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TECHNICAL NOTE

Some Observations on: Improving Production Policy for a Deteriorating Item Under Permissible Delay in Payments with Stock-Dependent Demand Rate

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Abstract Das et al. (Comput Math Appl 60(7):1973–1985, 2010) proposed a production inventory model for a deteriorating item under permissible delay in payments assuming that the demand is stock dependent. In the production inventory model, the production rate is partially constant and dependent upon on both on-hand inventory and demand. The production inventory model assumes that the supplier gives a price discount and permissible delay in payment. In this paper, some shortcomings in the solutions of the numerical example given in Das et al. (2010) are identified, discussed and corrected. Moreover, this paper presents the optimal solutions to the numerical example as well as the correct sensitivity analysis.

Keywords Inventory · Trade credit · Delay in payment · Stock dependent demand · Deteriorating items

Introduction

Deterioration is a significant factor in inventory analysis and it cannot be ignored its effect in the inventory. Many researchers have been doing their research in both EPQ and EOQ models by considering deterioration effect in inventory. In this connection, the reader can study the works of Cárdenas-Barrón and Sarkar [1], Sarkar [3], Sett et al. [5], Sarkar et al. [4] and others.

Das et al. [2] developed a production inventory model for a deteriorating item under permissible delay in payments considering that the demand is stock dependent. In the production inventory model, the production rate is partially constant and partially dependent upon on both on-hand inventory and demand. The production inventory model considers that the

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supplier offers price discount and permissible delay in payment. Das et al. [2] formulated a single objective optimization problem that maximizes the total profit. Then they solved the optimization problem with a real-coded genetic algorithm (GA) with rank-based selection and arithmetic crossover. They illustrated the production inventory model with a numerical example, and a sensitivity analysis was done.

Das et al. [2] said that they found the optimal solutions. However, it is worth mentioning that a genetic algorithm (GA) cannot guarantee to obtain the optimal solution.

Discussion

We have read Das et al. [2]'s paper with a high interest and after going through the paper very carefully, we identified two shortcomings in their paper. The shortcomings of their paper are as follows:

1. The solutions are not optimal
2. The solutions are infeasible

Das et al. [2] stated that they obtained the optimal solution. However, in fact, their solutions are not optimal because they solved the numerical example with a genetic algorithm. It is important to mention that the solutions have another problem because the optimization problem contains two decision variables and the results in all tables only show one decision variable, which is the production time-period (t_1). There is missing the solution to the cycle time (T). Additionally, we identified that all solutions are wrong because all solutions have inconsistencies. The inconsistencies are as follows:

Considering the results reported in Table 1 in Das et al. [2]'s paper, the following discussion is stated: The results for Case I are wrong because the cash discount is negative for the five solutions. It is worth mentioning that a negative value for cash discount is impossible, therefore, the solutions are incorrect. The results of Case II in the last three solutions the cash discount values are negative, and the payable interest values for the five solutions are negative. It is important to note that the payable interest cannot be negative therefore all five solutions are incorrect. In addition, it was found that all these solutions are infeasible. In Case III both cash discount and interest payable are positive but the solutions are infeasible.

Taking into account the results shown in Table 2 in Das et al. [2] paper, the following argument is given: The results for Case IV are incorrect due to the fact that the cash discount value is negative in all five solutions. A negative value for cash discount does not make sense. With respect to Case V, it was found that all solutions the cash discount and the interest payable are negative. These inconsistencies produce invalid solutions. Because both cash discount and interest payable must be positive. Finally, in Case VI both cash discount and interest payable are positive but the solutions are infeasible.

Das et al. [2]'s production inventory model

The production inventory model considers the following notation, which was given by Das et al. [2].

