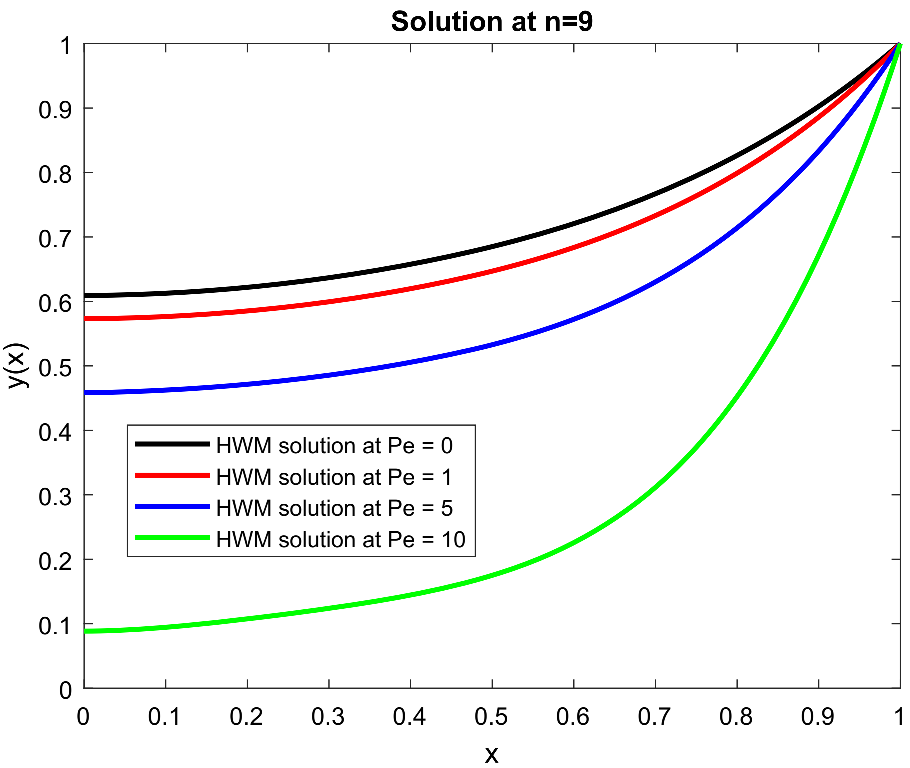

Fig. 7 The effect of Pe on $y(x)$

inverse transform of $Y(k)$ is defined as follows:

$$y(x) = \sum_{k=0}^{\infty} \left(\frac{x}{\mathrm{M}}\right)^k Y(k)$$

The solution $y(x)$ is articulated by a finite power series, and the above equation can also be expressed as:

$$y(x) = \sum_{k=0}^{j} \left(\frac{x}{\mathrm{M}}\right)^k Y(k).$$

The basic operations of DTM are given in below table.

New function	Transformed function
$y(x) = ay_1(x) \pm by_2(x)$	$Y(k) = aY_1(k) \pm bY_2(k)$
$y(x) = \frac{dy_1(x)}{dx}$	$Y(k) = (k+1)\, Y_1(k+1)$
$y(x) = \frac{d^2y_1(x)}{dx^2}$	$Y(k) = (k+1)(k+2)\, Y_1(k+2)$

New function	Transformed function
$y(x) = y_1(x)y_2(x)$	$Y(k) = \sum_{l=0}^{k} Y_1(l)Y_2(k-l)$
$y(x) = x^r$	$Y(k) = \delta(k-r) = \begin{cases} 0 & r \neq k \\ 1 & r = k \end{cases}$
$y(x) = [y_1(x)]^r$	$Y^r(k) = Y^{r-1}(k) \otimes Y(k) = \sum_{l=0}^{k} Y^{r-1}(l)Y(k-l)$

Method of Solutions

Solutions with HWM

Now, assume that

$$y^{''}(x) = A^T\phi(x) \tag{4.1}$$

where $A^{\mathrm{T}} = \left[C_{1,0}, \ldots C_{1,M-1}, C_{2,0}, \ldots C_{2,M-1}, \ldots C_{2^{k-1},0}, \ldots C_{2^{k-1},M-1}\right]$, $\phi(x) = \left[\phi_{1,0}, \ldots \phi_{1,M-1}, \phi_{2,0}, \ldots \phi_{2,M-1}, \ldots \phi_{2^{k-1},0}, \ldots \phi_{2^{k-1},M-1}\right]^{\mathrm{T}}$.

