Int. J. Appl. Comput. Math (2020) 6:160
https://doi.org/10.1007/s40819-020-00906-2

ORIGINAL PAPER

Fundamental Solutions for the Generalised Third-Order Nonlinear Schrödinger Equation

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Accepted: 28 September 2020 / Published online: 22 October 2020

Abstract
In this article, we establish exact solutions for the generalised third-order nonlinear Schrödinger equation. The elliptic function expansion and He's semi-inverse techniques are employed to establish exact solutions for this equation. These solutions are so important and vital for mathematicians and physicists to prescribe some complex physical phenomena. Using Matlab 18, we plot 2D and 3D graphs of acquired solutions for certain values of the parameters. The proposed techniques are direct, sturdy and efficient tools to solve different types of nonlinear partial differential equations arising in engineering and physics.

Keywords Elliptic functions · Variational principle · Solitons · Physical phenomena · Generalised third-order nonlinear Schrödinger equation

Mathematics Subject Classification 35A08 · 35Q40 · 35Q60

Introduction

Nonlinear wave is one of the main area of focus for researchers and scientists doing research in applied science, such as biology, engineering, electromagnetic theory, optics, chemical physics, fluid mechanics, ecology, deep water, plasma physics, elastic media, [1–10]. Nonlinear complex features in plasma physics, quantum mechanics, optoelectronics hydrodynamics, semiconductors and bimolecular dynamical modes can be formulated in the form of the nonlinear Schrödinger equations (NLSEs). These equations are the nonlinear partial differential equations (NLPDEs), which have been studied in various areas of applied science [11–13].

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Studying of their soliton solutions is so important in explaining various interesting physical phenomena in nature, and this branch turns into one of the most vital area of scientific research [14–20].

Because of the complexity structure of the nonlinear wave equations, there is no unified technique to extract all solutions of these equations. Different analytical and numerical methods are used to find solutions of these equations, such as tanh-sech method [21], sine-cosine method [22], Jacobi elliptic function method [23], $(\frac{G'}{G})-$ expansion method [24], F-expansion method [25], exp-function method [26], extended tanh-method [27], homogeneous balance method [28] and Riccati-Bernoulli sub-ODE technique [29,30]. Recently, there are a great development in analytical and numerical techniques for extracting solutions for NLPDEs, see for example [31–42].

The generalised third-order nonlinear NLSE given as follows [43]

$$i\,(\psi_t + \psi_{xxx}) + (\alpha_1\,\psi + i\alpha_2\,\psi_x)\mid\psi\mid^2 + i\alpha_3\mid\psi^2\mid_x\ \psi = 0\,, \tag{1}$$

where $\alpha_1, \alpha_2, \alpha_3$, are real-valued parameters and ψ is a complex-valued function. Eq. (1) has been utilized to prescribe the propagation of ultra-short pulses in nonlinear optical fibres. The present work is motivated to find new and more general solutions to (1). Namely, the main interest is to introduce some new exact solution of the generalised third-order nonlinear Schrödinger equation, using extended Jacobian elliptic function expansion method (JEFEM) and He's variational principle. To the best of our knowledge, no previous article has been done utilizing the techniques in this paper for solving this equation.

This paper is arranged as follows. In Sect. 2, some new exact solutions for the generalised third-order nonlinear Schrödinger equation are presented. Indeed, some 2D and 3D graphs of acquired solutions are illustrated. The conclusions are reported in Sect. 3.

Soliton Solutions of the Generalised Third-Order NLSE

$$\psi(x,t) = e^{i\varphi(x,t)}q(\xi), \qquad \varphi(x,t) = px + \mu t + \epsilon, \xi = kx + wt, \tag{2}$$

k, p, w and μ are constants and ϵ is the phase constant.

