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ORIGINAL PAPER

A Comprehensive Study of EOQ (Economic Order Quantity) System for Spoilage Commodities with Stock–Sensitive Demand and Trade Credits

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Abstract
Prior EOQ models under trade credits generally assumed that the demand of the commodities was either stable or purely dependent on the selling price. This paper develops an economic order control model of deteriorating commodities with inventory associated demand under different conditions. It is assumed that the permissible delay period is permitted till specific threshold. The comparative study of order quantity with threshold quantity is also discussed. Optimal solutions are obtained with the help of differential calculus and optimality condition. Numerical examples and sensitive analyses are provided to validate the optimal solution. It is also shown that the total cost function is U-shaped.

Keywords Inventory · Deterioration · Economic order quantity · Trade credits · Stock-dependent demand

Introduction

Researchers presume in the standard EOQ models, that the value of inventory commodities is unaltered by the length of time. In practice, however many items depreciate during the usual storage stage. Volatile liquids, green vegetables, blood accumulated in blood banks, chemicals and electronic gears decline considerably. Deterioration means decompose, damage, spoilage, fading, or aeration out of foodstuffs. Therefore, the ideal case visualizes by the conventional model remains a perfect one.

At present scenario, researchers are facing a major problem to analyze the practical and actual work problem in the correct environment. In this environment supplier has to offer a number of discounts for their clients for concession in the permissible delay period. Delayed payment might be considered to enhance sales. Large number of elements disturbed the smooth transaction and business process as like, lock off, labour problem, earth quake etc.

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Most of the commodities lose their originality at the time passes as like, bread, seasonal vegetable, fruits, food stuffs etc. having more deterioration is comparison to other items. Some commodities like medicine, electronic products, machine with fixed life span.

Classical inventory model was established in the year 1915 in which demand was constant, while in actual practice, it is in a dynamic state. Some internal factor such as selling price and availability for a particular product made by manufacturer affect the smooth business transaction. In this study, demand rate is considered stock-sensitive. Goh [1] designed the unbroken, unlimited horizon inventory scheme where the demand in stock-dependent. Tripathi [2] established an EOQ model for price-linked demand and different carrying cost function. Affares [3] designed an EOQ inventory model for stock-associated demand and shortage with time—linked carrying cost. Taleizadeh and Noori-daryan [4] designed pricing, industrialized and inventory strategies for unprocessed objects in a three-stage supply chain. Tripathi [5] studied a model for spoilage commodities with linearly time-linked demand rate under permitted delay. Goyal and Change [6] pointed out an ordinary-transfer an EOQ model for determining the vendor's optimal order quantity and a number of transfers from the warehouse to the exhibit area. Soni and Shah [7] considered an optimal ordering policy for retailer when customers' demand is stock-sensitive and supplier suggest progressive credit period. Khanra et al. [8] designed an EOQ model for deteriorating item and quadratic time-linked demand under trade credits. Other related studies based on variable demand are: Kumar and Sana [9], Chowdhury [10], Tripathi and Mishra [11].Tripathi [12], Guchhait et al. [13], Ghiami and Williams [14], Hsieh and Dye [15], Ouyang et al. [16] etc.

The delay in payment is an actual part is business transaction to enhance demand for a short run span, it dominates the cost of the business system like: holding cost, ordering cost and total cost. Tripathi and Chaudhary [17] developed an EOQ model for Weibull distribution decline with inflation and permitted delay in payments. Teng et al. [18] established an EOQ model under trade credit financing by means of growing demand. A number of related studies have been presented by; Chakrabarty et al. [19], Taleizadeh et al. [20], Mishra [21], Dave [22], Liao et al. [23], Tripathi et al. [24], Jiangtao et al. [25], Soni [26], Wu et al. [27] in this direction.

The problem of deteriorating inventory has taken strong concentration in the present scenario. Several researchers have considered constant deterioration rate is their inventory models. However, in real life it is not always steady. One simple example of variable deterioration is Weibull distribution which differs with time. Geetha and Udayakumar [28] presented an EPQ model for deteriorating items with price and advertisement dependent demand under partial backlogging. Yang [29] designed an EOQ model for deteriorating item with inflation and two level of storage. Min [30] presented a model for spoilage commodities under inventory-sensitive demand and two-level trade credits. Tsao and Sheen [31] established dynamic pricing promotion problem for deteriorating items. Some articles related to deterioration like: Chen et al. [32], Taleizadeh et al. [33], Chang et al. [34], Taleizadeh and Noori-daryan [35], Taleizadeh et al. [36].

In this paper, EOQ models are presented for items with stock-linked demand under deterioration. The specific threshold is compared with order quantity. We also compare threshold with permissible delay. The main idea of the model is to settle on least total cost.

The rest part of the study is arranged as follows: in the next portion 2, the assumption and notations used are given. In Sect. 3, mathematical model is obtained to decide minimum total cost/unit time. Numerical examples and sensitivity examination are provided in Sects. 4 and 5 respectively. We establish future research direction in Sect. 6.

Assumptions and Notations

Assumptions

The succeeding assumptions are made to buildup the model:

(i) Deterioration rate is invariable.
(ii) In case of order size exceeds as pre-determined quantity complete permissible delay period is offered.
(iii) Demand is considered stock—sensitive (i.e. $D(t) = \alpha + \beta\ I(t)$, $\alpha > 0, 0 < \beta < 1\}$.
(iv) Shortages are not acceptable.
(v) Time horizon is never-ending.

Notations

The following notations are used in the whole manuscript:

A	Ordering cost
$D(t)$	Stock-dependent demand/year
W	Specific threshold
θ	Deterioration rate, $(0<\theta<1)$
P	Selling price
T	Cycle time
C	Purchasing cost
h	Holding cost/unit/year
$I(t)$	Inventory level at any time 't'
I_e and I_c	Interest earned and charges/ year
M	Permissible delay period
μ	Permitted fraction of delay in payment
Q	Order quantity
T_w	Threshold time-period
T	Cycle time
T_0	Time during $(1-\mu)Q$ units are depleted
IC	Total cost on interest
TE	Total interest earned
$NTC_i(T)$	Annual non-identical part of cost function, $i = 1 - 3$
$ATC_i(T)$	Annual total cost, $i = 1 - 3$

Mathematical Formulation

The following differential equation represents the change of $I(t)$

$$\frac{dI(t)}{dt} + \gamma I(t) = -\{\alpha + \beta I(t)\}; \quad 0 \le t \le T, \quad \mathrm{I}(T) = 0 \tag{1}$$

The, order quantity:

$$Q = I(0) = \alpha T\left\{1 + \frac{\beta T}{2} + \frac{T^2}{6}\left(\beta^2 + \gamma\right)\right\} \tag{2}$$

From (2), the time throughout which W units are exhausted is attained as follows:

$$T_w = \frac{\sqrt{1 + \frac{\beta W}{\alpha}} - 1}{\beta} \tag{3}$$

If $Q > W(T > T_w)$, completely permissible delay is allowed. Consumer must pay $(1-\mu)CQ$ and pay off μCQ at the closing stages of delay phase.