Integrate Eq. (4.1) integrating x form 0 to x and using second initial condition $y'(0) = 0$, we get,

$$y^{'}(x) = A^{\mathrm{T}}\left[H\phi(x) + \overline{\phi}(x)\right]. \tag{4.2}$$

Integrate (4.2) with reverence to x form 0 to x,we get

$$y(x) = y(0) + A^{\mathrm{T}}\left[H^{'}\phi(x) + \overline{\phi}^{'}(x)\right] \tag{4.3}$$

put $x = 1$ in (4.3)

$$y(0) = 1 - A^{\mathrm{T}}\left[H^{'}\phi(x) + \overline{\phi}^{'}(x)\right]\Big|_{x=1} \tag{4.4}$$

substitute (4.4) in (4.3)

$$y(x) = 1 + A^{\mathrm{T}}\left[H^{'}\phi(x) + \overline{\phi}^{'}(x)\right] - A^{\mathrm{T}}\left[H^{'}\phi(x) + \overline{\phi}^{'}(x)\right]\Big|_{x=1} \tag{4.5}$$

Fit (4.1), (4.3), and (4.5) in (2.1), and collocate using the following grid points $x_i = \frac{2i-1}{2N}$, $i = 1, 2, 3, \ldots, N$. Solve these equations using the Newton's-Raphson method that yields unknown coefficients. Then substitute these coefficients in (4.5) that contribute us the numerical solution of (2.1).

Solution with DTM

Now considering M = 1, $m = 0$ and be valid DTM into Eq. (2.1). Now applying the differential transform of Eq. (2.1) with respect to x we get:

$$\mathrm{B}_1(k+1)(k+2)Y(k+2) - NcY(k) + \mathrm{B}_2Y(k) - Pe(k+1)Y(k+1) + \mathrm{B}_3$$

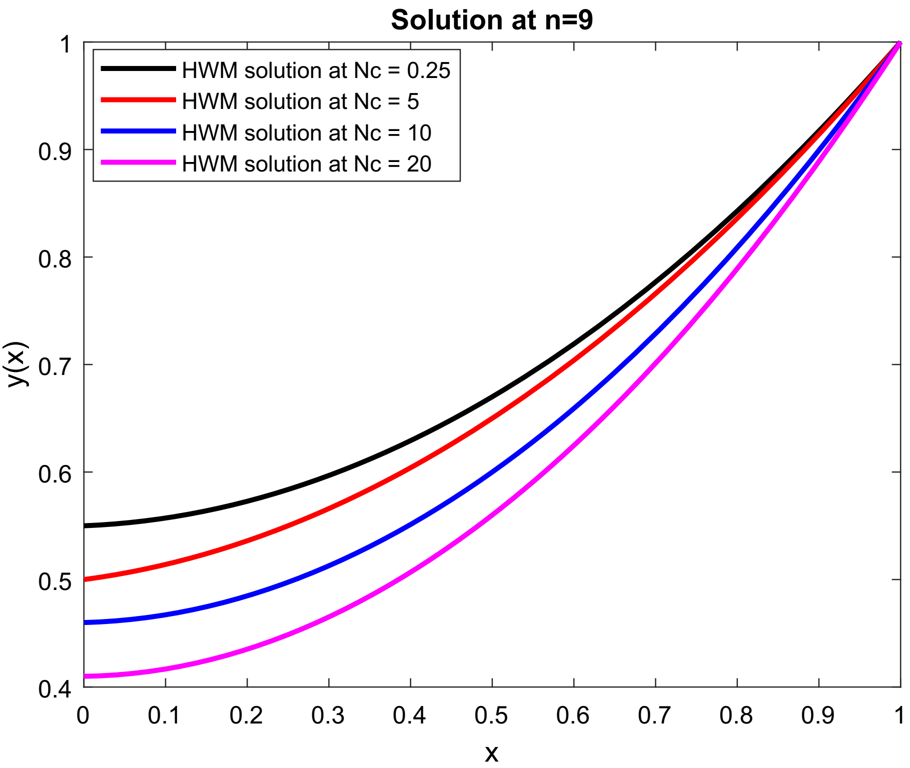

Fig. 8 The effect of Nc on $y(x)$

$$+\beta\sum_{i=0}^{k}[(k-i+1)(k-i+2)Y(i)Y(k-i+2)+(i+1)(k-i+1)Y(i+1)Y(k-i+1)]$$
$$-Np\sum_{i=0}^{k}Y(i)Y(k-i)-Nr\sum_{i=0}^{k}Y^{3}(i)Y(k-i)=0 \tag{4.6}$$

with $\mathrm{B}_1=1-\beta Y_a, \mathrm{B}_2=2NpY_a,\ \mathrm{B}_3=NcY_a-NpY_a^2+NrY_a^4$.