Symbol	Description	Units
C_3	Replenishment cost per order	\$/order
C_1	Holding cost excluding interest charges	\$/unit/unit time
P_0	Regular production rate	units/unit time
C_p	Regular production cost	\$/unit
C_o	Overtime production cost	\$/unit
$q(t)$	On hand inventory level	units
$D(t)$	Demand rate dependent on inventory level	units/unit time
s	Selling price	\$/unit
I_c	Interest paid by the retailer	%/unit time
I_d	Interest earn by the retailer	%/ unit time
r	Cash discount rate	%/ unit
t_1	Production time period (time at which the inventory level reaches its maximum level)	unit time
M_1	The period of cash discount for which supplier cannot charge the interest	unit time
M_2	Last time of permissible delay period	unit time
T	The length of the cycle	unit time
θ	Deterioration rate	%
α, β	Demand parameters ($\alpha, \beta > 0$)	
γ, δ	Overtime production parameters (where $\delta > 0$ and $0 \le \gamma \le 1$)	

Notation

Das et al. [2] considered six cases. For each case, they stated that profit function is a function of the length of the cycle (T). But, actually, the production inventory model has two decision variables: one is the production time period (t_1) and another one is the length of cycle (T).

Thus, mathematical formulation of Das et al. [2] model is given as follows. The selling price is determined as

$$S_T(t_1, T) = s\left[P_0 t_1 + k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right) - \frac{\mu\theta}{\lambda}\left\{t_1 + \frac{1}{\lambda}\left(e^{-\lambda t_1} - 1\right)\right\}\right.$$
$$\left. + \frac{\alpha\theta}{\theta+\beta}\left\{\frac{1}{\theta+\beta}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) + (T - t_1)\right\}\right]$$
$$k_1 = \alpha\delta + \frac{\mu(\beta\delta-\gamma)}{\lambda}, \quad k_2 = \frac{\mu(\beta\delta-\gamma)}{\lambda^2}, \lambda = \theta + \gamma - \beta(\delta - 1) \text{ and}$$
$$\mu = P_0 + \alpha(\delta - 1)$$

The total cost for each case is given below

$TC_i(t_1, T)$ = Ordering cost + Holding cost + Production cost + Interest charged − Interest earned − Cash discount

$$
\begin{aligned}
TC_i(t_1, T) &= C_3 + \frac{\mu C_1}{\lambda}\left\{t_1 + \frac{1}{\lambda}\left(e^{-\lambda t_1} - 1\right)\right\} \\
&- \frac{\alpha C_1}{\theta+\beta}\left\{\frac{1}{\theta+\beta}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) + (T - t_1)\right\} \\
&+ C_p P_0 t_1 + C_o\left\{k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right)\right\} \\
&+ C_p(1-r_i)I_c\left\{\begin{array}{l} \frac{1}{2}(\alpha + \frac{\beta\mu}{\lambda})\left(t_1^2 - M_i^2\right) + \frac{\beta\mu}{\lambda}\left\{\frac{1}{\lambda}\left(e^{-\lambda t_1} - e^{-\lambda M_i}\right) + \left(t_1 e^{-\lambda t_1} - M_i e^{-\lambda M_i}\right)\right\} \\ + \frac{\alpha\theta}{2(\theta+\beta)}\left(T^2 - t_1^2\right) - \frac{\alpha\beta}{(\theta+\beta)^2}\left(T - t_1 e^{(\theta+\beta)(T-t_1)}\right) - \frac{\alpha\beta}{(\theta+\beta)^3}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) \end{array}\right\} \\
&- sI_d\left\{\frac{1}{2}\left(\alpha + \frac{\beta\mu}{\lambda}\right)M_i^2 + \frac{\beta\mu}{\lambda^2}M_i e^{-\lambda M_i} + \frac{\beta\mu}{\lambda^3}\left(e^{-\lambda M_i} - 1\right)\right\} \\
&- r_i C_p\left\{k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right)\right\}
\end{aligned}
$$

for $i = 1$ or 4 and $M_1 = M_1, M_4 = M_2, r_1 = r, r_4 = 0$.

$TC_j(t_1, T)$ = Ordering cost + Holding cost + Production cost + Interest charged − Interest earned − Cash discount