T_0 is defined as the time in which $(1-\mu)Q$ units are exhausted, from (2), we get

$$T_0 = \frac{\beta(1-\mu)}{2\alpha^2} \quad \text{(approx.)} \tag{4}$$

It should be noted that parameters M, T_w, T_0 and T are constraints that have affects the capital cost. We can consider only cases.

Identical Costs

Identical related costs are: ordering cost (OC), carrying cost (HC) and purchasing cost (PC);

$$OC = \frac{A}{T} \tag{5}$$

$$HC = \frac{h}{T}\int_0^T I(t)dt = \frac{h\alpha T}{6}\left\{3 + \beta T + \left(\beta^2 - \gamma\right)T^2\right\}, \quad \text{(approx.)} \tag{6}$$

$$PC = \frac{CQ}{T} = C\alpha\left\{1 + \frac{\beta T}{2} + \frac{T^2}{6}\left(\beta^2 + \gamma\right)\right\}, \quad \text{(approx.)} \tag{7}$$

The identical cost of the system is:

$$ITC(T) = OC + \text{HC} + \text{PC} \quad \text{(see in Appendix I)} \tag{8}$$

Non-Identical Terms

As stated, because of unlike values of T_w and M, two group of $M \geq T_w$ and $M < T_w$ may take place. Now for every case of the opening set, where $M \geq T_w$, IC and IE will be derived.

Case 1 $T_w \leq M \leq T$.

In this consideration, the credit period is shorter than cycle time. The pictorial representation is as follows:

From the Fig. 1, we get

$$\begin{aligned} IC &= \frac{I_k C}{T}\int_M^T I(t)dt \\ &= \frac{I_k C\alpha}{T}\Bigg[\frac{T^2}{2} + \frac{\beta T^3}{6}\left\{3 + \left(\beta^2 + \gamma\right)\right\} - \frac{MT}{3}\left\{6 + T^2\left(5\gamma - 4\beta^2\right)\right\} \\ &\quad - \frac{M^2}{2}\left\{2 - \beta T + T^2\left(3\gamma - 2\beta^2\right) + \frac{\beta T^3}{6}\left(2\left(\beta^2 + \gamma\right) + 9\beta\left(\gamma - \beta^2\right)\right)\right\} \\ &\quad + \frac{M^3}{12}\left\{-6\beta + 6T\left(\gamma - \beta^2\right) + 3\beta T^2\left(\gamma - \beta^2\right) + T^3\right\}\Bigg] \end{aligned} \tag{9}$$

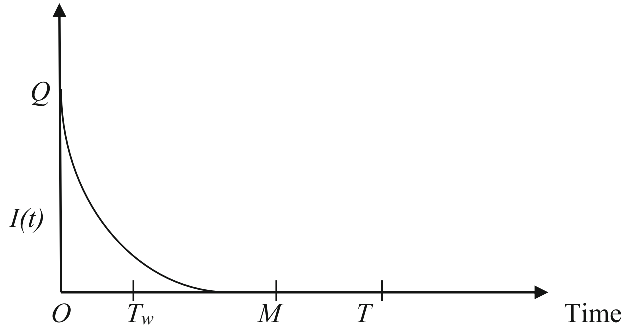

Fig. 1 Graph between time and $I(t)$ $(T_w \le M \le T)$

and

$$IE = \frac{I_e P}{T}\int_0^M \{\alpha + \beta I(t)\}dt$$
$$= \frac{I_e P\alpha}{6T}\left[6 + \frac{BM^2}{2}\left\{6T + 3\beta T^2 + T^3\left(\beta^2+\gamma\right)\right\}\right.$$
$$\left. - \frac{\beta M^3}{3}\left\{6 + 6\beta T + 3\beta^2T^2 + \beta T^3\left(\beta^2+\gamma\right)\right\}\right] \tag{10}$$

now,

$$NTC_1(T) = \frac{I_k C}{T}\int_M^T I(t)dt - \frac{I_e P}{T}\int_0^M \{\alpha + \beta I(t)\}dt \tag{11}$$

Therefore, the annual total cost is:

$$ATC_1(T) = ITC + NTC_1(T) \quad \text{(see Appendix I)} \tag{12}$$

Case 2 $T_w \le T \le M$.

In this case, the permissible delay period is longer than cycle time, then interest charged and interest earned both will considered in the calculation of total inventory cost.

From Fig. 2, we getand

$$IC = \frac{I_e P}{T}\int_0^T \{\alpha + \beta I(t)\}\, t\, dt = \frac{I_e P\alpha T}{12}\left[6 + 2\beta T - \beta^2 T^2\right]$$
$$IE = \frac{I_e P}{T}\int_M^T \{\alpha + \beta I(t)\}dt$$
$$= I_e P\alpha\left[\left\{1 - \frac{M}{T} + \beta T + \beta^2T^2 + \frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\}\right.$$
$$- \beta M\left\{1 + \frac{\beta T}{2} + \frac{T^2}{6}\left(\beta^2+\gamma\right)\right\} - \frac{\beta T}{2}\left\{1 + \beta T + \frac{\beta T^2}{2}\right\}$$
$$- \frac{\beta M^2}{2}\left\{\frac{1}{T} + \beta + \frac{\beta^2 T}{2} + \frac{\beta T^2}{6}\left(\beta^2+\gamma\right)\right\} + \frac{\beta T^2}{6}\left\{\beta - \left(\gamma - \beta^2\right)T\right\}$$
$$\left. + \frac{\beta M^3}{3}\left\{-\frac{\beta}{2T} + \left(\frac{1}{2} + \frac{\beta T}{4} + \frac{T^2}{12}\left(\beta^2+\gamma\right)\right)\left(\gamma - \beta^2\right)\right\}\right] \tag{13}$$

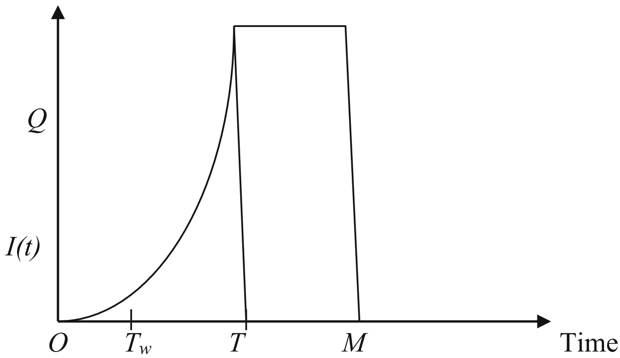

Fig. 2 Graph between time and $I(t)$ ($T_w \le T \le M$)

Therefore,

$$NTC_2(T) = -\frac{I_e P}{T}\int_0^T \{\alpha + \beta I(t)\}t\,dt - \frac{I_e P}{T}\int_M^T \{\alpha + \beta I(t)\}dt \tag{14}$$

and,

$$ATC_2(T) = ITC + NTC_2(T) \quad \text{(see Appendix I)} \tag{15}$$

Case 3 $T \le T_w \le M$.

