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ORIGINAL PAPER

A Production: Inventory Model for Defective Items with Shortages Incorporating Inflation and Time Value of Money

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Abstract This paper develops a production-inventory model of a single product with imperfect production process in which inflation and time value of money are considered under shortages. Demand rate has been considered to be a function of quadratic decreasing and exponential decreasing of selling price. The selling price of a unit is determined by a mark-up over the production cost. Unit production cost is considered incorporating several features like energy and labour cost, raw material cost, replenishment rate and other factors of the manufacturing system. The defective items which is a certain fraction of the total production or a random number are either reworked or refunded if those reach to the customer. Two scenarios have been considered in which defective items are refunded from the customer with penalty in scenario (a) and the defective items are repaired and sold to the customer as good items in scenario (b). Based on these two scenarios, three models have been developed in which defective items are certain fraction of the produced quantity in Model-I, a random number in Model-II, and are dependent in reliability parameter and time in Model-III. Considering all these phenomena optimum production of the product has been evaluated to have maximum profit. Finally, numerical examples are given to illustrate the results along with graphical analysis. Sensitivity analysis has also been carried out for different values of the parameter.

Keywords Inventory · Defective items · Shortages · Inflation · Quadratic and exponential price dependent demand

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Introduction

In the manufacturing system, a production process is not always completely perfect and as a result of which some defective items may be produced from the very beginning of the production. In that case defective items are certain fraction of the total production. Again on the other hand, all the produced items may be non-defective at the beginning of the production process but as long as the production continues, the production process deteriorates with time. In that case, defective items are random number. These defective items are either repaired or refunded if they reach to the customer. Lee and Park [1], Urban [2], Lin [3], Rosenblatt and Lee [4], Lee and Rosenblatt [5] developed an EPL model of this type of production process. Sana et al. [6,7] developed an EMQ model in an imperfect production system in which defective items are sold at a reduced price. Then several research works have been done on imperfect production process and defective items [8–11].

Recently, Mondal et al. [12] developed an inventory model for defective items with variable production cost. But in this paper shortages and time-value of money were not taken into account. So, in this paper we have developed an EPL model of defective items considering shortages, inflation and time-value of money (Table 1).

In this model demand has been considered as quadratic and exponential decreasing function of selling price. The selling price of a product is one of the important factors in the present competitive market situation. It has been seen in case of defective goods whose demand is mainly price dependent that higher selling price negates the demand whereas lower selling price has a reverse effect. Whitin [13] first considered the effect of price dependent demand in an inventory model. Then many researchers have worked in this area [14–17]. Recently, Maiti et al. [18] developed a production-inventory with stochastic lead-time where price dependent demand was considered. Different types of demand like stock dependent and time varying demand have also been considered in several research work [19–23].

Table 1 Brief literature review

Reference no.	Defective items	Price dependent demand	Inflation and time value of money	Shortage	Deterioration
[3]	√				
[4]	√				
[12]	√	√			
[25]			√		
[27]			√		
[28]			√	√	
[31]			√		√
[35]			√	√	√
[37]			√	√	√
[38]			√		√
[39]			√	√	
[41]			√	√	
[18]		√		√	
This model	√	√	√	√	

Again many EOQ models do not take into account the effects of inflation and time-value of money. So the time-value of money which plays an important role, can not be ignored in the present economic situation. Buzacott [24] was the first who had included the idea of inflation in inventory literature. Misra [25], Van Hees and Monhemius [26], Bierman and Thomas [27], Sarkar and Pan [28] also have worked in this direction. Other notable paper in this direction are Hariga [29], Cheung [30], Chung, Liu and Tsai [31].

Again, in the present economic situation, shortage of the items takes an important role. Chandra and Bahner [32] established an inventory model for deteriorating items with shortages and linear time-dependent demand in which time-value of money was considered. Again several research work in the direction of probabilistic deterioration have been done by many researcher [33,34]. Bose, Goswami and Chaudhuri [35], Dohi, Kaio and Osaki [36], Chen [37], Wee and Law [38] also developed the inventory model in which shortages were taken into consideration. Datta and Pal [39], Bose et al. [40] developed inventory model considering effect of inflation and shortages. Roy and Chaudhuri [41] analysed a finite time-horizon deterministic EOQ model with stock dependent demand and effect of inflation and allowing shortages in all cycles. Sarkar et al. [42] developed an inventory model with finite replenishment rate where price discount offer was considered. Some recent works in this area are given by [43–50] (Table 2).

Table 2 Some recent works in the area of defective items and imperfect production

Reference no.	Authors and published year	Defective items
[43]	Barzoki and Jahanbazi and Bijari (2011)	Yes (imperfect product)
[44]	Pal and Sana and Chaudhuri (2012)	Yes (reworkable items)
[45]	Sarkar and Gupta and Chaudhuri and Goyal (2014)	Yes (defective units)
[46]	Pal and Sana and Chaudhuri (2013)	Yes (imperfect production system)
[47]	Pal and Sana Chaudhuri (2014)	Yes (imperfect Production)
[48]	Das Roy and Sana and Chaudhuri (2011)	Yes (imperfect items)
[49]	Sarkar and Sana and Chaudhuri (2011)	Yes (imperfect production process)

Notations and Assumptions

This paper is developed with the following *Notations* and *Assumptions*.