Superseding (2) into Eq. (1) yields:

$$3pk^2q'' + (\alpha_2\,p - \alpha_1)q^3 + (\mu - p^3)q = 0\,, \tag{3}$$

$$k^3\,q''' + (w - 3p^2k)q' + k(\alpha_2 + 2\alpha_3)q^2q' = 0. \tag{4}$$

Integrating Eq. (4) and taking the integration constant as zero, we have

$$k^3q'' + \frac{k}{3}(\alpha_2 + 2\alpha_3)q^3 + (w - 3p^2k)q = 0. \tag{5}$$

Eqs. (3) and (5) are the same under the constraint conditions

$$\alpha_1 = -2\alpha_3\,p, \quad \mu = \frac{3pw - 8p^3k}{k}. \tag{6}$$

In the sequel we use the extended JEFEM and He's principle for solving Eq. (1).

The Extended JEFEM

According to the extended JEFEM [24,44–46], the solution of Eq. (1) is

$$q = a_0 + a_1 sn(\xi, m) + b_1 cn(\xi, m). \tag{7}$$

Eq. (7) yields

$$q' = a_1 cn(\xi, m)\, dn(\xi, m) - b_1 sn(\xi, m)\, dn(\xi, m), \tag{8}$$

$$q'' = -m^2 sn(\xi, m)\, a_1 + 2\, a_1 sn(\xi, m)^3 m^2 + 2\, m^2 sn(\xi, m)^2 cn(\xi, m)\, b_1 - a_1 sn(\xi, m) - b_1 cn(\xi, m). \tag{9}$$

Superseding Eqs. (7)-(9) into Eq. (3) and putting the coefficients of sn^3, $sn^2 cn$, sn^2, $sncn$, sn, cn, sn^0 with zero, gives

$$6pk^2\, m^2 a_1 + (\alpha_2\, p - \alpha_1)\left(a_1^3 - 3\, a_1 b_1^2\right) = 0, \tag{10}$$

$$6pk^2\, m^2 b_1 + (\alpha_2\, p - \alpha_1)\left(3\, a_1^2 b_1 - b_1^3\right) = 0, \tag{11}$$

$$a_0\left(a_1^2 - b_1^2\right) = 0, \tag{12}$$

$$a_0 a_1 b_1 = 0, \tag{13}$$

$$-3pk^2\, a_1\,\left(1 + m^2\right) + (\alpha_2\, p - \alpha_1)\left(3\, a_0^2 a_1 + 3\, a_1 {b_1}^2\right) + (\mu - p^3) a_1 = 0, \tag{14}$$

$$-3pk^2\, b_1 + (\alpha_2\, p - \alpha_1)\left(3\, a_0^2 b_1 + b_1^3\right) + (\mu - p^3) b_1 = 0, \tag{15}$$

$$(\alpha_2\, p - \alpha_1)\left(a_0^3 + 3\, a_0 b_1^2\right) + (\mu - p^3) a_0 = 0. \tag{16}$$

Solving these algebraic equations, gives:

Family 1.

$$a_0 = 0, a_1 = \pm\frac{\sqrt{6p}\, km}{\sqrt{\alpha_1 - \alpha_2\, p}}, b_1 = 0, w = k^3(1 + m^2) + 3kp^2.$$

The first family of solutions is

$$q_1(x, t) = \pm\frac{\sqrt{6p}\, km}{\sqrt{\alpha_1 - \alpha_2\, p}}\, sn(kx + wt)\, e^{i(px + \mu t + \epsilon)}. \tag{17}$$

When $m \to 1$, Eq. (17) becomes

$$q_1(x, t) = \pm\frac{\sqrt{6p}\, k}{\sqrt{\alpha_1 - \alpha_2\, p}}\, tanh(kx + wt)\, e^{i(px + \mu t + \epsilon)},\quad w = 2k^3 + 3kp^2. \tag{18}$$

Family 2.

$$a_0 = 0, a_1 = \pm\sqrt{\frac{3}{2}}\frac{km\,\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}, b_1 = i\sqrt{\frac{3}{2}}\frac{km\,\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}, w = -\frac{1}{2}k^3(m^2 - 2) + 3kp^2.$$