$$
\begin{aligned}
TC_j(t_1, T) &= C_3 + \frac{\mu C_1}{\lambda}\left\{t_1 + \frac{1}{\lambda}\left(e^{-\lambda t_1} - 1\right)\right\} \\
&- \frac{\alpha C_1}{\theta+\beta}\left\{\frac{1}{\theta+\beta}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) + (T - t_1)\right\} \\
&+ C_p P_0 t_1 + C_o\left\{k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right)\right\} \\
&+ C_p(1-r_j)I_c\left\{\frac{\alpha\theta}{2(\theta+\beta)}\left(T^2 - M_j^2\right)\right. \\
&\left. - \frac{\alpha\beta}{(\theta+\beta)^2}\left(T - M_j e^{(\theta+\beta)(T-M_j)}\right) - \frac{\alpha\beta}{(\theta+\beta)^3}\left(1 - e^{(\theta+\beta)(T-M_j)}\right)\right\} \\
&- sI_d\left\{\begin{array}{l} \frac{1}{2}\left(\alpha + \frac{\beta\mu}{\lambda}\right)t_1^2 + \frac{\beta\mu}{\lambda^2}t_1 e^{-\lambda t_1} + \frac{\beta\mu}{\lambda^3}\left(e^{-\lambda t_1} - 1\right) + \frac{\alpha\theta}{2(\theta+\beta)}\left(M_j^2 - t_1^2\right) \\ - \frac{\alpha\beta}{(\theta+\beta)^2}\left(M_j e^{(\theta+\beta)(T-M_j)} - t_1 e^{(\theta+\beta)(T-t_1)}\right) - \frac{\alpha\beta}{(\theta+\beta)^3}\left(e^{(\theta+\beta)(T-M_j)} - e^{(\theta+\beta)(T-t_1)}\right) \end{array}\right\} \\
&- r_j C_p\left\{k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right)\right\}
\end{aligned}
$$

for $j = 2$ or 5 and $M_2 = M_1, M_5 = M_2, r_2 = r, r_5 = 0$.

$TC_k(t_1, T)$ = Ordering cost + Holding cost + Production cost − Interest earned − Cash discount

$$
\begin{aligned}
TC_k(t_1, T) &= C_3 + \frac{\mu C_1}{\lambda}\left\{t_1 + \frac{1}{\lambda}\left(e^{-\lambda t_1} - 1\right)\right\} \\
&- \frac{\alpha C_1}{\theta+\beta}\left\{\frac{1}{\theta+\beta}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) + (T - t_1)\right\} \\
&+ C_p P_0 t_1 + C_o\left\{k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right)\right\} \\
&- sI_d\left\{\begin{array}{l} \frac{1}{2}\left(\alpha + \frac{\beta\mu}{\lambda}\right)t_1^2 + \frac{\beta\mu}{\lambda^2}t_1 e^{-\lambda t_1} + \frac{\beta\mu}{\lambda^3}\left(e^{-\lambda t_1} - 1\right) + \frac{\alpha\theta}{2(\theta+\beta)}\left(T^2 - t_1^2\right) \\ - \frac{\alpha\beta}{(\theta+\beta)^2}\left(T - t_1 e^{(\theta+\beta)(T-t_1)}\right) - \frac{\alpha\beta}{(\theta+\beta)^3}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) + (M_k - T)\left\{\left(\alpha + \frac{\beta\mu}{\lambda}\right)t_1 + \frac{\beta\mu}{\lambda^2}\left(e^{-\lambda t_1} - 1\right)\right\} \\ + (M_k - T)(T - t_1)\frac{\alpha\theta}{(\theta+\beta)} - (M_k - T)\frac{\alpha\beta}{(\theta+\beta)^2}\left(1 - e^{(\theta+\beta)(T-t_1)}\right) \end{array}\right\} \\
&- r_k C_p\left\{k_1 t_1 + k_2\left(e^{-\lambda t_1} - 1\right)\right\}
\end{aligned}
$$

for $k = 3$ or 6 and $M_3 = M_1, M_6 = M_2, r_3 = r, r_6 = 0$.

Therefore, the total average profit is expressed as follows

$$TP_i(t_1, T) = \frac{[S_T(t_1, T) - TC_i(t_1, T)]}{T}$$

Table 1 Optimal solution of the numerical example for Cases I, II and III

M_1	α	β	θ	Case I			Case II			Case III		
				TP	t_1	T	*TP*	t_1	T	*TP*	t_1	T
4.5	55	0.25	0.1	108.6551	4.50	5.6053	141.4926	3.4462	4.50	145.3574	2.0378	2.9017
4.7	55	0.25	0.1	112.6669	4.70	5.8133	146.2126	3.6357	4.70	150.6524	2.0777	2.9508
4.9	55	0.25	0.1	116.6859	4.90	6.0209	150.876	3.8257	4.90	155.9632	2.1179	3.0014
5.1	55	0.25	0.1	120.7147	5.10	6.2281	155.4933	4.0163	5.10	161.2902	2.1594	3.0534
5.3	55	0.25	0.1	124.7554	5.30	6.4347	160.0732	4.2073	5.30	166.6338	2.2022	3.1069