Notations:

p:	Selling price per unit item.
$D(p)$:	Demand rate which is a function of selling price.
P:	Production rate (a decision variable).
$f(P)$:	Unit production cost.
A:	Advertisement cost per unit item.
c_r:	Raw material cost.
L:	Labour charges.
S:	Maximum stock level.
S_1:	Maximum shortage.
c_h:	Inventory carrying cost per unit quantity per unit time.
c_0:	Set up cost which is known and constant.
$q(t)$:	Stock level at time t.
Q:	Number of produced units(a decision variable).
$M(Q, P)$:	Average profit per unit time for a cycle.
P_1:	Total number of defective items.
μ:	Scaling parameter for defective items.
$\frac{1}{m}$:	Mean of exponential distribution.
t_1:	Time upto which the production is made i.e. after $t = t_1$ the production is discontinued.
t_2:	Time at which stock level falls to zero due to demand.
t_3:	Time at which shortages reach to the level S_1.
T:	Time at which stock level is again zero i.e. one cycle time.
γ:	r-i, r is the interest rate per unit currency and i is the inflation rate per unit currency.
ψ:	Product reliability parameter.

Assumptions:

(a) The demand rate $D(p)$ is deterministic function of selling price p. It is either quadratic decreasing or exponential decreasing function of selling price p. $D(p) = a - bp - cp^2$, where $a, b, c > 0$ and $D(p) = d \times p^{-k}$, $d, k > 0$.
(b) The unit production cost $f(P) = c_r + A + \frac{L}{P^\alpha} + KP^\beta$, where K is a positive constant and α, β are chosen to provide the feasible solution of the model.
(c) The defective items are fraction of the produced items in first and third model and a random number for the second model.
(d) Selling price p is determined by a mark-up over the unit production cost $f(P)$. i.e. $p = \lambda f(P)$, $\lambda > 1$ where λ is the mark-up.
(e) Lead time is assumed to be zero.

Development of the Model and Analysis

The defective items are either reworked or refunded if those are sold to the customer. Under these circumstances, we investigate the following two scenarios:

Scenario (a): Q units are to be produced. All produced items including the defective items are sold to the customers at the rate of D units as good units and later P_1 defective items are refunded from the customer with penalty at a cost of c_v per unit.

Scenario (b): Q units are produced and P_1 defective items are spotted just after the production. Those are repaired against the cost of c_θ per unit and sold as good items to the customer.

At $t = 0$, the stock level is zero and then the variable production starts to produce items at a rate P units per unit time. The production stops at $t = t_1$. As the production rate is greater than demand rate, some units are accumulated during the interval $0 \le t \le t_1$. At $t = t_1$, the inventory level reaches to the maximum stock level S. After $t = t_1$, the stock level decreases due to demand only and at $t = t_2$ it falls to zero. Then shortages start and are accumulated to the level S_1 at $t = t_3$. After $t = t_3$ the production starts again. Fresh production and supply to the consumers occur simultaneously during the interval $t_3 \le t \le t_4$. The whole backlog is cleared by the time $t = t_4$ and the stock level is again zero at $t = t_4$. The graphical representation of the model is given by Fig. 1.

Hence under the above assumptions, the differential equation satisfied by $q(t)$ at time t can be represented as:

$$\frac{dq(t)}{dt} = P - D, \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dq(t)}{dt} = -D, \quad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dq(t)}{dt} = -D, \quad t_2 \le t \le t_3 \tag{3}$$

$$\frac{dq(t)}{dt} = P - D, \quad t_3 \le t \le T \tag{4}$$

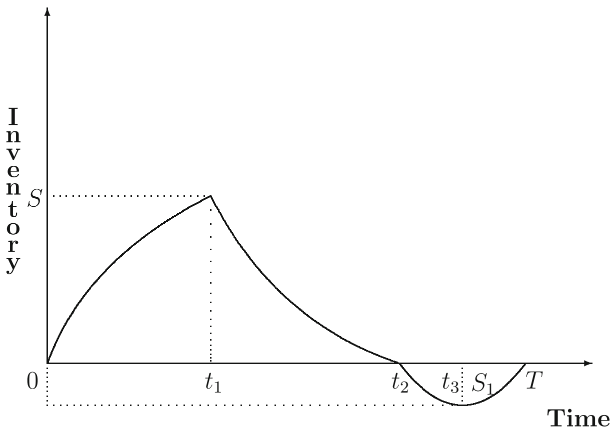

Fig. 1 Graphical representation of Model

with initial and boundary condition

$$q(0) = q(t_2) = q(T) = 0, \quad q(t_1) = S, \quad q(t_3) = -S_1. \tag{5}$$

From (1) and (2), we have

$$S = Q - D\frac{Q}{P} \quad \left[since \quad t_1 = \frac{Q}{P}\right] \tag{6}$$

and

$$t_2 = \frac{Q}{D} \tag{7}$$

From (3) and (4), we have

$$S_1 = Q - D\frac{Q}{P} \quad [since \quad t_3 = T - t_1] \tag{8}$$

and

$$T = 2\frac{Q}{D} \tag{9}$$

The present-value of total revenue is

$$\begin{aligned} C_{REV} &= \int_0^T pDe^{-\gamma t}dt \\ &= \frac{1}{\gamma}pD(1 - e^{-\gamma T}) \end{aligned} \tag{10}$$