The second family of solutions is

$$q_2(x, t) = \left(\sqrt{\frac{3}{2}}\frac{km\,\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}\, sn(kx + wt) + i\sqrt{\frac{3}{2}}\frac{km\,\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}\, cn(kx + wt)\right)\; e^{i(px + \mu t + \epsilon)}. \tag{19}$$

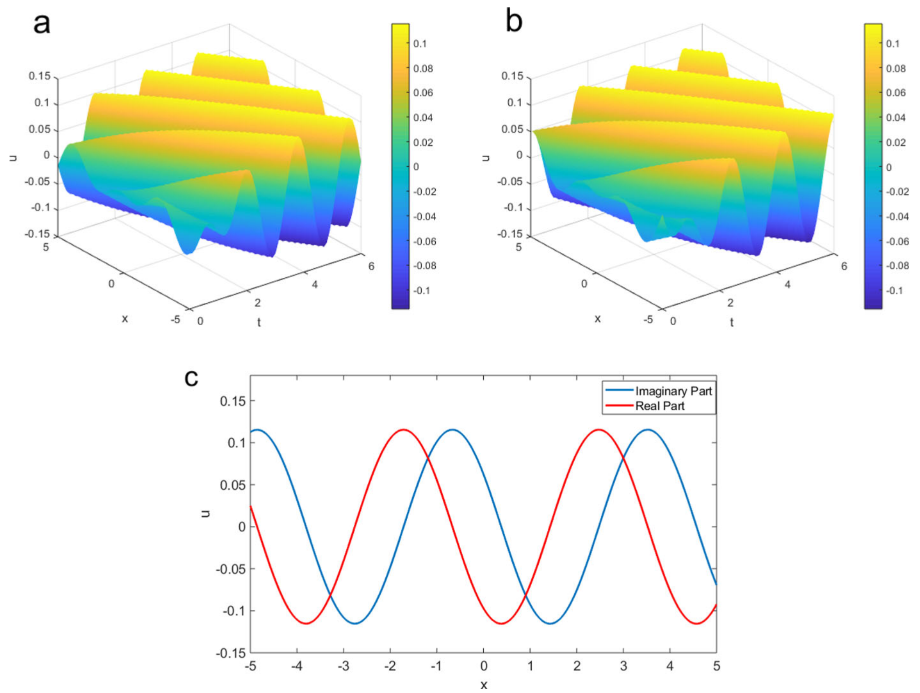

Fig. 1 Real and imaginary travelling wave solutions for Eq. (18) are illustrated: real solutions in (**a**), imaginary solutions in (**b**). **c** presents 2D real and imaginary solutions for Eq. (18). The parameter values are given by $\alpha_2 = 2.1,\ \alpha_3 = 1.2,\ p = 1.5,\ k = 0.1,\ \epsilon = 2.2$. $x \in [-5,\ 5]$ and $t = 0 \rightarrow 6$. The values of α_1 and μ are taken by Eq. (6). **c** plotted at $t = 6$.

When $m \rightarrow 1$, Eq. (19) becomes

$$\begin{aligned} q_2(x,t) &= \left(\sqrt{\frac{3}{2}} \frac{k\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}\, tanh(kx + wt) \right. \\ &\quad \left. + i \sqrt{\frac{3}{2}} \frac{k\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}\, sech(kx + wt) \right)\ e^{i(px + \mu t + \epsilon)}, \\ w &= \frac{1}{2} k^3 + 3kp^2. \end{aligned} \tag{20}$$

Family 3.

$$a_0 = 0,\ a_1 = \pm \sqrt{\frac{3}{2}} \frac{km\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}},\ b_1 = -i \sqrt{\frac{3}{2}} \frac{km\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}},\ w = -\frac{1}{2} k^3 (m^2 - 2) + 3kp^2.$$