In this case, the trade credit phase is shorter than specific threshold time.

From Fig. 3, we search out the total cost for this case is as follows:

$$NTC_3(T) = \frac{I_k C}{T}\int_0^{T_0} I(t)dt - \frac{I_k C}{T}\mu Q T_0 - \frac{I_e P}{T}\int_0^M \{\alpha + \beta I(t)\}\,\mathrm{t}\,dt - \frac{I_e P}{T}\int_M^T \{\alpha + \beta I(t)\}dt \tag{16}$$

and,

$$ATC_3(T) = ITC + NTC_3(T) \text{ (see Appendix I)} \tag{17}$$

Optimal value of T_i is obtained on putting $\frac{dTAC_i}{dT} = 0, i = 1 - 3$ (See Appendix I).

It is difficult to find $\frac{d^2TAC_i}{dT^2} > 0$, for i $= 1-3$, we can show numerically in the numerical examples and sensitivity analysis.

Note The remaining mathematical calculations are given in the Appendix.

Numerical Examples

In this part, the model is demonstrated by numerical explanations for three unlike cases with Mathematica 9.0 software.

Example 1. (Case 1) $T_w \le M \le T$.

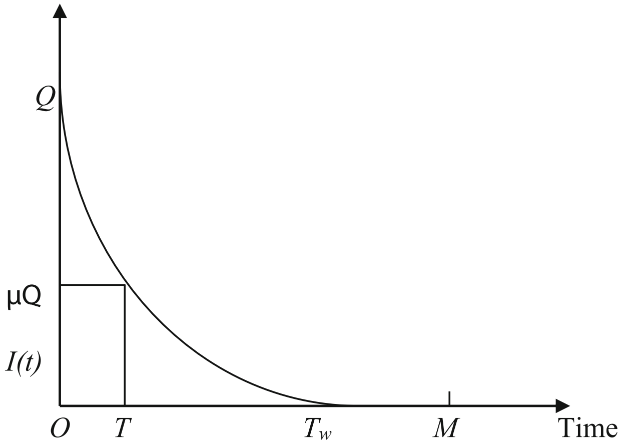

Fig. 3 Graph between time and inventory level ($T < T_w \leq M$)

Let us consider A = 50, h = 5, α = 1000, $\beta = 0.1$, γ = 0.5, $M = 1$, $I_k = 0.1$, $I_e = 0.01$, $C = 5$, $P = 100$ in appropriate units. Putting these in (A8) and calculating for T, we get, $T_1^* = 1.7355$, corresponding, $Q^* = 2330.41$,$ATC_1^* = 6269.93$ and $\frac{d^2ATC_1}{dT} = 212165.0 > 0$.

Example 2. (Case 2) $T_w \leq T < M$.

On considering $A = 50$, $h = 5$,$\alpha = 1000$, $\beta = 0.1$, γ = 0.5,$M = 1$, $I_k = 0.1$, $I_e = 0.01$, $C = 5$, $P = 100$ in proper units. On substituting these in (A9) and calculating for T, we obtain $T_2^* = 0.69123$, corresponding, $Q^* = 743.193$, $ATC_2^* = 7316.46$ and $\frac{d^2ATC_2}{dT} = 6011.97 > 0$.

Example 3. (Case 3) $T < T_w \leq M$.

Let us take $A = 40$, $h = 8$, $\alpha = 800$, $\beta = 0.1$, γ = 0.5, M = 1,$I_k = 0.1$, $I_e = 0.01$, $C = 5$, $P = 80$,$\mu = 0.2$ in suitable units. Substituting these in (A10) and solving for T, we get $T_3* = 0.47872$, corresponding $Q* = 399.603$, $ATC_3* = 6413.49$, and $\frac{d^2ATC_3}{dT^2} = 12386.9 > 0$.

Sensitivity Analysis

Sensitivity analysis is a significant component of any kind of business transaction for both vendors and buyers; it affects the whole transaction system. We know that any type of business is a long taking process, depends on nature and future planning, while future planning's are unsure and estimated by their elements like weather, lock off, strikes etc.

Case 1 Taking all numerical values discussed in Ex. 1, shifting one constraints at a time, preserving residual constraints identical.

From Table 1, the following deductions can be made:

(i) Boost of c, β and α resulting, increase of T_1^*, Q^* and ATC_1^*.It mean that the optimal values move in the identical direction with c, β and α.
(ii) Argument of A, r and h leading, decrease of T_1^*, Q^* and ATC_1^*.It shows that T_1*, $Q*$ and ATC_1* move in the toward the back direction with A, r and h.
(iii) Boost of I_e and p resulting, increase in T_1^*, Q^* and decrease in ATC_1^*.

Table 1 (a) Variation of T_1*, Q_1* and ATC_1* with different parameters

C	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$	α	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$
08	1.81637	2490.7	8887.17	229,889.0	1500	1.73674	3499.2	9347.35	702,863.0
10	1.86908	2598.8	10,669.7	209,787.0	2000	1.73735	4668.0	12,453.6	1.6475×10^6
12	1.92104	2708.2	12,466.8	169,470.0	2500	1.73772	5836.8	15,559.8	3.19411×10^6
14	1.97238	2819.1	14,278.5	112,399.0	3000	1.73797	7005.6	18,666.1	5.49072×10^6
16	2.02321	2931.8	16,104.6	41,325.7	3500	1.73814	8174.4	21,772.3	8.68581×10^6
A	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$	β	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$
60	1.73476	2329.0	6246.86	212,872.0	0.01	1.58609	1931.2	5488.92	51,343.7
70	1.73401	2327.5	6252.59	213,589.0	0.02	1.59995	1967.1	5560.04	82,762.5
80	1.73327	2326.1	6258.33	214,297.0	0.03	1.61444	2004.8	5633.91	110,923.0
90	1.73252	2324.6	6264.07	215,017.0	0.04	1.62957	2044.4	5710.69	135,756.0
100	1.73177	2323.2	6269.82	215,737.0	0.05	1.64537	2086.1	5790.50	157,199.0
γ	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$	h	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$
1.0	1.214340	1589.5	4591.75	936,532.0	10	1.62764	2126.6	8740.51	324,768.0
1.5	0.995069	1292.5	3765.77	1.91687×10^6	15	1.58762	2053.8	11,277.70	372,684.0
2.0	0.867066	1123.0	3217.90	3.07729×10^6	20	1.56659	2016.1	13,825.40	398,343.0
2.5	0.780553	1010.0	2809.80	4.37817×10^6	25	1.55359	1993.0	16,377.60	413,664.0
3.0	0.716957	927.54	2486.17	5.79991×10^6	30	1.54475	1977.4	18,392.10	423,326.0
M	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$	P	T_1^*	Q^*	ATC_1^*	$\frac{d^2ATC_1}{dT^2}$
2.0	1.53725	1964.2	3060.56	4.89357×10^6	50	1.69964	2261.4	6552.30	121,542.0
2.5	1.53420	1958.8	1807.41	1.01503×10^7	100	1.73550	2330.4	6241.12	212,165.0
3.0	1.60212	2080.0	1002.48	1.55969×10^6	150	1.76807	2394.2	5935.05	275,945.0
3.5	1.77447	2406.8	938.571	1.77613×10^7	200	1.79800	2453.7	5633.47	318,578.0
4.0	2.17978	3297.7	2241.87	1.25257×10^6	250	1.82574	2509.7	5335.89	34,107.0

Case 2 Taking all numerical values from example 2. On fluctuating one constraint at a time and keeping rest parameters unvarying.