Table 2 Optimal solution of the numerical example for Cases IV, V and VI

M_2	α	β	θ	Case IV			Case V			Case VI		
				TP	t_1	T	*TP*	t_1	T	*TP*	t_1	T
5.0	55	0.25	0.1	118.6417	5.0	6.1245	153.1604	3.9209	5.0	157.8276	2.8081	3.8215
5.2	55	0.25	0.1	122.6757	5.2	6.3314	157.7576	4.1117	5.2	163.4382	2.8559	3.8727
5.4	55	0.25	0.1	126.7226	5.4	6.5379	162.3216	4.3031	5.4	169.0706	2.9043	3.9243
5.6	55	0.25	0.1	130.7835	5.6	6.7441	166.8579	4.4948	5.6	174.7247	2.9531	3.9764
5.8	55	0.25	0.1	134.8595	5.8	6.9498	171.3728	4.6871	5.8	180.4005	3.0023	4.0288

for $i = 1$ or 4 and $M_1 = M_{1,} M_4 = M_2, r_1 = r, r_4 = 0$.

$$TP_j(t_1, T) = \frac{\left[S_T(t_1, T) - TC_j(t_1, T)\right]}{T}$$

for $j = 2$ or 5 and $M_2 = M_{1,} M_5 = M_2, r_2 = r, r_5 = 0$.

$$TP_k(t_1, T) = \frac{[S_T(t_1, T) - TC_k(t_1, T)]}{T}$$

for $k = 3$ or 6 and $M_3 = M_{1,} M_6 = M_2, r_3 = r, r_6 = 0$.

Thus, the optimization problem is expressed as follows:

Maximize $TP_i(t_1, T) \quad \forall \quad i = 1, \ldots, 6$

The above optimization problem can be solved optimally using Lingo 10. In next section, the optimal solutions to the numerical example are provided.

Optimal solution for numerical example in Das et al. [2]'s production inventory model.

The parameters for the numerical example are: $C_3 = 55$, $C_1 = 0.25$, $C_p = 2$, $C_o = 2.5$, $s = 3.5$, $I_c = 0.2$, $I_d = 0.15$, $P_0 = 75$, $\gamma = 0.03$, $\delta = 0.3$, $r = 0.001$ in appropriate units. Thus, the optimal solutions to the six cases are presented in Tables 1 and 2. The sensitivity analysis are reported in Tables 3, 4, 5 and 6.

Table 3 Sensitivity analysis with respect demand parameters when $\theta = 0.1$ and $M_1 = 4.9$

α	β	Case I			Case II			Case III		
		TP	t_1	*T*	*TP*	t_1	*T*	*TP*	t_1	*T*
50	0.20	102.5230	4.90	6.1239	135.8054	3.7405	4.90	135.3199	2.0009	3.0527
	0.25	107.8384	4.90	6.1064	140.5561	3.7542	4.90	140.2853	2.2257	3.2921
	0.30	113.0934	4.90	6.0819	145.1538	3.7732	4.90	145.6520	2.4062	3.4627
55	0.20	111.473	4.90	6.0415	146.2056	3.8082	4.90	151.4841	1.9275	2.7999
	0.25	116.6859	4.90	6.0209	150.8760	3.8257	4.90	155.9632	2.1179	3.0014
	0.30	121.8335	4.90	5.9947	155.4611	3.8477	4.90	160.856	2.2744	3.1518
60	0.20	120.2938	4.90	5.9729	156.4345	3.8658	4.90	168.0386	1.8584	2.5784
	0.25	125.3656	4.90	5.9496	160.9697	3.8864	4.90	172.0262	2.0171	2.7446
	0.30	130.3688	4.90	5.9208	165.3986	3.9106	4.90	176.4270	2.1497	2.8731

Table 4 Sensitivity analysis with respect deterioration parameters when $\alpha = 55$ and $M_1 = 4.9$