The present-value of production cost is

$$\begin{aligned} C_{PRO} &= \int_0^T f(P)De^{-\gamma t}dt \\ &= \frac{f(P)D}{\gamma}(1 - e^{-\gamma T}) \end{aligned} \tag{11}$$

The present-value of holding cost is

$$\begin{aligned} C_{HOL} &= \frac{1}{2}\int_0^{t_2} c_h Se^{-\gamma t}dt \\ &= \frac{c_h S}{2\gamma}(1 - e^{-\gamma t_2}) \end{aligned} \tag{12}$$

The present-value of set-up cost is

$$\begin{aligned} C_{SET} &= \int_0^T \frac{c_0 D}{Q}e^{-\gamma t}dt \\ &= \frac{c_0 D}{\gamma Q}(1 - e^{-\gamma T}) \end{aligned} \tag{13}$$

The present-value of shortage cost is

$$\begin{aligned} C_{SHO} &= \frac{1}{2}\int_0^{T-t_2} c_s(-S_1)e^{-\gamma t}dt \\ &= -\frac{c_s S_1}{2\gamma}(1 - e^{-\gamma (T-t_2)}) \end{aligned} \tag{14}$$

The present-value of refund cost is

$$C_{REF} = \int_0^T c_v \mu P^{\delta-1} D e^{-\gamma t} dt = \frac{c_v \mu P^{\delta-1} D}{\gamma}(1 - e^{-\gamma T}) \tag{15}$$

The present-value of rework cost is

$$C_{REW} = \int_0^T c_\theta \mu P^{\delta-1} D e^{-\gamma t} dt = \frac{c_\theta \mu P^{\delta-1} D}{\gamma}(1 - e^{-\gamma T}) \tag{16}$$

Model-I : Defective items are a certain fraction of the produced quantity:
Scenario (a): The total profit incorporating inflation and time-value of money is given by

$$M(Q,P) = \frac{1}{\gamma}\left(pD - f(P)D - c_0\frac{D}{Q} - \mu c_v P^{\delta-1} D\right)\left(1 - e^{-\gamma T}\right) - \frac{1}{2\gamma}c_h Q \left(1 - \frac{D}{P}\right)\left(1 - e^{-\gamma t_2}\right) + \frac{1}{2\gamma}c_s Q\left(1 - \frac{D}{P}\right)\left(1 - e^{-\gamma (T - t_2)}\right) \tag{17}$$

Scenario (b): The total profit incorporating inflation and time-value of money is given by

$$M(Q,P) = \frac{1}{\gamma}\left(pD - f(P)D - c_0\frac{D}{Q} - \mu c_\theta P^{\delta-1} D\right)\left(1 - e^{-\gamma T}\right) - \frac{1}{2\gamma}c_h Q \left(1 - \frac{D}{P}\right)\left(1 - e^{-\gamma t_2}\right) + \frac{1}{2\gamma}c_s Q\left(1 - \frac{D}{P}\right)\left(1 - e^{-\gamma (T - t_2)}\right) \tag{18}$$

Model-II : Number of defective items is random:
Let the time τ at which in-control state changes to a out-control state is a random variable and follows exponential distribution with mean $\frac{1}{m}$. So the number of defective items is a random variable and is given by

$$X(t_1) = 0 \quad if \quad \tau \geq t_1 \\ = \alpha_1 P(t_1 - \tau) \quad if \quad \tau < t_1 \tag{19}$$

So the expected number of total defective item is given by

$$P_1 = E[X(t_1)] = \alpha_1 P\left\{\left(t_1 + \frac{1}{m}e^{-m t_1}\right) - \frac{1}{m}\right\}, t_1 = \frac{Q}{P} \tag{20}$$

Scenario (a): The expected average profit M(Q, P) is given by

$$M(Q,P) = \frac{1}{\gamma}\left[pD - f(P)D - c_0\frac{D}{Q} - c_v\left\{\alpha_1 P\left(\frac{Q}{P} + \frac{1}{m}e^{-\frac{mQ}{P}} - \frac{1}{m}\right)D\right\}/Q\right] \left(1 - e^{-\gamma T}\right) - \frac{1}{2\gamma}c_h Q\left(1 - \frac{D}{P}\right)\left(1 - e^{-\gamma t_2}\right) + \frac{1}{2\gamma}c_s Q\left(1 - \frac{D}{P}\right) \left(1 - e^{-\gamma(T - t_2)}\right) \tag{21}$$

Scenario (b): The expected average profit M(Q, P) is given by

$$M(Q,P)=\frac{1}{\gamma}\left[pD-f(P)D-c_0\frac{D}{Q}-c_\theta\left\{\alpha_1 P\left(\frac{Q}{P}+\frac{1}{m}e^{-\frac{mQ}{P}}-\frac{1}{m}\right)D\right\}/Q\right]\left(1-e^{-\gamma T}\right)-\frac{1}{2\gamma}c_h Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma t_2}\right)+\frac{1}{2\gamma}c_s Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma(T-t_2)}\right) \tag{22}$$

Model-III : Defective items are dependent on reliability parameter ψ and time t:

The amount of defective items produced at time t is $\eta e^{\psi t}P$ where $\eta e^{\psi t} < 1$. Since the fraction $\eta e^{\psi t}$ increases with time t and ψ simultaneously, so in this system the production of defective items increase with increase of time. It has been seen, after a certain time almost all manufacturing system undergoes unsatisfactory performance. So, in long production run process, the system shifts in-control state to a out-control state during malfunctioning. As a result percent of defective items increase with time t. Again, lower value of ψ decrease the percent of defective items. For that reason, the defective items at time t has been considered as $\eta e^{\psi t}P$.