From Table 2 the following deductions can be reviewed.

(i) Increase of c, β and h lead decrease of T_2^*, Q^* and increase of ATC_2^*. It means that T_2* and Q*, move in opposite direction with c, β and h, while total cost parallel to c, β and h.

(ii) Increase of M, r, A and P causes, augment in T_2^*, Q^* and ATC_2^* It shows that all optimal values move in the similar direction.

Case 3 Taking all numerical values discussed in Ex. 3, varying one parameter at a time and keeping rest parameters same.

Table 2 Variation of T_2*, Q_2* and ATC_2* with different parameters

C	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$	α	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$
10	0.594520	630.05	12,648.7	10,906.2	1500	0.685385	1104.4	10,938.4	9141.75
15	0.538513	566.29	17,925.4	15,513.1	2000	0.682455	1465.5	14,560.1	12,272.4
20	0.499379	522.43	23,169.0	20,039.9	2500	0.680694	1826.7	18,181.8	15,403.6
25	0.469564	489.39	28,389.5	24,549.6	3000	0.679520	2192.0	21,803.5	18,420.9
30	0.445653	463.11	33,592.6	29,069.1	3500	0.678680	2557.3	25,425.1	21,666.2
A	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$	β	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$
100	0.708662	764.023	7387.90	5771.15	0.2	0.655234	723.485	7697.98	7979.19
200	0.743155	805.655	7525.67	5316.32	0.3	0.625799	708.642	8104.84	10,149.1
300	0.777345	847.485	7657.23	4890.72	0.4	0.601121	697.284	8539.41	12,527.8
400	0.811459	889.799	7783.13	4481.10	0.5	0.579968	688.444	9004.19	15,127.0
500	0.845732	932.913	7903.84	4102.92	0.6	0.561474	681.421	9501.82	17,962.9
γ	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$	h	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$
0.5	0.691230	743.19	7316.46	6011.97	10	0.484643	506.06	8692.95	18,167.8
1.0	0.693689	773.94	7372.48	5082.55	15	0.392299	405.13	9759.30	34,958.1
1.5	0.697770	807.61	7427.78	4082.05	20	0.337902	346.89	10,658.40	55,307.8
2.0	0.705313	847.73	7482.28	2934.36	25	0.301143	308.00	11,449.80	78,664.2
2.5	0.726450	913.21	7535.68	1254.86	30	0.274207	279.72	12,164.60	104,675.0
M	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$	P	T_2^*	Q^*	ATC_2^*	$\frac{d^2ATC_2}{dT^2}$
1.3	0.805867	882.82	7792.38	4567.73	50	0.463582	482.80	7006.13	11,441.5
1.5	0.884172	982.01	8079.28	3714.19	60	0.510328	534.65	7105.01	10,017.1
1.7	0.968021	1092.0	8344.80	2884.79	70	0.555767	585.80	7182.63	8820.12
1.9	1.064740	1224.0	8589.47	2009.69	80	0.600638	637.10	7242.31	7780.91
2.1	1.205160	1426.6	8811.20	853.285	90	0.645582	689.3	7286.37	6855.08

Note Some graphs on sensitivity investigations are provided in the Appendix II.

From the above Table 3 following inferences can be summarized.

(i) Increase of c, β and h lead, decrease in T_3^*, Q^* and increase in ATC_3^*.It shows that T_3* and Q* move opposite to c, β and h, while ATC_3* move alike to c, β and h.
(ii) On increasing α, diminish in T_3*, while increase in Q*and ATC_3*.
(iii) On increasing γ, A, P and M, increase in T_3^*, Q^* and ATC_3^*. It shows that parameters and optimal values move the similar way.

Conclusion and Future Research

In this study EOQ models are developed for deteriorating commodities with stock-linked demand considering three dissimilar situations. We assumed that spoilage rate is constant.

Table 3 Variation of T_3*, Q* and ATC_3* with various parameters

C	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$	α	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$
10	0.443033	368.191	10,577.5	16,582.1	1000	0.475894	496.38	7995.92	15,600.50
15	0.416500	345.052	14,726.1	20,685.7	1200	0.472106	744.57	11,951.70	23,638.60
20	0.395477	326.844	18,863.2	24,751.9	1400	0.470204	992.76	15,907.30	3678.800
25	0.378143	311.911	22,991.4	28,806.8	1600	0.469059	1222.1	19,862.90	39,720.20
30	0.363449	299.308	27,112.3	32,865.2	1800	0.468295	1466.5	2388.40	47,761.90
A	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$	β	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$
100	0.499549	418.098	6356.11	11,724.2	0.2	0.467544	398.882	6670.40	14,420.6
200	0.533008	448.067	6729.74	10,746.8	0.3	0.457343	398.499	6948.54	16,689.7
300	0.565203	477.218	6911.81	9892.40	0.4	0.498020	398.445	7249.60	19,197.7
400	0.596417	505.789	7083.94	9131.47	0.5	0.439456	398.676	7575.36	21,953.2
500	0.626880	533.975	7247.39	8443.25	0.6	0.431533	399.134	7927.71	23,127.5
γ	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$	h	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3^*}{dT^2}$
1.0	0.482570	410.505	6423.87	6011.97	10	0.428150	355.189	6768.65	17,437.9
1.5	0.487307	422.643	6433.73	5082.55	15	0.348708	286.714	7529.01	32,672.7
2.0	0.493353	436.600	6442.97	4082.05	20	0.301285	246.519	8171.73	51,020.7
2.5	0.501545	453.520	6451.42	2934.36	25	0.269004	219.421	8738.29	72,011.3
3.0	0.514038	476.312	6458.76	1254.86	30	0.245242	199.602	9250.51	95,339.8
M	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$	P	T_3^*	Q^*	ATC_3^*	$\frac{d^2ATC_3}{dT^2}$
0.5	0.341089	280.22	5541.35	18,360.3	10	0.189170	153.23	5231.30	36,991.80
1.0	0.478720	399.60	6413.49	12,386.9	20	0.246177	200.38	5536.03	28,054.40
1.5	0.596806	506.15	7108.68	9149.72	30	0.293502	239.97	5763.63	2312.500
2.0	0.709844	612.35	7752.45	6829.61	40	0.335466	275.44	5944.93	19,795.00
2.5	0.827482	727.90	8199.58	4875.97	50	0.374040	308.39	6094.05	17,329.50

Mathematical formulations are provided for three models to find optimal solution and corresponding examples are conversed to validate the EOQ models.