The third family of solutions is

$$q_3(x,t) = \left(\sqrt{\frac{3}{2}} \frac{km\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}\, sn(kx + wt) - i \sqrt{\frac{3}{2}} \frac{km\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\, p}}\, cn(kx + wt) \right)\ e^{i(px + \mu t + \epsilon)}. \tag{21}$$

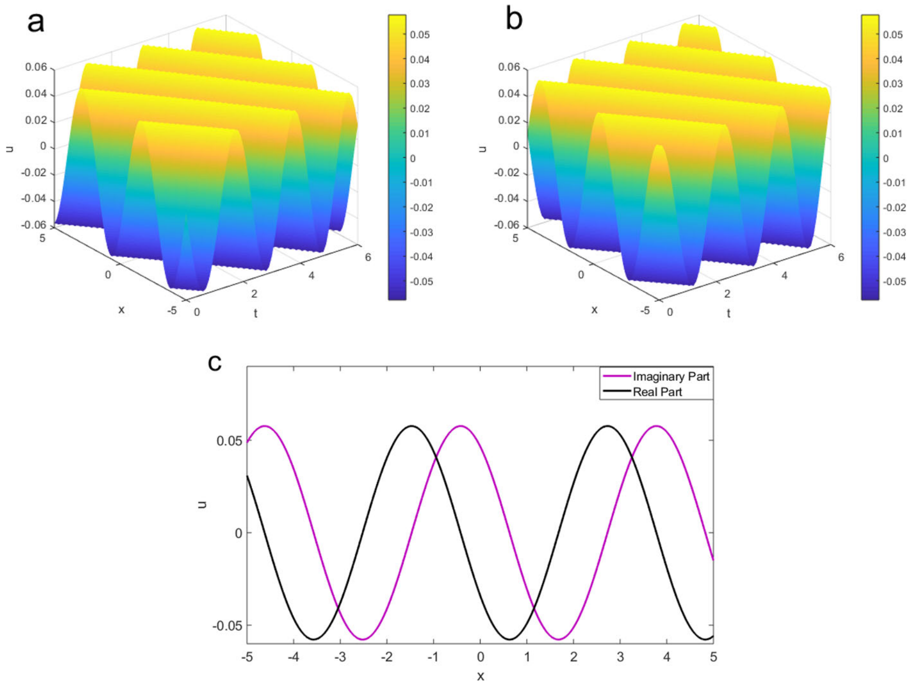

Fig. 2 3D real and imaginary travelling wave solutions for Eq. (20) are plotted: real solutions in (**a**), imaginary solution in (**b**). 2D real and imaginary solutions are presented in (**c**) at $t = 6$. The parameter values are taken by $\alpha_2 = 2.1,\ \alpha_3 = 1.2,\ p = 1.5,\ k = 0.1,\ \epsilon = 2.2$. $x \in [-5,\ 5]$ and $t = 0 \to 6$. The values of α_1 and μ are taken by Eq. (6)

When $m \to 1$, Eq. (21) becomes

$$\begin{aligned} q_3(x,t) = &\left(\sqrt{\frac{3}{2}}\frac{k\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\,p}}\,tanh(kx+wt)\right. \\ &\left. -i\sqrt{\frac{3}{2}}\frac{k\sqrt{p}}{\sqrt{\alpha_1 - \alpha_2\,p}}\,sech(kx+wt)\right)\ e^{i(px+\mu t+\epsilon)}, \\ w = &\ \frac{1}{2}k^3 + 3kp^2. \end{aligned} \tag{22}$$

Family 4.

$$a_0 = 0, a_1 = 0, b_1 = \pm\frac{\sqrt{6p}\,km}{\sqrt{\alpha_2\,p - \alpha_1}}, w = k^3(1-2m^2) + 3kp^2.$$

The fourth family of solutions is

$$q_1(x,t) = \pm\frac{\sqrt{6p}\,km}{\sqrt{\alpha_2\,p - \alpha_1}}\,cn(kx+wt)\,e^{i(px+\mu t+\epsilon)}. \tag{23}$$