The boundary-condition in Eq. (2.2), with the intention of we have exerting transformation

$$Y(1)=0 \tag{4.7}$$

The other boundary conditions are considered as follow:

$$Y(0)=b \tag{4.8}$$

where b is the constant, and we will find it with allowing for an additional boundary condition in Eq. (2.2) in point $x=1$. Our motivation has:

$$\mathrm{Y}(2)=\frac{-\mathrm{B}_1+b\left(-\mathrm{B}_3+Nc+bNp+b^3Nr\right)}{(\mathrm{B}_2+b\beta)2!}$$
$$\mathrm{Y}(3)=\frac{Y(2)Pe}{3(\mathrm{B}_2+b\beta)}$$

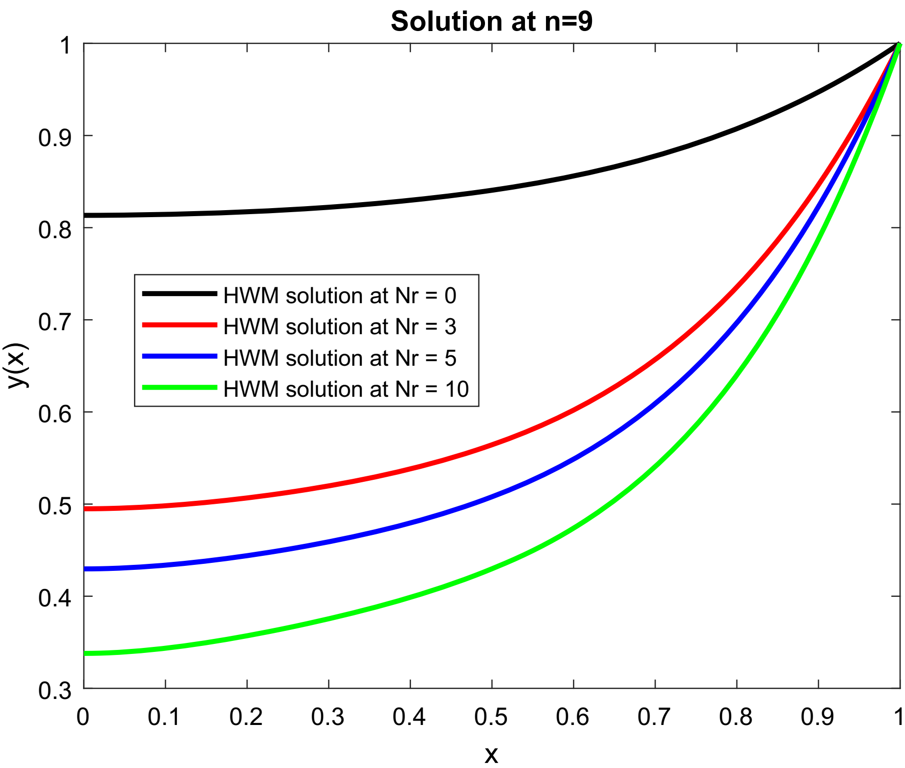

Fig. 9 The effect of Nr on $y(x)$

$$\mathrm{Y}(4) = \frac{\mathrm{Y}(2)\left(-\mathrm{B}_3 + Nc + 2bNp + b^3Nr - 6\beta\mathrm{Y}(2) + \mathrm{bNrY}^2(2)\right) + 3Pe\mathrm{Y}(3)}{12(\mathrm{B}_2 + b\beta)}$$

$$\mathrm{Y}(5) = \frac{Y(3)\left[-\mathrm{B}_3 + Nc + 2bNp + b^3Nr - 20\beta Y(2) + bNrY^2(3)\right] + 4PeY(4)}{20(\mathrm{B}_2 + b\beta)}$$

$$\mathrm{Y}(6) = \frac{(-\mathrm{B}_3 + Nc)Y(4) + Np\left(Y^2(2) + 2bY(4)\right) - 15\beta\left(Y^2(3) + 2Y(2)Y(4)\right) + Nr\left(Y^4(2) + b^3Y(4) + bY^3(4)\right) + 5PeY(5)}{30(\mathrm{B}_2 + b\beta)}$$