This paper also offered a realistic submission illustration, in which the projected EOQ model is employed to hold up trade assessment. Mainly, the model presented in the learning could be helpful in the area of supply string administration. Our consequence demonstrates that this model can be relatively functional in determining the best possible ordering strategy, in which the permitted delay period is being investigated. The planned model can be used in inventory control of convinced decomposing commodities such as pictorial film, electronic components, and radioactive equipments which display stock-sensitive utilization.

Sensitivity examination with the variation of several parameters is also conversed from managerial point of view. Following main point will be kept in mind for vendors and buyers.

(i) Purchase cost, optimal cycle time, order quantity and total cost do not move in the same direction, it means that vendor and buyer has to take precaution during transaction process.

(ii) Deterioration rate, optimal cycle time and total cost of all time cases are uniform.

A usual generalization of the model would be to study that the case of gaps in the trade credit periods. In addition, this research can be generalized for deteriorating items with a two-parameter Weibull distribution. The planned model can further integrate more sensible suppositions, such as stochastic demand and trapezoidal fuzzy demand. The study can be extended by taking into account discount, shortage and time value money. The consideration of price-sensitive demand is another fulltime research direction. We may also generalized by taking deterioration is variable etc.

Appendix I

Solution of (1), using condition $I(T) = 0$ is given by:

$$\begin{aligned} I(t) &= T\alpha\left\{1 + \frac{\beta T}{2} + \frac{T^2}{6}\left(\beta^2 + \gamma\right)\right\} - \alpha\left\{1 + \beta T + \frac{\beta^2 T^2}{2} + \frac{\beta T^3}{6}\left(\beta^2 + \gamma\right)\right\}t \\ &\quad - \frac{\alpha}{2}\left\{-\beta + T\left(\gamma - \beta^2\right) + \frac{\beta T^2}{2}\left(\gamma - \beta^2\right) + \frac{T^3}{6}\left(\gamma^2 - \beta^4\right)\right\}t^2 \\ &\quad - \alpha\left\{\frac{\beta^2}{6} + \frac{2}{3}\gamma - \frac{\beta\gamma T}{2} - \frac{\beta^2\gamma T^2}{4} - \frac{\beta\gamma T^3}{12}\left(\gamma + \beta^2\right)\right\}t^3, \end{aligned} \tag{A1}$$

$$\begin{aligned} NTC_1(T) &= \frac{I_k C\alpha}{T}\left[\frac{T^2}{2} + \frac{\beta T^3}{6}\left\{3 + \left(\beta^2 + \gamma\right)\right\} - \frac{MT}{3}\left\{6T + T^3\left(5\gamma - 4\beta^2\right)\right\}\right. \\ &\quad - \frac{M^2}{2}\left\{2 - \beta T + T^2\left(3\gamma - 2\beta^2\right) + \frac{\beta T^3}{6}\left(2\left(\beta^2 + \gamma\right) + 9\beta\left(\gamma - \beta^2\right)\right)\right\} \\ &\quad \left. + \frac{M^3}{12}\left\{-6\beta + 6T\left(\gamma - \beta^2\right) + 3\beta T^2\left(\gamma - \beta^2\right) + T^3\right\}\right] \\ &\quad - \frac{I_e P\alpha}{6T}\left[6 + \frac{BM^2}{2}\left\{6T + 3\beta T^2 + T^3\left(\beta^2 + \gamma\right)\right\}\right. \\ &\quad \left. - \frac{\beta M^3}{3}\left\{6 + 6\beta T + 3\beta^2 T^2 + \beta T^3\left(\beta^2 + \gamma\right)\right\}\right], \end{aligned} \tag{A2}$$

$$\begin{aligned} ATC_1(T) &= \frac{A}{T} + \frac{h\alpha}{6}\left[3T + \beta T^2 + \left(\beta^2 - \gamma\right)T^3\right] + \frac{C\alpha}{T}\left[T + \frac{\beta T^2}{2} + \frac{T^3}{6}\left(\beta^2 + \gamma\right)\right] \\ &\quad + \frac{I_k C\alpha}{T}\left[\frac{T^2}{2} + \frac{\beta T^3}{6}\left\{3 + \left(\beta^2 + \gamma\right)\right\} - \frac{M}{6}\left\{12T + T^3\left(10\gamma - 8\beta^2\right)\right\}\right. \\ &\quad - \frac{M^2}{2}\left\{2 - \beta T + T^2\left(3\gamma - 2\beta^2\right)\right. \\ &\quad \left. + \frac{\beta T^3}{6}\left(2\left(\beta^2 + \gamma\right) + 9\beta\left(\gamma - \beta^2\right)\right)\right\} \\ &\quad \left. + \frac{M^3}{12}\left\{-6\beta + 6T\left(\gamma - \beta^2\right) + 3\beta T^2\left(\gamma - \beta^2\right) + T^3\right\}\right] \\ &\quad - \frac{I_e P\alpha}{6T}\left[6 + \frac{BM^2}{2}\left\{6T + 3\beta T^2 + T^3\left(\beta^2 + \gamma\right)\right\}\right. \\ &\quad \left. - \frac{\beta M^3}{3}\left\{6 + 6\beta T + 3\beta^2 T^2 + \beta T^3\left(\beta^2 + \gamma\right)\right\}\right], \end{aligned} \tag{A3}$$

$$
\begin{aligned}
NTC_2(T) = & -\frac{I_e P\alpha T}{12}\left[6+2\beta T-\beta^2T^2\right] \\
& -I_eP\alpha\left[\left\{1-\frac{M}{T}+\beta T+\beta^2T^2+\frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\}\right. \\
& -\beta M\left\{1+\frac{\beta T}{2}+\frac{T^2}{6}\left(\beta^2+\gamma\right)\right\}-\frac{\beta T}{2}\left\{1+\beta T+\frac{\beta T^2}{2}\right\} \\
& -\frac{\beta M^2}{2}\left\{\frac{1}{T}+\beta+\frac{\beta^2 T}{2}+\frac{\beta T^2}{6}\left(\beta^2+\gamma\right)\right\} \\
& +\frac{\beta}{3}\left\{\frac{\beta}{2}T^2-\frac{T^3}{2}\left(\gamma-\beta^2\right)\right\} \\
& \left.+\frac{\beta M^3}{3}\left\{-\frac{\beta}{2T}+\left(\frac{1}{2}+\frac{\beta T}{4}+\frac{T^2}{12}\left(\beta^2+\gamma\right)\right)\left(\gamma-\beta^2\right)\right\}\right], \qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
ATC_2(T) = & \frac{A}{T}+\frac{h\alpha}{6}\left[3T+\beta T^2+\left(\beta^2-\gamma\right)T^3\right]+\frac{C\alpha}{T}\left[T+\frac{\beta T^2}{2}+\frac{T^3}{6}\left(\beta^2+\gamma\right)\right] \\
& -\frac{I_e P\alpha T}{12}\left[6+2\beta T-\beta^2T^2\right]-I_eP\alpha\left[\left\{1-\frac{M}{T}+\beta T+\beta^2T^2+\frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\}\right. \\
& -\beta M\left\{1+\frac{\beta T}{2}+\frac{T^2}{6}\left(\beta^2+\gamma\right)\right\}-\frac{\beta T}{2}\left\{1+\beta T+\frac{\beta T^2}{2}\right\} \\
& -\frac{\beta M^2}{2}\left\{\frac{1}{T}+\beta+\frac{\beta^2 T}{2}+\frac{\beta T^2}{6}\left(\beta^2+\gamma\right)\right\} \\
& +\frac{\beta}{3}\left\{\frac{\beta}{2}T^2-\frac{T^3}{2}\left(\gamma-\beta^2\right)\right\} \\
& \left.+\frac{\beta M^3}{3}\left\{-\frac{\beta}{2T}+\left(\frac{1}{2}+\frac{\beta T}{4}+\frac{T^2}{12}\left(\beta^2+\gamma\right)\right)\left(\gamma-\beta^2\right)\right\}\right], \qquad \text{(A5)}
\end{aligned}
$$