When $m \to 1$, Eq. (23) becomes

$$q_1(x,t) = \pm\frac{\sqrt{6p}\,k}{\sqrt{\alpha_2\,p - \alpha_1}}\,sech(kx+wt)\,e^{i(px+\mu t+\epsilon)},\quad w = -k^3 + 3kp^2. \tag{24}$$

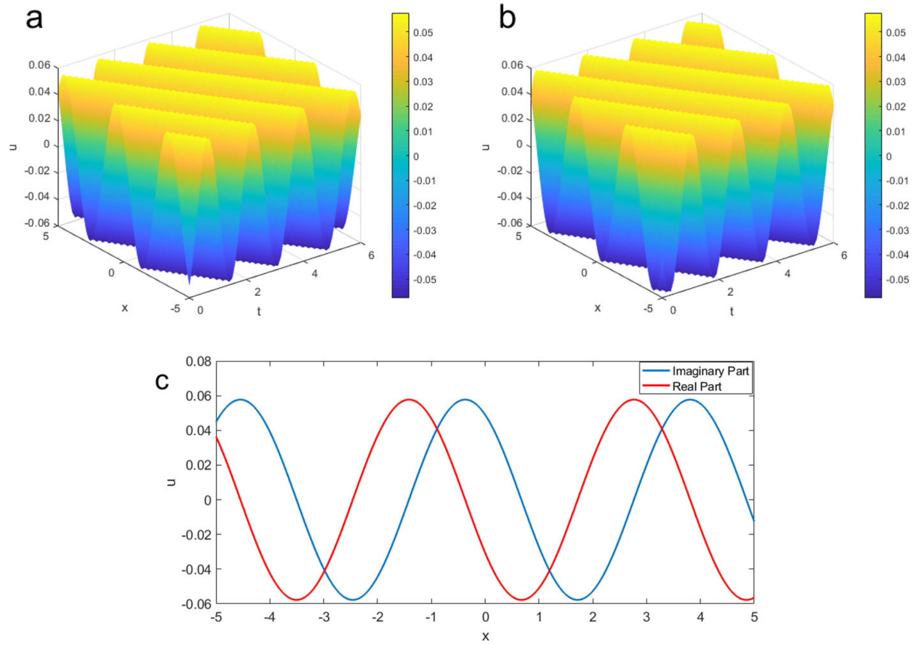

Fig. 3 3D real and imaginary travelling wave solutions for Eq. (22) are depicted: real travelling wave solutions in (**a**), imaginary travelling wave solutions in (**b**). **c** 2D graph for the real and imaginary solutions of Eq. (22) at $t = 6$.. The considered parameters are given by the values $\alpha_2 = 2.1,\ \alpha_3 = 1.2,\ p = 1.5,\ k = 0.1,\ \epsilon = 2.2.$ $x \in [-5,\ 5]$ and $t = 0 \to 6$. The values of α_1 and μ are taken by Eq. (6)

The He's Semi-inverse Technique

Based on the He's semi-inverse technique [47–49], the variational formulation corresponding to Eq. (3) is

$$J = \int_0^\infty \left[-\frac{3}{2} pk^2 (q'^2) + \frac{1}{4}(\alpha_2 p - \alpha_1) q^4 + \frac{1}{2}(\mu - p^3) q^2 \right] d\xi. \tag{25}$$