Therefore, the present-value of refund cost is

$$C_{REF}=\int_0^T c_v \eta e^{\psi t} D e^{-\gamma t}dt = \frac{c_v \eta D}{\gamma-\psi}(1-e^{(\psi-\gamma)T}) \tag{23}$$

Therefore, the present-value of rework cost is

$$C_{REF}=\int_0^T c_\theta \eta e^{\psi t} D e^{-\gamma t}dt = \frac{c_\theta \eta D}{\gamma-\psi}(1-e^{(\psi-\gamma)T}) \tag{24}$$

Scenario (a): The total profit M(Q, P) is given by

$$M(Q,P)=\frac{1}{\gamma}\left[pD-f(P)D-c_0\frac{D}{Q}\right]\left(1-e^{-\gamma T}\right)-\frac{1}{2\gamma}c_h Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma t_2}\right)+\frac{1}{2\gamma}c_s Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma(T-t_2)}\right)-\frac{c_v\eta D}{\gamma-\psi}\left(1-e^{(\psi-\gamma)T}\right) \tag{25}$$

Scenario (b): The total profit M(Q, P) is given by

$$M(Q,P)=\frac{1}{\gamma}\left[pD-f(P)D-c_0\frac{D}{Q}\right]\left(1-e^{-\gamma T}\right)-\frac{1}{2\gamma}c_h Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma t_2}\right)+\frac{1}{2\gamma}c_s Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma(T-t_2)}\right)-\frac{c_\theta\eta D}{\gamma-\psi}\left(1-e^{(\psi-\gamma)T}\right) \tag{26}$$

Numerical Examples

To illustrate the proposed model-1, model-2 and model-3 we consider the following examples given below.

Example-1 of model-1: Let us take $D(p) = a - bp - cp^2$ and the parameter values in the inventory system are $c_0 = \$100$, $c_h = \$3$, $c_s = \$2$, $c_r = \$50$, $a = 100$, $b = 0.3$ $\alpha = 0.7$, $\beta = 1.5$, $c_v = 200$, $\lambda = 1.2$, $A = \$50$, $\mu = 0.08$, $\gamma = 0.02$, $L = \$1500$, $K = 0.01$, $\delta = 0.8$, $c = 0.001$, in appropriate units. The optimal solution is $P^* = 159.811$, $Q^* = 80.2984$, and maximum expected average profit is $M = 1637.74$ (Fig. 2).

Example-2 of model-1: Let us take $D(p) = d \times p^{-k}$ and the parameter values in the inventory system are c_0 =\$100, $c_h = \$3$, $c_s = \$2$, $c_r = \$50$, $d = 20000$,$k = 1.6$ $\alpha = 0.7$, $\beta = 1.5$, $c_v = 200$, $\lambda = 1.2$, $A = \$50$, $\mu = 0.08$, $\gamma = 0.02$, $L = \$1500$, $K = 0.01$, $\delta = 0.8$, in appropriate units. The optimal solution is $P^* = 478.359$, $Q^* = 88.9129$, and maximum expected average profit is $M = 2678.58$ (Fig. 3).

Example-1 of model-2: Let us take $D(p) = a - bp - cp^2$ and the parameter values in the inventory system are $c_0 = \$100$, $c_h = \$3$, $c_s = \$2$, $c_r = \$50$, $a = 200$,$b = 0.7$ $\alpha = 0.7$, $m = 0.08$, $\alpha_1 = 0.001$, $\beta = 1.5$, $c_v = 200$, $\lambda = 1.2$, $A = \$50$, $\gamma = 0.01$, $L = \$1500$, $K = 0.01$, $\delta = 0.8$, $c = 0.001$, in appropriate units. The optimal solution is $P^* = 154.575$, $Q^* = 1081.71$, and maximum expected average profit is $M = 30469.9$ (Fig. 4).
Example-2 of model-2: Let us take $D(p) = d \times p^{-k}$ and the parameter values in the inventory system are $c_0 = \$100$, $c_h = \$3$, $c_s = \$2$, $c_r = \$50$, $d = 20000$, $k = 1.6$, $\alpha = 0.7$, $m = 0.08$, $\alpha_1 = 0.001$, $\beta = 1.5$, $c_v = 200$, $\lambda = 1.2$, $A = \$50$, $\gamma = 0.01$, $L = \$1500$, $K = 0.01$, in appropriate units. The optimal solution is $P^* = 64.704$, $Q^* = 156.406$, and maximum expected average profit is $M = 4879.64$ (Fig. 5).