$$
\begin{aligned}
& NTC_3(T) = \frac{I_k C\beta}{2T\alpha^2}\left[3(1-\mu)+\frac{\beta^2(1-\mu)^2}{2\alpha^2}+\frac{\beta^2\left(\beta^2-\gamma\right)(1-\mu)^3}{4\alpha^4}\right] \\
& -\frac{I_k C\mu\beta(1-\mu)}{2}\left[1+\frac{\beta T}{2}+\frac{T^2}{6}\left(\beta^2+\gamma\right)\right]-\frac{I_e P\alpha T}{12}\left(6+2\beta T-\beta^2T^2\right) \\
& -I_eP\alpha\left[\left\{1-\frac{M}{T}+\beta T+\beta^2T^2+\frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\}-\beta M\left\{1+\frac{\beta T}{2}+\frac{T^2}{6}\left(\beta^2+\gamma\right)\right\}\right. \\
& -\frac{\beta T}{2}\left\{1+\beta T+\frac{\beta T^2}{2}\right\}-\frac{\beta M^2}{2}\left\{\frac{1}{T}+\beta+\frac{\beta^2 T}{2}+\frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\}+\frac{\beta T^2}{6}\left\{\beta-\left(\gamma-\beta^2\right)T\right\} \\
& \left.+\frac{\beta M^3}{2}\left\{-\frac{\beta}{2T}+\left(\frac{1}{2}+\frac{\beta T}{4}+\frac{T^2}{12}\left(\beta^2+\gamma\right)\right)\left(\gamma-\beta^2\right)\right\}\right], \qquad \text{(A6)}
\end{aligned}
$$

$$
\begin{aligned}
ATC_3(T) &= \frac{A}{T} + \frac{h\alpha}{6}\left[3T + \beta T^2 + \left(\beta^2 - \gamma\right)T^3\right] + \frac{C\alpha}{T}\left[T + \frac{\beta T^2}{2} + \frac{T^3}{6}\right.\\
&\left.\left(\beta^2 + \gamma\right)\right] + \frac{I_k C}{T}\left[\frac{3\beta(1-\mu)}{2\alpha^2} + \frac{\beta^3(1-\mu)^2}{4\alpha^4} + \frac{\beta^3\left(\beta^2-\gamma\right)(1-\mu)^3}{8\alpha^6}\right]\\
&- \frac{I_k C\mu\beta(1-\mu)}{2T}\left[T + \frac{\beta T^2}{2} + \frac{T^3}{6}\left(\beta^2+\gamma\right)\right] - \frac{I_e P\alpha T}{12}\left[6 + 2\beta T - \beta^2 T^2\right]\\
&- I_e P\alpha\left[\left\{1 - \frac{M}{T} + \beta T + \beta^2 T^2 + \frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\} - \beta M\left\{1 + \frac{\beta T}{2} + \frac{T^2}{6}\left(\beta^2+\gamma\right)\right\}\right.\\
&- \frac{\beta T}{2}\left\{1 + \beta T + \frac{\beta T^2}{2}\right\} - \frac{\beta M^2}{2}\left\{\frac{1}{T} + \beta + \frac{\beta^2 T}{2} + \frac{\beta T^3}{6}\left(\beta^2+\gamma\right)\right\}\\
&\left.+ \frac{\beta T^2}{6}\left\{\beta - \left(\gamma-\beta^2\right)T\right\} + \frac{\beta M^3}{2}\left\{-\frac{\beta}{2T} + \left(\frac{1}{2} + \frac{\beta T}{4} + \frac{T^2}{12}\left(\beta^2+\gamma\right)\right)\left(\gamma-\beta^2\right)\right\}\right].
\end{aligned} \tag{A7}
$$

Differentiating (A3), (A5) and (A7), we get

$$
\begin{aligned}
\frac{dATC_1}{dT} &= -\frac{A}{T^2} + \frac{h\alpha}{6}\left\{3 + 2\beta T + 3T^2\left(\beta^2-\gamma\right)\right\}\\
&+ \frac{C\alpha}{6}\left\{3\beta + 2T\left(\beta^2+\gamma\right)\right\} + \frac{I_k C\alpha}{12}\left[6 + 4\beta T\right.\\
&\left\{3 + \left(\beta^2+\gamma\right)\right\} - 4MT\left(10\gamma - 8\beta^2\right)\\
&- 6M^2\left\{\frac{-2}{T^2} + \left(3\gamma - 2\beta^2\right)\right\} + \frac{\beta T}{3}\left\{2\left(\beta^2+\gamma\right) + 9\beta\left(\gamma-\beta^2\right)\right\}\\
&\left.+ M^3\left\{\frac{6\beta}{T^2} + \left(3\gamma - 2\beta^2\right) + 2T\right\}\right] - \frac{I_e P}{6}\left[-\frac{6\alpha}{T^2} + \frac{\beta M^2}{2}\left\{3\alpha\beta + 2\alpha T\left(\beta^2+\gamma\right)\right\}\right.\\
&\left.- \frac{\beta M^3}{2}\left\{-\frac{6\alpha}{T^2} + 3\alpha\beta^2 + 2\alpha\beta T\left(\beta^2+\gamma\right)\right\}\right],
\end{aligned} \tag{A8}
$$