By Ritz-like method, we search for the following solitary wave solution

$$\psi(\xi) = A\, sech(B\xi). \tag{26}$$

superseding Eq. (26) into Eq. (25), gives

$$\begin{aligned} J &= \int_0^\infty \left[-\frac{3}{2} pk^2 A^2 B^2 sech^2(B\xi) tanh^2(B\xi) \right. \\ &\quad \left. + \frac{A^4}{4}(\alpha_2 p - \alpha_1) sech^4(B\xi) + \frac{A^2}{2}(\mu - p^3) sech^2(B\xi) \right] d\xi \\ &= -\frac{3}{2B} pk^2 A^2 B^2 \int_0^\infty sech^2(\theta) tanh^2(\theta) d\theta \\ &\quad + \frac{A^4}{4B}(\alpha_2 p - \alpha_1) \int_0^\infty sech^4(\theta) d\theta + \frac{A^2}{2B}(\mu - p^3) \int_0^\infty sech^2(\theta) d\theta \\ &= -\frac{1}{2} pk^2 A^2 B + \frac{A^4}{6B}(\alpha_2 p - \alpha_1) + \frac{A^2}{2B}(\mu - p^3). \end{aligned}$$

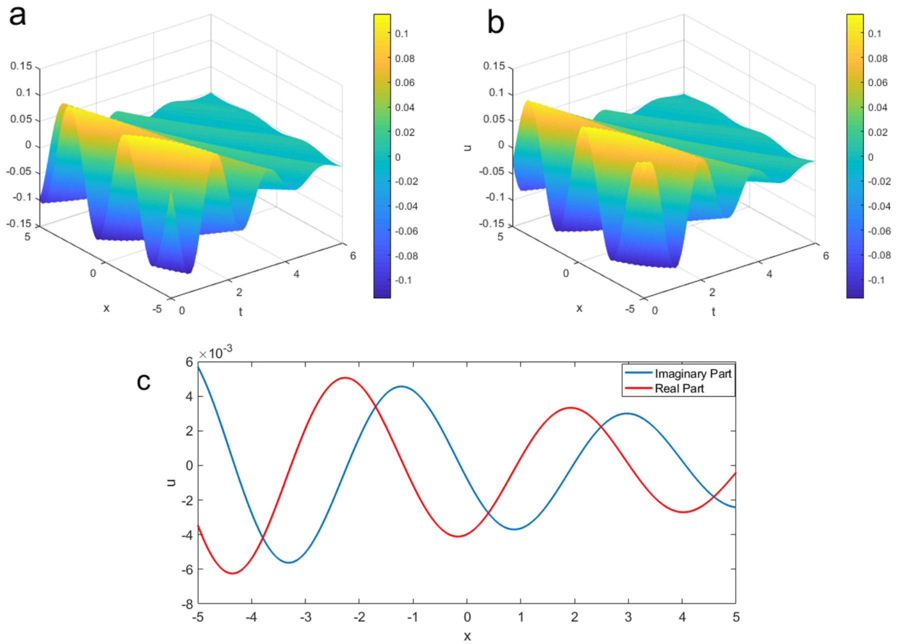

Fig. 4 **a**, **b** show the real and imaginary travelling wave solutions for Eq. (24), respectively. 2D graph for the real and imaginary travelling wave solutions of Eq. (24) are demonstrated in (**c**) at $t = 6$. The parameter values are given by $\alpha_2 = 2.1,\ \alpha_3 = 1.2,\ p = 1.5,\ k = 0.1,\ \epsilon = 2.2$. $x \in [-5,\ 5]$ and $t = 0 \to 6$. The values of α_1 and μ are taken by Eq. (6)

For getting stationary with respect to A and B results in

$$\frac{\partial J}{\partial A} = -pk^2AB + \frac{2A^3}{3B}(\alpha_2\,p - \alpha_1) + \frac{A}{B}(\mu - p^3) = 0, \tag{27}$$

$$\frac{\partial J}{\partial B} = -\frac{1}{2}pk^2A^2 - \frac{A^4}{6B^2}(\alpha_2\,p - \alpha_1) - \frac{A^2}{2B^2}(\mu - p^3) = 0. \tag{28}$$

Solving Eqs. (27) and (28), we can easily obtain the following relations:

$$A = \pm\frac{\sqrt{2}\sqrt{\mu - p^3}}{\sqrt{\alpha_1 - \alpha_2\,p}}, \quad B = \pm\frac{\sqrt{p^3 - \mu}}{\sqrt{3pk}}. \tag{29}$$