β	θ	Case-I			Case-II			Case-III		
		TP	t_1	*T*	*TP*	t_1	*T*	*TP*	t_1	*T*
0.20	0.075	115.4974	4.90	6.0550	149.8316	3.7977	4.90	151.5449	2.0657	3.0081
	0.10	111.4730	4.90	6.0415	146.2056	3.8082	4.90	151.4841	1.9275	2.7999
	0.125	107.8126	4.90	6.0263	142.7774	3.8203	4.90	151.4374	1.8206	2.6369
0.25	0.075	120.4767	4.90	6.0379	154.3986	3.8123	4.90	156.4104	2.2661	3.2165
	0.10	116.6859	4.90	6.0209	150.8760	3.8257	4.90	155.9632	2.1179	3.0014
	0.125	113.2251	4.90	6.0027	147.5371	3.8403	4.90	155.6043	1.9985	2.8267
0.30	0.075	125.4182	4.90	6.0143	158.8951	3.8321	4.90	161.6024	2.4241	3.3627
	0.10	121.8335	4.90	5.9948	155.4611	3.8477	4.90	160.8560	2.2744	3.1518
	0.125	118.5488	4.90	5.9744	Infeasible	Infeasible	Infeasible	160.2370	2.1497	2.9750

Table 5 Sensitivity analysis with respect demand parameters when $\theta = 0.1$ and $M_2 = 5.4$

α	β	Case IV			Case V			Case VI		
		TP	t_1	*T*	*TP*	t_1	*T*	*TP*	t_1	*T*
50	0.20	111.1316	5.40	6.6461	145.8686	4.2118	5.40	152.3727	2.8200	3.9118
	0.25	117.2579	5.40	6.6269	151.3805	4.2266	5.40	156.8600	2.9373	4.0248
	0.30	123.2323	5.40	6.6006	infeasible	infeasible	infeasible	161.6489	3.0252	4.1008
55	0.20	120.7641	5.40	6.5599	156.9547	4.2847	5.40	164.6500	2.8034	3.8342
	0.25	126.7226	5.40	6.5379	162.3213	4.3031	5.40	169.0706	2.9043	3.9243
	0.30	132.5304	5.40	6.5102	infeasible	infeasible	infeasible	173.7745	2.9765	3.9800
60	0.20	130.2603	5.40	6.4882	167.8569	4.3465	5.40	176.7808	2.7851	3.7652
	0.25	136.0094	5.40	6.4637	173.0186	4.3679	5.40	181.1054	2.8706	3.8347
	0.30	141.6110	5.40	6.4349	178.0008	4.3933	5.40	185.6994	2.9282	3.8719

Table 6 Sensitivity analysis with respect deterioration parameters when $\alpha = 55$ and $M_2 = 5.4$

β	θ	Case-I			Case-II			Case-III		
		TP	t_1	T	TP	t_1	T	TP	t_1	T
0.20	0.075	125.0761	5.40	6.5746	160.9186	4.2732	5.40	166.8288	2.9624	4.0139
	0.10	120.7641	5.40	6.5599	156.9547	4.2847	5.40	164.6500	2.8034	3.8342
	0.125	116.8630	5.40	6.5434	153.2253	4.2978	5.40	162.7274	2.6652	3.6754
0.25	0.075	130.7987	5.40	6.5564	166.1841	4.2884	5.40	171.3214	3.0585	4.1012
	0.10	126.7226	5.40	6.5379	162.3213	4.3031	5.40	169.0706	2.9043	3.9243
	0.125	123.0179	5.40	6.5184	Infeasible	Infeasible	Infeasible	167.0868	2.7678	3.7655
0.30	0.075	136.3988	5.40	6.5312	171.3021	4.3094	5.40	176.0501	3.1254	4.1529
	0.10	132.5304	5.40	6.5102	Infeasible	Infeasible	Infeasible	173.7745	2.9765	3.9800
	0.125	128.9989	5.40	6.4886	Infeasible	Infeasible	Infeasible	171.7663	2.8428	3.8228

Conclusion

This paper identifies some shortcomings in the solutions of the numerical example in Das et al. [2]'s model. The paper shows that all solutions reported by Das et al. [2] are incorrect and infeasible. Additionally, this paper provides the optimal solutions to all cases of the production inventory model. Now, the Das et al. [2]'s research is valuable and significant because it is corrected.

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