$$
\begin{aligned}
\frac{dATC_2}{dT} &= -\frac{A}{T^2} + \frac{h\alpha}{6}\left\{3 + 2\beta T + 3T^2\left(\beta^2-\gamma\right)\right\}\\
&+ \frac{C\alpha}{6}\left\{3\beta + 2T\left(\beta^2+\gamma\right)\right\} - \frac{I_e P\alpha}{12}\left\{6 + 4\beta T - 3\beta^2 T^2\right\}\\
&- I_e P\alpha\left[\left\{\frac{M}{T^2} + \beta + 2\beta^2 T + \frac{\beta T^2}{2}\left(\beta^2+\gamma\right)\right\} - \beta M\left\{\frac{\beta}{2} + \frac{T}{3}\left(\beta^2+\gamma\right)\right\}\right.\\
&- \frac{\beta}{2}\left\{1 + 2\beta T + \frac{3\beta T^2}{2}\right\} - \frac{\beta M^2}{2}\left\{-\frac{1}{T^2} + \frac{\beta^2}{2} + \frac{\beta T}{3}\left(\beta^2+\gamma\right)\right\}\\
&+ \frac{\beta}{3}\left\{\beta T - \frac{3T^2}{2}\left(\gamma-\beta^2\right)\right\} + \frac{\alpha\beta M^3}{2}\left\{\frac{\beta}{2T^2}\right.\\
&\left.\left.+ \left(\frac{\beta}{4} + \frac{T}{6}\left(\beta^2+\gamma\right)\right)\left(\gamma-\beta^2\right)\right\}\right],
\end{aligned} \tag{A9}
$$

and

$$
\begin{aligned}
\frac{dATC_3}{dT} &= -\frac{A}{T^2} + \frac{h\alpha}{6}\left\{3 + 2\beta T + 3T^2\left(\beta^2-\gamma\right)\right\}\\
&+ \frac{C\alpha}{6}\left\{3\beta + 2T\left(\beta^2+\gamma\right)\right\} - \frac{I_e P\alpha}{12}\left\{6 + 4\beta T - 3\beta^2 T^2\right\}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{I_k C}{T^2}\left[\frac{3\beta(1-\mu)}{2\alpha^2} + \frac{\beta^3(1-\mu)^2}{4\alpha^4} + \frac{\beta^3\left(\beta^2-\gamma\right)(1-\mu)^3}{8\alpha^6}\right] \\
&- \frac{I_k C\mu\beta(1-\mu)}{2}\left[\frac{\beta}{2} + \frac{T}{3}\left(\beta^2+\gamma\right)\right] \\
&- I_e P\alpha\left[\left\{\frac{M}{T^2} + \beta + 2\beta^2 T + \frac{\beta T^2}{2}\left(\beta^2+\gamma\right)\right\}\right. \\
&- \beta M\left\{\frac{\beta}{2} + \frac{T}{3}\left(\beta^2+\gamma\right)\right\} - \frac{\beta}{2}\left\{1 + 2\beta T + \frac{3\beta T^2}{2}\right\} \\
&- \frac{\beta M^2}{2}\left\{-\frac{1}{T^2} + \frac{\beta^2}{2} + \frac{\beta T}{3}\left(\beta^2+\gamma\right)\right\} + \frac{\beta}{3}\left\{\beta T - \frac{3T^2}{2}\left(\gamma-\beta^2\right)\right\} \\
&\left.+ \frac{\beta M^3}{2}\left\{\frac{\beta}{2T^2} + \left(\frac{\beta}{4} + \frac{T}{6}\left(\beta^2+\gamma\right)\right)\left(\gamma-\beta^2\right)\right\}\right]. \qquad \text{(A10)}
\end{aligned}
$$

Again differentiating (A8), (A9) and (A10), we get

$$
\begin{aligned}
\frac{d^2TAC_1}{dT^2} &= \frac{2A}{T^3} + \frac{h\alpha}{3}\left[\beta + 3T\left(\beta^2-\gamma\right)\right] + \frac{C\alpha}{3}\left(\beta^2+\gamma\right) \\
&+ \frac{I_k C\alpha}{12}\left[4\beta\left\{3 + \left(\beta^2+\gamma\right)\right\} - 4M\left(10\gamma - 8\beta^2\right)\right. \\
&\left.- 6M^2\left\{\frac{4}{T^3} + \frac{\beta}{3}\left(2\left(\beta^2+\gamma\right) + 9\beta\left(\gamma-\beta^2\right)\right) + M^3\left(-\frac{12\beta}{T^3} + 2\right)\right\}\right] \\
&- \frac{I_e P\alpha}{6}\left[\frac{12\alpha}{T^3} + \frac{\beta M^2}{2}\left\{2\alpha\left(\beta^2+\gamma\right)\right\} - \frac{\beta M^3}{3}\left\{\frac{12\alpha}{T^3} + 2\alpha\beta\left(\beta^2+\gamma\right)\right\}\right], \qquad \text{(A11)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d^2TAC_2}{dT^2} &= \frac{2A}{T^3} + \frac{h\alpha}{3}\left[\beta + 3T\left(\beta^2-\gamma\right)\right] + \frac{C\alpha}{3}\left(\beta^2+\gamma\right) \\
&+ \frac{I_e P\alpha}{12}\left(4\beta - 6\beta^2 T\right) - I_e P\alpha\left[\left(-\frac{2M}{T^3} + 2\beta^2\right) - \frac{\beta M}{3}\left(\beta^2+\gamma\right)\right. \\
&- \frac{\beta}{2}(2\beta + 3\beta T) - \frac{\beta M^2}{2}\left\{\frac{2}{T^3} + \frac{\beta}{3}\left(\beta^2+\gamma\right)\right\} \\
&\left.+ \frac{\beta}{3}\left\{\beta - 3T\left(\gamma-\beta^2\right)\right\} + \frac{\beta M^3}{3}\left\{-\frac{\beta}{T^3} + \frac{1}{6}\left(\gamma^2-\beta^4\right)\right\}\right], \qquad \text{(A12)}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{d^2TAC_3}{dT^2} &= \frac{2A}{T^3} + \frac{h\alpha}{3}\left[\beta + 3T\left(\beta^2-\gamma\right)\right] + \frac{C\alpha}{3}\left(\beta^2+\gamma\right) \\
&+ \frac{2I_k C}{T}\left[\frac{3\beta(1-\mu)}{2\alpha^2} + \frac{\beta^3(1-\mu)^2}{4\alpha^4} + \frac{\beta^3\left(\beta^2-\gamma\right)(1-\mu)^3}{8\alpha^6}\right] \\
&- \frac{I_k C\mu\beta(1-\mu)}{6}\left(\beta^2+\gamma\right) - \frac{I_e P\alpha}{6}\left(2\beta - 3\beta^2 T\right) \\
&- I_e P\alpha\left[\left\{-\frac{2M}{T^3} + 2\beta^2 + \beta T\left(\beta^2+\gamma\right)\right\} - \frac{\beta M}{3}\left(\beta^2+\gamma\right)\right. \\
&- \frac{\beta}{2}(2\beta + 3\beta T) - \frac{\beta M^2}{2}\left\{\frac{2}{T^3} + \beta T\left(\beta^2+\gamma\right)\right\}
\end{aligned}
$$

$$+\frac{\beta}{3}\left\{\beta-3T\left(\gamma-\beta^{2}\right)\right\}+\frac{\beta M^{3}}{2}\left\{-\frac{\beta}{T^{3}}+\frac{1}{6}\left(\gamma^{2}-\beta^{4}\right)\right\}\Bigg]. \tag{A13}$$

Appendix II

Graph of Case 1

See Figs. 4 and 5.