Example-1 of model-3: Let us take $D(p) = a - bp - cp^2$ and the parameter values in the inventory system are $c_0 = \$100$, $c_h = \$3$, $c_s = \$2$, $c_r = \$50$, $a = 200$,$b = 0.7$, $\alpha = 0.7$,

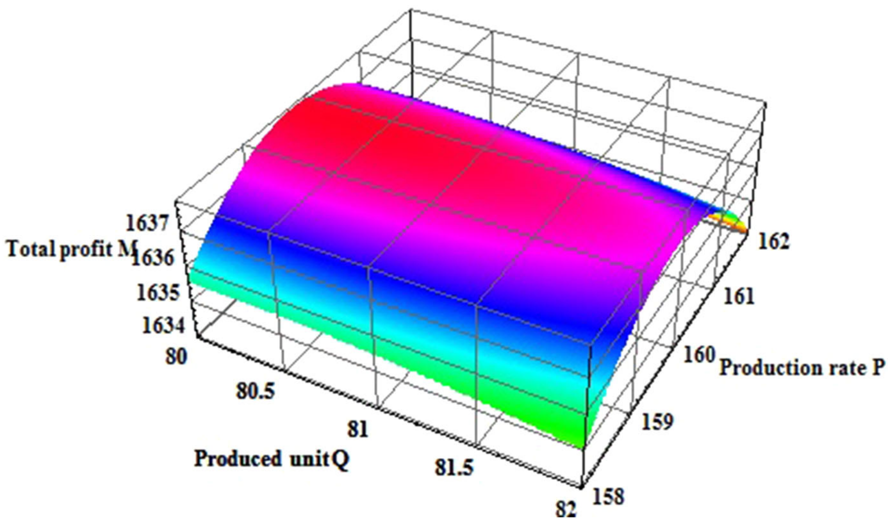

Fig. 2 Maximum total profit M(Q, P) versus Q and P of Example-1(Model-1)

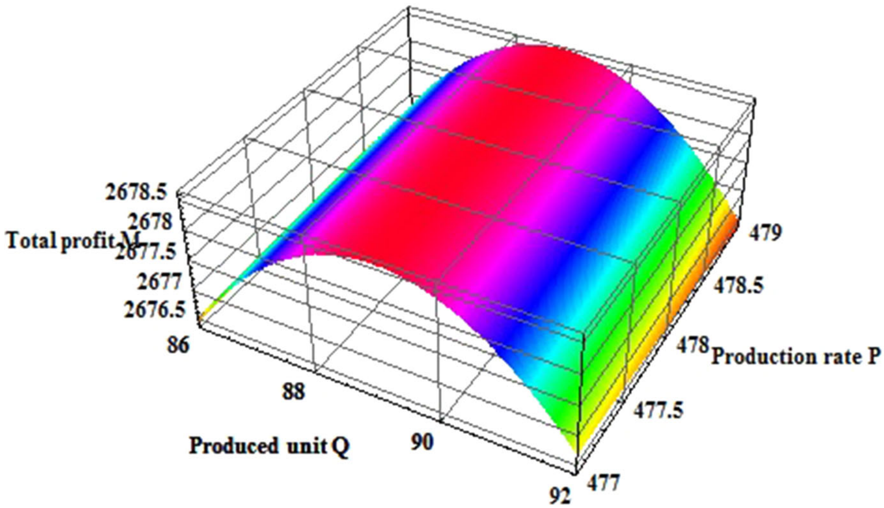

Fig. 3 Maximum total profit M(Q, P) versus Q and P of Example-2(Model-1)

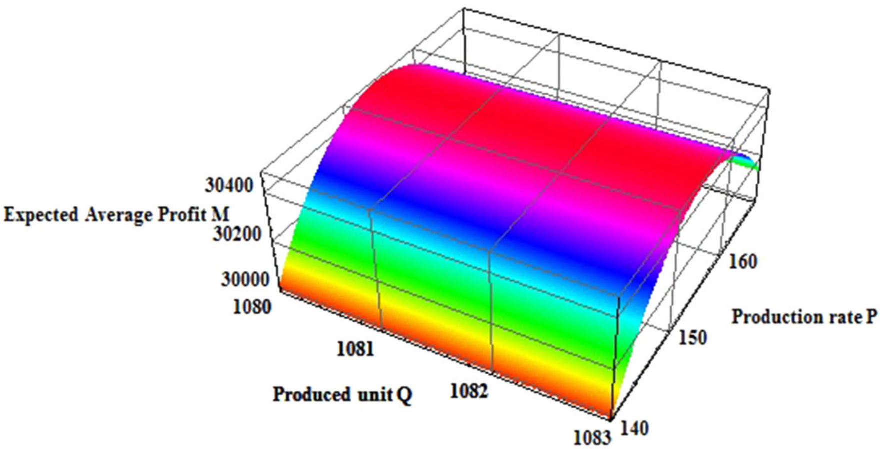

Fig. 4 Maximum expected average profit M(Q, P) versus Q and P of Example-1(Model-2)

$\beta = 1.5$, $c_v = 200$, $\lambda = 1.2$, $A = \$50$, $\eta = 0.09$, $\psi = 0.05$, $\gamma = 0.01$, $L = \$1500$, $K = 0.01$, $c = 0.001$, in appropriate units. The optimal solution is $P^* = 159.276$, $Q^* = 57.6545$, and maximum expected average profit is $M = 1608.07$ (Fig. 6).