The above procedure is incessant. Substituting the values of $Y(0)$ to $Y(n)$ into the most important equation based on DTM, it can be obtained that the series solution in powers of x:

$$y(x) = Y(0) + Y(1)x + Y(2)x^2 + Y(3)x^3 + Y(4)x^4 + Y(5)x^5 + Y(6)x^6 + \cdots. \tag{4.9}$$

To find the numerical value of b, we alternative the boundary-condition from Eq. (2.2) into Eq. (4.9) in point $x = 1$. Then

$$y(1) = 1. \tag{4.10}$$

Solving Eq. (4.10) gives the constant value of b. By substituting the calculated value of b addicted to Eq. (4.9), we can obtain the polynomial expressions of $y(x)$.

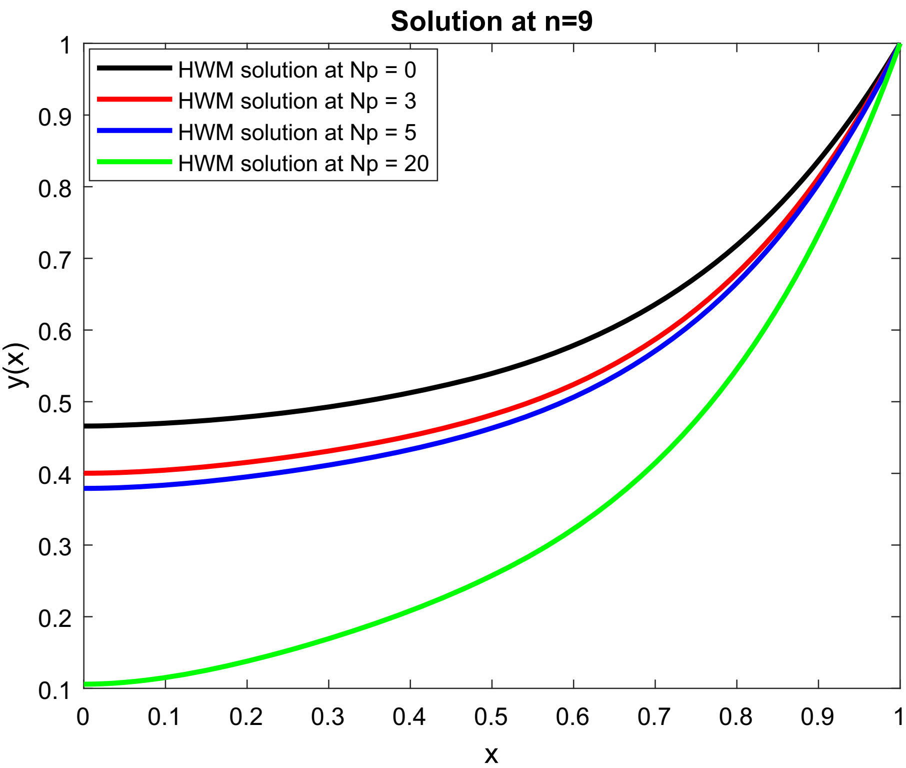

Fig. 10 The effect of Np on $y(x)$

Results and Discussions

The HWM and DTM are applied to resolve the highly nonlinear differential equation arising in the heat transfer fin problem, the disadvantages and advantages of HWM are discussed. In the absence of thermal-conductivity gradient ($\beta = 0$), solutions of the current problem are tabulated vs. the analytical solution [26] in Tables 1 and 2. An enormously good-looking agreement between the solutions is obtained, which concludes the strength of HWM and DTM. Subsequently, in Figs. 1 and 2, the assessment of the results between exact solution, HWM, and DTM is shown. The presence of thermal-conductivity gradient ($\beta \neq 0$), for $\beta = 0.4$ (Table 3), $\beta = 0.2$(Table 4), numerical solution calculated by 4th-order Runge–Kutta of the current problem are given in Tables 3 and 4. In this case, an extremely motivating concurrence connecting the solutions is observed too, which confirms the outstanding strength of the HWM and DTM. After that, in Figs. 3 and 4, the numerical solutions, HWM, and DTM results are compared. Evaluation of the absolute errors of two techniques revealed in Tables 1, 2, 3, 4 exhibit the higher accuracy of HWM than DTM. For this motivation, HWM has been applied to re-examine the unique properties of the governing parameters of the fin problem.