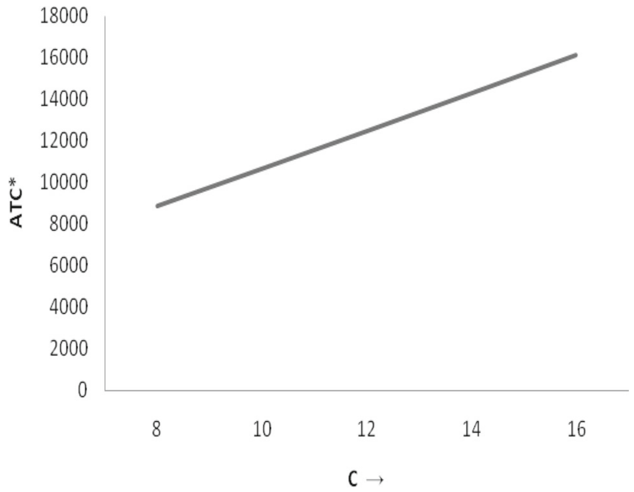

Fig. 4 Graph between C and ATC^*

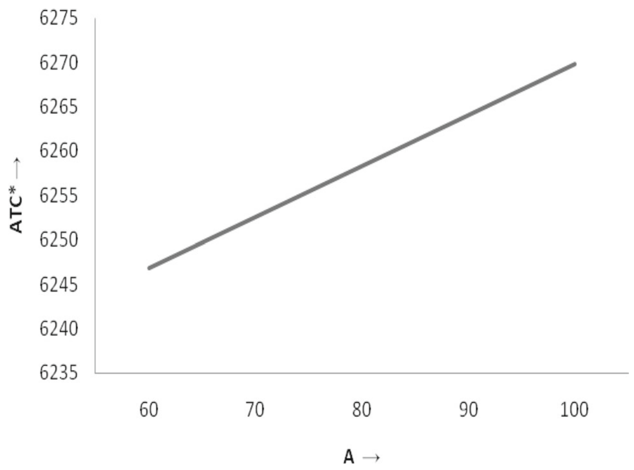

Fig. 5 Graph between A and ATC^*

Graphs of Case 2

See Figs. 6 and 7.

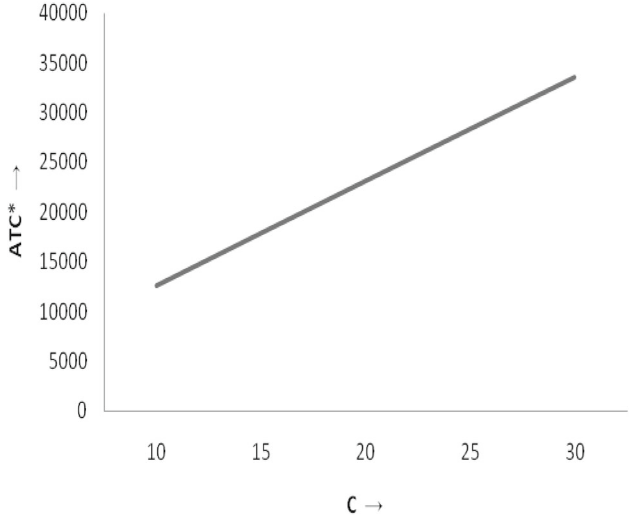

Fig. 6 Graph between C and ATC^*

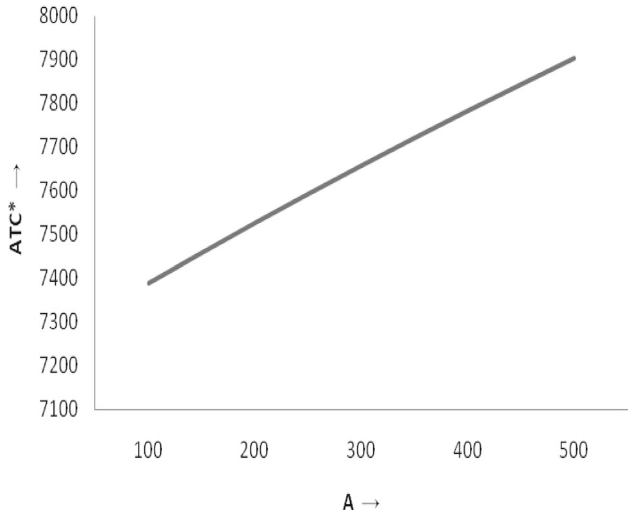

Fig. 7 Graph between A and ATC^*

Graphs of Case 3

See Figs. 8 and 9.

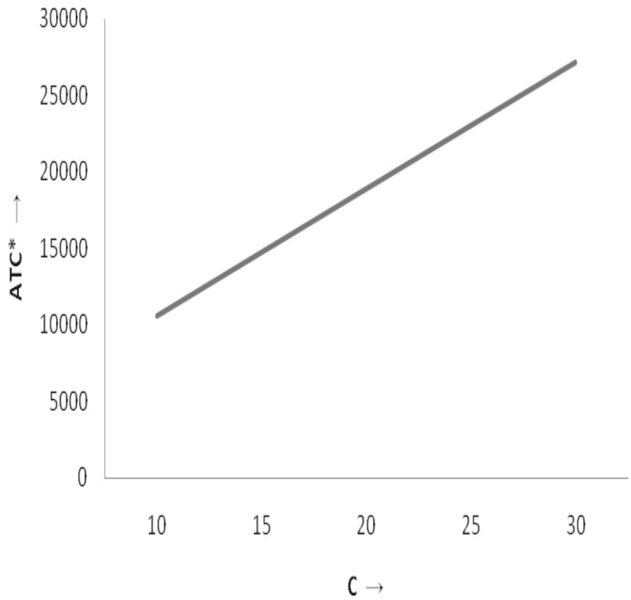

Fig. 8 Graph between C and ATC^*

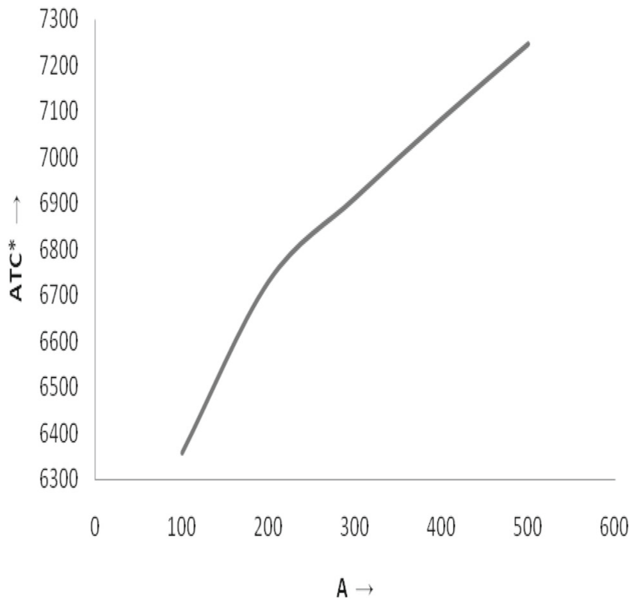

Fig. 9 Graph between A and ATC^*

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