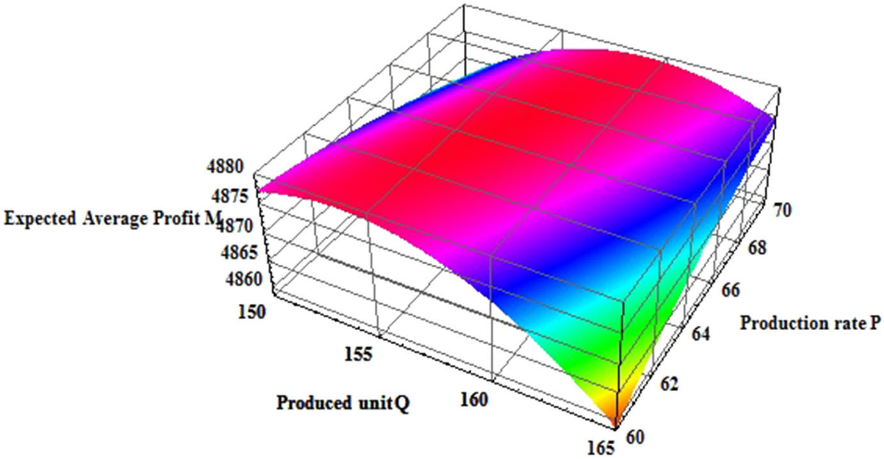

Fig. 5 Maximum expected average profit M(Q, P) versus Q and P of Example-2 (Model-2)

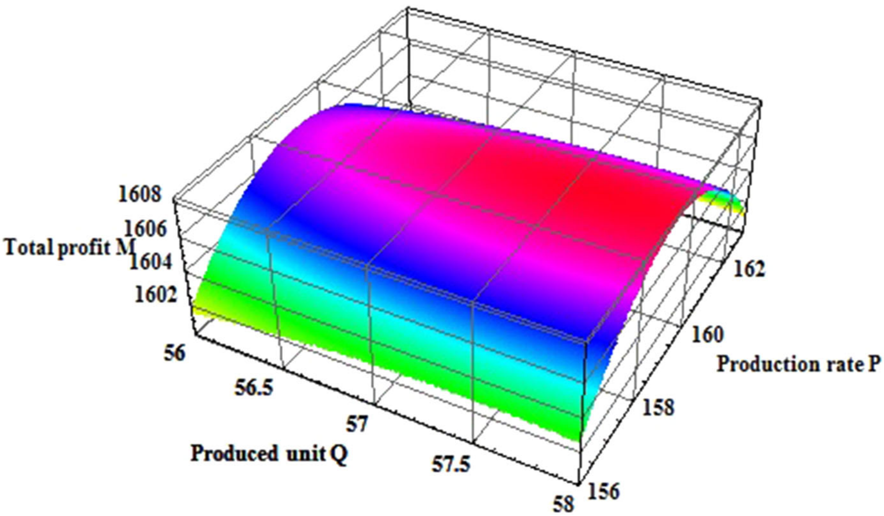

Fig. 6 Maximum total profit M(Q, P) versus Q and P of Example-1(Model-3)

Discussion

In scenario (a) and scenario (b) of model-1, the system with free from defective items ($\mu = 0$) gives more profit than the system with defective items ($\mu \neq 0$). Again in defective production system, the amount of profit decreases as δ changes from 0.8 to 1.0 (Table 3)

In case of model-2 with random defective items, for all scenarios, profit is less when mean of the exponential distribution is less i.e. profit with $m = 0.08$ is more than the profit with $m = 0.4$. But the change in profit with mean is very slow (Table 4).

In case of model-3 for all scenarios the profit decreases as the reliability parameter η changes from 0.05 to 0.08. Therefore lower value of η gives more profit (Table 5).

Table 3 The optimal solution of model-1 for different values of μ and δ in Example-1 and Example-2

Example	Scenario	μ	δ	Q (max lot size)	P (max prod. rate)	M (average profit)
1.	(a)	0.08	0.8	80.2984	159.811	1637.74
		0.08	1.0	60.5379	159.24	718.814
		0.00	–	89.4166	159.269	2216.73
1.	(b)	0.08	0.8	85.0098	159.509	1923.11
		0.08	1.0	76.5067	159.258	1427.55
		0.00	–	89.4166	159.269	2216.73
2.	(b)	0.08	0.8	88.9129	478.359	2678.58
		0.08	1.0	–	–	–
		0.00	–	132.727	131.429	3382.07
2.	(b)	0.08	0.8	117.911	276.082	2958.3
		0.08	1.0	72.3388	21.3201	2464.81
		0.00	–	132.727	131.429	3382.07

Table 4 The optimal solution of model-2 for different values of m in Example-1 and Example-2

Example	Scenario	m	Q (max lot size)	P (max prod. rate)	M (average profit)
1.	(a)	0.08	1081.71	154.575	30469.9
		0.40	1079.35	154.606	30346.3
1.	(b)	0.08	1082.86	154.531	30503.6
		0.40	1081.68	154.547	30441.7
2.	(a)	0.08	156.406	64.704	4879.64
		0.40	156.655	65.6804	4868.59
2.	(b)	0.08	156.283	64.3799	4881.52
		0.40	156.397	64.8412	4875.98

Table 5 The optimal solution of model-3 for different values of η and ψ in Example-1

Example	Scenario	η	ψ	Q (max lot size)	P (max prod. rate)	M (average profit)
1.	(a)	0.09	0.05	57.6545	159.276	1608.07
		0.09	0.08	37.9048	159.297	1048.49
		0.09	0.05	94.8979	159.264	3554.85
1.	(b)	0.09	0.08	63.3979	159.29	2520.52

Sensitivity Analysis

The sensitivity of the maximum total profit is examined due to changes in production rate and price mark-up. To illustrate the result, it has been shown only for Model-1 (Example-1), scenario-(a).