The solitary wave solution is, therefore, given as follows

$$\hat{q}(x,t) = \pm\frac{\sqrt{2}\sqrt{\mu - p^3}}{\sqrt{\alpha_1 - \alpha_2\,p}}\, sech(\pm\frac{\sqrt{p^3 - \mu}}{\sqrt{3pk}}(kx + wt))\, e^{i(px+\mu t+\epsilon)}. \tag{30}$$

Shortly, it has been investigated that the exact solutions of the generalised third-order nonlinear NLSE were obtained in the explicit form. The behavior of solutions for this equation being solitons, rouge, periodic, breather or shock, is a significance for the values of the physical parameters of the generalised third-order NLSE. Moreover, the obtained solutions may be interpreted the telecommunications experiments, nuclear physics, chaotic pulses laser, capillary profiles and transistor [50–55]. Finally, the gained solutions is so vital in the developments of quantum mechanics, namely in quantum hall effect, entire computer industry and nuclear medicine.

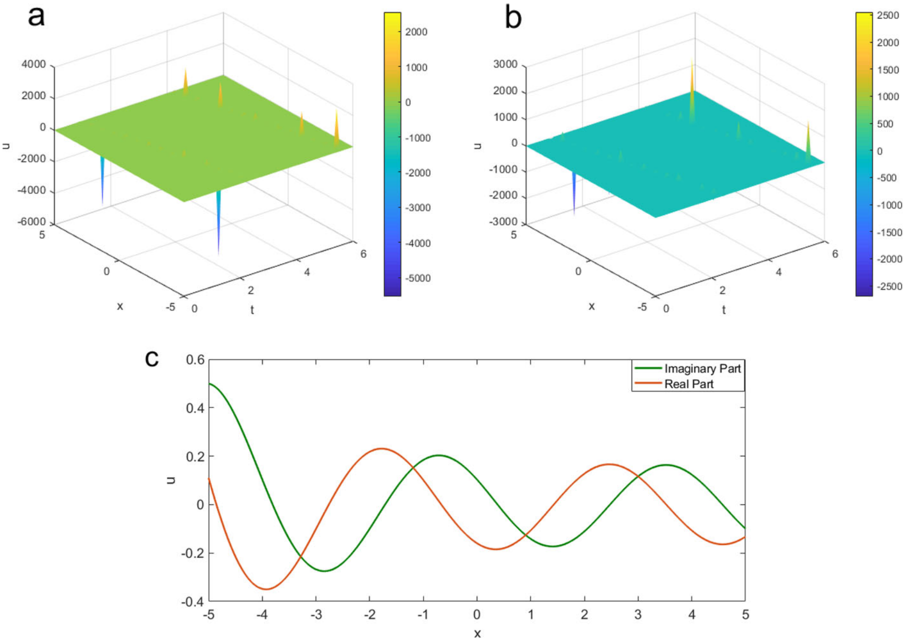

Fig. 5 3D real and imaginary travelling wave solutions for Eq. (30) are illustrated in (**a**, **b**), respectively. **c** depicts 2D travelling wave solutions for Eq. (30) at $t = 6$. The used parameters are given by the values $\alpha_2 = 2.1$, $\alpha_3 = 1.2$, $p = 1.5$, $k = 0.1$, $\epsilon = 2.2$. $x \in [-5, 5]$ and $t = 0 \rightarrow 6$. The values of α_1 and μ are taken by Eq. (6)

Conclusions

In this work, we have developed a new type of solution for the generalised third-order nonlinear Schrödinger equation, utilizing the extended Jacobian elliptic function expansion and variational principle techniques. These solutions are very vital for explaining some complex physical phenomena. All gained solutions have been plotted in 2D and 3D surfaces, using Matlab 18, for appropriate values of the parameters. The proposed techniques in this study are efficient for establishing vital solutions for other NPDEs, which are involved in physics and engineering.

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