The effect of governing parameters β, y_a, Pe, Nc, Nr, and Np on temperature distribution as a function of x is established in Figs. 5, 6, 7, 8, 9, and 10, respectively. The results exhibited in these figures are for $\beta = y_a = Nc = Np = 0.25,\ m = 2,\ Nr = 4$ and $Pe = 3$ except the varying parameter in any concerned figure. Figures 5 and 6 show the respective effects of β

and y_a, on the temperature distribution $y(x)$. It is easy to see that $y(x)$ increases as β and y_a, increases progressively. Figures 7, 8, 9, 10 give the opposite performance of the temperature distribution $y(x)$ under the influence of Pe, Nc, Nr, and Np, respectively.

Conclusion

In the present article, the natural convection fin flow problem is resolved using HWM and DTM. The flow contains the nonlinear temperature distribution in a rectangular moving permeable fin. We resolved a highly nonlinear ODE by using DTM and HWM. The outcome process of HWM and DTM show the competence for solving highly nonlinear ODE, which is exposed in the table and figures. Comparison between calculated solutions showed that HWM is more suitable and correct than DTM. Moreover, the HWM is more proficient than numerical methods to solve this type of coupled highly nonlinear ODEs.

Author Contribution Both authors contributed to developing the model and performing the study. KRR conceived the core concepts to be used in the model, and KNS developed the method of solution and related the parameter study to applications in robotics.

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Data Accessibility The data that support the findings of this study are available within the article.

Declarations

Conflict of interest The authors declare that they have no competing interests.

References

1. Hoshyar, H.A., Rahimipetroudi, I., Ganji, D.D., Majidian, A.R.: Thermal performance of porous fins with temperature-dependent heat generation via the homotopy perturbation method and collocation method. J. Appl. Math. Comput. Mech. **14**(4), 53–65 (2015)
2. Ma, J., Sun, Y., Li, B.: Simulation of combined conductive, convective, and radiative heat transfer in moving irregular porous fins by spectral element method. Int. J. Thermal Sci. **118**, 475–487 (2017)
3. Motsa, S.S., Sibanda, P., Shateyi, S.: A new spectral-homotopy analysis method for solving a nonlinear second-order BVP. Commun. Nonlinear Sci. Numer. Simul. **15**(9), 2293–2302 (2010)
4. Esmaeilpour, M., Ganji, D.D.: Solution of the Jeffery-Hamel flow problem by optimal homotopy asymptotic method. Comput. Math. Appl. **59**(11), 3405–3411 (2010)
5. Ndlovu, P.L., Moitsheki, R.J.: Steady-state heat transfer analysis in a rectangular moving porous fin. Propul. Power Res. **9**(2), 188–196 (2020)
6. Mosayebidorcheh, S., Rahimi-Gorji, M., Ganji, D.D., Moayebidorcheh, T., Pourmehran, O., Biglarian, M.: Transient thermal behavior of radial fins of rectangular, triangular and hyperbolic profiles with temperature-dependent properties using DTM-FDM. J. Central South Univ. **24**(3), 675–682 (2017)
7. Alkam, M. K., Al-Nimr, M. A.: Solar collectors with tubes partially filled with porous substrates (1999)
8. Kiwan, S., Al-Nimr, M.A.: Using porous fins for heat transfer enhancement. J. Heat Transf. **123**(4), 790–795 (2001)
9. Kiwan, S.: Effect of radiative losses on the heat transfer from porous fins. Int. J. Thermal Sci. **46**(10), 1046–1055 (2007)
10. Kiwan, S., Zeitoun, O.: Natural convection in a horizontal cylindrical annulus using porous fins. Int. J. Numer. Methods Heat Fluid Flow (2008)
11. Patel, T., Meher, R.: A study on temperature distribution, efficiency, and effectiveness of longitudinal porous fins by using the Adomian decomposition sumudu transform method. Proc. Eng. **127**, 751–758 (2015)