Figure 7 shows that total profit increases with the production rate P and it attains maximum value \$1637.74 at $P = 159.811$ when $Q = 80.3$.

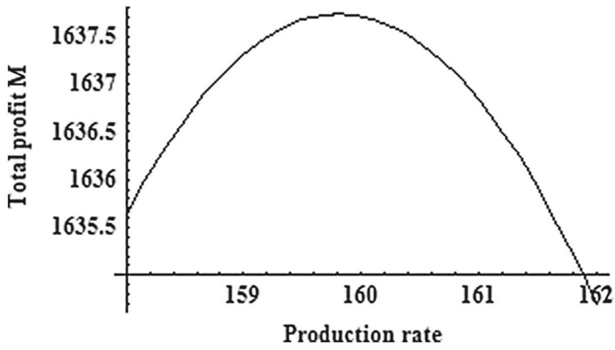

Fig. 7 Maximum total profit M versus production rate P

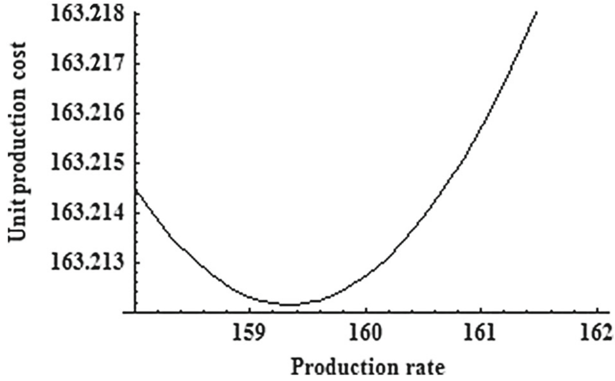

Fig. 8 Unit production cost f versus production rate P

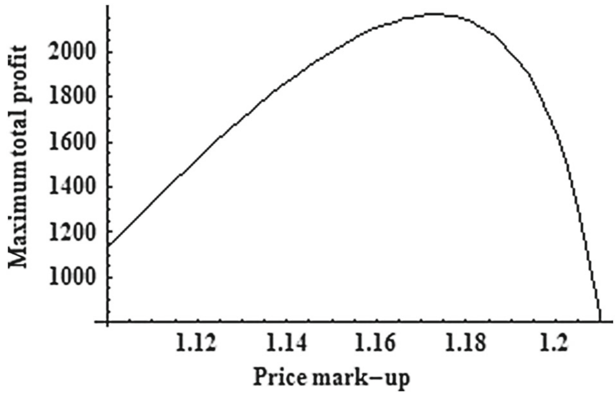

Fig. 9 Maximum total profit M versus price mark-up

From Fig. 8 it is observed that unit production cost is minimum i.e. Rs. \$163.212 at production rate $P = 159.337$. It is interesting to note that at $P = 159.811$ unit production cost is not minimum.

Figure 9 represents maximum total profit versus price mark-up λ. Normally, profit increases with the increase of price mark-up. From Fig. 9, it is observed that the profit

Table 6 Effects of c_0, c_h, c_s, γ and L on profit in Example-1 (Model-I)

Parameter	% change in the parameter	Average profit	% change in the average profit
c_0	+50	1577.57	−3.67
	+20	1613.63	−1.47
	−20	1661.88	1.47
	−50	1698.2	3.69
c_h	+50	908.444	−44.53
	+20	1248.83	−23.75
	−20	2382.25	45.46
	−50	–	–
c_s	+50	–	–
	+20	2063.72	26.01
	−20	1357.26	−17.13
	−50	1073.71	−34.44
γ	+50	1282.32	−21.70
	+20	1473.53	−10.03
	−20	1845.56	12.69
	−50	2290.43	39.85
L	+50	–	–
	+20	–	–
	−20	5593.63	241.55
	−50	10529.3	542.92

is maximum when the price mark-up is 1.17 and the profit decreases as price mark-up is more than 1.17 because demand decreases with increase of selling price. Again sensitivity to the different changes of parameters are observed in Table 6.

Conclusions

In this paper, we have extended Mondal et al. [7] EPL model for defective items considering shortages, inflation and time-value of money. Again, in this model different types of demand like quadratic decreasing and exponential decreasing function of selling price have been considered.

This model could be extended in fuzzy and fuzzy-stochastic environment taking demand, defective items and other inventory parameters to be imprecise.

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Appendix

Theorem: *The profit function $M(Q, P)$ possess a maximum* solution.

Proof:

$$M(Q,P) = \frac{1}{\gamma}\left(pD - f(P)D - c_0\frac{D}{Q} - \mu c_v P^{\delta-1}D\right)\left(1-e^{-\gamma T}\right) - \frac{1}{\gamma}c_h Q \left(1-\frac{D}{P}\right)\left(1-e^{-\gamma t_2}\right) + \frac{1}{\gamma}c_s Q\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma(T-t_2)}\right)$$

$$\frac{\partial M(Q,P)}{\partial Q} = \frac{c_0 D}{\gamma Q^2}\left(1-e^{-\gamma T}\right) + \left(pD - f(P)D - c_0\frac{D}{Q} - \mu c_v P^{\delta-1}D\right)\frac{2e^{-\gamma T}}{D} - \frac{1}{\gamma}c_h\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma t_2}\right) - \frac{1}{D}c_h Q\left(1-\frac{D}{P}\right)e^{-\gamma t_2} + \frac{1}{\gamma}c_s\left(1-\frac{D}{P}\right)\left(1-e^{-\gamma(T-t_2)}\right) + \frac{1}{D}c_s Q\left(1-\frac{D}{P}\right)e^{-\gamma(T-t_2)} = 0 \quad (27)$$

We first obtain second order derivative of $M(Q, P)$ and using (36) we have

$$\begin{aligned}\frac{\partial^2 M(Q,P)}{\partial Q^2} &= -2\frac{c_0 D}{\gamma Q^3}\left(1-e^{-\gamma T}\right) + \frac{4c_0}{Q^2}e^{-\gamma T} - \frac{4\gamma e^{-\gamma T}}{D^2}\left(pD - f(P)D - c_0\frac{D}{Q} - \mu c_v P^{\delta-1}D\right) - \frac{2c_h}{D}\left(1-\frac{D}{P}\right)e^{-\gamma t_2} + \frac{\gamma c_h Q}{D^2}\left(1-\frac{D}{P}\right)e^{-\gamma t_2} + \frac{3c_s}{D}\left(1-\frac{D}{P}\right)e^{-\gamma(T-t_2)}\Bigg) - \frac{\gamma c_s Q}{D^2}\left(1-\frac{D}{P}\right)e^{-\gamma(T-t_2)}\\ &= -\frac{2c_0}{Q^2}\left(\frac{D}{\gamma Q} - \frac{D}{\gamma Q}e^{-\gamma T} - e^{-\gamma T}\right) - \frac{1}{D}\left(1-\frac{D}{P}\right)\left(2c_h + \frac{\gamma c_h Q}{D}e^{-\gamma t_2} - c_s\left(2+e^{-\gamma t_2}\right) - \frac{\gamma c_s Q}{D}e^{-\gamma(T-t_2)} - \frac{2c_0 D}{Q^2\left(1-\frac{D}{P}\right)}\right)\\ &= -\frac{2c_0}{Q^2}X - \frac{1}{D}\left(1-\frac{D}{P}\right)Y < 0\end{aligned}$$

provided $X = (\frac{D}{\gamma Q} - \frac{D}{\gamma Q}e^{-\gamma T} - e^{-\gamma T}) > 0$ and $Y = (2c_h + \frac{\gamma c_h Q}{D}e^{-\gamma t_2} - c_s(2+e^{-\gamma t_2}) - \frac{\gamma c_s Q}{D}e^{-\gamma(T-t_2)} - \frac{2c_0 D}{Q^2(1-\frac{D}{P})}) > 0$

$$\frac{\partial M(Q,P)}{\partial P} = \frac{1}{\gamma}\left(-\frac{\alpha L D}{P^{\alpha+1}} + K\beta D P^{\beta-1} - \mu c_v(\delta-1)P^{\delta-2}D\right)\left(1-e^{-\gamma T}\right) - \frac{c_h Q D}{\gamma P^2}\left(1-e^{-\gamma t_2}\right) + \frac{1}{\gamma}\frac{c_s Q D}{P^2}\left(1-e^{-\gamma(T-t_2)} = 0\right) \quad (28)$$

$$\frac{\partial^2 M(Q,P)}{\partial P^2} = \frac{1}{\gamma}\left(-\frac{\alpha(\alpha+1)LD}{P^{\alpha+2}} - K\beta(\beta-1)DP^{\beta-2} - \mu c_v(\delta-1)(\delta-2)P^{\delta-3}D\right)(1-e^{-\gamma T}) - \frac{2c_h QD}{\gamma P^3}(1-e^{-\gamma t_2}) - \frac{2c_s QD}{\gamma P^3}(1-e^{-\gamma(T-t_2)})$$

$$= -\frac{1}{\gamma}K\beta(\beta-1)DP^{\beta-2}(1-e^{-\gamma T}) - \frac{1}{\gamma}\mu c_v(\delta-1)(\delta-2)P^{\delta-3}D$$
$$(1-e^{-\gamma T}) - \frac{2c_s QD}{\gamma P^3}(1-e^{-\gamma(T-t_2)}) - \frac{D}{\gamma P^3}\left\{\frac{\alpha(\alpha+1)L}{P^{\alpha-1}} - \right.$$
$$\left. 2c_h Q(1-e^{-\gamma t_2})\right\} < 0$$

provided $B = \frac{\alpha(\alpha+1)L}{P^{\alpha-1}} - 2c_h Q(1-e^{-\gamma t_2}) > 0$

$$\frac{\partial^2 M(Q,P)}{\partial Q \partial P} = -\frac{D}{\gamma P^2}\left\{c_h(1-e^{-\gamma t_2}) - c_s(1-e^{-\gamma(T-t_2)})\right\} < 0$$

provided $C = c_h(1-e^{-\gamma t_2}) - c_s(1-e^{-\gamma(T-t_2)}) > 0$

Hence $M(Q,P)$ has a maximum with respect to Q and P if $\frac{\partial^2 M(Q,P)}{\partial Q^2} < 0$ and $\frac{\partial^2 M(Q,P)}{\partial Q^2}\frac{\partial^2 M(Q,P)}{\partial P^2} - \frac{\partial^2 M(Q,P)}{\partial Q \partial P} > 0$

For our numerical data the above conditions are satisfied and therefore the profit function has a maximum solution.

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