12. Saedodin, S., Sadeghi, S.: Temperature distribution in long porous fins in natural convection condition. Middle-East J. Sci. Res. **13**(6), 812–817 (2013)
13. Gorla, R.S.R., Bakier, A.Y.: Thermal analysis of natural convection and radiation in porous fins. Int. Commun. Heat Mass Transf. **38**(5), 638–645 (2011)
14. Kundu, B., Bhanja, D.: An analytical prediction for performance and optimum design analysis of porous fins. Int. J. Refrig. **34**(1), 337–352 (2011)
15. Taklifi, A., Aghanajafi, C., Akrami, H.: The effect of MHD on a porous fin attached to a vertical isothermal surface. Transp. Porous Media **85**(1), 215–231 (2010)
16. Hussain, M., Ghaffar, A., Ali, A., Shahzad, A., Nisar, K.S., Alharthi, M.R., Jamshed, W.: MHD thermal boundary layer flow of a Casson fluid over a penetrable stretching wedge in the existence of nonlinear radiation and convective boundary condition. Alexandria Eng. J. **60**(6), 5473–5483 (2021)
17. Lund, L.A., Omar, Z., Khan, I., et al.: Dual similarity solutions of MHD stagnation point flow of Casson fluid with the effect of thermal radiation and viscous dissipation: stability analysis. Sci. Rep. **10**, 15405 (2020). https://doi.org/10.1038/s41598-020-72266-2
18. Nisar, K.S., Mohapatra, R., Mishra, S.R., Ganeswar Reddy, M.: Semi-analytical solution of MHD free convective Jeffrey fluid flow in the presence of heat source and chemical reaction. Ain Shams Eng. J. **12**(1), 837–845 (2021)
19. Ramzan, M., Gul, H., Baleanu, D., et al.: Role of Cattaneo-Christov heat flux in an MHD Micropolar dusty nanofluid flow with zero mass flux condition. Sci. Rep. **11**, 19528 (2021). https://doi.org/10.1038/s41598-021-98988-5
20. Ali, K., Ahmad, S., Nisar, K.S., Faridi, A.A., Ashraf, M.: Simulation analysis of MHD hybrid $CuAl_2O_3/H_2O$ nanofluid flow with heat generation through a porous media. Int. J. Energy Res. (2021). https://doi.org/10.1002/er.7016
21. Ghanbari, B., Nisar, K.S.: Determining new soliton solutions for a generalized nonlinear evolution equation using an effective analytical method. Alexandria Eng. J. **59**(5), 3171–3179 (2020)
22. Kumbinarasaiah, S., & Raghunatha, K. R. (2021). The applications of Hermite wavelet method to nonlinear differential equations arising in heat transfer. Int. J. Thermofluids, 9, 100066.
23. Kumbinarasaiah, S., Raghunatha, K. R., Rezazadeh, M., Mustafa, Inc.: A solution of coupled nonlinear differential equations arising in a rotating micropolar nanofluid flow system by Hermite wavelet technique. Eng. Comput. pp 1–22 (2021)
24. Kumbinarasaiah, S., Raghunatha, K.R.: A novel approach on micropolar fluid flow in a porous channel with high mass transfer via wavelet frames. Nonlinear Eng. **10**(1), 39–45 (2021)
25. Raghunatha, K. R., & Kumbinarasaiah, S. (2021). Laguerre wavelet numerical solution of micropolar fluid flow in a porous channel with high mass transfer. J. Interdiscip. Math., 1–14.
26. Joneidi, A.A., Ganji, D.D., Babaelahi, M.: Differential transformation method to determine fin efficiency of convective straight fins with temperature-dependent thermal conductivity. Int. Commun. Heat Mass Transf. **36**(7), 757–762 (2009)
27. Saeed, U.: Hermite wavelet method for fractional delay differential equations. J. Differ. Equ. (2014)
28. Gupta, A.K., Ray, S.S.: An investigation with Hermite Wavelets for accurate solution of Fractional Jaulent-Miodek equation associated with energy-dependent Schrödinger potential. Appl. Math. Comput. **270**, 458–471 (2015)
29. Shiralashetti, S.C., Kumbinarasaiah, S.: Hermite wavelets operational matrix of integration for the numerical solution of nonlinear singular initial value problems. Alexandria Eng. J. **57**(4), 2591–2600 (2018)
30. Kumbinarasaiah, S., Adel, W.: Hermite wavelet method for solving nonlinear Rosenau–Hyman equation. Part. Differ. Equ. Appl. Math. 100062 (2021)
31. Kumbinarasaiah, S., Mundewadi, R.A.: The new operational matrix of integration for the numerical solution of integro-differential equations via Hermite wavelet. SeMA J. 1–18 (2021)
32. Kumbinarasaiah, S.: Hermite wavelets approach for the multi-term fractional differential equations. J. Interdiscip. Math. 1–22 (2021)

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